Conventionality of Simultaneity and Reality

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Abstract
An important epistemological lesson can be learned from the impossibility to determine the one-way velocity of light and the immediate implication that simultaneity is conventional. The vicious circle – to determine whether two distant events are simultaneous we need to know the one-way velocity of light between them, but to determine the one-way velocity of light we need to know that the two events are simultaneous – is an indication of the need for a profound change of our view on reality.

1 Introduction
After all that has been written on the conventionality in determining the one-way velocity of light and the conventionality of simultaneity (see for instance [1]-[15]) one wonders what more can be added to this issue. It turns out, however, that an important aspect has not been sufficiently explored – the link between conventionality of simultaneity and reality. In 1972 Weingard first made this link but only barely mentioned it by devoting a single sentence to it: “But while distant simultaneity is a matter of convention, being real, I take it, cannot be merely a matter of convention” [11, p. 120]. As that link has remained unexplored so far, excluding my own attempts to go a little further [12, Sec. 3], [16, Sec. 5.6.1], [17, Sec. 2.2], the purpose of this paper is to examine the implications of the issue of conventionality of simultaneity for our view on reality.

Section 2 analyzes the origin and physical meaning of the vicious circle in attempting to determine the one-way velocity of light and any one-way velocity. Section 3 demonstrates that since the pre-relativistic view on reality is formulated in terms of absolute simultaneity it is directly affected by
the relativistic view on simultaneity – that simultaneity of distant events is both conventional and relative.

2 Any One-Way Velocity and Simultaneity are Conventional

In 1898 Poincaré first realized that any measurement of the velocity of light is based on an implicit assumption, namely “that light has a constant velocity, and in particular that its velocity is the same in all directions. That is a postulate without which no measurement of this velocity could be attempted. This postulate could never be verified directly by experiment” [1, p. 220]. Seven years later Einstein arrived at the same conclusion. In the section ‘Definition of Simultaneity’ of his 1905 paper he discussed the introduction of a common time at two distant points A and B: “We have not defined a common ‘time’ for A and B, for the latter cannot be defined at all unless we establish by definition that the ‘time’ required by light to travel from A to B equals the ‘time’ it requires to travel from B to A” [2, p. 40].

It is clear that “by definition” Einstein meant “by convention”. Therefore, according to Poincaré and Einstein the magnitude of the one-way velocity of light cannot be discovered by experiment and should be determined by convention.

To see why the one-way velocity of light cannot be determined experimentally assume that we are trying to do just that – to measure the velocity of light from a point A to another point B. To do that we obviously need to know the distance between A and B and the time for which light propagates from A to B. In order to measure that time the clocks at A and B should show the same readings simultaneously, i.e. they should be synchronized.

But how can that be done? One can use two methods to synchronize the clocks at A and B. The first is to send a light (or any other) signal from A to B whose one-way velocity is known. Hence we arrive at a vicious circle – to determine the one-way velocity of light propagating from A to B the clocks at these points should be synchronized, but to synchronize the clocks the one-velocity of light should be known beforehand.

The second method to synchronize the clocks at A and B is the so called slow transport of a third clock C from A to B – the C-clock is initially synchronized with the A-clock and then slowly transported to point B where the B-clock is synchronized with the third clock. It is called “slow transport” to imply that the time dilation that the C-clock undergoes should be neglected. However, neglecting it would mean missing the whole point in the
synchronization of distant clocks by a third clock. No matter how small the
time dilation might be, if we attempt to calculate it we arrive at the same
vicious circle as in the case of the first method: to determine the magnitude
of time dilation in order to synchronize the A and B clocks we should know
the one-way velocity of the C-clock, but to measure that velocity the A and
B clocks should be synchronized in advance.

One might object that in the case of the second method of synchronizing
two distant clocks the vicious circle can be avoided if an observer at rest
in clock C’s reference frame uses the C-clock itself to measure the time of
its journey from A to B, not the clocks at A and B. Then by knowing the
distance between A and B one can calculate the one-way velocity of C. I
believe the problem with this objection is obvious – the distance between A
and B is relativistically contracted for the observer in C’s reference frame.
In order to determine the magnitude of the length contraction the one-way
velocity of C should be known and we again arrive at the vicious circle.

Two main conclusions can be drawn from here. First, not only the one-
way velocity of light but any one-way velocity is a matter of convention
since it cannot be directly measured. Second, the conventionality of the
one-way velocity of light implies conventionality of simultaneity of distant
events as well; this was evident to both Poincaré [1, p. 222] and Einstein.

These conclusions raise difficult questions about their physical meaning. An
obvious question is “If in reality the velocity of light in one direction has
an objective value, how can it depend on human choices and be a matter
of definition (convention)?” Obviously, this question is based on the as-
sumption that the concept of velocity, and therefore the one-way velocity of
light as well, has a counterpart in the objective world. Questioning this as-
sumption amounts to questioning the fact that objects are in motion, which
means that they move with some (definite) velocities with respect to an
inertial reference frame. Poincaré does not seem to have been bothered by
this type of questions: “So for the velocity of light a value is adopted, such
that the astronomic laws compatible with this value may be as simple as
possible” [1, p. 221]. Neither Poincaré nor Einstein appear to have asked the deep questions: “Why can the one-way velocity of light not be measured?” and “Why is simultaneity conventional?” In any case their position on the physical meaning of conventionality of the one-way velocity of light and simultaneity is not known.

As we will see below the profound physical meaning of the vicious circle in determining the one-way velocity of light and the resulting conventionality of simultaneity is that the world cannot be three-dimensional. Had Poincaré and Einstein tried to reveal what causes the conventionality of the one-way velocity of light and simultaneity they might have arrived at the idea that reality is a four-dimensional world before Minkowski [19]. This especially applies to Poincaré who wrote in 1906 that the Lorentz transformations are a rotation in a four-dimensional space with time as the forth dimensions [20]. It seems, however, he regarded that space merely as a mathematical space that does not represent anything real.

At first sight, the tough questions posed by the conventionality thesis are not immediately obvious. Any one-way velocity is determined by convention but this does not seem to be a big deal. Velocity is a frame-dependent concept which means that absolute velocity does not represent anything objective; for this reason it is not so surprising that it does not have an objective (absolute or frame-independent) value. Similarly, simultaneity is conventional but special relativity showed that it is also frame-dependent (relative) and therefore nothing in the objective world corresponds to the concept of absolute simultaneity.

But this is not the whole story. One may argue that the concept of velocity does represent something objective for the following reason. Velocity is relative, but that frame-dependency does not appear to undermine the belief that with respect to a single observer the velocity of a particle reflects an objective fact – the motion of the particle relative to the observer. So, how can the one-way velocity of a particle be conventional in one reference frame? In a given reference frame the one-way velocity of the particle appears to be an objective fact that should not depend on the choice made by an observer in that frame. However, as we will see in Sec. 3 this argument and this question turn out to be based on our pre-relativistic intuition. The fact is that the concept of velocity (as the measure of the motion of a three-dimensional object) does not reflect anything real due to its frame-dependency. And if this concept does not have an objective counterpart we

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3There would be a problem with the conventionality thesis only if the value of a frame-independent physical quantity would be a matter of convention.
are free to assign a value of our choice to any velocity. So it does follow from the relativity of velocity that velocity is also conventional.

The situation with simultaneity is the same. The frame-dependency of simultaneity demonstrates that no class of absolutely simultaneous events exists. This means that we are indeed free to choose which events to regard as simultaneous since no class of events is objectively or absolutely privileged. Therefore relativity of simultaneity implies conventionality of simultaneity. The opposite is also true – conventionality of simultaneity implies relativity of simultaneity. As distant simultaneity is conventional it follows that no class of events is objectively privileged as being simultaneous; if such a class of privileged simultaneous events existed, then simultaneity could not be conventional. But as no class of events is absolutely (objectively) simultaneous, different observers in relative motion are not forced (due to the lack of a class of objectively privileged simultaneous events) to share the same class of simultaneous events, which means that simultaneity is not absolute and is therefore relative\(^4\). I think the fact that relativity of simultaneity implies conventionality of simultaneity and vice versa should be specifically emphasized since any claim that simultaneity is relative but not conventional amounts to a contradiction in terms: there is no objectively privileged class of simultaneous events (due to relativity of simultaneity), but there is an objectively privileged class of simultaneous events (due to the non-conventionality of simultaneity).

The direct link between relativity of simultaneity and conventionality of simultaneity follows from the fact that “in special relativity, the causal structure of space-time defines a notion of a ‘light cone’ of an event, but does not define a notion of simultaneity” [21]. No class of events lying outside of the light cone at an event \(P\) can be defined as simultaneous on the basis of causal relations in spacetime, which means that no class of events is objectively privileged or objectively distinct from the other events in that region. For comparison consider the light cone at \(P\). Its three regions – past, future, and outside – are, in terms of causal relations\(^5\), objectively distinct from one another; that is why the light cone is a frame-independent

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\(^4\)If two observers in relative motion choose either the standard (Einstein) convention \(\epsilon = \frac{1}{2}\) or the same non-standard convention \(\epsilon \neq \frac{1}{2}\) they will have different classes of simultaneous events, which means that simultaneity will be relative for them. There will be no relativity of simultaneity only in one special case when they choose the same class of events to be simultaneous for each of them; this choice corresponds to different values of \(\epsilon\) for each of the observers.

\(^5\)All events in the past light cone can influence the event at \(P\). The event at \(P\) can affect all events in the future light cone. No event in the spacetime region lying outside of the light cone can influence the event at \(P\) and vice versa.
concept. However, in relativity the concept of simultaneity is still used – for example, length contraction and time dilation cannot be formulated if that concept is not employed. As a result of our insistence on using the concept of simultaneity where it does not reflect anything objective we arrive at the conclusions that simultaneity is relative when two observers in relative motion are considered and that simultaneity is conventional if just one observer is involved. Both conclusions follow from the fact that the status of all events in the area outside of the light cone is the same. That is why if simultaneity is relative it is also conventional and vice versa – both relativity and conventionality of simultaneity imply that there is no objectively privileged class of events among all events in the area lying outside of the light cone.

So the conventionality of the one-way velocity of light or the one-way velocity of any particle and of simultaneity is inescapable in the framework of relativity. Despite this, however, the tough questions mentioned above remain. It is sufficient to point out the conventionality of the one-way velocity of light. Unlike the velocities of particles, the velocity of light is absolute (frame-independent). This suggests that the concept of velocity of light reflects something objective. Then, how can the one-way velocity of light be a matter of convention? As we will see in the next section, an even more

6In this connection it is worthwhile to point out that Malament’s theorem [7] merely proves that the standard synchronization $\epsilon = \frac{1}{2}$ is the simplest one. His result did not disprove the conventionality thesis. If we assume it did, it would follow that the class of simultaneous events (determined by the choice $\epsilon = \frac{1}{2}$) would be objectively privileged (no conventionality!) and therefore observers in relative motion would share the same class of objectively privileged events. Hence simultaneity would turn out to be absolute, if distant simultaneity were not conventional. That is why it is indeed a contradiction in terms to say that simultaneity is not conventional, but is relative.

7It is natural to ask whether conventionality of any one-way velocity and simultaneity is a feature of only the theory of relativity. In Newtonian physics the first method of synchronizing distant clocks through light (or any other) signals also leads to a vicious circle. However, the second method (slow transport of a third clock) does not lead to such problems since there is no time dilation and length contraction in Newtonian physics. Hence the one-way velocity of light and simultaneity of distant events are not conventional in classical (pre-relativistic) physics. This conclusion also follows from the fact that simultaneity is absolute in Newtonian physics. At any moment of the absolute time there is one class of absolutely simultaneous events that is shared by all observers in relative motion. Due to the privileged status of this class the observers are not free to choose different classes of simultaneous events. By the same argument every single observer is not free to choose which events to be regarded as simultaneous, which means that simultaneity is not conventional. As we will see in the next section what makes the only class of absolutely simultaneous events privileged is the fact that according to the pre-relativistic world view what exists at the present moment is namely the class of absolutely simultaneous events.
difficult question is raised when it is taken into account that conventionality of simultaneity would imply conventionality with respect to what exists if reality were a three-dimensional world. The link between conventionality of simultaneity and reality will be explored in the next section which will allow us to arrive at a view on reality that is fully consistent with the conventionality thesis and that provides natural answers to all difficult questions raised by the conventionality thesis.

3 Simultaneity and Reality

The real challenge of the conventionality thesis is fully manifested when the issue of what is real is explicitly addressed. Since the time of Aristotle reality has been regarded as a three-dimensional world [22]. At that time the concept of reality could have been formulated only in terms of what is directly perceived – the observable world. Therefore, until the seventeenth century the three-dimensional world could have been defined as ‘everything that we see (or can in principle see) simultaneously at the present moment’. However, after Rømer determined in 1675 that the velocity of light was finite it became clear that what we see is all past. Then the second view on reality could have defined the three-dimensional world as ‘everything that exists simultaneously at the present moment’. This second view on reality, called presentism, was fully consistent with the pre-relativistic physics, but is incompatible with relativity.

I believe the reason is obvious – the pre-relativistic view on reality is defined in terms of absolute simultaneity, but according to special relativity simultaneity is both relative and conventional which means that no class of simultaneous events, that can be identified with the three-dimensional world, is objectively privileged. If reality were a three-dimensional world, i.e. a single class of simultaneous events, conventionality of simultaneity would imply that what exists is also a matter of convention which is clearly unacceptable. Therefore the message of the vicious circle involved in any attempt to

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8On the pre-relativistic (presentist) view everything that exists, exists at the present moment. Therefore it is natural to consider absolute simultaneity at the moment ‘now’ in order to understand more clearly its physical meaning and why simultaneity is not conventional in the pre-relativistic physics. When it is taken into account that the concept of absolute simultaneity in Newtonian physics implies a three-dimensional world (defined as everything that exists simultaneously at the present moment) it becomes clear that it is the existence of the simultaneous events at the present moment that makes them absolutely simultaneous or objectively privileged: according to the pre-relativistic world view reality is a single three-dimensional world (the present); therefore there exists a single
determine any one-way velocity and simultaneity of distant events is truly profound – reality is not a three-dimensional world. The same conclusion follows from relativity of simultaneity – as two observers in relative motion have different classes of simultaneous events it follows that they have different three-dimensional worlds, which is possible only if these worlds are three-dimensional cross-sections of a real four-dimensional world represented by Minkowski spacetime. Here I would like to emphasize again the link between relativity of simultaneity and conventionality of simultaneity – both relativity of simultaneity and conventionality of simultaneity imply that reality is a four-dimensional world; neither relativity of simultaneity nor conventionality of simultaneity are possible in a three-dimensional world.

To see this and also why the pre-relativistic world view is in unsurmountable contradiction with special relativity, assume that reality were indeed a three-dimensional world. Then it would follow that

- No relativity of simultaneity would be possible since all observers in relative motion would share the same three-dimensional world and therefore would have the same class of simultaneous events.

- No conventionality of simultaneity would be possible since the three-dimensional world would be the only thing that exists and therefore would be objectively privileged and not a matter of convention.

It is explicit that the conclusion ‘neither relativity of simultaneity nor conventionality of simultaneity are possible in a three-dimensional world’ follows from the definition of a three-dimensional world – the class of events that are absolutely\textsuperscript{9} simultaneous at the present moment. In such a desperate situation when the widely accepted presentist view is strongly challenged\textsuperscript{9}:

\textsuperscript{9}Strictly speaking, it is not even necessary to add ‘absolutely’ to the definition of a three dimensional world for two reasons:

- On the presentist view there exists a single class of simultaneous events at the present moment, which as the only class of simultaneous events are absolute because they are common to all observers in relative motion.

- When a three-dimensional world is regarded as a sub-space in Minkowski spacetime one cannot use absolute simultaneity. In this case a three-dimensional world (a three-dimensional cross-section of Minkowski spacetime) is defined merely as ‘the class of events that are simultaneous at a given moment of an observer’s time’. As we saw above due to relativity of simultaneity two observers in rel-
by special relativity, it appears natural to ask “Is it possible to define the 
three-dimensional world in such a way that simultaneity is not involved?” 
The answer is “No”. On the presentist view it is only the three-dimensional 
world that exists at the present moment. If it is not defined as ‘all events 
that are simultaneous at the present moment’, the only option for another 
definition is that the three-dimensional world contains events occurring at 
different moments of time. This is clearly impossible in the framework of 
the presentist view since such a three-dimensional world would contain past, 
present, and future events, not only the existing present events.

Another possibility to challenge the conclusion that special relativity 
is impossible in a three-dimensional world is to point out that the 1905 
formulation of relativity given by Einstein was in terms of the ordinary 
three-dimensional (presentist) view. It is true that special relativity can 
be equally formulated in the usual three-dimensional language as well as 
in the four-dimensional language of Minkowski spacetime. However, while 
both representations of relativity correctly describe the relativistic phenom-
ena, they are obviously not equivalent in terms of the dimensionality of the 
world. Therefore only one of them adequately represents the world’s di-

mensionality. So the original formulation of special relativity was in three-
dimensional language, but it does not mean that it was possible in a three-
dimensional world. To see this let us ask what the physical meaning of the 
kinematical relativistic effects is. It becomes immediately evident that they 
are impossible in a three-dimensional world. Take as an example relativ-
ity of simultaneity and assume that reality were a three-dimensional world 
(the present). As such a world is defines as the class of events that are 
simultaneous at the present moment it follows that all observers in relative 
motion would share the same class of simultaneous events since only this 
class of events would exist at the moment ‘now’. This means that simul-
taneity would be absolute in contradiction with special relativity. Therefore 
special relativity is indeed impossible in a three-dimensional world.

As the causal structure of spacetime defines a notion of a light cone, not 
a notion of simultaneity, the most rigorous approach to the issue of reality in 
the framework of relativity is to ask what is real in terms of the light cone. 
What is immediately clear is that reality cannot be a three-dimensional 
world since it is defined in terms of simultaneity – as everything that exists 
simultaneously at the moment ‘now’ of an observer’s time – whereas the 

ative motion have different three-dimensional worlds which are simply different 
three-dimensional cross-sections of Minkowski spacetime.
Figure 1: Two observers $A$ and $B$ in relative motion, who meet at event $O$, are represented by their worldlines. If it is assumed that what is real is represented by the area outside the light cone at $O$, it follows that the observers will have the freedom to choose different classes of simultaneous events (represented by their $x$-axes). The events in the past and future light cone also turn out to be real since they fall in the area lying outside a second light cone at event $P$ which is space-likely separated from event $O$.

spacetime causal structure does not define such a concept. As a first attempt one can identify the spacetime area lying outside the light cone at event $O$ (Fig. 1). This choice is dictated by both relativity of simultaneity and conventionality of simultaneity – the area outside of the light cone at $O$ must exist in order that (i) two observers $A$ and $B$ in relative motion could have different instantaneous three-dimensional spaces, i.e. different classes of simultaneous events (which are represented by the $x$-axes of the observers in Fig. 1), and (ii) each of the observers could choose (by convention) his own instantaneous three-dimensional space, i.e. his class of simultaneous events from that spacetime area. Put another way, the area outside the light cone at $O$ must exist in order that relativity of simultaneity and conventionality of simultaneity be possible.

The next step would be to ask: what is the status of the events in the past and future light cone? As Weingard [11] demonstrated, if reality is represented by the area lying outside the light cone at $O$, by the same criterion events in the past and future light cone are also real since they lie in the area outside a second light cone at $P$ (Fig. 1). So it follows that all spacetime events are real. This conclusion is inevitable when one asks what the impact of each of the relativistic changes of the concept of simultaneity – relativity of simultaneity and conventionality of simultaneity – on the view
Figure 2: The worldlines of an observer and a body form an angle $\alpha$. The time axis is chosen along the worldline of the observer. In a three-dimensional language the two worldlines can be interpreted to mean that the observer and the body are in relative motion. Although their relative velocity can be expressed in terms of the angle $\alpha$, it does not mean that the relative velocity is completely determined by $\alpha$. The observer is free to determine by convention whether his instantaneous three-dimensional space (depicted here only by the $x$ axis) is orthogonal to his worldline ($x$) or not ($x'$). The relative velocity between the observer and the body depends on this choice.

The relativistic view, according to which reality is represented by the four-dimensional Minkowski world, provides complete answers to the difficult questions raised by the conventionality thesis.

In Minkowski world the whole history of every signal or every body is entirely realized in the signal’s worldline or the body’s worldtube. There are no three-dimensional objects in spacetime and no motion of such objects. That is why the concept of velocity does not have an ontological counterpart. For this reason we are indeed free to choose the value of velocity when we describe Minkowski spacetime in terms of our three-dimensional language. When the motion of a body is described with respect to a given observer what corresponds to the body’s velocity is the angle $\alpha$ between the worldlines of the observer and the body (Fig. 2). However, $\alpha$ is not the ontological counterpart of the relative velocity since the relative velocity depends not only on $\alpha$, but also on whether the instantaneous three-dimensional space of the observer is chosen to be orthogonal to the observer’s worldline or not.
That is why the velocity of the body with respect to the observer is a matter of convention.

This is clearly seen in Fig. 2. Imagine that the observer decides to measure the instantaneous velocity of the body at event $P$. Depending on whether or not the observer’s instantaneous three-dimensional space is orthogonal to his worldline, the time it takes the body to reach event $P$ will be either $OB$ or $OA$. Therefore, the body’s velocity is indeed determined by convention. The velocity of a light signal is conventional for the same reason – the observer is free to choose his instantaneous three-dimensional space to be either orthogonal to his worldline or to form an angle with it. The fact that the velocity of light is not frame-dependent is a result of the frame-independency of the concept of a light cone but that does not affect the conventionality of the light velocity due to the observer’s freedom to choose the angle between his worldline and his instantaneous three-dimensional space.

**Conclusion**

The epistemological lesson that can be learned from the impossibility to determine the one-way velocity of light and the immediate implication that simultaneity is conventional demonstrates that every time when we arrive at a vicious circle some of our views should be drastically changed. And indeed the fact that the one-way velocity of light and simultaneity of distant events are conventional has turned out to have a profound meaning - reality is a four-dimensional world represented by Minkowski spacetime. There are no moving light signals or three-dimensional bodies in this four-dimensional world and when we describe it in our three-dimensional language in terms of motions, the velocities of these signals and bodies are determined by convention since they do no represent anything real.

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