A DIRECT INTERPRETATION OF QUANTUM MECHANICS

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Abstract. The Quantum Mechanics is interpreted, in this article, in a simple and direct way. By combining the unitary evolution with the quantum condition that observations require the state vector to be an eigenstate of the observable, a discontinuity in evolution (the state vector reduction) seems to be mandatory. Thus, for each such discontinuity, new initial conditions for the time evolution state vector are needed, and they are obtained by measurements. Delayed-choice experiments suggest that these new initial conditions are specified after the discontinuity takes place. Consequently, because it needs initial conditions that can be specified with a delay, the time evolving state vector is semi-realistic (in the sense that it is not completely specified until the measurement is performed), and not entirely realistic. The collapse of the wave function, especially when it is combined with the entanglement, seems to be a non-local phenomenon. In fact, the non-locality is present only as a consistency requirement for the initial conditions needed to select a solution of the evolution equation.

The Direct Interpretation is intended to provide to our intuition a physical background, for helping us thinking about quantum phenomena. It identifies the main counterintuitive parts of the Quantum Mechanics in the discontinuity and the delayed initial conditions. Because it makes minimal assumptions, it is compatible with the main interpretations of Quantum Mechanics.

Two principal unclear points of Quantum Mechanics are identified in the discontinuities, and the measurement problem. Both problems will be approached in subsequent articles.

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Contents

1. Introduction

Quantum Mechanics is one of the two most successful theories in physics[1], because of its capacity of explanation and prediction. Because the quantum phenomena look so counterintuitive, some of the physicists try from time to time to interpret them in more intuitive terms. At the opposite side, other physicists are content with the abstract (although inconsistent) mathematical form, and consider that the efforts to interpret the quantum world are useless.

In this paper, I will present the central ideas of Quantum Mechanics in a way that will allow us to extract some key elements. I use these elements to provide a portrait of Quantum Mechanics, which can be resumed like this:

(1) We start with the space $\mathcal{S}_d(H)$ of functions representing the time evolution of a state vector from the Hilbert space, $\psi : \mathbb{R} \to \mathcal{H}$. The functions of $\mathcal{S}_d(H)$ are taken to be piecewise solutions of the Schrödinger equation given by the Hamiltonian H. By this we mean that they are smooth and continuous solutions of the Schrödinger equation on an interval (t_i, t_{i+1}), then, at t_i, they have a discontinuity, then again on an interval (t_{i+1}, t_{i+2}) evolve smoothly and continuously according to the same equation, and so on, for i $\in \mathbb{Z}$ (or a subset). They are waves providing the full description of reality. The piecewise continuity condition allows the wave to suffer discontinuous jumps, which seem to be required by the Quantum Mechanics [vN55].

(2) Having an equation and a space of solutions requires also extra conditions in order to identify the actual solution. In our case, the discontinuities require restatements of the initial conditions. From time to time, we add such conditions, by performing observations. The conditions in general come together with the state vector, forcing it to jump in one of the observable's eigenstates.

(3) The state vector reduction must respect the Born probabilistic rule.

(4) The conditions needed to reduce the space of solutions are not necessary "initial", but they can be "delayed": they seem to apply also to instants previous to the moment of observation. This behavior can be emphasized with "delayed-choice" experiments, and it is perhaps the key of the counterintuitive nature of Quantum Mechanics.

The present description will be used to present several non-classical aspects of the Quantum Mechanics. A quantum system is described by waves that evolves in general in a continuous manner, suffering from time to time discontinuities. The discontinuities lead to a loss of the initial information, and new measurements provide a way to replace this information. Actually, it seem that the measurement themselves are responsible for the discontinuities, hence they simply replace the old information describing the solution with the new one. Most of the paradoxical aspects of Quantum Mechanics are in general traceable to seemingly non-causal and non-local behaviors. These behaviors are nothing but *delayed initial conditions* [Wei31, Sch49, Whe77, Whe78, WZ83], combined with quantum entanglement [Sch35, EPR35]. Despite various efforts to find an underlying, more classical, or at least more local and causal explanation, and despite all the efforts

[1]The other one being, of course, the Theory of General Relativity.

to deny it, this type of behavior exists and is irreducible. What I want to point out in this paper is that, by accepting that choosing a time evolution function requires "initial conditions", and by understanding that these conditions can be specified with a delay, the strange behavior of quantum phenomena become more intuitive.

2. Wave vs. quanta

In this section we will review a central incompatibility in Quantum Mechanics. On the one hand, the Schrödinger equation describes very well the evolution of quantum systems (the wave-like behavior). On the other hand, the observation forces the system to be in an eigenstate of the observable (the quantum-like behavior). We have a partial differential equation (the Schrödinger equation), and a set of initial conditions, provided by the observations. The core problem is that it seems that there are too many initial conditions, this leading to incompatibilities.

We can understand easily the problem we have by thinking at a linear function $f : \mathbb{R} \to \mathbb{R}$. Generally, there exist two real numbers a and b, such that $f(x) = ax + b$, for all $x \in \mathbb{R}$. We can determine a and b by knowing two distinct points on the line, (t_1, f_1) and (t_2, f_2), where $t_1 \neq t_2$. We construct a system of two linear equations in a and b and we solve it. But if we know more than two points, we have more than two linear equations, with only two unknowns. If we have more than two independent conditions, the system is necessarily inconsistent. It is like we want to determine a line by more than two points that are not collinear.

Similarly, when we perform a quantum measurement, we are free to choose observables that have only eigenstates that are incompatible with the previously established initial conditions. This inconsistency is usually solved by admitting that the solution has discontinuities.

Admitting that the quantum system may evolve with discontinuities, combined with the fact that each observable let the system choose among more possible initial conditions, has as consequence a certain randomness. The Born rule [Born26] gives the probability distribution governing this randomness.

2.1. The wave-like behavior

A quantum system is described by a vector (named a *state vector*) $|\psi\rangle$ in a complex Hilbert space[2] $\mathcal{H}$, with the hermitian inner product of two vectors $|\psi_1\rangle$ and $|\psi_2\rangle$ denoted by $\langle\psi_1|\psi_2\rangle$. Any not null vector in $\mathcal{H}$ represents a possible state of the system under consideration. If one state vector $|\psi'\rangle$ is a multiple of $|\psi\rangle$, $|\psi'\rangle = z|\psi\rangle, z \in \mathbb{C} - \{0\}$, then both vectors represent the same quantum state. We will consider in general that the state vectors are unit vectors.

The Schrödinger equation describes how a quantum system evolves, by describing how its associate state vector ψ evolves:

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = H(t)|\psi(t)\rangle. \tag{1}$$

The Schrödinger equation has an infinity of solutions, forming a space $\mathcal{S}(H)$. For any initial state $|\psi_0\rangle$ at an initial moment t_0, it provides a unique continuous and smooth solution $|\psi\rangle : \mathbb{R} \to \mathcal{H}$. In order to specify the solution, it is enough to specify its value for a time t_0.

[2]Similar considerations can be made by considering a rigged complex Hilbert space instead.

The figure 1 represents, in a very symbolic manner, several solutions of the Schrödinger equation, and the way to select one of them by imposing an initial condition.

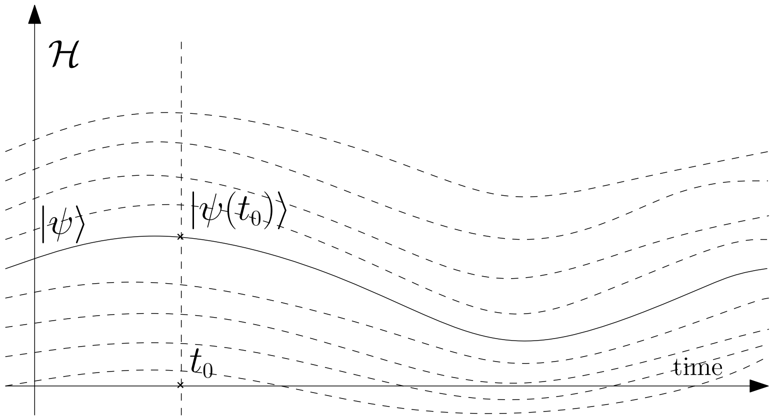

FIGURE 1. For each possible $|\psi_0\rangle \in \mathcal{H}$ there is a unique solution of the Schrödinger equation, satisfying the initial condition $|\psi(t_0)\rangle = |\psi_0\rangle$.

The Schrödinger equation is linear, it has the property of *superposition*. This means that if $|\psi_1\rangle$ and $|\psi_2\rangle$ are solutions, then their sum $|\psi_1\rangle + |\psi_2\rangle$ is also solution of the same equation, and any linear combination (with complex coefficients) is again a solution.

By solving the Schrödinger equation, we obtain a solution of the form

$$|\psi(t)\rangle = U(t, t_0)|\psi_0\rangle, \tag{2}$$

where $U(t, t_0)$ is a unitary operator (please refer to figure 2). In the case of H independent of time, the unitary operator has the form:

$$U(t, t_0) = \exp\left(-\mathrm{i}\frac{t - t_0}{\hbar}H\right). \tag{3}$$

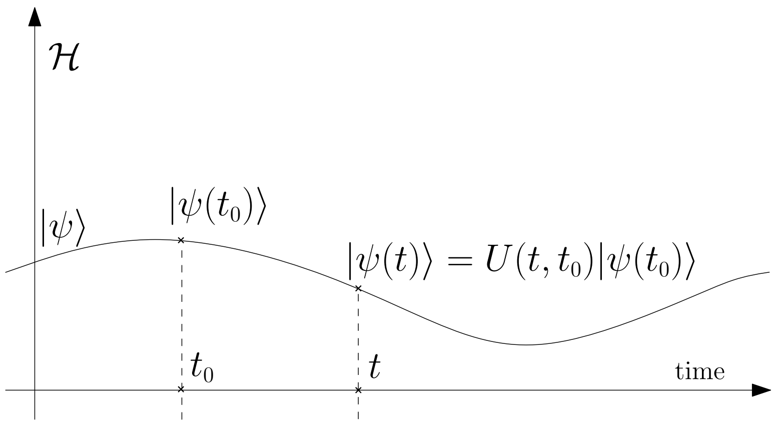

FIGURE 2. By knowing the evolution operator U and the state vector $|\psi(t_0)\rangle$ at a time t_0, we can predict the state at another time t.

Because the evolution operator U is unitary, the hermitian inner product on the Hilbert space is preserved. This means that, for two solutions $|\psi_1\rangle$ and $|\psi_2\rangle$ of the Schrödinger equation, $\langle\psi_1(t_0)|\psi_2(t_0)\rangle = \langle\psi_1(t)|\psi_2(t)\rangle$ for any $t_0, t \in \mathbb{R}$.

2.2. The quantum behavior

In order to find out the state of a quantum system, we have to perform a measurement. We don't know how to measure the system itself, but only its observables. An *observable* is a linear operator O on the Hilbert space $\mathcal{H}$. By measuring the observable O of a quantum system $|\psi\rangle$, we obtain a number λ. This number is an eigenvalue of the operator O, and if it is obtained, the system is found to be in an eigenstate $|\psi_\lambda\rangle$ corresponding to λ. Because λ is usually a real number, it is customary to impose to the observables the condition of being hermitian.

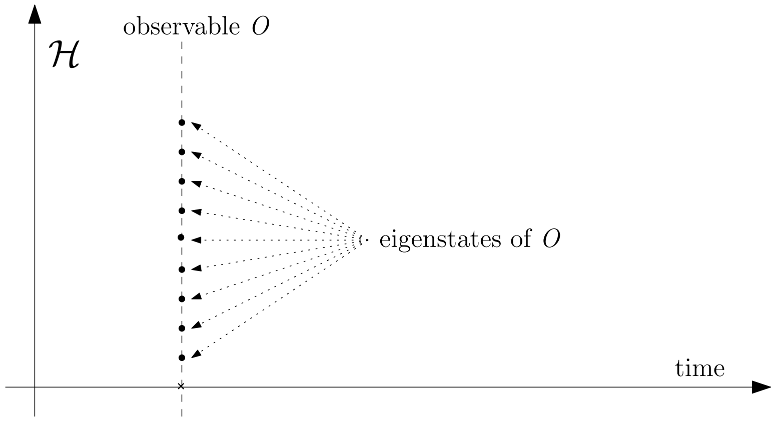

FIGURE 3. The eigenstates of the observable O form a subset of the Hilbert space $\mathcal{H}$; we can depict them as a subset of points of the vertical axis representing the Hilbert space.

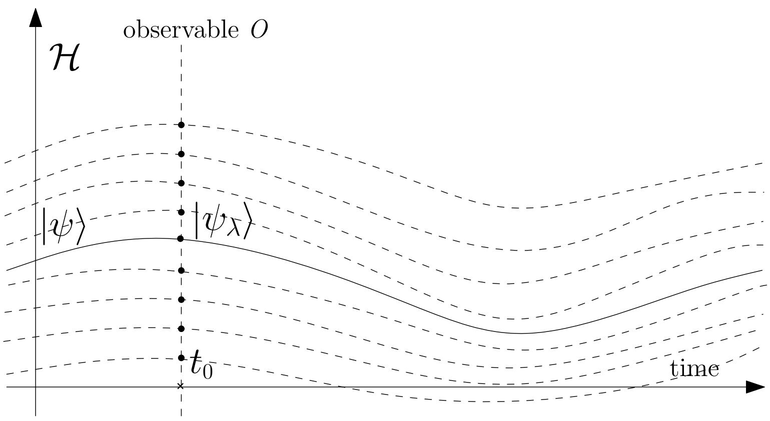

FIGURE 4. The quantum system is found to be in an eigenstate $|\psi_\lambda\rangle$ of the eigenvalue λ.

The linear operators (an hence the hermitian operators too) are characterized by their eigenvalues and the corresponding eigenstates. The only effect an observable has to the measured system is given by the eigenstates, hence we can forget, for the moment, about the eigenvalues and represent the observable only by its eigenstates, as in the figure 3.

In the figure 4 we can see that the condition imposed by the observable to the quantum system $|\psi\rangle$ at the time t_0 takes the form $|\psi(t_0)\rangle \in \{\text{eigenstates of } O\}$.

2.3. **Two incompatible laws**

The (unitary) evolution of any quantum system is governed very accurately by the Schrödinger equation. If at t_0 the state is observed and found to be $|\psi(t_0)\rangle$, at the time t_1 it evolves to $|\psi(t_1)\rangle$, according to equation (2). On the other hand, by measuring an observable O_1 we find the quantum system as being only in one of a specific set of states, namely the eigenstates of the observable. The problem is that it is very likely that $|\psi(t_1)\rangle$ is not an eigenvalue of O_1.

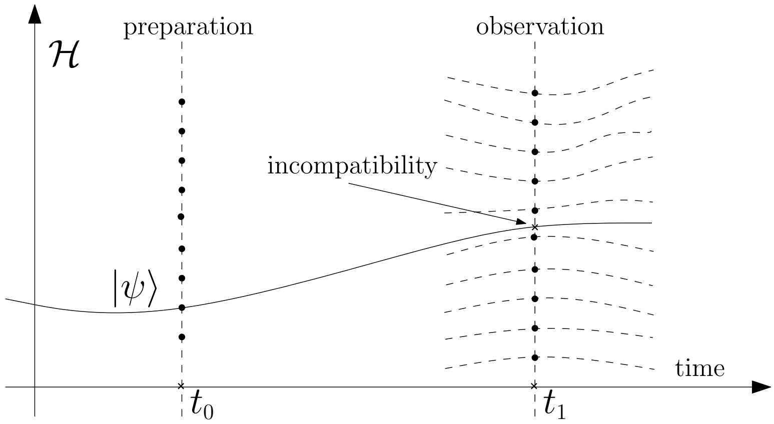

FIGURE 5. The first measurement in the figure, named *preparation*, measures the observable O_0 at t_0 and finds the state vector to be $|\psi(t_0)\rangle$. At a subsequent moment the state will evolve to $|\psi(t_1)\rangle$. On the other hand, the observation at t_1 will find $|\psi(t_1)\rangle$ to be an eigenstate of the observable O_1. The trouble is that, in most cases, $|\psi(t_1)\rangle$ will not be an eigenstate of O_1.

The wave behavior, as described by the Schrödinger equation, has been well tested experimentally and it describes very precisely the quantum systems. If we have a quantum system whose Hamiltonian is known, we can write down its associated Schrödinger equation. Let's say that, by measuring an observable at t_0, we find a nondegenerate eigenvalue λ_0. Because the multiplicity of λ_0 is one, any of the corresponding eigenstates will describe the same quantum state. Hence, we have identified precisely the state, and we can choose any of the equivalent state vectors describing it, say $|\psi_0\rangle$. The unitary evolution allows us to define a function

$$|\psi\rangle : \mathbb{R} \to \mathcal{H}, |\psi(t)\rangle = U(t, t_0)|\psi_0\rangle,$$

and it also predicts that, at a subsequent time t_1, the state vector will be $|\psi(t_1)\rangle$. If we measure at t_1 an observable O_1 having $|\psi(t_1)\rangle$ as an eigenstate, we will obtain precisely the eigenvalue λ_1 that corresponds to $|\psi(t_1)\rangle$. We can say that our measurement confirmed the Schrödinger equation. Such an experiment will always confirm the wave behavior described by the Schrödinger equation.

On the other hand, if we decide to measure an observable O_1 that does not admit $|\psi(t_1)\rangle$ as an eigenstate, then the things change. The state vector is no longer found to be $|\psi(t_1)\rangle$, but one of the eigenstates of O_1. The quantum behavior has been tested as well, and it was found no exception to the rule of obtaining eigenvalues of the measured observables.

Therefore, the two rules appear to be incompatible in an irreducible manner.

The solution proposed is that, as a result of the measurement, the state vector jumps from $|\psi(t_1)\rangle$ to an eigenstate of O_1. Of course, this implies that a discontinuity appears in the evolution of $|\psi\rangle$.

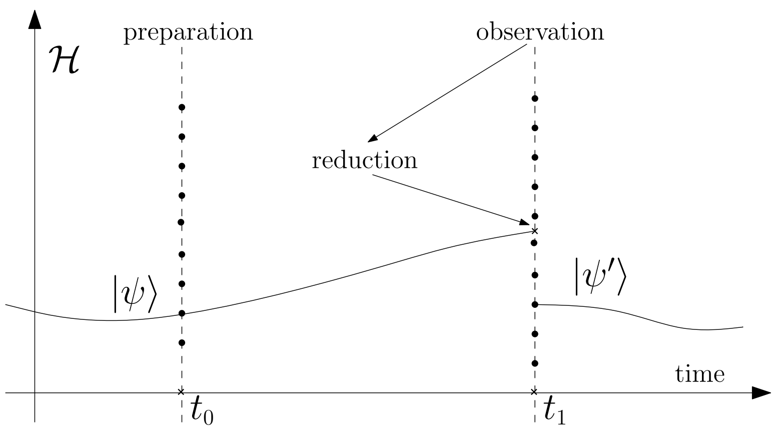

FIGURE 6. It appears as the state vector $|\psi\rangle$ jumps to another state $|\psi'\rangle$, such that $|\psi'(t_1)\rangle$ is an eigenstate of the observable O_1 measured at t_1.

We are forced to consider a broader space, say $\mathcal{S}_d(H)$, by allowing solutions of the Schrödinger equation (1) on $\mathbb{R} - D$, where D is a discrete subset of $\mathbb{R}$ having a finite number of elements in any closed interval. Thus, we admit functions that are solutions of the Schrödinger equation only piecewise, having discontinuities at the moments of time $t \in D$.

Not all the solutions contained by $\mathcal{S}_d(H)$ are acceptable, as we shall see.

2.4. **The probabilistic behavior**

When we measure the observable O_1 of a quantum system, the result is always an eigenvalue of O_1. The result is obtained by taking an element $|\psi\rangle \in \mathcal{S}_d(H)$ satisfying both the Schrödinger equation and the initial conditions $|\psi(t_0)\rangle = |\psi_0\rangle$ and $|\psi(t_1)\rangle = |\psi'_1\rangle$. In fact, the first initial condition gives a solution $|\psi\rangle$, and the second one gives a solution $|\psi'\rangle$. As a result of the measurement, the system, which was described by $|\psi\rangle$, becomes described by $|\psi'\rangle$. The probability to obtain a given eigenvalue is given by the Born rule [Born26]:

$$P_{|\psi(t_1)'\rangle}(\psi(t_1)) = |\langle\psi(t_1)|\psi'(t_1)\rangle|^2, \tag{4}$$

for normed state vectors $|\psi(t_1)\rangle$ and $|\psi'(t_1)\rangle$.

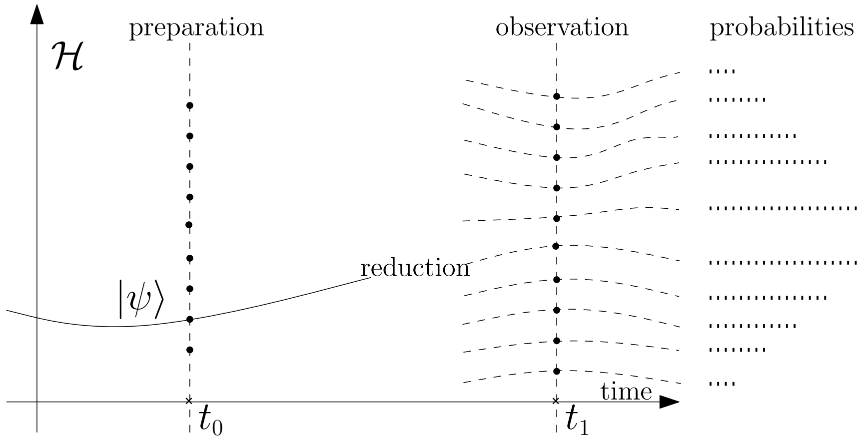

FIGURE 7. At the moment t_1 the state vector will be found not to be $|\psi(t_1)\rangle$, as predicted by the Schrödinger equation, but one of the eigenstates of O_1. The probability to find the state to be $|\psi'(t_1)\rangle$ is given by $|\langle\psi(t_1)|\psi'(t_1)\rangle|^2$.

The Born rule is a principle that completes the wave-like behavior and the quantum behavior presented above.

Although the equation (4) is probabilistic, it has a definite consequence: the state vector $|\psi(t_1)\rangle$ cannot jump to a state which is orthogonal to it. As a corollary, if we measure an observable for which $|\psi(t_1)\rangle$ is an eigenstate, we obtain the same state vector $|\psi(t_1)\rangle$. That is, there is no jump. This happens because the eigenstates corresponding to different eigenvalues are orthogonal. This special case of the probabilistic law ensures us that the Schrödinger equation is respected.

According to the Born rule, the state vector cannot jump to an orthogonal state, because the associated probability is zero. Therefore, we have to exclude the solutions having this kind of discontinuities. The remaining elements of $\mathcal{S}_d(H)$ will be denoted by $\mathcal{S}_q(H)$.

All the elements of $\mathcal{S}_q(H)$ can describe a quantum system. They are defined independent of the observables to be measured. We can study them as a space of piecewise solutions of the Schrödinger equation, with additional conditions that allows us to select a solution. Indeed, these conditions can, and actually do come from observations, but we can separate the measurements from the description of the quantum system.

Given two consecutive measurements O_i and O_{i+1}, such that between them the quantum system don't interact with another system, we will allow solutions from $\mathcal{S}_q(H)$ that contain at most one jump between O_i and O_{i+1}.

3. Delayed "initial" conditions

We actually don't know what happens between two consecutive incompatible measurements. If the measurements are compatible, then we expect that the system evolves according to the Schrödinger equation. If the measurements are incompatible, then we assume that a discontinuity occurs. First we will see that the precise moment of discontinuity cannot be inferred from the Schrödinger equation, the observable's eigenstates and the Born rule. It can happen at any time between the measurements, and the predictions are the same.

But when we arrange the measurements so that the choice of the observable is made after the observed quantum system has left the preparation device, as in the delayed-choice experiments, then it becomes clear that the state vector reduction took place prior to the current measurement. Somehow, the choice we have made selects the history already happened, as long as this history has not yet been recorded. The causality is not broken, but it has some flexibility, in that it let us choose the initial conditions with a delay.

What happened to the time also happens to the space. The Einstein, Podolsky and Rosen's experiment emphasize the fact that the choice of the initial conditions need to be made in a non-local fashion.

3.1. When exactly takes place the state vector reduction?

Since the evolution operator U is unitary, it preserves the inner products. This means that for any two state vectors $|\psi\rangle$ and $|\psi'\rangle$, the inner product $\langle\psi(t)|\psi'(t)\rangle$ is independent of the time t. Suppose that we performed two observations, at the moment t_0 finding the state vector to be $|\psi(t_0)\rangle$, and at t_1 finding it to be $|\psi'(t_1)\rangle$. The corresponding probability is $|\langle\psi(t_1)|\psi'(t_1)\rangle|^2$, and the unitarity ensures us that the probability would be the same if we consider that the state vector reduction took place at any other intermediate moment $t_0 \leq t \leq t_1$. We can check this directly:

$$\begin{aligned}\langle\psi(t)|\psi'(t)\rangle &= \langle U(t,t_0)\psi(t_0)|U(t,t_1)\psi'(t_1)\rangle = \langle U(t,t_1)^{-1}U(t,t_0)\psi(t_0)|\psi'(t_1)\rangle \\ &= \langle U(t_1,t_0)\psi(t_0)|\psi'(t_1)\rangle = \langle\psi(t_1)|\psi'(t_1)\rangle.\end{aligned}$$

We can see that the probability for $|\psi\rangle$ to jump to $|\psi'\rangle$ is independent on the time $t \in [t_0, t_1]$:

$$P_{|\psi'\rangle}(\psi) = |\langle\psi(t)|\psi'(t)\rangle|^2, \tag{5}$$

for any $t \in [t_0, t_1]$.

Because we cannot distinguish between the different situations provided by the choice of t between t_0 and t_1, it is possible to consider the jump as taking place between the two observations, without referring to t. It is natural to simply write a formula that does not make use of the particular choice of the time t of the collapse:

$$P_{|\psi'\rangle}(\psi) = |\langle\psi(t_0)|U(t_0,t_1)|\psi'(t_1)\rangle|^2, \tag{6}$$

thus being "invariant at the choice of the reduction moment".

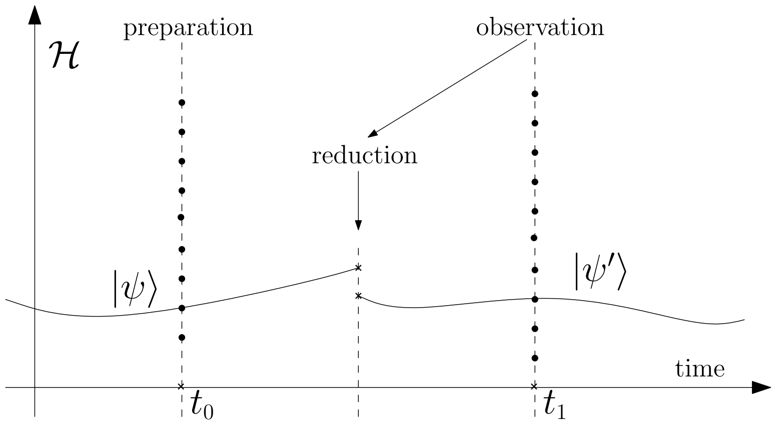

FIGURE 8. There is no way to distinguish whether the state vector reduction happened at t_0, t or t_1, because all the probabilities and values obtained as a result of an observation are the same.

To the observable O_1 corresponds an observable $U(t,t_1)O_1U(t,t_1)^{-1}$ at t which has the same eigenvalues and the corresponding eigenstates. If $|\psi'_\lambda(t_1)\rangle$ is an eigenstate of O_1, at t we have the corresponding observable with the eigenstate $|U(t,t_1)\psi'_\lambda(t_1)\rangle$:

$$\begin{aligned}(U(t,t_1)O_1U(t,t_1)^{-1})U(t,t_1)|\psi'_\lambda(t_1)\rangle &= U(t,t_1)O_1|\psi'_\lambda(t_1)\rangle \\ = U(t,t_1)\lambda|\psi'_\lambda(t_1)\rangle = \lambda U(t,t_1)|\psi'_\lambda(t_1)\rangle &= \lambda|\psi'_\lambda(t)\rangle.\end{aligned}$$

Hence, the unitary evolution operator U also preserves the eigenvalues and the property of a state vector to be an eigenstate of an observable.

The only thing the observer knows is that at two moments of time t_0 and t_1 she measured the observables O_0 and O_1, obtaining $|\psi(t_0)\rangle$ and $|\psi'(t_1)\rangle$. She don't know when exactly between t_0 and t_1 the state vector reduction occurred. She may only presume that the state vector reduction took place at t_1, because this is when she performed the measurement of O_1. But we will see that there are situations when it is more natural to presume that the reduction happened at t_0.

3.2. Delayed-choice experiments

Feynman considered that the two-slit experiment contains the full mystery of the Quantum Mechanics [Fey85]. A lot of variations, meant to emphasize various aspects of this mystery, were proposed during the times. The mystery consists in nonseparability in space and time. Experiments manifesting entanglement, like the one proposed by Einstein, Podolsky and Rosen, expose the space nonseparability. Delayed-choice experiments, like the one proposed by Wheeler, emphasize the time nonseparability. These two aspects are related, and they can be combined in a sophisticate manner, as in the delayed-choice quantum eraser experiment [SD82, SMKYKS00]. The truth is that all these aspects are present in a more or less visible form in the two-slit experiment. And the strange aspects of the two-slit experiment are, in turn, contained in a simpler form in the Mach-Zehnder quantum experiment.

3.2.1. **The Mach-Zehnder experiment**

The Mach-Zehnder interferometer consists in two beam splitters (that usually are half silvered mirrors) and two mirrors. A beam of light is divided by the first beam splitter in two components, one transmitted and one reflected. Each of the two components is reflected such that the transmitted beam encounters one face of the second beam splitter, and the reflected one the other face. If we decide to detect whether a photon passed through one of the two ways, by removing the second beam splitter, then we will find out that it passes either through one way, or through the other (figure 9 a). If we leave the beam splitter in place, then it recomposes the beam, and the interferometer behaves as the beam traveled both ways (figure 9 b). Note that if we try to cheat and detect in any way whether the photon passed through one way or the other, then the situation falls back to the first case and the interference is destroyed.

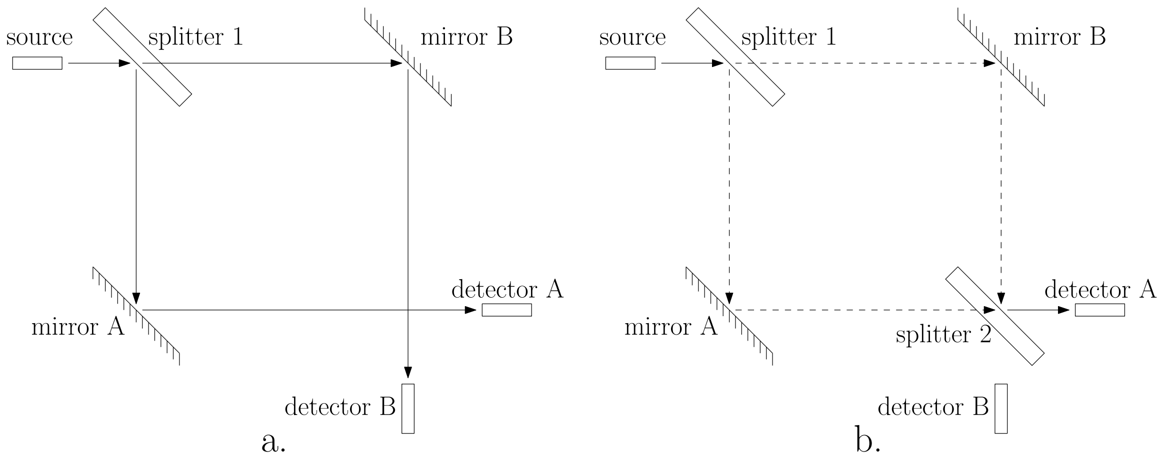

FIGURE 9. The first figure shows the *which way* type of observation. In this case, the electron is detected either by the detector **A**, or by **B**, and the conclusion is that the photon passes through one way or the other. In the second figure, the second beam splitter recomposes the photon and the interference allows only the detector **A** to report. The photon traveled *both ways*.

As in the case of other experiments like the two-slit experiment, there are two small modification that we can made to Mach-Zehnder experiment, in order to emphasize the meaning of its results. The first modification is to filter the beam of light to make sure that there is at most one photon passing through the interferometer at a given moment. This modification proves that the interference can happen between the two components (reflected and transmitted) of the wave describing one and the same photon. The photon definitely hasn't passed through one way, nor to the other. Well, it hasn't passed through both, nor to neither, but what it really did is usually referred as "it passed *both ways*".

The second modification is to delay the choice between the two types of measurements, the *which way* or the *both ways*. This kind of variation of a quantum experiment is usually named *delayed-choice experiment*. Its purpose is to emphasize the idea that the physicist can, by choosing what to measure, make the photon walk through both ways or through only one of them, after the photon passed the first beam splitter. It is like choosing one version of history, after this already happened.

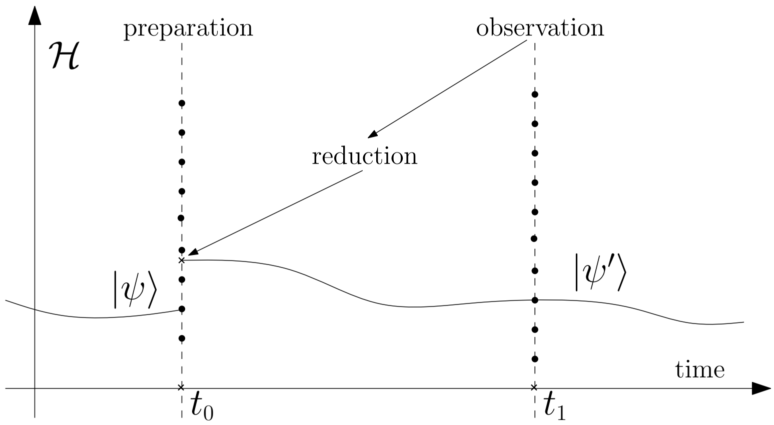

FIGURE 10. When we measure an observable O_1 at the moment t_1, the reduction seems to "already happen" at the moment t_0. The reduction seems to take place in advance, anticipating the experimenter's choice of the observable O_1.

3.2.2. **An explanation**

Let's say that the photon in the Mach-Zehnder experiment was emitted at t_0, and detected at t_1. When exactly did the state vector reduction happened between t_0 and t_1? It seems to happen when the photon interacts with the first beam splitter, or even earlier. It depends on this interaction if it travels *both ways* or through one of the two ways. But perhaps we may infer that this interaction also depends on the state of the photon when it was emitted. In any case, the observation at t_1 added a condition that allowed us to reduce the bunch of solutions allowed by the condition at t_0, and the condition applied to a time interval that, chronologically, contains moments previous to t_1.

This seems to defy our common sense belief in the impossibility of changing the past history. I think that this is the key difficulty in understanding and accepting the Quantum Mechanics. But this is not that paradoxical as it may seem, and certainly it does not break the causality. The choice of the observable at t_1 don't change the already happened events. It only selects one solution among a set of solutions that were not specified completely. If someone is curious to check the state of the system at an intermediate moment $t_{\frac{1}{2}} \in (t_0, t_1)$, then the corresponding observation $O_{\frac{1}{2}}$ will add a condition and the evolution of the state will be fixed by $O_{\frac{1}{2}}$. No further observation will change it.

In the Classical Physics, the parameters identifying the state are determined experimentally, but it is assumed that they are already "out there", in the reality. In Quantum Mechanics, the parameters describing the evolution of a quantum system also may not be known since the beginning, as in the classical case, but something more happens.

The world may go on with them incompletely specified, and the measurement may add extra information that determine the state. The proof that the world evolved with the parameters unspecified is that we can make decisions about them, by choosing what experiment to perform. The randomness of the result, together with our choice, selects the history that actually happened.

The time evolution of the system between t_0 and t_1 is determined by the values taken by $|\psi\rangle$ at both moments. Until the value at t_1 is not defined, there is no preferred solution for the time evolution. Each discontinuity in the evolution of the quantum system requires new initial parameters, new initial conditions for the jump moment. But the required initial conditions may be delayed.

The selection of the alternative history consists of two conditions. One of them happened when the physicist made her choice of what to measure, the *which way* or the *both ways*. Perhaps she decided this spontaneously, or perhaps her choice was determined by some chemistry in her brain, or by an event in her childhood, this is out of our understanding and of our purpose. Possibly it resulted from the very evolution of a state vector describing the physicist herself. What matters is that it resulted in the choice of an observable.

The other condition, dependent on the first one, imposes the selection of the eigenstate of that observable. According to our current understanding, this happens randomly, with a probability governed by the Born rule.

3.2.3. The two-slit experiment

Having discussed the distilled version provided by Mach-Zehnder interferometer, it is now easier to approach the two-slit experiment. It consists in sending a beam of particles (usually photons or electrons, but they can also be atoms or even "buckyball" molecules of C_{60}) towards a screen. Before reaching the screen, they have to encounter a wall having two slits. By placing a detector in one of the two slits we are inhibiting the *both ways* aspect and exhibiting the *which way* behavior. By contrary, by allowing the both slits to be available, we let the way open for the *both ways* behavior, on the expense of the *which way* behavior. If the choice is delayed, then it becomes clear that it acts like a condition that selects the time evolution after it happened.

The main difference from the Mach-Zehnder experiment is that, in the case of a *both ways* measurement, there are more possible eigenstates, given by the positions where the particles may hit the screen. But the main idea is the same, namely that the already passed evolution can be selected after it happened.

3.3. The Einstein-Podolsky-Rosen experiment

In the Einstein-Podolsky-Rosen experiment, two particles are considered to interact in such a manner that they remain entangled. In their original paper [EPR35], the measurements referred to position and momentum, but for simplicity we will prefer Bohm's version [Bohm51], where the observables under consideration represent the spin. One particle of spin zero decays in two particles of opposite non-zero spins, for example two electrons. We don't know their spins, we only know that the spins cancel one another. If we measure the spin of the first electron, say along the z axis, we obtain $\pm\frac{1}{2}$, with equal probabilities. A measurement of the second particle's spin on the same axis yields exactly opposite values, with probability 1. If we measure the spin of the second particle along a direction of space that makes an angle α with the direction of the spin obtained by for the first particle, the probability of obtaining the spin in that direction is $\frac{1}{2}(1-\cos\alpha)$. The opposite direction corresponds to the angle $\pi+\alpha$, and its associated probability is given, of course, by $\frac{1}{2}(1+\cos\alpha)$.

The probabilities of the results of the measurement of the second particle depends on the results in measuring the first one. They are in fact correlated in a symmetric way, such that it doesn't matter what particle was measured first.

In this experiment, the states of the two electrons are entangled, and they are described by the same state vector. If two quantum systems are represented by $|\psi_1\rangle \in \mathcal{H}_1$ and $|\psi_2\rangle \in \mathcal{H}_2$, the composed system is represented by a state vector $|\psi_1\rangle\otimes|\psi_2\rangle \in \mathcal{H}_1\otimes\mathcal{H}_2$. If they interact one another, the state vector representing them is still a vector in $\mathcal{H}_1\otimes\mathcal{H}_2$, but which in general is not of the form $|\psi_1\rangle\otimes|\psi_2\rangle$. Even after the interaction ceased, it is possible to have a state that cannot be expressed in the form $|\psi_1\rangle\otimes|\psi_2\rangle$, and this is an *entangled* state.

In the case of the two electrons, the initial state, of total spin zero, is the spin singlet state

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle\otimes|\downarrow\rangle - |\downarrow\rangle\otimes|\uparrow\rangle), \tag{7}$$

which is represented in the basis given by $(|\uparrow\rangle, |\downarrow\rangle)$ in each of the two spin spaces. The expression of this formula is independent of the choice of this basis.

The entangled states respects the same quantum rules described in the previous sections. The weirdness of the EPR experiment resides in the fact that the two quantum systems may become separated by a large distance. When we measure one of them, we learn things about the other one. But the measurement implies a state vector reduction, and the reduction depends on the observable we choose to measure. The result of the measurement of an observable of the first electron should be correlated with the result of the measurement of the second electron, and this looks a bit like an instantaneous communication between the two particles. But it is not.

What really happens is that the only possible states in which the total state vector can collapse are correlated in such a manner. If we measure the spin of the second electron along a spatial direction $\searrow$, the possible results are $|\searrow\rangle$ and $|\nwarrow\rangle$, each one with a probability of $\frac{1}{2}$. The spin singlet state can be expressed in this basis (and its correspondent basis for the second electron) by

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|\searrow\rangle\otimes|\nwarrow\rangle - |\nwarrow\rangle\otimes|\searrow\rangle). \tag{8}$$

If the result of the measurement performed on the second electron is $|\searrow\rangle$, then the first electron follows to be in $|\nwarrow\rangle$, and reciprocally. But if we measure the spin of the first electron along a different direction $\uparrow$, the probability to obtain $|\uparrow\rangle$ or $|\downarrow\rangle$ depends on the angle made with $|\nwarrow\rangle$. Bell showed that the correlations predicted by the Quantum Theory are incompatible with the ones predicted by the local hidden variables theories.

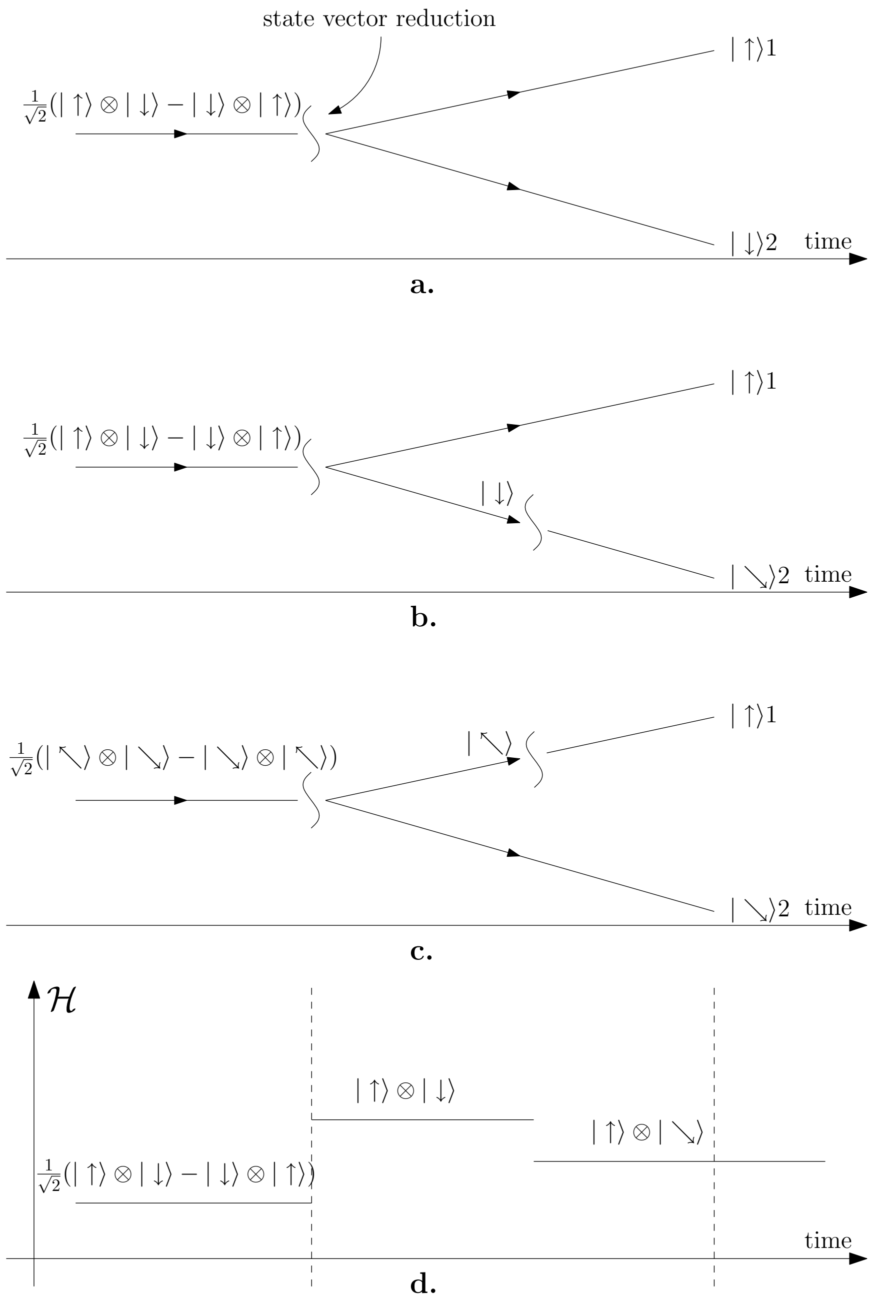

FIGURE 11. Measuring the spins of the two electrons along the same direction of space $\uparrow$ gives correlated results (**a**). One measurement is a confirmation measurement of the other. The reduction occurs during the decay. If the two electrons are measured along different directions, there are two incompatible measurements, hence two reductions. The first occurs at the decay, and the other somewhere between the decay and one of the measurements, as in **b** and **c**. **d** is another way to represent the case in **b**.

The spin states belong to Hilbert spaces that don't include the description of the positions, but we can complete this description by tensoring the spin Hilbert spaces with the Hilbert spaces representing the others degrees of freedom. This way, the entanglement involves also the positions of the two particles. It is the inseparability in space what makes this experiment so strange. If we ignore this feature, what remains is the plain quantum behavior: the state vector reduces as a result of a measurement. When we consider the positions, we emphasize the inseparability involved by the entangled systems. We emphasize the weird character of the collapse.

If we measure only the spin of the first electron along the direction of space $\uparrow$, the initial spin singlet state (equation 7) of the two particles reduces either to $|\uparrow\rangle \otimes |\downarrow\rangle$ or $|\downarrow\rangle \otimes |\uparrow\rangle$. If the second measurement is performed along the same direction, the measurement can be viewed as a measurement of confirmation, since we obtain a result which is precisely the one predicted by the measurement performed on the first electron (please refer to the figure 11). The state vector reduction can be thought as taking place during the decay. If the second measurement is along a different direction in space, then we have two different reductions. The first reduction is the one that collapses the initial state (equation 7). The second reduction takes place somewhere between the decay and one of the two measurements, it doesn't matter which one, from the observation point of view (please refer to the discussion in 3.1).

In the EPR experiment, there are two distinct measurements, two "delayed initial conditions", for two entangled systems. These two conditions are statistically correlated in the way that two consecutive measurements are correlated. The difference is that they may be separated by a spatial interval, and the correlation seems to defy the speed limit imposed in the Special Relativity. But what matters is that the two conditions, together with the initial condition of the spin singlet state, can be respected by at least a solution from the space $\mathcal{S}_q(H)$. And this should respect also the rule of not having two state vector reductions without measurements between them.

4. The Direct Interpretation of Quantum Mechanics

I will summarize in a more formal way what we have discussed so far.

4.1. The time evolution of a quantum system

Let $\mathcal{H}$ be a complex Hilbert space, named the *state space*, with the hermitian inner product between two state vectors $|\psi\rangle, |\varphi\rangle$ denoted by $\langle\psi|\varphi\rangle$. We denote by $\mathrm{Herm}(\mathcal{H})$ the space of hermitian operators acting on the state space $\mathcal{H}$. The *Hamiltonian* is a hermitian operator $H : \mathbb{R} \to \mathrm{Herm}(\mathcal{H})$, depending smoothly on a real parameter representing the time.

Let $\mathcal{S}(H)$ be the space of the solutions $|\psi\rangle : \mathbb{R} \to \mathcal{H}$ of the Schrödinger equation associated to the Hamiltonian H:

$$\mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}|\psi(t)\rangle = H(t)|\psi(t)\rangle. \tag{9}$$

The *space of piecewise solutions* of the Schrödinger equation (9), $\mathcal{S}_d(H)$, is defined as the space of the functions $|\psi\rangle : \mathbb{R} \to \mathcal{H}$ which are solutions of the Schrödinger equation (9) on $\mathbb{R} - D$, where D is a discrete subset of $\mathbb{R}$ having a finite number of elements in any closed interval. D depends on the function $|\psi\rangle$ under consideration, and this dependency will be expressed by denoting it by $D_{|\psi\rangle}$.

It follows that the elements of $\mathcal{S}_d(H)$ are allowed to have discontinuities at the instants $t \in D$. It also follows that $\mathcal{S}(H) \subset \mathcal{S}_d(H)$.

Let's consider a function $|\psi\rangle \in \mathcal{S}_d(H)$. We denote the left and right limits of $|\psi\rangle$ at $t_0 \in \mathbb{R}$ by:

$$|\psi_l(t_0)\rangle = \lim_{t \nearrow t_0} |\psi(t)\rangle$$

and

$$|\psi_r(t_0)\rangle = \lim_{t \searrow t_0} |\psi(t)\rangle,$$

We define $\mathcal{S}_q(H)$ as the subspace of $\mathcal{S}_d(H)$ satisfying the conditions

$$\langle\psi_l(t_i)|\psi_r(t_i)\rangle \neq 0 \tag{10}$$

for all $t_i \in D_{|\psi\rangle}$ (hence for all $t \in \mathbb{R}$), and

$$\langle\psi_l(t_i)|\psi_r(t_i)\rangle \neq 1. \tag{11}$$

The second condition signifies that we will not allow discontinuous jumps from a state vector to another state vector differing only by a phase. The identity will be respected, of course, for all t not in $D_{|\psi\rangle}$.

The evolution (including the reductions) of a quantum system is thus represented by an element of $\mathcal{S}_q(H)$. From the point of view of the system itself, it seems that the description is complete.

What about the probability of each discontinuity to occur during the time evolution described by a function $|\psi\rangle \in \mathcal{S}_q(H)$? Couldn't the Born rule improve this description, by predicting the probabilities? The formula (4) saids that the probability depends only on the state vector before and after the reduction, and not on the observable itself. But at a closer look we understand that, although the probability depends only on the inner product between the state before reduction and the state after reduction, the final state is an eigenstate of a certain operator (the observable). It is therefore meaningless to introduce the Born rule before introducing the observables, because we don't know the probability of each observable to occur.

Hence, the description provided by the requirement that the evolution of a quantum state is given by elements of $\mathcal{S}_q(H)$ is complete from the point of view of the quantum system itself, which is not aware about the external observations.

On the other hand, from the point of view of an external observer that performs the measurement, the description can be completed by adding two requirements. The first one is that the result of an observation to be an eigenstate of the operator representing the observable. The second one is the Born rule.

4.2. External conditions imposed to a quantum system

An *observation*, or a *measurement*, performed at a time t_α to the system represented by $|\psi\rangle$, can be thought as a *condition* imposed to the function $|\psi\rangle$. This condition is that $|\psi(t_\alpha)\rangle$ is an eigenstate of the observable O_α under consideration. We can express the eigenstates of O_α as a complete set of mutually orthogonal subspaces of $\mathcal{H}$. These subspaces, in turn, can be viewed as projectors. A way of generalizing the idea of representing the observables by projectors at different times is provided by the consistent histories approach [Gri84, Omn88, GH90a, GH90b, GH90c, Omn92, Omn94, Ish94]. For our purposes, it will be enough to consider that at each time t_α a condition of the form $|\psi(t_\alpha)\rangle \in \sigma_\alpha$ is fulfilled, where σ_α is a subset of $\mathcal{H}$. We require that the sets σ_α include, together with a state vector, all other vectors representing the same state (hence, not null multiples of itself).

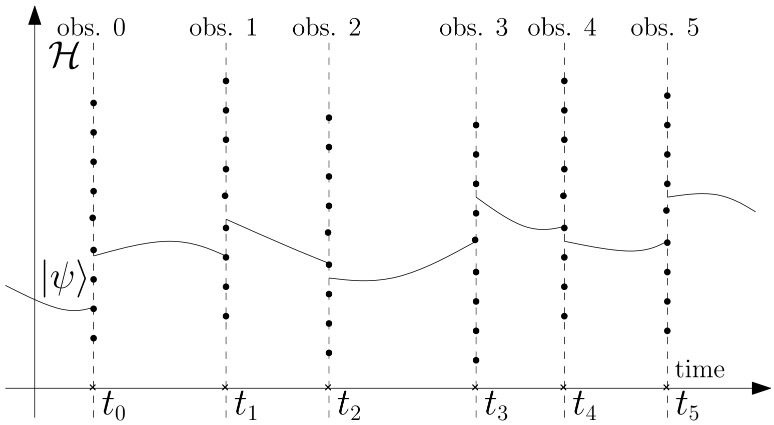

FIGURE 12. The evolution of a quantum state is described by a state vector rotating smoothly in the Hilbert space, and jumping discontinuously from time to time. This figure represents the reductions as taking place at the preparation time for each observation.

An *external condition* imposed to $|\psi\rangle \in \mathcal{S}_q(H)$ is given by a pair $(t_\alpha, \sigma_\alpha)$, which is required to satisfy the rules:

(1) $0 \notin \sigma_\alpha \subset \mathcal{H}$.
(2) For any $0 \neq \lambda \in \mathbb{C}$, for any $|\varphi\rangle \in \sigma_\alpha$, $\lambda|\varphi\rangle \in \sigma_\alpha$.
(3) $|\psi(t_\alpha)\rangle \in \sigma_\alpha$.

The *external history* of a quantum system will be a nonempty set of external conditions $\sigma = \{(t_\alpha, \sigma_\alpha)|t_\alpha \in \mathbb{R}, \sigma_\alpha \subseteq \mathcal{H}\}$. The external history together with the function $|\psi\rangle \in \mathcal{S}_q(H)$ give a *history* $(|\psi\rangle, \sigma)$ of the system, if they satisfy:

(1) Between any two consecutive $t_\alpha < t_{\alpha+1}$, there is at most one $t_i \in D_{|\psi\rangle}$.
(2) Any discontinuity of $|\psi\rangle$ should take place between two conditions σ_α and $\sigma_{\alpha+1}$.

We will denote the set of all such pairs $(|\psi\rangle, \sigma)$ by $\mathcal{Q}(H)$.

A *simple external condition* is an external condition $(t_\alpha, \sigma_\alpha)$ such that σ_α is a ray (the set of all not null complex multiples of a given not null vector in $\mathcal{H}$). A *simple external history* of a quantum system is an external history σ with all σ_α simple external conditions.

To each pair $(|\psi\rangle, \sigma)$ we can associate the actual history of the system (which is a simple history), by taking instead of σ the set $\sigma_* = \{(t_\alpha, (\mathbb{C} - \{0\}) \cdot |\psi(t_\alpha)\rangle)\}$, obtained from the conditions of σ. The space $\mathcal{Q}_*(H) \subset \mathcal{Q}(H)$ will denote the space of all histories having all σ_α simple external conditions (hence generated by $|\psi(t_\alpha)\rangle$).

A set of conditions σ for $|\psi\rangle \in \mathcal{S}_q(H)$ is said to be an *set of independent conditions*, or a *minimal set of conditions* if any two consecutive conditions $(t_\alpha, \sigma_\alpha)$ and $(t_{\alpha+1}, \sigma_{\alpha+1})$ are disjoint in the Heisenberg representation, or equivalently, $\sigma_{\alpha+1} \cap U(t_{\alpha+1}, t_\alpha)\sigma_\alpha = \emptyset$.

Each simple external condition specifies the value of the state vector up to a complex factor. A condition that specifies exactly the value will be named *elementary external condition*. An *elementary external history* is a set of elementary external conditions imposed at different times. If we want to specify a time evolution $|\psi\rangle \in \mathcal{S}(H)$, it is enough to have an elementary external condition σ_α, and this one will be the initial condition. To specify a time evolution $|\psi\rangle \in \mathcal{S}_q(H) - \mathcal{S}(H)$, two types of information are needed. On the one hand, the set $D_{|\psi\rangle}$ of the discontinuities. On the other hand, an elementary external history of $|\psi\rangle$. These two types of information specify uniquely the solution $|\psi\rangle$. It is not enough to have only the external history.

On the other hand, from the point of view of the external observer, it is enough to know a simple external history of $|\psi\rangle$, and the Hamiltonian. The possible solutions will differ in the way analyzed in (3.1), and by scalar factors. These differences are not observable from outside the quantum system represented by $|\psi\rangle$.

Let $|\psi\rangle \in \mathcal{S}_q$, and σ_α a condition occurring in the interval (t_i, t_{i+1}), where $t_i, t_{i+1} \in D_{|\psi\rangle}$ are consecutive times. If $t_i \lneq t_\alpha$, then σ_α is named *delayed initial condition.* The delayed-choice versions of various quantum experiments stress that the initial conditions can be indeed delayed.

4.3. **Probabilities**

The space $\mathcal{Q}_\perp(H)$ will denote all the histories for which each σ_α is given by eigenspaces of a hermitian operator O_α.

Each history from $\mathcal{Q}_\perp(H)$ have an associated probability obtained by applying the Born rule.

The Born rule expresses the probability that the state vector jumps from $|\psi_l(t_i)\rangle$ to an eigenstate $|\psi_r(t_i)\rangle$ of the observable measured at the instant t_i:

$$P_{|\psi_l(t_i)\rangle}(\psi_r(t_i)) = |\langle\psi_l(t_i)|\psi_r(t_i)\rangle|^2. \tag{12}$$

This rule is independent on the requirements introduced so far. To check the independence, we can consider instead another probability distribution, with the conditions that

(1) the probability is 1 if and only if the state vector before the measurement is an eigenstate of the observable, and that
(2) the probability to obtain after the reduction a state vector orthogonal to the state vector before the reduction is 0.

Any probability distribution satisfying these conditions will also be compatible with all the functions from the space $\mathcal{S}_q(H)$.

4.4. Conclusions

The time evolution of a quantum system can be described as an element $|\psi\rangle$ of $\mathcal{S}_q(H)$. From the system's point of view, this description is complete. Observations performed to $|\psi\rangle$ can be thought as external conditions of the form $|\psi(t_\alpha)\rangle \in \sigma_\alpha \subset \mathcal{H}$. The history of a quantum system $|\psi\rangle$ contains, together with the time evolution, a set of conditions imposed to it. These conditions are usually "delayed", being specified at different instants of times. For an interval (t_i, t_{i+1}) between two consecutive discontinuities of $|\psi\rangle$, the function $|\psi\rangle$ is specified by a condition of the form $|\psi(t_\alpha)\rangle \in \sigma_\alpha$ for a time $t_\alpha \in [t_i, t_{i+1}]$. The quantum experiments analyzed in this paper seem to suggest that there are situations in which the conditions are delayed in the sense that $t_i \lneq t_\alpha$.

5. Reality, locality and completeness

It is well known the Einstein-Podolsky-Rosen argument [EPR35] proving that Quantum Mechanics cannot in the same time:

(1) describe reality,
(2) respect the principle of locality,
(3) be complete.

They defined an *element of physical reality* as being associated to any physical quantity that can be predicted with absolute certainty without disturbing the system. Accordingly, a *complete physical theory* is characterized to account for every element of physical reality. After exposing their famous experiment, they concluded that the Quantum Mechanics provides an incomplete description. Their paper also pointed out the incompatibility between reality and locality, because by measuring a system we can disturb the predictions made for the outcome of the measurement of another system.

They suggested that perhaps there exist some hidden variables able to complete the description of reality provided by the Quantum Mechanics. Later, J.S. Bell [Bel64] showed that the Quantum Mechanics is in conflict with the locally hidden variables theories. Subsequent experiments [CHSH69, CS78, ADR82, Asp99] ruled out the locally hidden variables. Hence, the only allowed hidden variables theories ought to be non-local. David Bohm already provided an example of non-local hidden variables theory [Bohm52].

We will see what conditions of the EPR argument satisfies the Direct Interpretation.

5.1. The inside view

Sometimes the Born rule is applied to give the state vector a probabilistic interpretation. The state vector before reduction is thought to be a superposition of states, and the state vector after the reduction is thought to be the real state. This view has the disadvantage of considering the state vector to have two different meanings, as a physical state and as a superposition of physical states.

In the Direct Interpretation of the Quantum Mechanics, the state vector describing the system has physical existence. Before or after reduction, it is simply a wave propagating through space according to the Schrödinger equation. This wave is indeed different than the classical waves, in two aspects. First, it is valued in a large complex space. This complex space increases its dimension as the quantum system to be represented gets more complex. The set of all such wave functions form the Hilbert space $\mathcal{H}$. The second difference resides in the discontinuities. Except for these two aspects, $|\psi\rangle$ looks very classical.

When the system consists in one particle, this one is described by a wave. And the wave contains everything we can know about the particle. Sometimes, the particles are viewed as being point-like, and the wave as describing the probabilities to find the particles in a position or another. But if we consider the particle as being the wave, we understand that the particle is point-like only if it is localized in a very small region of space. This can happen as a result of a measurement of its position. But it doesn't mean that the particle has been found to be point-like, and the wave is an unphysical function governing the probabilities to find the particles in different places. In fact, not only the positions, but any other basis in the Hilbert space, can be used to describe the particle. In all these cases, the probability wave will be not about positions, but about other state vectors. It is therefore natural to consider that there is no preferred basis in the Hilbert space, from the Quantum Mechanics point of view. The positions are sometimes viewed as more natural because the wave functions are functions over the space. From this point of view, indeed, the positions should be privileged. But the fundamental principles of Quantum Mechanics makes no use of this preference, being independent on the orthonormal basis we choose for representing the phenomena.

The time evolution of the state vector, $|\psi\rangle \in \mathcal{S}_q$, contains all the information that can be determined about the system. It is *real*, describing the physical reality. The description is *complete*, because there are no other physical quantities except the ones that can be computed from $|\psi\rangle$. The *locality* is clear, because $|\psi\rangle$ is, piecewise, just like a classical wave. What is missing is the determinism. The state vector evolves deterministically on some intervals, but since from inside the system the moment of the next collapse cannot be predicted, nor the state after that, the system is not deterministic.

We conclude that, from the point of view of the system itself, there is nothing that contradicts the reality, completeness and locality.

5.2. The outside view

The things change when we try to describe the quantum system by observations. The observation causes the system to be found in an eigenstate of the observable. For example, if we choose to measure the momentum, we will find the system in a totally different state than if we would measure the position. But we decide what to measure, and we can make this decision in such a way that the preparation of the measurement devices don't interact with the system under consideration. It is like the system anticipates our decision, or like its past is chosen by our decision. Therefore, its state is not established until we determine it by an observation. We can see the word "determine" as a two-way road. "To determine" can mean to detect the state of the system, but also it can signify to influence the state.

According to Niels Bohr [Bohr28, WZ83], the phenomena does not exist until they are observed. But this don't mean that there is no reality prior to the observation. By contrary, all possible realities available are real, and by the mean of the experiment, one is chosen.

Should there be an underlying level that contains extra information to help us? The quantum system is completely described by $|\psi\rangle$. The only thing is that we have to determine it (in the both senses I mentioned).

Determining by observation the reality must have a non-local character, but not the reality itself. The reality is local. But when we determine it we have to take care of all possible implications, so that we don't break the physical laws. Therefore, the choice cannot be local. For example, let's consider a wave function which extends through all the space. If we measure its position, the wave function will collapse. The state vector reduces to an eigenstate of the position, a Dirac function. The position can be anywhere, because the wave function extends everywhere. But once the position was determined to be at a place A, instantaneously the possibility to find it at another place disappears.

5.3. Particles vs. waves

It is often stated that the Quantum Mechanics exhibits a duality between waves and particles. Sometimes, this duality is presented as being the core of the quantum phenomena. The Direct Interpretation considers that the duality is between the wave-like behavior (expressed by the unitary evolution) and the quantum behavior (expressed by the eigenstate condition). Nevertheless, there is a duality between the position operators and the momentum operators. They are canonical conjugated observables. The duality refers to two ways to describe the state vector, but there are more possible bases in the Hilbert space (actually, an infinity). The preference for the positions and momenta resides in the privileged role of the space, and in the classical descriptions in terms of point-like particles.

I consider that, although the point-like particles provide an intuitive image for many situations, it is simpler to consider the quantum systems as being waves, subject to the delayed initial conditions. Thinking in point-like particles entails complications, such as rejecting the existence of the particles until they are observed [Bohr28, WZ83], integrating over all the possible paths, changing the logic to allow contradictory statements, evoking hidden trajectories that interact instantaneously etc. Each of these solutions proved to be very valuable, leading to the path integral formalism, various types of logics (quantum logics [BvN36, Lup47], consistent logics [Omn88, Omn92, Omn94, Ish94]), and causal interpretations of the Quantum Mechanics [Bohm52, BH93]. The contribution of these approaches to our understanding of the world is inestimable. They may have started as means of accommodating the classical paradigm with the quantum revolution, but they did much more than this.

The Direct Interpretation presented in this article is intended as a simple way of viewing the quantum world, complementing all these views already fertile. I name this interpretation "direct" because it seems to me as arising without many assumptions from the postulates of the Quantum Mechanics. It is the simplest way I could find to represent to myself in a physical way what the mathematics of Quantum Mechanics expresses. In this sense, it can be considered minimal. It is compatible with other interpretations, or at least it can be made compatible with them.

5.4. The nature of the wave function

There were several attempts to provide the wave function with a physical meaning. Louis de Broglie proposed that to each particle, such as the electron, to be associated a wave function. He believed that this wave really exists in the physical world. Schrödinger himself, who provided the equation governing these waves, believed in their reality. He was unhappy when Bohr tried to convince him that the collapse of the wave function really happens. Bohr insisted to convince him that the wave function has no physical meaning, but only a probabilistic one, as seemed to result from Born's probabilistic interpretation. Louis de Broglie believed that both the wave function and the point-like particle have physical existence, idea developed by Vigier and Bohm [Bohm51, Bohm52, BH93]. In Bohm's approach the point-like particle is related to the wave by a quantum potential. The probabilistic aspect is recovered by statistical methods, reaching de Broglie's dream of a hidden (deterministic) "thermodynamics" behind the quantum randomness.

The difference between the image presented by the Direct Interpretation and Bohr's view is that in the former, the wave really is physical. In contrast to other approaches, such as the hidden variables one, there is no need for a point-like particle. We don't need, in the Direct Interpretation, a point-like particle to be guided by the pilot wave or by a quantum potential, nor do we need the wave function to have a singularity representing the particle.

But how is it possible for the wave functions to be physical? The interference doesn't occur between any two wave functions, but rather between one and the same. As Dirac pointed out, it is only the particle's wave function that interferes with itself, and not to other wave functions. He based his assumption on the observation that the interference occurs, for example in the case of the two-slit experiment, even when there is only one photon.

The well known answer is that the wave functions are valued in different vector spaces. The interference occurs, but only between the components living in the same vector space, and there is no contradiction with the experiments. This argument against the physicality of the wave function holds only if we consider that all the wave functions are valued in a vector space with a few number of dimensions. But when each particle has it's own dimensions occupied in the Hilbert space, there is enough place for the physical particles to interfere or not, according to the predictions of the Quantum Mechanics.

There are, though, some questions that the direct view fail to address. For example, we don't know when, why and how the collapse takes place. Also the measurement problems remain. The present article don't try to answer these problems.

5.5. Many Worlds

Taking the wave function as real in a different way led Everett to the Relative State Interpretation. He considered that the collapse does not occur, or rather that all the possible collapses occur. This seem paradoxically, but here is the point. The wave function evolves all the time governed by the Schrödinger equation. After a measurement is performed, the state vector evolves as a superposition of all possible collapsed states. The coefficients in the superposition can be taken such that the Schrödinger equation is not violated by a state vector reduction. Everett interpreted the state vector reductions as a split of the world in one world for each possible outcome of the measurement. This way, the unitary evolution governs all the history of the Universe, but in the same time each possibility occurs, such that their superposition gives the state vector according to Schrödinger equation. But each possibility in the superposition entails that the measurement device obtained the corresponding eigenvalue. The total unitary evolution contains all the alternative histories.

Each alternative world is similar to ours, in that the people who lives in it have the illusion of the collapse. For them, the collapse is not an illusion, but their reality. It would be nice if we can answer to their own questions about the occurrence of the state vector reduction. These questions are the same as in other interpretations, and they are not answered. We cannot simply dismiss them as unappropriate by saying that there is no collapse at the level of the *multiverse*, because at the level of each universe the collapse has a meaning.

In the view proposed by the Direct Interpretation, the Many Worlds Interpretation has its place. A world is defined starting from the space $\mathcal{S}_q(H)$. Each of its sections is, piecewise, a local solution of the Schrödinger equation. Each solution is selected from the possibilities provided by the space $\mathcal{S}_q(H)$, by the mean of the "initial conditions", which may be delayed. We can see each of these possibilities as having its own reality, as being a world. The Many Worlds Interpretation, in Everett's vision [Eve57, Eve73], or in DeWitt's view [dW71, dWG73], David Deutsch's Multiverse variation [Deu85, Deu91], and the Many Minds Interpretation, are all naturally conceivable in the framework provided by the Direct Interpretation. And, of course, the problems enumerated above still remain unsolved.

But I want to emphasize that the Direct Interpretation is not a warrant of the Many Worlds Interpretation (nor of its variations). It simply is a minimal formalism which uses the delayed initial conditions to select a piecewise solution of the Schrödinger equation. The Direct Interpretation provides a roof for many interpretations of the Quantum Mechanics, but not a proof.

6. The probabilities in the Direct Interpretation

6.1. Born's rule

Born's principle tells us the probability that, during a collapse, a system in a state represented by the vector $|\psi\rangle$ jumps on another state represented by $|\psi'\rangle$:

$$|\langle\psi|\psi'\rangle|^2. \tag{13}$$

This probability is independent on the specific observable which is measured, depending only on the chosen eigenstate. It is the core of the probabilistic interpretation of the wave function, or generally, of the state vector.

In the Direct Interpretation, the state vector has a physical meaning, and not a probabilistic one. The probabilistic behavior occurs only during a state vector reduction, but the probabilities are related to the initial and final physical states. Because in the theory presented in the current paper the wave function is entirely physical, it makes no sense to consider that its nature is probabilistic. Only the selection of the eigenstate during the quantum jump is probabilistic. Assigning a probabilistic meaning to the state vector itself makes the same sense as assigning a probabilistic meaning, in Classical Mechanics, to a coin, based on the possibility that we can toss it and obtain head or tail.

6.2. Heisenberg's relations

Heisenberg's relations can be derived from the non-commutativity of the operators, or from Fourier analysis. But the meaning is the same: they express a minimum of the product of the dispersions of the state vector in two canonically conjugated bases. For example, the less the wave is dispersed in the momenta space, the more it is dispersed in the positions space.

We see that there is nothing probabilistic here. I haven't employed any word like "uncertainty" or "indeterminacy", simply because there was nothing like this in what I said. These relations are valid whether or not a measurement is performed to the state vector, whether or not a collapse, or a state vector reduction, takes place. Heisenberg's relations are simply about the dispersions of the state vector's coordinates in two complementary bases.

But why we name them usually *the uncertainty principle* or *relations*, or the *principle of indeterminacy*? Because, when we combine Heisenberg's relations with Born's rule, we indeed obtain a probabilistic meaning for them. But this probabilistic meaning manifests only during a state vector reduction, while Heisenberg's relations are valid at any instant, and most of the time we have a dynamical evolution. Hence, most of the time the system evolves according to the Schrödinger equation, and respects Heisenberg's relations. When a collapse takes place, Born's principle enters the game, and Heisenberg's relations acquire a probabilistic meaning.

7. Some problems of Quantum Mechanics

The description of Quantum Mechanics provided in the previous sections allows us to state clearly some problems. These problems were pointed out at various times after the discovery of the quanta.

7.1. Problem 1: The delayed-choice initial conditions

The possibility that the initial conditions be delayed is a strange behavior which seems to contain all the mystery of Quantum Mechanics. Choosing later the parameters describing the evolution of a quantum system which already took place is very strange for the common sense. But if we can accommodate with this phenomenon, then the Quantum Mechanics become more friendly. There still remain some unexplained problems, but I think that one of the main mysteries is this one. It was emphasized by Wheeler [Whe77, Whe78, WZ83] when he revived [3] the idea of delayed-choice experiments.

The nonlocal character of the collapse of the wave function is due to a conjunction between the delayed initial conditions behavior and the discontinuities representing the state vector reduction. I think that the delayed initial conditions phenomenon is one face of the "only mystery" indicated by Feynman [Fey85] to be find in the two-slit experiment. Another face being given by the discontinuities.

7.2. Problem 2: The discontinuities

During the evolution described by the Schrödinger equation, there are some quantities, like the momentum, energy, electric charge etc., associated to the quantum system, that are conserved. What is interesting is that these quantities are to be conserved also during the collapse caused by the measurement. We don't know of any process during which the conservation laws break down. Even the experiments with entangled particles respect them. While in the case of the dynamical evolution described by the Schrödinger equation the quantities are conserved because of Noether's theorem, in the case of the collapse there is a discontinuity that requires another explanation for the conservation laws. Moreover, although the collapse has a non-local character, there is no experimental evidence that the conservations are also non-local. On the theoretical side, the relativity of the simultaneity don't cope well with a non-local conservation law, because it would imply that for some reference frames the conservation is broken. These considerations entail that we should complete the list of the principles of Quantum Mechanics with the strange requirement:

Principle. The collapse must take place in such a manner that it conserves the quantities conserved by the Schrödinger equation. At least, it should happen in a way that prevents the experimental detection of a violation of the conservation laws in their local form.

[3] It seems that similar suggestions were made before by Weiszäcker [Wei31] and Bohr [Sch49].

In fact, not the requirement is strange, but it is strange that we need to add it. It is an unusual kind of principle, because it states that the system evolving discontinuously should behave like there is no discontinuity.

A second problematic aspect of the discontinuities is the question about the time when they occur. On the one hand, it is the possibility that the initial conditions be specified not when the discontinuity occurs, but later. On the other hand, there have been emitted various hypotheses about when exactly the collapse happens as a result of a measurement: when the interaction takes place, when the effect to the apparatus becomes irreversible, when the apparatus records, when the conscious observer acknowledges the result, or even never. These possibilities were suggested by various physicists during the discussions about the measurement problem.

7.3. Problem 3: The measurement

This problem has been stated in many forms. Various definitions, explanations, analyses were proposed during the time. Its paradoxical status has been exposed by the mental experiment known as "The Schrödinger's cat".

I will state this problem like this:

The measurement problem. Why the observations find always the quantum state to be an eigenstate of the observables?

According to the discussion at the beginning of this paper, there are two behaviors, the wave-like one and the quantum one. The wave-like behavior states that the evolution of a system should obey the Schrödinger equation. According to this principle, the Schrödinger equation being linear, any two possible solutions can be linearly combined in a third one. On the other hand, there is the quantum behavior: the observations find the state vector to be an eigenstate of the observable.

Why cannot linearly combine, according to the wave-like behavior, two eigenstates of the observable? Well, if we would be allowed to do this, there would be no quantum behavior, and the space of solutions $\mathcal{S}(H)$ will contain every possible time evolution. There would be no discontinuities. This is because an orthonormal frame provides a basis of solutions for the Schrödinger equation, and the observable admits an orthonormal frame of eigenstates. If we would allow superpositions of the outcomes of the measurements, then any outcome would be allowed, and there would be no quantum behavior. We would have only the wave behavior.

The measurement problem is then related as well to the wave vs. quantum incompatibility. The rule of obtaining eigenvalues would not be that strange, if there would not be the superposition principle to contradict it.

Maybe the idea that the Schrödinger equation should be everything led to the idea that in fact no reduction takes place, idea which is at the core of the Many Worlds Interpretation. In my opinion, MWI is very interesting and plausible, enlightening some crucial aspects of the Quantum Mechanics. But this interpretation needs also to be completed with an explanation of the illusion of the state vector reduction, which seems to have such convincingly reality for the observers confined to only one of these parallel worlds. We cannot simply say that the reduction doesn't happen, we have to explain why it appears to happen. Therefore, the measurement problem cannot be dismissed as not real.

A promising approach in understanding how the classical world emerged from the quantum world, and of the measurement problem, is the study of environmental induced decoherence and selection [Zeh96, Zur98, Zur02, Zur03a, Zur03b, Zur04].

7.4. Three related problems

The three problems exposed so far are all different faces of the incompatibility between the wave-like behavior and the quantum behavior. It is the quantum behavior that, when the eigenstate condition is imposed to the unitary time evolution, adds the delayed initial conditions. It is again this incompatibility that seems to require the discontinuities. And it is the wave-like behavior that makes us to expect a superposition between "dead cat" and "alive cat", while the quantum behavior allows us to find only one of the possible eigenstates, and not a superposition.

The central problem of Quantum Mechanics is the incompatibility between wave and quanta, from which all other problems, paradoxes and mysteries follow.

7.5. Other problems

The three problems presented above are not necessarily all, nor the most important problems of Quantum Mechanics. An important one concerns the emergence of the classical level from the quantum one, considering that the latter is more fundamental than the former.

The various directions taken in the development of Quantum Mechanics raise continuously new interesting problems, and continuously some of them are solved. For example, the difficulties of dealing with the Special Relativity led to the Quantum Field Theory. A big problem is the apparent incompatibility between Quantum Mechanics and General Relativity, although new important progresses in reconciling them are reported permanently.

8. Conclusions

The Schrödinger equation (1) describes the evolution of a quantum system. All solutions form a space $\mathcal{S}(H)$, where H is the Hamiltonian. In order to know a specific solution of such an equation, we need to know some initial conditions. The measurements are ways to specify the initial data which allows us to know which one of the possible solutions of (1) is the one under observation. Because two consecutive measurements may provide inconsistent sets of initial data, it seems that we have to enlarge the space of solutions $\mathcal{S}(H)$ to a space $\mathcal{S}_q(H)$ containing discontinuous solutions of the Schrödinger equation.

We can consider that the evolution of a quantum system is simply a solution from $\mathcal{S}_q(H)$. This gives some physical intuition to the Quantum Mechanics. On the other hand, the delayed-choice experiments allows us to understand that the solution is dependent on the observer, in fact, on the observable she choose to measure. More precisely, we have some freedom in choosing the initial data of the solution even after the evolution described by that solution took place. This makes us to see that the reality may exist, but it is not yet fixed, even for already happened events.

When we determine (in both meanings of the word) what is the history of a system, our choices seem to have non-causal and non-local consequences. In fact, each evolution contained in $\mathcal{S}_q(H)$ is causal and local (being piecewisely a wave), but the initial conditions imposed by observations need to be consistent, even if they are separated in spacetime by spatial intervals, so here resides the apparent non-locality.

Three main problems are enumerated: the delayed initial conditions, the discontinuity, the measurement problem. In subsequent articles, I will propose a solution to the problem of discontinuities and discuss the measurement problem.

References

[ADR82] A. Aspect, J. Dalibard, and G. Roger, *Experimental Test of Bell's Inequalities Using Time-Varying Analyzers*, Physical Review Letters (1982), no. 49.

[Asp99] A. Aspect, *Bell's Inequality Test: More Ideal than Ever. Nature 398*, 1999.

[BH93] D. Bohm and B. Hiley, *The Undivided Universe: An Ontological Interpretation of Quantum Mechanics. Routledge and Kegan*, 1993.

[Bel64] J. S. Bell, *On the Einstein-Podolsky-Rosen Paradox.*

[Bohm51] D. Bohm, *The Paradox of Einstein, Rosen, and Podolsky*, Quantum Th. (1951), 611–623.

[Bohm52] D. Bohm, *A Suggested Interpretation of Quantum Mechanics in Terms of "Hidden Variables", I and II*, Phys. Rev (1952), No. 85.

[Bohr28] N. Bohr, *Das Quantenpostulat und die neuere Entwicklung der Atomistik*, Naturwissenschaften **16** (1928), 245–257.

[Born26] M. Born, *Zur Quantenmechanik der Stoßvorgänge, Z. Phys.* **37**, 863 (1926). Reprinted and translated in J. A. Wheeler and W. H. Zurek (eds.), *Quantum Theory and Measurement* (Princeton University Press, Princeton, NJ, 1963), p. 52.

[BvN36] G. Birkhoff and J. von Neumann, *The Logic of Quantum Mechanics*, Ann. Math. Phys. (1936), no. 37, 823–843.

[CHSH69] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, *Proposed Experiment to Test Local Hidden-Variable Theories*, Physical Review Letters (1969), no. 23.

[CS78] J. F. Clauser and A. Shimony, *Bell's Theorem: Experimental Tests and Implications*, Reports in the Progress of Physics, 1978.

[Deu85] D. Deutsch, Int. J. Theor. Phys. (1985), no. 24, 1–41.

[Deu91] D. Deutsch, *Quantum theory of probability and decisions, Proc. Roy. Soc. London* **A455**, 3129 (1999).

[dW71] B. S. de Witt, *The Many-Universes Interpretation of Quantum Mechanics.* in B. d'Espagnat (ed.), *Foundations of Quantum Mechanics* (Academic Press, New York, NY, 1971).

[dWG73] B. S. de Witt and N. Graham, *The Many-Worlds Interpretation of Quantum Mechanics*, 1973.

[EPR35] A. Einstein, B Podolsky, and N Rosen, *Can Quantum-Mechanical Description of Physical Reality be Considered Complete?, Physical Review 47:777–780*, 1935.

[Eve57] Hugh Everett, *"Relative State" Formulation of Quantum Mechanics*, Rev. Mod. Phys. **29** (1957), no. 3, 454–462.

[Eve73] Hugh Everett, *The Theory of the Universal Wave Function*, in The Many-Worlds Hypothesis of Quantum Mechanics, Press, 1973, pp. 3–137.

[Fey85] R. P. Feynman, *QED: The Strange Story of Light and Matter*, Princeton Science, 1985.

[GH90a] M. Gell-Mann and J. Hartle. *Alternative decohering histories in quantum mechanics*, In K.K. Phua and Y. Yamaguchi, editors, *Proceedings of the 25th International Conference on High Energy Physics, Singapore, August, 2–8, 1990*, Singapore, 1990. World Scientific.

[GH90b] M. Gell-Mann and J. Hartle. *Quantum mechanics in the light of quantum cosmology*, In S. Kobayashi, H. Ezawa, Y. Murayama, and S. Nomura, editors, *Proceedings of the Third International Symposium on the Foundations of Quantum Mechanics in the Light of New Technology*, pages 321–343. Physical Society of Japan, Tokyo, 1990.

[GH90c] M. Gell-Mann and J. Hartle. *Quantum mechanics in the light of quantum cosmology*, In W. Zurek, editor, *Complexity, Entropy and the Physics of Information, SFI Studies in the Science of Complexity, Vol. VIII*, pages 425–458. Addison-Wesley, Reading, 1990.

[Gri84] R. B. Griffiths, *Consistent Histories and the Interpretation of Quantum Mechanics*, Journal of Statistical Physics (1984), no. 36, 219–272.

[Ish94] C. J. Isham, *Quantum Logic and the Histories Approach to Quantum Theory*, J.Math.Phys. (1994), no. 35, 2157–2185.

[Lup47] Stéphane Lupasco, *Logique et contradiction*, 1947.

[Omn88] R. Omnés, *Logical Reformulation of Quantum Mechanics*, Journal of Statistical Physics (1988), no. 53, 893–932.

[Omn92] R. Omnès. *Consistent interpretations of quantum mechanics*, Rev. Mod. Phys., 64:339–382, (1992).

[Omn94] R. Omnés, *The Interpretation of Quantum Mechanics*, 1994.

[Sch35] E. Schrödinger, *Die Gegenwärtige Situation in der Quantenmechanik. Naturwissenschaften 23*, pp. 152–167, 1935.

[Sch49] Paul Arthur Schilpp Ed. *Albert Einstein : Philosopher-Scientist. The library of living philosophers, vol VII.* Cambridge University Press. 1949.

[SD82] M.O. Scully and K. Drühl, *Quantum eraser: A proposed photon correlation experiment concerning observation and "delayed choice" in quantum mechanics*, Phys. Rev. A **25**, 2208–2213 (1982).

[SMKYKS00] Scully, Marlan O.; Yoon-Ho Kim, R. Yu, S.P. Kulik, and Y.H. Shih. *A Delayed Choice Quantum Eraser.* Physical Review Letters **84**: 15, eprint quant-ph/9903047, (2000).

[vN55] J. von Neumann, *Mathematical Foundations of Quantum Mechanics*, 1955.

[Wei31] K. F. Weiszäcker, Zeit. F. Phys. (1931), no. 70, 114.

[Whe77] J. A. Wheeler, *Include the Observer in the Wave Function*, Quantum Mechanics a Half Century Later, 1977, pp. 1–18.

[Whe78] J. A. Wheeler, *The "Past" and the "Delayed-Choice" Experiment*, In A.R. Marlow, ed., *Mathematical Foundations of Quantum Theory*, 1978, p. 30.

[WZ83] J. A. Wheeler and W. H. Zurek, *Quantum Theory and Measurement*, 1983.

[Zeh96] H. D. Zeh, *What is Achieved by Decoherence?*, eprint quant-ph/9610014 (1996).

[Zur98] W. H. Zurek, *Decoherence, Einselection, and the Existential Interpretation (The Rough Guide), Phil. Trans. Roy. Soc. Lond.* **A356**, 1793 (1998).

[Zur02] W. H. Zurek, *Decoherence, Einselection, and the Quantum Origins of the Classical*, *Rev. Mod. Phys.* **75**, 715 (2003).

[Zur03a] W. H. Zurek, *Environment-assisted Invariance, Entanglement, and Probabilities in Quantum Physics*, *Phys. Rev. Lett.* **90** (12), 120404 (2003).

[Zur03b] W. H. Zurek, *Quantum Darwinism and Envariance*, eprint quant-ph/0308163 (2003).

[Zur04] W. H. Zurek, *Probabilities from Envariance*, eprint quant-ph/0405161 (2004).