The relativity of inertia and reality of nothing

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Abstract

We first see that the inertia of Newtonian mechanics is absolute and troublesome. General relativity can be viewed as Einstein's attempt to remedy, by making inertia relative, to matter—perhaps imperfectly though, as at least a couple of freedom degrees separate inertia from matter in his theory. We consider ways the relationalist (for whom it is of course unwelcome) can try to overcome such undetermination, dismissing it as physically meaningless, especially by insisting on the right transformation properties.

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1. Introduction

The indifference of mechanical phenomena and the classical laws governing them to absolute position, to translation has long been known. This 'relativity' extends to the first derivative, velocity, but not to the second, acceleration, which—together with its 'opposite,'[1] *inertia*—has a troubling absoluteness, dealt with in §2.1. General relativity can be seen as Einstein's attempt to overcome that absoluteness (§2.2), by making inertia relative, to matter. But we can wonder about the extent and nature of the 'relativisation.'

Following Einstein we take matter, in §3.1, to be represented by the energy-momentum tensor T^a_b—rather than by $U^a_b = T^a_b + t^a_b$, which includes the gravitational energy-momentum t^a_b whose transformation properties make it too subjective and insubstantial to count. The relevance of *distant* matter is considered in §3.2. Inertia can be identified with affine or projective structure, as we see in §3.3. In §3.4 matter appears to underdetermine inertia by ten degrees of freedom,

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[1] In the sense that motion is inertial when acceleration vanishes. This can also be understood, in more Aristotelian terms, as a contraposition of "natural" (inertial) and "violent" (accelerated) motion. Weyl (2000, p. 138) has a corresponding *dualism between inertia and force*: "gravity, in the dualism between inertia and force, belongs to inertia, not to force. *In the phenomena of gravitation therefore the inertial- or*, as I prefer to say, *the guidance-field* [...]." *Cf.* Weyl (1924, p. 198): "Dualism between guidance and force." (The translations from German are all ours.)

eight of which are made, in §3.5, to 'disappear into the coordinates.' The status of coordinate transformations has been dealt with abundantly, especially in the literature on the hole argument; we take them to be physically meaningless and concentrate on the significance of the remaining (double) freedom instead, representing the polarisation of gravitational waves.

But gravitational waves are not generally covariant (§3.6): their generation, energy and detection can all be 'transformed away' if the full range of transformations, on which general relativity was built, is available. Belief in the production of gravitational radiation is bound up with the binary star PSR 1913 +16 (§3.13), which is supposed to lose kinetic energy as it spirals inwards; if energy is conserved, the energy lost in one form must be converted, into a perturbation of the surrounding spacetime in this case. But the conservation law is flawed (§3.8), involving, in its integral form, a distant comparison of directions which cannot be both generally covariant and unambiguously integrable. Even the 'spiral' behaviour itself, the loss of kinetic energy, and the oscillation on which detection (§3.14) is based can be transformed away; as can the mass-energy of the gravitational field, which is customarily assigned using the pseudotensor t_{ab}, whose transformation properties make it a matter of opinion: while an observer in free fall sees nothing at all, an acceleration would produce 'mass' out of nowhere, out of a mere transformation to another 'point of view' or rather state of motion. To exploit this vulnerability of gravitational waves to coordinate substitutions, the relationalist wanting inertia to be completely determined by matter will be mathematically intransigent and attribute physical significance only to notions with the right transformation behaviour—and none to those that can be transformed away—thus allowing him to dispute the reality of the unwelcome freedoms separating matter and inertia, which he can dismiss as mere opinion, as meaningless decoration. If general covariance *has to hold*, matter would seem to determine inertia rather strongly.

In the early years of general relativity, Hilbert, Levi-Civita, Schrödinger and others attributed physical meaning only to objects, like tensors, with the right transformation behaviour. Einstein was at first less severe, extending reality to notions with a more radical dependence on the observer's state of motion. With a mathematical argument (§3.9) giving a favourable representation of the integral conservation law's transformation properties he persuaded the community to share his tolerance; but he would soon, having meanwhile read a manuscript by Cassirer (§3.11), change his mind (§3.10), and also require general covariance for physical significance. The relationalist can wonder about an argument, and of a widespread indulgence it helped produce, whose proponent and advocate soon adopted the intransigence of his previous opponents.

Meyerson (§3.15) provides a further principle—the requirement of conservation for genuine causality and explanation—the relationalist can invoke to attenuate belief in gravitational waves: their bare detection can be dismissed as mere *légalité*, as a 'legal' correlation between binary star and detector; to speak of *causalité*, of *explication* (which surely correspond to a stronger form of belief) Meyerson would require the gravitational radiation to persist 'robustly' on its way to the detector. And the relationalist can reasonably demand appropriate transformation behaviour for such persistence.

But rather than as a defence of relationalism—for we have no axe to grind—this should be viewed as an exploration of certain logical gaps or possibilities the relationalist can exploit, especially one that (perhaps unwisely) takes fundamental principles like general covariance more seriously than the lenient pragmatics of day-to-day practice, computation, prediction and success.

The various ways we help or hinder the relationalist may sometimes seem arbitrary; to some extent they are arbitrary, or rather influenced by our tastes and interests; but they also take account of the literature and the very full treatments it contains, to which in many cases we have nothing to add.[2]

2. Absolute inertia

2.1. Newtonian mechanics

Newton distinguished between an "absolute" space he also called "true and mathematical," and the "relative, apparent and vulgar" space in which distances and velocities are measured. Absolute position and motion were not referred to anything. Leibniz identified unnecessary determinations, excess structure[3] in Newton's 'absolute' kinematics with celebrated arguments resting on the *principium identitatis indiscernibilium*: as a translation of everything, or an exchange of east and west, produces no observable effect, the situations before and after must be the same, for no difference is discerned. But there were superfluities even with respect to Newton's own dynamics,[4] founded as it was on the proportionality of force and acceleration. With his *gran navilio*, Galileo (1632, Second day) had already noted the indifference of various *effetti* to inertial transformations; the invariance[5] of Newton's laws would more concisely express the indifference of all the *effetti* they governed.[6]

Modern notation, however anachronistic, can help sharpen interpretation. The derivatives $\dot{\mathbf{x}} = d\mathbf{x}/dt$ and $\ddot{\mathbf{x}} = d\dot{\mathbf{x}}/dt$ are quotients of differences; already the position difference

$$\begin{aligned}\Delta\mathbf{x} &= \mathbf{x}(t+\varepsilon) - \mathbf{x}(t)\\ &= \mathbf{x}(t+\varepsilon) + \mathbf{u} - [\mathbf{x}(t) + \mathbf{u}]\end{aligned}$$

is indifferent to the addition of a constant $\mathbf{u}$ (which is the same for $\mathbf{x}$ both at t and at $t+\varepsilon$). The velocity

$$\dot{\mathbf{x}} = \lim_{\varepsilon\to 0}\frac{\Delta\mathbf{x}}{\varepsilon}$$

is therefore unaffected by the three-parameter group $\mathcal{S}$ of translations $\mathbf{x} \mapsto \mathbf{x} + \mathbf{u}$ acting on the three-dimensional space E. The difference

$$\begin{aligned}\Delta\dot{\mathbf{x}} &= \dot{\mathbf{x}}(t+\varepsilon) - \dot{\mathbf{x}}(t)\\ &= \dot{\mathbf{x}}(t+\varepsilon) + \mathbf{v} - [\dot{\mathbf{x}}(t) + \mathbf{v}]\end{aligned}$$

of velocities is likewise indifferent to the addition of a constant velocity $\mathbf{v}$ (which is the same for $\dot{\mathbf{x}}$ both at t and at $t+\varepsilon$). The acceleration

$$\ddot{\mathbf{x}} = \lim_{\varepsilon\to 0}\frac{\Delta\dot{\mathbf{x}}}{\varepsilon}$$

is therefore invariant under the six-parameter group $\mathcal{S}\times\mathcal{V}$ which includes, alongside the translations, the group $\mathcal{V}$ of the inertial transformations $\mathbf{x} \mapsto \mathbf{x} + \mathbf{v}t$, $\dot{\mathbf{x}} \mapsto \dot{\mathbf{x}} + \mathbf{v}$ acting on the space-time $\mathbb{E} = E \times \mathbb{R}$.

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[2]For instance we hesitate to help the relationalist with the distant masses (which of course constrain inertia) whose influence in the initial-value formulation has been so abundantly considered by Wheeler and others.

[3]For a recent treatment see Ryckman (2003, pp. 76-80).

[4]*Cf.* Dieks (2006, p. 178).

[5]Newton (1726), *Corollarium V* (to the laws).

[6]On this distinction and its significance in relativity see Dieks (2006), where the *effetti* are called "factual states of affairs."

But there are other symmetries; Newton's second law is 'covariant' with respect to the group $\mathcal{R} = \mathrm{SO}(\mathcal{S})$ of rotations $R : E \to E$, which turn the "straight line along which the force is applied" with the "change of motion," in the sense that the two rotations $(F \mapsto RF, \ddot{\mathbf{x}} \mapsto R\ddot{\mathbf{x}})$, taken together, maintain the proportionality: $[F \sim \ddot{\mathbf{x}}] \Leftrightarrow [RF \sim R\ddot{\mathbf{x}}]$. Including the group $\mathcal{T}$ of temporal translations $t \mapsto t + a \in \mathbb{R}$, we can say the second law is indifferent[7] to the action of the ten-parameter Galilei group[8] $\mathcal{G} = (\mathcal{S} \times \mathcal{V}) \rtimes (\mathcal{T} \times \mathcal{R})$ with composition

$$(\mathbf{u}, \mathbf{v}, a, R) \rtimes (\mathbf{u}', \mathbf{v}', a', R') = (\mathbf{u} + R\mathbf{u}' + a'\mathbf{v}, \mathbf{v} + R\mathbf{v}', a + a', RR'),$$

$\rtimes$ being the semidirect product. So the *absolute* features of Newtonian mechanics—acceleration, force, inertia, the laws—emerge as invariants of the Galilei group, whose transformations change the *relative* ones: position, velocity and so forth. A larger group would undermine the laws, requiring generalisation with other forces.

Cartan (1923)[9] undertook such a generalisation, with a larger group, new laws and other forces. The general covariance of his Newtonian formalism (with a flat connection) may seem to make inertia and acceleration relative, but in fact the meaningful acceleration in his theory is not d^2x^a/dt^2, which can be called *relative*[10] (to the coordinates), but the *absolute*

$$A^a = \frac{d^2x^a}{dt^2} + \sum_{b,c=1}^{3} \Gamma^a_{bc} \frac{dx^b}{dt}\frac{dx^c}{dt} \tag{1}$$

($a = 1, 2, 3$ and the time t is absolute). Relative acceleration comes and goes as coordinates change, whereas absolute acceleration is generally covariant and transforms as a tensor: if it vanishes in one system it always will. The two accelerations coincide with respect to inertial coordinates, which make the connection components $\Gamma^a_{bc} = \langle dx^a, \nabla_{\partial_b}\partial_c\rangle$ vanish (where $\langle \alpha, v\rangle$ is the value of the covector α at the vector v, and $\partial_a = \partial/\partial x^a$ is the partial derivative operator representing the unit vector tangent to the ath coordinate line). The absolute acceleration of inertial motion vanishes however it is represented—the connection is there to cancel the acceleration of noninertial coordinates.

So far, then, we have two formal criteria of inertial motion:

- $\ddot{\mathbf{x}} = \mathbf{0}$ in Newton's theory
- $A^a = 0$ in Cartan's.

But Newton's criterion doesn't really get us anywhere, as the vanishing acceleration has to be referred to an inertial frame in the first place; to Cartan's we shall return in a moment.

Einstein (1916, p. 770; 1988, p. 40) and others have appealed to the *simplicity of laws* to tell inertia apart from acceleration: inertial systems admit the simplest laws. Condition $\ddot{\mathbf{x}} = \mathbf{0}$, for

[7] For Newton's forces are superpositions of $\mathcal{G}$-covariant fundamental forces $F = f(|\mathbf{x}_2 - \mathbf{x}_1|, |\dot{\mathbf{x}}_2 - \dot{\mathbf{x}}_1|, |\ddot{\mathbf{x}}_2 - \ddot{\mathbf{x}}_1|, \dots)$ exchanged by pairs of points.

[8] See Lévy-Leblond (1971, pp. 224-9).

[9] See also Friedman (1983, §III), Penrose (2005, §17.5).

[10] In Baker (2005) there appears to be a confusion of the two accelerations as they arise—in much the same way—in general relativity. The acceleration $d^2x^a/d\tau^2 \neq 0$ Baker sees as evidence of the causal powers possessed by an ostensibly empty spacetime with $\Lambda \neq 0$ is merely *relative*; even with $\Lambda \neq 0$ free bodies describe geodesics, which are wordlines whose *absolute* acceleration vanishes. The sensitivity of *projective structure* to the cosmological constant would seem to be more meaningful, and can serve to indicate similar causal powers.

instance, is simpler than $\ddot{\mathbf{y}} + \mathbf{a} = \mathbf{0}$, with a term $\mathbf{a}$ to compensate the acceleration of system y. But we have just seen that Cartan's theory takes account of possibile acceleration *ab initio*, thus preempting subsequent complication—for accelerated coordinates do not appear to affect the syntactical form[11] of (1), which is complicated to begin with by the connection term. One could argue that the law simplifies when that term disappears, when the coefficients Γ^a_{bc} all vanish; but then we're back to the Newtonian condition $\ddot{\mathbf{x}} = \mathbf{0}$. And just as that condition requires an inertial system in the first place, Cartan's condition $A^a = 0$ requires a connection, which is equivalent: it can be seen as a convention stipulating how the three-dimensional simultaneity surfaces are 'stitched'[12] together by a congruence of (mathematically) arbitrary curves *defined* as geodesics. The connection would then be determined, *a posteriori* as it were, by the requirement that its coefficients vanish for those inertial curves. Once one congruence is chosen the connection, thus determined, provides all other congruences that are inertial with respect to the first. So the initial geodesics, by stitching together the simultaneity spaces, first provide a notion of rest and velocity, then a connection, representing inertia and acceleration. The Newtonian condition $\ddot{\mathbf{x}} = \mathbf{0}$ presupposes the very class of inertial systems given by the congruence and connection in Cartan's theory. So we seem to be going around in circles: *motion is inertial if it is inertial with respect to inertial motion*.

We should not be too surprised that purely formal criteria are of little use on their own for the identification of something as physical as inertia. But are more physical, empirical criteria not available? Can inertial systems not be characterised[13] as free and far from everything else? Even if certain bodies may be isolated enough to be almost entirely uninfluenced by others, the matter remains troublesome. For one thing we have no direct access to such roughly free bodies, everything around us gets pulled and accelerated. And the absence of gravitational force is best assessed with respect to an inertial system, which is what we were after in the first place.

Just as the *absence* of force has been appealed to for the identification of inertia, its *presence* can be measured in an attempt to characterize acceleration; various passages[14] in the *scholium* on absolute space and time show that Newton, for instance, proposed to tell apart inertia and acceleration through *causes*, *effects*, *forces*.[15] In the two experiments described at the end of the *scholium*, involving the bucket and the rotating globes, there is an interplay of local causes and effects: the rotation of the water causes it to rise on the outside; the forces applied to opposite sides of the globes cause the tension in the string joining them to vary. But this doesn't get us very far either. For rectilinear acceleration our problem remains, as we see using the distinction we drew between absolute acceleration A^a and relative acceleration d^2x^a/dt^2, which surprisingly corresponds to a distinction Newton himself is groping for in the following passage from the *scholium*:

> The causes by which true and relative motions are distinguished, one from the other, are the forces impressed upon bodies to generate motion. True motion is neither

[11] *Cf.* Dieks (2006, p. 186).

[12] See Earman (1989, §§1,2), for instance.

[13] Einstein (1916, p. 772; 1988, p. 40)

[14] "Distinguuntur autem quies et motus absoluti et relativi ab invicem per proprietates suas et causas et effectus"; "Causæ, quibus motus veri et relativi distinguuntur ab invicem, sunt vires in corpora impressæ ad motum generandum"; "Effectus, quibus motus absoluti et relativi distinguuntur ab invicem, sunt vires recedendi ab axe motus circularis"; "Motus autem veros ex eorum causis, effectibus, et apparentibus differentiis colligere, et contra ex motibus seu veris seu apparentibus eorum causas et effectus, docebitur fusius in sequentibus."

[15] *Cf.* Rynasiewicz (1995).

> generated nor altered, but by some force impressed upon the body moved; but relative motion may be generated or altered without any force impressed upon the body. For it is sufficient only to impress some force on other bodies with which the former is compared, that by their giving way, that relation may be changed, in which the relative rest or motion of this other body did consist. Again, true motion suffers always some change from any force impressed upon the moving body; but relative motion does not necessarily undergo any change by such forces. For if the same forces are likewise impressed on those other bodies, with which the comparison is made, that the relative position may be preserved, then that condition will be preserved in which the relative motion consists. And therefore any relative motion may be changed when the true motion remains unaltered, and the relative may be preserved when the true suffers some change. Thus, true motion by no means consists in such relations.[16]

The translators, Motte and Cajori, render *motus* as "motion" throughout, but the passage only makes sense (today) if we use *acceleration*, for most occurrences at any rate: Newton first speaks explicitly of the generation or alteration of motion, to establish that 'acceleration' is at issue; having settled that he abbreviates and just writes *motus*—while continuing to mean acceleration. And he distinguishes between a true acceleration and a relative acceleration which can be consistently interpreted, however anachronistically, as A^a and d^2x^a/dt^2. Of course Newton knows neither about connections nor affine structure, nor even matrices; but he is clearly groping for something neither he nor we can really pin down using the mathematical resources then available. It may not be pointless to think of a 'Cauchy convergence' of sorts towards something which at the time is unidentified and alien, and only much later gets discovered and identified as the goal towards which the intentions, the gropings were tending.

That leaves force. When Newton states, in the second law, that the *mutationem motus* is proportional to force, he could mean either the true acceleration or the relative acceleration; indeed it is in the spirit of the passage just quoted to distinguish correspondingly—pursuing our anachronism—between a *true force* $F^a = mA^a$ and a *relative force* $f^a = m\, d^2x^a/dt^2$. This last equation represents one condition for two unknowns, of which one can be fixed or measured to yield the other. But the relative force f^a is the wrong one. The 'default values' for both force and acceleration, the ones Newton is really interested in, the ones he means when he doesn't specify, the ones that work in his laws, are the 'true' ones: true force and true acceleration. And even if $F^a = mA^a$ also looks like one condition for two unknowns, the true acceleration A^a in fact conceals *two* unknowns, the relative acceleration d^2x^a/dt^2 and the difference $A^a - d^2x^a/dt^2$ representing the absolute acceleration of the coordinate system. Nothing doing then, we're still going around in circles: the inertia of Newtonian mechanics remains absolute, and cannot even be 'made relative' to force.

But what's wrong with absolute inertia? In fact it can also be seen as 'relative,' but to something—affine structure or the *sensorium Dei* or absolute space—that isn't really there, that's

[16] "Causæ, quibus motus veri et relativi distinguuntur ab invicem, sunt vires in corpora impressæ ad motum generandum. Motus verus nec generatur nec mutatur nisi per vires in ipsum corpus motum impressas: at motus relativus generari et mutari potest absq; viribus impressis in hoc corpus. Sufficit enim ut imprimantur in alia solum corpora ad quæ fit relatio, ut ijs cedentibus mutetur relatio illa in qua hujus quies vel motus relativus consistit. Rursus motus verus a viribus in corpus motum impressis semper mutatur, at motus relativus ab his viribus non mutatur necessario. Nam si eædem vires in alia etiam corpora, ad quæ fit relatio, sic imprimantur ut situs relativus conservetur, conservabitur relatio in qua motus relativus consistit. Mutari igitur potest motus omnis relativus ubi verus conservatur, et conservari ubi verus mutatur; et propterea motus verus in ejusmodi relationibus minime consistit."

too tenuous, invisible, mathematical, ætherial, unmeasurable or theological to count as a cause, for most empiricists at any rate.[17] The three unknowns of $F^a = mA^a$ are a problem because in Newtonian mechanics affine structure, which determines $A^a - d^2x^a/dt^2$, is unobservable. By relating it to matter Einstein would make affine structure measurable, tangible, empirically accessible.

2.2. Einstein

Having considered the epistemological *problem*—absolute inertia—which gave rise to general relativity, we can now wonder how complete a solution it represented.

We have just seen that Newton proposed to find absolute acceleration throught its causes and effects. Einstein also speaks of cause and effect—and practically seems to be addressing Newton and his efforts to sort out absolute and relative *motus*—in his analysis of the thought experiment described towards the beginning of *Die Grundlage der allgemeinen Relativitätstheorie* (1916, p. 771). There he brings together elements of Newton's two experiments—rotating fluid, two rotating bodies: Two fluid bodies of the same size and kind, S_1 and S_2, spin with respect to one another around the axis joining them while they float freely in space, far from everything else and at a considerable, unchanging distance from each other. Whereas S_1 is a sphere S_2 is ellipsoidal.

Einstein's analysis betrays positivist zeal and impatience with metaphysics. Newton, who could be metaphysically indulgent to a point of mysticism, might—untroubled by the absence of a manifest local cause—have been happy to view the deformation of S_2 as the effect of an absolute rotation it would thus serve to reveal. Einstein's epistemological severity makes him more exacting; he wants the observable cause[18] of the differing shapes; seeing no *local* cause, within the system, he feels obliged to look elsewhere and finds an *external* one in distant masses which rotate with respect to S_2. General relativity, which he goes on to formulate, does away with absolute inertia (to some extent at any rate) by spelling out its dependence on *matter*, which has a more obvious physical presence than merely mathematical spacetime structure or the *sensorium Dei*.

In 1918 Einstein goes so far as to claim that inertia[19] in his theory is entirely determined[20] by matter, which he uses T_{ab} to represent.[21] He explains in a footnote (p. 241) that this *Machsches Prinzip* is a generalisation of Mach's requirement (Mach, 1988) that inertia be derivable from interactions between bodies.[22]

So we seem to be wondering about what Einstein calls *Mach's principle*,[23] which provides a convenient label, and is something along the lines of *matter determines inertia*. We have

[17] *Cf.* Einstein (1916, pp. 771-2; 1917, p. 49), Cassirer (1921, pp. 31, 38, 39), Rovelli (2007, §2.2.2).

[18] Einstein (1916, p. 771); *cf.* footnote 17 above. Einstein wants visible effects to have visible causes; *cf.* Poincaré (1908, pp. 64-94), who sees "chance" when "large" effects have "small" causes—which can even be too small to be observable; and Russell (1961, p. 162): "[...] a very small force might produce a very large effect. [...] An act of volition may lead one atom to this choice rather than that, which may upset some very delicate balance and so produce a large-scale result, such as saying one thing rather than another."

[19] In fact he speaks of the "G-field" (1918a, p. 241), "the state of space described by the fundamental tensor [...]," by which inertia is represented: "Inertia and weight are essentially the same. From this, and from the results of the special theory of relativity, it follows necessarily that the symmetrical 'fundamental tensor' ($g_{\mu\nu}$) determines the metrical properties of space, the inertial behaviour of bodies in it, as well as gravitational effects."

[20] *Ibid.* p. 241: "*Mach's principle:* The G-field is *completely* determined by the masses of bodies."

[21] *Ibid.* 241-2: "Since mass and energy are the same according to special relativity, and energy is formally described by the symmetric tensor ($T_{\mu\nu}$), the G-field is determined by the energy tensor of matter."

[22] See Hoefer (1995) on "Einstein's formulations of Mach's principle."

[23] Barbour & Pfister (1995) is full of excellent accounts; see also Earman (1989, pp. 105-8), Mamone Capria (2005) and Rovelli (2007, §2.4.1).

seen what a nuisance absolute inertia can be; to remedy Einstein made it relative, to matter; we accordingly consider the extent and character of his ‘relativisation,’ of the *determination of inertia by matter*.

3. The relativity of inertia

3.1. Matter

To begin with, what is matter? Einstein, we have seen, used T_b^a to characterise it, but maybe one should be more permissive and countenance less substantial stuff as well. Einstein proposed

$$t_b^a = \frac{1}{2}\delta_b^a g^{mn}\Gamma_{mr}^l\Gamma_{nl}^r - g^{mn}\Gamma_{mr}^a\Gamma_{nb}^r \qquad (2)$$

for the representation of *gravitational* mass-energy; matter without mass, or mass away from matter, is hard to imagine; so perhaps we can speak of gravitational *matter*-mass-energy.[24] How about $U_b^a = T_b^a + t_b^a$ then, rather than just T_b^a? Several drawbacks come to mind. The right-hand side[25] of (2) shows how such ‘matter’ would be related to the notoriously untensorial connection components. In free fall, when they vanish, the pseudotensor t_b^a does too, which means that gravitational matter-mass-energy would be a *matter of opinion*,[26] its presence depending on the state of motion of the observer. The distribution of matter-mass-energy in apparently empty spacetime would accordingly depend on the choice of coordinates. To be extremely liberal one could even fill the whole universe, however empty or flat, on grounds that matter-mass-energy is potentially present everywhere: an appropriate acceleration could introduce it anywhere.

A superabundance of matter would help constrain inertia and hence make Mach’s principle—indeed any relationalist claim or principle—easier to satisfy, perhaps to a point of vacuity. The relationalist would also be brought uncomfortably close to his ‘absolutist’ opponent, who believes there is more to inertia than one may think, that it goes beyond and somehow transcends determination by matter. Indeed if we spread matter too liberally we hardly leave the relationalist and absolutist room to differ. Their debate has already been called outmoded (Rynasiewicz, 1996); the surest way to hasten its complete and regrettable demise is to impose agreement, by a questionable appeal to a dubious object, which can cover the universe with slippery coordinate-dependent matter that disappears in free fall and reappears under acceleration. We began with Newton, Leibniz and Galileo, have been guided by a continuity connecting their preoccupations with Einstein’s, and accordingly adopt a notion of matter that differs as little as possible (within general relativity) from theirs: hence T_b^a, rather than $U_b^a = T_b^a + t_b^a$.

3.2. Distant matter

Mach and Einstein both speak of *distant* matter (or masses), which can indeed affect inertia, but in two very different ways.

[24] *Cf.* Russell (1927, p. 82): “We do not regard energy as a “thing,” because it is not connected with the qualitative continuity of common-sense objects: it may appear as light or heat or sound or what not. But now that energy and mass have turned out to be identical, our refusal to regard energy as a “thing” should incline us to the view that what possesses mass need not be a “thing.””

[25] Its convenient form is assumed with respect to coordinates satisfying $1 = \sqrt{-\mathbf{g}}$, where $\mathbf{g}$ is the determinant of the metric.

[26] *Cf.* Earman & Norton (1987, p. 519).

Einstein's equation $G_{ab}(P) = T_{ab}(P)$ seems to express a 'direct' determination—be it excessive, exact or insufficient—of inertia at point P by the matter there; inertia would thus be directly governed by 'local' matter. The matter-energy-momentum tensor $T^{ab}(P) = \rho(P)V^aV^b$, for instance, describing a 'dust' with density ρ and four-velocity V^a, would directly determine inertia at P, not at other points far away. But much as in electromagnetism, the 'continuity' of ρ is deceptive. Once the scale gives a semblance of continuity to the density ρ, almost all the celestial bodies contributing to the determination of $\rho(P)$ will be very far, on any familiar scale, from P.

Turning to the second way, general relativity is of course a field theory, and fields tend to be smooth, holistic entities, which can undulate, drag, propagate and so forth. Needless to say, all values of a field cannot be assigned freely and independently; fixing certain values or features on a boundary or at infinity will constrain others, which could be distant. If we know that a wave is a plane harmonic, its whole history will be fixed by the wave vector at any point; or if the Ricci tensor vanishes within a certain region we know it will at most contain a gravitational wave, which will be constrained by circumstances on the boundary. *Inertia* is significantly constrained by distant values of the matter field; so significantly that even the matter on a single simultaneity slice can be enough to fix inertia everywhere on that slice; the determination becomes even stronger and more general if matter in the rest of the universe is appealed to as well.[27] But such determination has been dealt with abundantly;[28] and again, if too much matter is made available to the relationalist, satisfaction of his principles becomes uninterestingly easy. Rather than appealing to *all* values of T^a_b, everywhere-everywhen, or even just to T^a_b everywhere-now, we will chiefly consider how inertia is constrained by matter here-now.

3.3. Inertia

Inertial motion is free and not forced by alien influences to deviate from its natural course. The characterisation is general, its terms take on specific meaning in particular theories; in general relativity, inertial motion is subject only to gravity and not to electromagnetic or other forces; we accordingly identify *inertia* with the structures that guide the free fall of bodies (perhaps the hands of clocks too) by determining the (possibly parametrised) geodesics they describe.

We have seen that Einstein identifies inertia with the metric g, which in general relativity, where ∇g vanishes (along with torsion), corresponds to the affine structure given by the Levi-Civita connection $\nabla = \Pi_{\mathbf{0}}$, with twenty degrees of freedom. It gives the *parametrised* geodesics $\sigma_{\mathbf{0}} : (a_{\mathbf{0}}, b_{\mathbf{0}}) \to M;\ s_{\mathbf{0}} \mapsto \sigma_{\mathbf{0}}(s_{\mathbf{0}})$ through $\nabla_{\dot{\sigma}_{\mathbf{0}}}\dot{\sigma}_{\mathbf{0}} = \mathbf{0}$, and represents the 'inertia' of the parameter, hence of time or the hands of clocks, along with that of matter. (M is the differential manifold representing the universe.)

But time and clocks may be less the point here than plain free fall. General relativity offers another candidate for inertia, namely projective structure[29] Π, which gives the 'generalised geodesics'[30] $\sigma : (a, b) \to M;\ s \mapsto \sigma(s)$, through $\nabla_{\dot{\sigma}}\dot{\sigma} = \lambda\dot{\sigma}$. Projective structure just represents free fall, in other words the inertia of bodies alone, not of bodies *and* the hands of accompanying clocks. One can say it is purely 'material,' rather than 'materio-temporal.'

[27] The *Machsches Prinzip* quoted in footnote 20 above is usually considered too strong to be right. But the determination Einstein has in mind is of a more holistic and '*univers*al' kind than one may imagine: (1918a, two pages on): "[...] all the masses of the universe will participate in the generation of the G-field." He seems to be thinking of the initial-value problem, boundary conditions *etc.* rather than of the tensor algebra at a single point.

[28] See for example Wheeler (1959), Choquet-Bruhat (1962), Ó Murchadha & York (1974), Isenberg & Wheeler (1979), Choquet-Bruhat & York (1980), Isenberg (1981), York (1982), Ciufolini & Wheeler (1995, §5).

[29] The distinction is due to Weyl (1921). See Malament (2006) for a more modern treatment.

[30] Or alternatively the unparametrised geodesics, in other words just the image $\Im(\sigma) = \Im(\sigma_0) \subset M$.

In the class $\Pi = \{\Pi_\alpha : \alpha \in \Lambda_1(M)\}$ of connections projectively equivalent to ∇, a particular connection Π_α is singled out by a one-form α, which fixes the parametrisations s of all the generalised geodesics σ. So projective structure has twenty-four degrees of freedom, four—namely $\alpha_0, \ldots, \alpha_3$—more than affine structure (where $\alpha_a = \langle \alpha, \partial_a \rangle$). We can write

$$\langle dx^a, \Pi_{\alpha\,\partial_b} \partial_c \rangle = \Gamma^a_{bc} + \delta^a_b \alpha_c + \delta^a_c \alpha_b,$$

where the Γ^a_{bc} are the components of the Levi-Civita connection. The most meaningful part of the added freedom would appear to be the 'acceleration'

$$\lambda = -2\langle \alpha, \dot{\sigma} \rangle = -2\,\alpha_a \frac{d\sigma^a}{ds} = -\left(\frac{ds}{ds_{\mathbf{0}}}\right)^2 \frac{d^2 s_{\mathbf{0}}}{ds^2}$$

of the parameter s along the generalised geodesic σ determined by Π_α.

3.4. The underdetermination of inertia by matter

We can now try to characterise and quantify the underdetermination of inertia by matter. The relationship between affine structure and curvature is given by

$$B^a_{bcd} = 2\Gamma^a_{b[d,c]} + \Gamma^e_{bd}\Gamma^a_{ec} - \Gamma^e_{bc}\Gamma^a_{ed}.$$

The curvature tensor B^a_{bcd} has ninety-six (6×4^2) independent quantities, eighty if the connection is symmetric, only twenty if it is metric, in which case B^a_{bcd} becomes the Riemann tensor R^a_{bcd}. Einstein's equation expresses the equality of the matter tensor T_{ab} and Einstein tensor

$$G_{ab} = R_{ab} - \frac{1}{2} R g_{ab},$$

where the Ricci scalar R is the contraction $g^{ab} R_{ab}$ of the Ricci tensor $R_{ab} = R^c_{acb}$. Many Riemann tensors therefore correspond[31] to the same Ricci tensor, to the same Einstein tensor. By removing the ten freedom degrees of a symmetric index pair, the contraction $R_{ab} = R^c_{acb}$ leaves the ten independent quantities of the Ricci tensor; the lost freedoms end up in the Weyl tensor

$$C_{abcd} = R_{abcd} - g_{a[c}R_{d]b} + g_{b[c}R_{d]a} + \frac{1}{3} R g_{a[c}g_{d]b}.$$

To the disappointment of the relationalist, local matter would therefore seem to underdetermine affine structure by ten, projective structure by fourteen degrees of freedom—some of which may prove less meaningful than others, however, as we shall soon see.

To understand how gauge choices eliminate eight degrees of freedom we can look at gravitational waves[32] in the linear approximation.

[31] The correspondence is complicated by the presence in the Einstein tensor of the metric—without which even T_{ab} has little meaning.

[32] For a recent and readable account see Kennefick (2007).

3.5. Inertia without matter

Through Einstein's equation, then, matter determines the rough curvature given by the Ricci tensor. The *absence* of matter, for instance, makes that curvature vanish identically—but not the finer Riemann curvature, which can oscillate nonetheless, and in many different ways. Here we will see the subtle freedom left by the absence of matter.

The weak perturbation $h_{ab} = g_{ab} - \eta_{ab}$ would (being symmetrical) first appear to maintain the ten freedoms of the Weyl tensor. It is customary to write $\gamma_{ab} = h_{ab} - \frac{1}{2}\eta_{ab}h$, where h is the trace h^a_a. A choice of coordinates satisfying the four continuity conditions $\partial_b \gamma_{ab} = 0$ allows us to set $\gamma_{a0} = 0$, which does away with the four 'temporal' freedoms. There remains a symmetric 'purely spatial' matrix

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_{11} & h_{21} & h_{31} \\ 0 & h_{21} & h_{22} & h_{32} \\ 0 & h_{31} & h_{32} & h_{33} \end{pmatrix}$$

(for now $\gamma_{ab} = h_{ab}$) with six degrees of freedom. We can also require h^a_a to vanish, which eliminates another freedom, leaving five. To follow the fates of these remaining freedoms we can consider the plane harmonic

$$h_{ab} = \text{Re}\{A_{ab}e^{i\langle k,x\rangle}\} \tag{3}$$

obeying $\Box h_{ab} = 0$. If the wave equation were $\Box_{\mathtt{c}} h_{ab} = (\partial_0^2 - \mathtt{c}^2\nabla^2)h_{ab} = 0$ instead, with arbitrary $\mathtt{c}$, the wave (co)vector k would have four independent components $k_a = \langle k, \partial_a\rangle$:

- the direction $k_1 : k_2 : k_3$, in other words $\mathbf{k}/|\mathbf{k}|$ (two)
- the length $|\mathbf{k}| = \sqrt{k_1^2 + k_2^2 + k_3^2}$ (one)
- the frequency $\omega = k_0 = \langle k, \partial_0\rangle = \mathtt{c}|\mathbf{k}|$ (one).

Since $\mathtt{c} = 1$ is a natural constant, the condition $\Box h_{ab} = 0$ reduces them to three, by identifying $|\mathbf{k}|$ and ω, which makes the squared length $\langle k, k^\sharp\rangle = k_0k^0 - |\mathbf{k}|^2$ vanish. And even these three degrees of freedom disappear into the coordinates if the wave is made to propagate along the third spatial direction, leaving two $(5 - 3)$ freedoms, of polarisation; for such an alignment—bearing in mind the three orthogonality relations[33] $A_{a1}k^1 + A_{a2}k^2 + A_{a3}k^3 = 0$—gets rid of the components h_{3b}, leaving a traceless symmetric matrix

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_{11} & h_{21} & 0 \\ 0 & h_{21} & -h_{11} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

with two independent components, $h_{11} = -h_{22}$ and $h_{12} = h_{21}$.

The above gauge choices therefore eliminate eight degrees of freedom:

- the four 'temporal' coordinates h_{a0} eliminated by the conditions $\partial_b \gamma_{ab} = 0$

[33] Which follow from $\partial_b h_{ab} = 0$ and situate the polarisation tensor A_{ab} in the plane $\mathbf{k}^\perp \subset k^\perp$ orthogonal to the three-vector $\mathbf{k} \in k^\perp$.

- the freedom eliminated by $\gamma^a_a = 0$
- the three freedoms of $\mathbf{k}$ eliminated by the conditions $A_{ab}k^b = 0$.

The physical meaning of coordinate transformations has been amply discussed, notably in the literature on the hole argument.[34] The relationalist will take the eight degrees freedom eliminated by the above gauge choices to be meaningless,[35] to lessen the underdetermination of inertia—and because as a relationalist he would anyway. We will too, and concentrate on the status of the double freedom of polarisation.

Matter still underdetermines inertia, then, by two degrees freedom, which obstruct the satisfaction of Mach's principle. But are they really there? Or do they in fact share the fate of the eight freedoms eliminated by gauge choices, which we have dismissed as physically meaningless? The relationalist may prefer to discard them too as an empty mathematical fiction without physical consequence; but we know their physical meaning is bound up with that of gravitational waves, whose polarisation they represent.[36]

3.6. Gravitational waves and general covariance

To deal with the polarisation obstructing a full determination of inertia the relationalist can insist on general covariance,[37] *which is not satisfied by gravitational waves*. He will argue that since the *generation, energy and detection* of gravitational waves can all be transformed away, they and the underdetermination of inertia by matter are just about as fictitious as the eight freedoms that recently disappeared into the coordinates.

If gravitational waves had mass-energy their reality could be hard to contest. We have seen that general relativity does allow the attribution of mass-energy to the gravitational field, to gravitational waves, through the pseudotensor t^a_b; but also that t^a_b has the wrong transformation behaviour.

Is the physical meaning of t^a_b really compromised by its troubling susceptibility to disappear, and reappear under acceleration?

A similar question arose, in §3.1, when we wondered what to count as matter. There we did not provide the relationalist with the 'gravitational matter' that would have favoured his agenda by making his principles easier to satisfy, on grounds that, being mere 'opinion,' it was too insubstantial and tenuous to count. To be fair to the relationalist we should perhaps dismiss t^a_b once more as mere opinion. But we have no reason to be fair, and are merely exploring certain logical possibilities. Perhaps 'matter' was something stronger, and required more; maybe a quantity that comes and goes with the accelerations of the observer can be real, despite being immaterial; so we shall treat the physical meaning of t^a_b—as opposed to its suitability for the representation of matter—as a further issue.

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[34] See Earman & Norton (1987), Butterfield (1987, 1989), Norton (1988), Earman (1989, §9), Maudlin (1993), Stachel (1993), Rynasiewicz (1994), Belot (1996), Ryckman (2005, pp. 19-23), Lusanna & Pauri (2006a), Rovelli (2007, §2.2.5), Esfeld & Lam (2008, §2) for instance.

[35] *Cf.* Rovelli (2007, §2.3.2).

[36] The four "ontic" freedoms of Lusanna & Pauri (2006a,b,c) include the momenta—conjugate to the polarisations—whose physical meaning is also bound up with that of gravitational waves.

[37] General covariance and invariance are rightly confused in much of the literature, and here too. Whether it is a number or the appearance of a law that remains unchanged is less the point than the generality—*complete* or *merely Lorentz*, for instance—of the transformations at issue.

General relativity has been at the centre of a tradition, conspicuously associated with Hilbert (1917),[38] Cassirer (1921), Einstein (1990) himself eventually, Langevin (1922, pp. 31, 54), Meyerson (1925, §48), Russell (1927, §VII) and Weyl (2000, §17), linking physical reality or objectivity to appropriate transformation properties, to something along the lines of invariance or covariance. Roots can be sought as far back as Democritus, who is said to have claimed that "sweet, bitter, hot, cold, colour" are mere opinion, "only atoms and void"—concerning which there ought in principle to be better agreement—"are real"; or more recently in Felix Klein's 'Erlangen programme' (1872), which based geometrical relevance on invariance under the groups he used to classify geometries. Bertrand Russell, in his version of neutral monism,[39] identified objects with the class of their appearances from different points of view—not really an association of invariance and reality, but an attempt to transcend the misleading peculiarities of individual perspectives nonetheless. Hilbert explicitly required invariance in the second part (1917) of *Die Grundlagen der Physik*; objects with the wrong transformation properties were physically meaningless in his scheme. Levi-Civita (1917, p. 382), Schrödinger (1918, pp. 6-7) and Bauer (1918, p. 165), who saw the relation of physical meaning to appropriate transformation properties as a central feature of relativity theory, likewise questioned[40] the reality of the energy-momentum pseudotensor. Schrödinger noted that appropriate coordinates make t^a_b vanish identically in a curved spacetime (containing only one body however); Bauer that appropriate coordinates would give energy-momentum to flat regions.

Einstein first seemed happy to ascribe physical meaning to objects with the wrong transformation properties. In January 1918 he upheld the reality of t^a_b in a paper on gravitational waves:

> [Levi-Civita] (and with him other colleagues) is opposed to the emphasis of equation $[\partial_\nu(\mathfrak{T}^\nu_\sigma + \mathfrak{t}^\nu_\sigma) = 0]$ and against the aforementioned interpretation, because the $\mathfrak{t}^\nu_\sigma$ do not make up a *tensor*. Admittedly they do not; but I cannot see why physical meaning should only be ascribed to quantities with the transformation properties of tensor components.[41]

3.7. Einstein's reply to Schrödinger

In February Einstein (1918c) responded to Schrödinger's objection, arguing that with more than one body the stresses t^a_b $(a, b = 1, 2, 3)$ transmitting gravitational interactions would not vanish: Take two bodies M_1 and M_2 kept apart by a rigid rod R aligned along ∂_1. M_1 is enclosed in a two-surface ∂V which leaves out M_2 and hence cuts R (orthogonally, we can add, for simplicity). Integrating over the three-dimensional region V, the conservation law $\partial_a U^a_b = 0$ yields[42]

$$\frac{d}{dx^0}\int_V U^0_b d^3V = \int_{\partial V}\sum_{a=1}^{3} U^a_b d^2\Sigma_a :$$

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[38] See also Brading & Ryckman (2008).

[39] Accounts can be found in Russell (1921, 1927, 1956). But see also Russell (1991, p. 14), which was first published in 1912. *Cf.* Cassirer (1921, p. 36).

[40] See Cattani & De Maria (1993).

[41] Einstein (1918b, p. 167): "[Levi-Civita] (und mit ihm auch andere Fachgenossen) ist gegen eine Betonung der Gleichung $[\partial_\nu(\mathfrak{T}^\nu_\sigma + \mathfrak{t}^\nu_\sigma) = 0]$ und gegen die obige Interpretation, weil die $\mathfrak{t}^\nu_\sigma$ keinen T e n s o r bilden. Letzteres ist zuzugeben; aber ich sehe nicht ein, warum nur solchen Größen eine physikalische Bedeutung zugeschrieben werden soll, welche die Transformationseigenschaften von Tensorkomponenten haben."

[42] We have replaced Einstein's cosines with the notation used, for instance, in Misner *et al*. (1973).

any change in the total energy $\int U_b^0 d^3V$ enclosed in volume V would be due to a flow, represented on the right-hand side, through the boundary ∂V. Since the situation is stationary and there are no flows, both sides of the equation vanish, for $b = 0, 1, 2, 3$. Einstein takes $b = 1$ and uses

$$\int_{\partial V} \sum_{a=1}^{3} U_1^a d^2\Sigma_a = 0.$$

He is very concise, and leaves out much more than he writes, but we are presumably to consider the intersection $R \cap \partial V$ of rod and enclosing surface, where it seems ∂_1 is orthogonal to ∂_2 and ∂_3, which means the off-diagonal components T_1^2 and T_1^3 vanish, unlike the component T_1^1 along R. Since

$$-\int_{\partial V} \sum_{a=1}^{3} t_1^a d^2\Sigma_a$$

must be something like T_1^1 times the sectional area of R, the gravitational stresses t_1^1, t_1^2, t_1^3 cannot all vanish identically. The argument is contrived and full of gaps, but the conclusion that gravitational stresses between two (or more) bodies cannot be 'transformed away' seems valid.

Then in May we again find Einstein lamenting that

> Colleagues are opposed to this formulation [of conservation] because $(\mathfrak{U}_\sigma^\nu)$ and $(\mathfrak{t}_\sigma^\nu)$ are not tensors, while they expect all physically significant quantities to be expressed by scalars or tensor components.[43]

In the same paper he defends his controversial energy conservation law,[44] which we shall soon come to.

3.8. *Conservation under coordinate substitutions*

Conservation is bound to cause trouble in general relativity. The idea usually is that even if the conserved quantity—say a 'fluid' with density ρ—doesn't stay put, even if it moves and gets transformed, an appropriate total over space nonetheless persists through time; a spatial integral remains constant:

$$\frac{d}{dt} \int \rho \, d^3V = 0. \tag{4}$$

So a *clean* separation into *space* (across which the integral is taken) and *time* (in the course of which the integral remains unchanged) seems to be presupposed when one speaks of conservation. In relativity the clean separation suggests a Minkowskian orthogonality

$$\partial_0 \perp \text{span}\{\partial_1, \partial_2, \partial_3\} \tag{5}$$

between time and space,[45] which already restricts the class of admissible transformations and hence the generality of any covariance. However restricted, the class will be far from empty; and

[43]Einstein (1918d, p. 447): "Diese Formulierung stößt bei den Fachgenossen deshalb auf Widerstand, weil $(\mathfrak{U}_\sigma^\nu)$ und $(\mathfrak{t}_\sigma^\nu)$ keine Tensoren sind, während sie erwarten, daß alle für die Physik bedeutsamen Größen sich als Skalare und Tensorkomponenten auffassen lassen müssen."

[44]See Hoefer (2000) on the difficulties of energy conservation.

[45]*Cf.* Einstein (1918d, p. 450).

what if the various possible integrals it produces give different results? Or if some are conserved and others aren't?

An integral law like (4) can typically be reformulated as a 'local' divergence law

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0,$$

which in four dimensions becomes $\partial_a J^a = 0$, where $\mathbf{j} = \rho\mathbf{v}$, $\mathbf{v}$ is the three-velocity of the fluid, $J^0 = \rho$ and $J^a = \langle dx^a, \mathbf{j}\rangle$ for $a = 1, 2, 3$. But the integral law is *primary*; the divergence law *derived from it* only really expresses conservation to the extent that it is fully equivalent to the more fundamental integral law. As Einstein puts it:

> From the physical point of view this equation $[\partial \mathfrak{T}_\sigma^\nu / \partial x_\nu + \frac{1}{2} g_\sigma^{\mu\nu} \mathfrak{T}_{\mu\nu} = 0]$ cannot be considered completely equivalent to the conservation laws of momentum and energy, since it does not correspond to integral equations which can be interpreted as conservation laws of momentum and energy.[46]

In flat spacetime, with inertial coordinates, the divergence law $\partial_a T_b^a = 0$ can be unambiguously integrated to express a legitimate conservation law. But the ordinary divergence $\partial_a T_b^a$ only vanishes in free fall (where it coincides with $\nabla_a T_b^a$), and otherwise registers the gain or loss seen by an accelerated observer. If such variations are to be viewed as exchanges with the environment and not as definitive acquisitions or losses, account of them can be taken with t_b^a, which makes $\partial_a(T_b^a + t_b^a)$ vanish by compensating the difference.[47] The generally covariant condition $\partial_a(T_b^a + t_b^a) = 0$, which is equivalent to $\nabla_a T_b^a = 0$ and $\partial_a T_b^a + \frac{1}{2}\partial_b g^{ca} T_{ca} = 0$, can also be unambiguously integrated in flat spacetime to express a legitimate conservation law. But integration will be less straightforward in curved spacetime, where it involves a distant comparison of direction which cannot be both generally covariant and integrable.

Nothing prevents us from comparing the values of a genuine scalar at distant points. But we know the density of mass-energy transforms according to

$$(\rho, \mathbf{0}) \mapsto \frac{\rho}{\sqrt{1 - |\mathbf{v}|^2}}(1, \mathbf{v}),$$

where $\mathbf{v}$ is the three-velocity of the observer. So the invariant quantity is not the mass-energy density, but (leaving aside the stresses that only make matters worse) the mass-energy-momentum density, which *is manifestly directional*. And how are distant directions to be compared? Comparison of components is not invariant: directions or rather component ratios that are equal with respect to one coordinate system may differ in another. Comparison by parallel transport will not depend on the coordinate system, but on the path followed.

3.9. Einstein's defence of energy conservation

Einstein tries to get around the problem in *Der Energiesatz in der allgemeinen Relativitätstheorie* (1918d). Knowing that conservation is not a problem in flat spacetime, where parallel transport

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[46] Einstein (1918d, p. 449): "Vom physikalischen Standpunkt aus kann diese Gleichung nicht als vollwertiges Äquivalent für die Erhaltungssätze des Impulses und der Energie angesehen werden, weil ihr nicht Integralgleichungen entsprechen, die als Erhaltungssätze des Impulses und der Energie gedeutet werden können."

[47] *Cf.* Brading & Ryckman (2008, p. 136).

is integrable, he makes the universe look as Minkowskian as possible by keeping all the mass-energy that undermines the flatness neatly circumscribed (which is already questionable, for matter may be infinite).

Einstein attributes an energy-momentum J to the universe, which he legitimates by imposing a kind of 'general' (but in fact restricted) invariance on each component J_a, defined as the spatial integral

$$J_a = \int \mathfrak{U}_a^0 d^3V$$

of the combined energy-momentum $\mathfrak{U}_a^0 = \mathfrak{T}_b^0 + \mathfrak{t}_b^0$ of matter and field (where $\mathfrak{U}_b^a = U_b^a \sqrt{-\mathbf{g}}$ *etc.*, and the stresses seem to be neglected). To impose it he separates time and space through (5), and requires the fields $\mathfrak{T}_b^a$ and $\mathfrak{t}_b^a$ to vanish outside a bounded region B. Einstein is prudently vague about B, which is first a subset of a simultaneity slice Σ_t, and then gets "infinitely extended in the time direction,"[48] to produce the world tube B_{∂_0} described by B along the integral curves of the "time direction" ∂_0. The supports $\overline{\mathfrak{T}}$ and $\overline{\mathfrak{t}}$ of $\mathfrak{T}_b^a$ and $\mathfrak{t}_b^a$ are contained in B_{∂_0} by definition; but $\overline{\mathfrak{T}}$ may be much smaller than $\overline{\mathfrak{t}}$ and hence B_{∂_0}: we have no reason to assume that $\overline{\mathfrak{T}}$ does not contain bodies that radiate gravitational waves—of which $\mathfrak{t}_b^a$ would have to take account—along the lightcones delimiting the causal future of $\overline{\mathfrak{T}}_t = \overline{\mathfrak{T}} \cap \Sigma_t$. Gravitational waves could therefore, by obliging B_{∂_0} to be much larger than $\overline{\mathfrak{T}}$, spoil the picture of an essentially Minkowskian universe barely perturbed by the 'little clump' of matter-energy it contains.

The generality of any invariance or covariance is already limited by (5); Einstein restricts it further by demanding Minkowskian coordinates $\eta_{ab} = g_{ab}$ (and hence flatness) outside B_{∂_0}. He then uses the temporal constancy $dJ_a/dx^0 = 0$ of each component J_a, which follows from $\partial_a \mathfrak{U}_b^a = 0$, to prove that J_a has the same value $(J_a)_1 = (J_a)_2$ on both three-dimensional simultaneity slices[49] $x^0 = t_1$ and $x^0 = t_2$ of coordinate system K; and value $(J'_a)_1 = (J'_a)_2$ at $x'^0 = t'_1$ and $x'^0 = t'_2$ in another system K'. A third system K'' coinciding with K around the slice $x^0 = t_1$ and with K' around $x'^0 = t'_2$ allows the comparison of K and K' across time. The invariance of each component J_a follows from $(J_a)_1 = (J'_a)_2$. Having established that, Einstein views the world as a 'body' immersed in an otherwise flat spacetime, whose energy-momentum J_a is covariant under the transformation laws—Lorentz transformations—considered appropriate[50] for that (largely flat) environment. Unusal mixture of transformation properties: four components, each one 'somewhat' invariant, which together make up a four-vector whose Lorentz covariance would be of questionable appropriateness even if the universe were *completely* flat.

Einstein's argument was nonetheless effective, and persuaded[51] the community, which became and largely remains more tolerant of objects (including laws and calculations) with dubious transformation properties.

3.10. Einstein's conversion

Einstein's own tolerance would not last, however; in 1921 he would say that

> With the help of speech, different people can compare their experiences to a certain extent. It turns out that some—but not all—of the sensory experiences of different

[48] Einstein (1918d, p. 450)

[49] For a recent treatment see Lachièze-Rey (2001).

[50] Despite Kretschmann (1917), who pointed out that even an entirely flat universe can be considered subject to general (and not just Lorentz) covariance. *Cf.* Rovelli (2007, §2.4.3).

[51] See Cattani & De Maria (1993), Hoefer (2000).

> people will coincide. To such sensory experiences of different people which, by coinciding, are superpersonal in a certain sense, there corresponds a reality. The natural sciences, and in particular the most elementary one, physics, deal with that reality, and hence indirectly with the totality of such experiences. To such relatively constant experience complexes corresponds the concept of the physical body, in particular that of the rigid body.[52]

Admittedly he only speaks of the "sensory experiences of different people" and not explicitly of the transformations that convert sensations between them, nor of general covariance for that matter. Not explicitly, but almost: he eventually mentions physics; *experiences* in physics can be called *measurements*, and they tend to yield numbers; theory provides the transformations converting the numbers measured by one person into those measured by another. For measurements yielding a single number, the interpersonal 'coincidence' at issue can be interpreted as numerical equality: only genuine *scalars*—which are the same for everyone—would belong to the 'superpersonal reality.' With measurements that produce *complexes* of numbers the notion of 'coincidence' upon which reality rests is less straightforward: it will no longer be a matter of numerical equality, for each component of the complex (which would be much too strong); it will have to be a more holistic kind of correspondence, to do with the way the components change together. Vanishing is an important criterion: a nonvanishing complex whose components can be 'transformed away' cannot be physically real; a complex whose components all vanish cannot 'coincide' with one whose components don't. Of course the characteristic class of transformations will not be the same in every theory; in general relativity it is the most general class (of transformations satisfying mimimal requirements of continuity and differentiability). So it does not seem unreasonable to interpret the above passage as saying that *only generally covariant notions represent reality in general relativity*.

Eight pages on Einstein speaks of geometry in a similar spirit:

> In Euclidean geometry it is manifest that only (and all) quantities that can be expressed as invariants (with respect to linear orthogonal coordinates) have objective meaning (which does not depend on the particular choice of the Cartesian system). It is for this reason that the theory of invariants, which deals with the structural laws of invariants, is significant for analytic geometry.[53]

Here "objective meaning" is explicitly attributed to invariance under the characteristic class of transformations.

The tension with the passages quoted in footnotes 41 and 43 above is not without its significance for the relationalist, who at this point can really question the legitimacy of a mathematical tolerance whose champion soon developed an intransigence surprisingly reminiscent of the severity expressed by his previous opponents.

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[52]Einstein (1990, p. 5): "Verschiedene Menschen können mit Hilfe der Sprache ihre Erlebnisse bis zu einem gewissen Grade miteinander vergleichen. Dabei zeigt sich, daß gewisse sinnliche Erlebnisse verschiedener Menschen einander entsprechen, während bei anderen ein solches Entsprechen nicht festgestellt werden kann. Jenen sinnlichen Erlebnissen verschiedener Individuen, welche einander entsprechen und demnach in gewissem Sinne überpersönlich sind, wird eine Realität gedanklich zugeordnet. Von ihr, daher mittelbar von der Gesamtheit jener Erlebnisse, handeln die Naturwissenschaften, speziell auch deren elementarste, die Physik. Relativ konstanten Erlebnis-komplexen solcher Art entspricht der Begriff des physikalischen Körpers, speziell auch des festen Körpers."

[53]"Offenbar haben in der euklidischen Geometrie nur solche (und alle solche) Größen eine objektive (von der besonderen Wahl des kartesischen Systems unabhängige) Bedeutung, welche sich durch eine Invariante (bezüglich linearer orthogonaler Koordinaten) ausdrücken lassen. Hierauf beruht es, daß die Invariantentheorie, welche sich mit den Strukturgesetzen der Invariante beschäftigt, für die analytische Geometrie von Bedeutung ist."

One can wonder what made Einstein change his mind, after Hilbert, Levi-Civita, Schrödinger and others had failed to persuade him. At the end of the foreword, dated 9 August 1920, to Cassirer's *Zur Einstein'schen Relativitätstheorie* (1921) we discover that Einstein had read the manuscript and made comments. There Einstein would have found the first thorough justification of the mathematical severity Einstein's opponents had expressed a few years before. We know how much the philosophical writings of Hume, Mach and Poincaré had influenced Einstein,[54] and can conjecture that even here he was finally persuaded by a philosopher after the best mathematical physicists of the day had failed.

Be that as it may, the damage had been done, the cause was already lost, and indeed the lenity Einstein promoted in 1918 continues to this day. General covariance[55] is often disregarded or violated in general relativity: if a calculation works in one coordinate system, too bad if it doesn't in another; if energy conservation is upset by peculiar coordinates, never mind.

3.11. Cassirer

Before going on we can briefly consider what Einstein would have found in Cassirer's manuscript.

Cassirer calls *unity* "the true goal of science."[56] It appears to have much to do with economy, of finding

> a minimum of assumptions, which are necessary and sufficient to provide an unambiguous representation of experiences and their systematic context. To preserve, deepen and consolidate this unity, which seemed threatened by the tension between the principle of the constancy of the velocity of light, and the mechanical principle of relativity, the theory of relativity abandoned the uniqueness of measurement results for space and time quantities in different systems.[57]

Introducing differences where there were none before would seem rather to undermine or disrupt unity than to produce it . . .

> But all these relativisations are so little in contradiction with the idea of the constancy and unity of nature, that they rather are required and carried out in the name of this very unity. The variation of space and time measurements represents the necessary *condition*, through which the new invariants of the theory are first found and established.[58]

[54] See Howard (2005).

[55] *Cf.* Norton (1993).

[56] Cassirer (1921, p. 28): "[die Einheit] ist das wahre Ziel der Wissenschaft. Von dieser Einheit aber hat der Physiker nicht zu fragen, o b sie ist, sondern lediglich w i e sie ist – d. h. welches das Minimum der Voraussetzungen ist, die notwendig und hinreichend sind, eine eindeutige Darstellung der Gesamtheit der Erfahrungen und ihres systematischen Zusammenhangs zu liefern [. . .]."

[57] *Ibid.* p. 28: "Um diese Einheit, die durch den Widerstreit des Prinzips der Konstanz der Lichtgeschwindigkeit und des Relativitätsprinzips der Mechanik gefährdet schien, aufrecht zu erhalten und um sie tiefer und fester zu begründen, hat die Relativitätstheorie auf die Einerleiheit der Maßwerte für die Raum- und Zeitgrößen in den verschiedenen Systemen verzichtet."

[58] *Ibid.* p. 29: "Aber alle diese Relativierungen stehen so wenig im Widerspruch zum Gedanken der Konstanz und der Einheit der Natur, daß sie vielmehr im Namen eben dieser Einheit gefordert und durchgeführt werden. Die Variation der Raum- und Zeitmaße bildet die notwendige B e d i n g u n g, vermöge deren die neuen Invarianten der Theorie sich erst finden und begründen lassen."

The foremost invariance is what we would typically call general covariance—which Cassirer considers "the fundamental principle of general relativity":[59]

> Above all there is the general *form* itself of the laws of nature, in which we must henceforth recognise the true invariant and as such the true logical basis of nature.[60]

Cassirer sees Einstein's theory as a fundamental step in the transition between a common sense world made of (apparently invariant) 'things,' to a more abstract and theoretical world of generally invariant mathematical objects, laws and relations.[61] Only relations that hold for *all* observers are genuinely objective,[62] they alone can be objectively real "natural laws."

> We should only apply the term "natural laws," and attribute objective reality, to relationships whose form does not depend on the peculiarity of our empirical measurement, on the special choice of the four variables x_1, x_2, x_3, x_4 which express the space and time parameters.[63]

Cassirer even associates *truth* with general covariance:

> The space and time measurements in each individual system remain relative: but the truth and generality of physical knowledge, which is nonetheless attainable, lies in the reciprocal correspondence of all these measurements, which transform according to specific rules.[64]

Though they are not *generally* covariant, U^a_b, t^a_b and Γ^a_{bc} are *Lorentz* covariant, in the sense that they transform as tensors with respect to the Lorentz group. But

> Measurement in *one* system, or even in an unlimited plurality of 'privileged' systems of some sort, would yield only peculiarities in the end, rather than the real 'synthetic unity' of the object.[65]

[59] *Ibid.* p. 39: "[...] den Grundsatz der allgemeinen Relativitätstheorie, daß die allgemeinen Naturgesetze bei ganz beliebigen Transformationen der Raum-Zeit-Variablen ihre Form nicht ändern [...]."

[60] *Ibid.* p. 29: "Vor allem aber ist es die allgemeine F o r m der Naturgesetze selbst, in der wir nunmehr das eigentlich Invariante und somit das eigentliche logische Grundgerüst der Natur überhaupt zu erkennen haben."

[61] *Ibid.* pp. 34-5: "Wahrhaft invariant sind niemals irgendwelche Dinge, sondern immer nur gewisse Grundbeziehungen und funktionale Abhängigkeiten, die wir in der symbolischen Sprache unserer Mathematik und Physik in bestimmten Gleichungen festhalten. D i e s e s Ergebnis der allgemeinen Relativitätstheorie ist aber vom Standpunkt der Erkenntniskritik so wenig paradox, daß es vielmehr als das logische Resultat und als der natürliche logische Abschluß einer Gedankenbewegung angesehen werden kann, die für das gesamte philosophische und naturwissenschaftliche Denken der neueren Zeit charakteristisch ist. Für die populäre Auffassung und ihre Denkgewohnheiten bleibt freilich die radikale Auflösung der „Dinge" in bloße Beziehungen nach wie vor bedenklich und befremdlich: – denn sie glaubt mit dem Dingbegriff zugleich auch den einzig sicheren Halt aller Objektivität, aller wissenschaftlichen Wahrheit überhaupt verlieren zu müssen. Und so hat man denn von dieser Seite her immer wieder nicht sowohl das positive, als vielmehr das negative Moment der Relativitätstheorie betont; – so hat man sich stets an das gehalten, was sie zerstört, nicht an das, was sie aufbaut."

[62] *Ibid.* p. 35: "Wahrhaft objektiv können nur diejenigen Beziehungen und diejenigen besonderen Größenwerte heißen, die dieser kritischen Prüfung standhalten – d. h. die sich nicht nur für e i n System, sondern für alle Systeme bewähren."

[63] *Ibid.* p. 39: "Wir dürfen eben nur diejenigen Beziehungen Naturgesetze n e n n e n, d. h. ihnen objektive Allgemeinheit zusprechen, deren Gestalt von der Besonderheit unserer empirischen Messung, von der speziellen Wahl der vier Veränderlichen $\mathbf{x}_1\, \mathbf{x}_2\, \mathbf{x}_3\, \mathbf{x}_4$, die den Raum- und Zeitparameter ausdrücken, unabhängig ist."

[64] *Ibid.* p. 36: "Die Raum- und Zeitmaße in jedem einzelnen System bleiben relativ: aber die Wahrheit und Allgemeinheit, die der physikalischen Erkenntnis nichtsdestoweniger erreichbar ist, besteht darin, daß alle diese Maße sich wechselseitig entsprechen und einander nach bestimmten Regeln zugeordnet sind."

[65] *Ibid.* p. 37: "Die Messung in e i n e m System, oder selbst in einer unbeschränkten Vielheit irgendwelcher „berechtigter" Systeme, würde schließlich immer nur Einzelheiten, nicht aber die echte „synthetische Einheit" des Gegenstandes ergeben."

And “overcoming the anthropomorphism of the natural sensory world view is,” for Cassirer, “the true task of physical knowledge,” whose accomplishment is advanced by general covariance.[66] It is not implausible that Einstein should have been more troubled by such “peculiarities” and “anthropomorphism” after reading Cassirer’s manuscript than before.

3.12. Consistency

One hesitates—with or without Cassirer—to attach objective reality, or even importance to things overly shaped by the peculiarities, point of view, state of motion or tastes of the subject or observer. Allowing him *no* participation would be rather drastic, leaving at most the meagrest ‘objective’ residue; but too much would make the object belong more to the observer than to the common reality. Appropriate transformation properties permit a moderate and regulated participation.

Is there an easy way of characterising how much participation would be too much? Of determining the ‘appropriateness’ of transformation properties? Again: vanishing, annihilation seems an important criterion, as to which the relationalist can demand agreement for physical significance. He will deny the reality of a quantity that can be ‘transformed away,’ that disappears for some observers but not others.

But perhaps there is more at issue than just opinion or perspective. Much as one can wonder whether the different narrators in Rashomon are *lying*, rather than expressing reasonable differences in perspective; whether their versions are *incompatible*, or just coloured by stance and prejudice—here the relationalist may even complain about something as strong as *inconsistency*, while his opponent sees no more than rival points of view.

Suppose observer Ω with four-velocity V attributes speed $w = \sqrt{|g(\mathbf{w}, \mathbf{w})|}$ to body β with four-velocity W, where the (spacelike) three-velocity $\mathbf{w}$ is the projection

$$P_{V^\perp} W = \sum_{a=1}^{3} \langle dx^a, W \rangle \partial_a = W - g(V, W)V$$

onto the three-dimensional simultaneity subspace $V^\perp = \mathrm{span}\{\partial_1, \partial_2, \partial_3\}$ orthogonal to V; and the projector $P_{V^\perp} = \langle dx^1, \cdot \rangle \partial_1 + \langle dx^2, \cdot \rangle \partial_2 + \langle dx^3, \cdot \rangle \partial_3$ is the identity minus the projector $P_V = g(V, \cdot)V$ onto the ray determined by V. Another observer Ω' moving at V' sees speed

$$w' = \sqrt{|g(\mathbf{w}', \mathbf{w}')|} = \|P_{V'^\perp} W\|$$

(all of this around the same event). The short statements

- β has speed w
- β has speed w'

are contradictory. If $w' = 0$ the contradiction becomes even more striking, for then β *is moving, and isn’t*. Consistency can of course be restored with longer statements specifying perspective, but the tension between the short statements is not without significance—if the number were a scalar even they would agree. Similar considerations apply, *mutatis mutandis*, to covariance; one would then speak of form or syntax being the same, rather than of numerical equality.

[66] *Ibid.* p. 37: “Der Anthropomorphismus des natürlichen sinnlichen Weltbildes, dessen Überwindung die eigentliche Aufgabe der physikalischen Erkenntnis ist, wird hier abermals um einen Schritt weiter zurückgedrängt.”

Consistency and reality are not unrelated. Consistency is certainly bound up with mathematical existence, for which it has long been considered necessary—perhaps even sufficient.[67] And in mathematical physics, how can the physical meaning of a mathematical structure not be compromised by its inconsistency? If inconsistency prevents part of a formalism from 'existing,' how can it represent reality? The relationalist will argue that an object, like t^a_b, whose existence is complicated—perhaps even compromised—by an 'inconsistency' of sorts, cannot be physically meaningful.

3.13. PSR 1913 + 16

We can now turn from the reality of gravitational waves to their very generation, about which the relationalist can also wonder, given the shortcomings of the conservation law: if a belief in the production of radiation rests on the conservation of energy, how can it remain indifferent to those shortcomings?

Belief in gravitational radiation rests chiefly on the binary star PSR 1913 +16, which loses kinetic energy as it spirals inwards (with respect to popular coordinates at any rate). If the kinetic energy is not to disappear without trace, it has to be converted, into radiation in this case. Since its disappearance is only ruled out by the conservation law, the very generation of gravitational waves must be subject to the doubts surrounding conservation.[68] If the conservation law is suspicious enough to make us wonder whether the lost energy is really radiated into the gravitational field, why take the polarization of that radiation—which corresponds to the underdetermination of inertia by matter—seriously? As we were wondering in §3.6, couldn't it be no more than a purely decorative gauge, without reality or physical meaning? The binary star's behaviour and emission of gravitational waves can admittedly be calculated with great accuracy, but the calculations are not *generally* covariant and only work in certain coordinate systems.

Even the 'spiral' behaviour, associated so intimately with the loss of kinetic energy, is not generally covariant. A coordinate system leaving the two pulsars at the constant positions $(t, 1, 0, 0)$ and $(t, 0, 0, 0)$ is easily found.[69] If the pulsars don't move, if they have no 'kinesis,' why should they lose a kinetic energy they never had in the first place?

To question the reality or generation of gravitational waves, the relationalist would demand general covariance—one of the central *principles* of general relativity—*as a matter of principle*, whereas his opponent will fall back on the more tolerant day-to-day pragmatism of the practising, calculating, approximating physicist, who views the theory more as an instrumental collection of recipes, perturbation methods, tricks and expedients, by which even the most sacred principles can be circumvented or even sacrificed, than as a handful of fundamental and inviolable axioms from which all is to be deduced. General covariance may well have provided useful guidance over eighty years ago, but surely general relativity has now outgrown it ...

3.14. A Doppler effect

The absolutist would be doubly satisfied by the discovery of gravitational waves, which would reinforce both his belief in the transcendence of inertia and in absolute motion.

We began with Newton's efforts to sort out absolute and relative *motus*, first took (certain occurrences of) *motus* to mean *acceleration*, and accordingly considered absolute acceleration;

[67] See Poincaré (1902, p. 59).
[68] *Cf.* Hoefer (2000), Baker (2005).
[69] *Cf.* Weyl (1924, p. 198).

but now, with gravitational waves, are in a position to countenance absolute motion in a more literal sense. Let us say that relative motion is motion referred to some*thing*—where by 'thing' we mean a material object that has mass whatever the state of motion of the observer (materiality, again, *is not an opinion*). Otherwise motion will be *absolute*.

Suppose an empty flat universe is perturbed by (3). Changes in the frequency ω measured by a roving observer would indicate absolute motion, and allow a reconstruction, through $\omega = \langle k, v\rangle$, of the observer's absolute velocity v.

Is this undulating spacetime is absolute, substantival,[70] Newtonian? It is absolute to the extent that according to the criterion adopted it admits absolute motion. But its absoluteness precludes its substantival reification, which would make the motion relative to some*thing* and hence not absolute. Newton, though no doubt approving on the whole, would disown it, for "Spatium absolutum [...] semper manet similare et immobile [...]," and our undulating spacetime is neither 'similar to itself' (R^a_{bcd} oscillates, though R_{bd} vanishes identically) nor immobile.

We may remember that Newton spoke of revealing absolute *motus* through its causes and effects, through forces. Absolute *motion* is precisely what our thought experiment would reveal, and through forces, just as Newton wanted: the forces, for instance, registered by a (most sensitive) dynamometer linking the masses whose varying tidal oscillations give rise to the described Doppler effect.

The absolutist will claim, then, that gravitational waves are so real they wiggle the detector, and in so doing reveal absolute motion. But wiggling, the relationalist will object, is not generally covariant: it can be transformed away. Let us continue to suppose, for simplicity, that the masses (two are enough) making up the detector are in the middle of nowhere, and not on the surface of the earth—whose gravitational field is not the point here. In what sense do they wiggle? As with the binary star, we can find coordinate systems that leave them at the constant positions $(t, 1, 0, 0)$ and $(t, 0, 0, 0)$. Both masses describe geodesics; how can objects wiggle if they stand still and don't even accelerate? The absolutist will reply that each mass, despite moving inertially, accelerates absolutely with respect to the other, for $d^2\xi^a/d\tau^2 = R^a_{0c0}\xi^c$ does not vanish (where the $\xi^a = \langle dx^a, \xi\rangle$ are the components of the separation ξ). But the relationalist, to whom we will give the last word, can say that

$$R^a_{bcd} = R(dx^a, \partial_b, \partial_c, \partial_d) \text{ and } R^a_{bcd}\xi^c = R(dx^a, \partial_b, \partial_c, \partial_d)\langle dx^c, \xi\rangle$$

may be the components of tensors ($R(\cdot\,,\cdot\,,\cdot\,,\cdot)$ and $R(\cdot\,,\cdot\,,\xi,\cdot)$), but the numbers

$$R^a_{0c0}\xi^c = R(dx^a, \partial_0, \partial_c, \partial_0)\langle dx^c, \xi\rangle$$

aren't, being the components of a projection onto the simultaneity subspace $\partial_0^\perp$ orthogonal to the four-velocity ∂_0 of the mass taken as reference. The relative character of the acceleration is expressed by the explicit reference to a particular state of motion.

3.15. Meyerson, belief and robust persistence

Émile Meyerson (1951) provides a further principle, a further requirement the relationalist can invoke to weaken belief in gravitational waves, even if they are 'detected.'

[70] Newton never seems to use words resembling 'substance' in reference to his absolute space, whereas the literature about it is full of them.

Meyerson argues that over and above mere *légalité*, science pursues *causalité* and *explication*, which have much to do with identity and its persistence. *Causalité* amounts, roughly, to the 'preservation' of the cause as it evolves—or rather doesn't evolve—into the effect. If the identity of the cause cannot be followed through to the effect, remaining 'the same,' there may be a 'legal' regularity, but the effect is not explained, in fact Meyerson would not even speak of cause and effect. His concern with identity makes him attach importance to conservation laws,[71] which after all assert persistence; indeed he requires conservation for causality. *Belief*, he doesn't explicitly speak of; but the relationalist can nonetheless add, quite reasonably, that Meyerson's *causalité* and *explication* correspond to a stronger form of belief than mere *légalité*; that belief can be bound up with a robust survival of the cause as it progresses towards the effect.

Suppose a gravitational wave detector seems to reveal a signal (resulting from an exchange with the gravitational field) that stands out well against the noise and appears to come from the decaying binary star. Meyerson would at least see *légalité* in the correlated behaviours of star and detector. But due to what exactly? To turn the mere correlation into a genuine causal relationship, into an explanation, Meyerson would require whatever is exchanged with the detector to be *the same*—for *everyone*, surely—as what left the star and crossed the gap in between. Otherwise there would be *légalité* without *causalité*, and more timid belief.

However strengthened by a 'detection,' belief in gravitational waves could long remain fragile; the separation of signal and noise is likely to be hard and controversial. The relationalist can aggravate the fragility by tying belief to explanation and explanation to a robust persistence, to invariant conservation.

But here we are on a level of abstract general principles too distant from direct experience for the empiricist to approve; principles uncomfortably reminiscent of the *a priori* system-building of recent centuries, imaginative and untroubled by the little imperfections of nature; principles that smack of an archaic, cavalier disdain for disagreeable niceties.[72] Indeed many physicists may consider Einstein and his general covariance[73] almost as suspiciously metaphysical as the obsolete elucubrations of Spinoza or of Kant.

3.16. Final remarks

Even the first 'detections,' then, may leave the exact physical meaning of the underdetermination of affine structure by matter unclarified. The relationalist can meanwhile insist on the disturbing limitations of the conservation law and even dismiss gravitational waves, perhaps their very generation, as mere *opinion*, claiming that physical significance must rest on appropriate transformation properties. We have to remember that if inertia is taken to concern only matter and not time as well, and is accordingly represented by projective rather than affine structure, there would remain another four degrees underdetermination to dismiss as physically meaningless. But as their significance is far from clear, the gauge choice $\alpha = \mathbf{0}$ (which takes us back to affine structure) is unlikely to trouble anyone.

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[71] *Cf.* Belot (2006).

[72] *Cf.* Lusanna & Pauri (2006a, p. 717): "[. . .] as soon as one leaves the rarefied atmosphere of full general covariance and soils his hands with the dirty facts of the empirical front of GR [. . .]."

[73] *Ibid.* p. 697: "[. . .] people (philosophers, especially) should free themselves from the bewitching fascination of manifest general covariance [. . .]."

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