

# The Frequency Hypothesis and Evolutionary Arguments

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## Abstract

Gerd Gigerenzer's views on probabilistic reasoning in humans have come under close scrutiny. Very little attention, however, has been paid to his evolutionary component of his argument. According to Gigerenzer, reasoning about probabilities as frequencies is so common today because it was favored by natural selection in the past. This paper presents a critical examination of this argument. It will show first, that, *pace* Gigerenzer, there are some reasons to believe that using the frequency format was not more adaptive than using the standard (percentage) format and, second, that Gigerenzer's evolutionary argument and his other arguments such as his historical description of the notion of probability are in tension with each other.

## 1 Introduction

Many psychologists and philosophers have discussed various kinds of psychological experiments which show that ordinary (or even learned) people often commit

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basic logical or probabilistic fallacies. Some psychologists (e.g., Tversky & Kahneman (1982)) have interpreted these results as showing that human thinking generally does not comply with logical and probabilistic laws and is therefore irrational. However Gerd Gigerenzer, a German experimental psychologist, argues that Kahneman and Tversky's claim is based on a false interpretation of norms of rationality and probability. Gigerenzer claims that the results are consistent with the *ecological rationality* — context-bound inferential and mathematical abilities— that human beings have acquired in the course of human evolution. Gigerenzer and his research group attribute the supposed 'errors' in question to condition of the experiments which make it difficult for human beings to exert their intellectual abilities properly — conditions very different from the sort of environmental context in which those abilities were acquired. Thus, it is no wonder that the subjects give wrong answers in these experiments; therefore, if an experimenter provides such settings that the subjects can exercise their intellectual abilities specific to the informational structure of a particular environment and a particular way of informational representation, then they can and do make a 'correct' answer (i.e., complying with the laws of logic and probability).

In the case of probability, psychological experiments have shown that the subject often make mistakes in the Bayesian inference quizzes<sup>1</sup> and psychologists and philosophers have taken this as showing that human beings generally do not comply with the probabilistic laws in probabilistic reasoning. Put it another way, when human beings make a probability calculation, they generally do not do along with the laws of probability such as the Bayes' theorem. Gigerenzer is against this interpretation. He argues that the problem lies in the

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<sup>1</sup>The quizzes where subjects are asked to calculate the probability of a hypothesis ( $H$ : you are infected with HIV) given data ( $D$ : a test says you are infected with HIV) — $P(H|D)$ — from other probabilities such as the probability of the hypothesis ( $P(H)$ ) and that of false alarm ( $P(D|\sim H)$ )— see appendices for details.

way in which probabilities are presented in those quizzes, namely as percentages. He argues further that presenting probabilities in the frequency format is more appropriate, and facilitates reasoning about those probabilities, because humans are more capable of reasoning about frequencies and in fact acquired that ability as an evolutionary adaptation: *frequency representation* (such as ‘1 out of 5’, in contrast to *percentage representation*, such as ‘20%’)<sup>2</sup> is the informational structure which our intellectual abilities on probability are specific for and they have information-processing psychological mechanisms specific for those abilities (for example, Gigerenzer (1991, 1993, 2000*b,a*), Gigerenzer & Hoffrage (1995), Hoffrage & Gigerenzer (1998)<sup>3</sup>). Therefore, if the probability quizzes such as the Bayesian inference quizzes are asked in the frequency format, then the subjects are better at them than in the percentage format. Gigerenzer’s hypothesis about probabilistic reasoning has caused much controversy in recent years. But few people have examined his *evolutionary argument*: having a probability-reasoning mechanism specific for the frequency representation is common among human beings today because it was more *adaptive* in the course of evolution than those for the percentage representation and was selected for. Although the evolutionary appeal of his overall hypothesis is more tentative and not always mentioned in his papers, Gigerenzer has maintained it for a decade (Gigerenzer 1993, Hoffrage et al. 2002). Furthermore Cosmides & Tooby (1996), leading evolutionary psychologists, also provide some support for this hypothesis. Moreover, others consider the evolutionary part of his hypothesis worth

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<sup>2</sup>Gigerenzer distinguishes *natural frequencies* and *relative frequencies*. Natural frequencies come from natural sampling. In natural sampling, one gets a frequency of a particular event-type from his or her experience (not systematic survey or experiments) and a natural frequency is the frequency which he or she acquires that way (e.g., ‘5 hunting successes out of 10 attempts’) and conveys information on the numbers of samples. Relative frequencies are normalized numbers and thereby convey no information on base rates (e.g., ‘success rate in hunting of 0.2’). Gigerenzer insists that the frequency representation by natural frequency rather than relative frequencies affects the subjects’ performance. Hereafter, we only refer to natural frequencies by ‘frequencies.’

<sup>3</sup>Gigerenzer makes claims on interpretation as well as representation of probability, but this paper does not discuss it.

pursuing. For example, Steven Pinker (1997) cites Gigerenzer’s hypothesis and says “the mind may have evolved to think of probabilities as relative frequencies in the long run, *not* as numbers expressing confidence in single event. [...] Our ancestors’ usable probabilities must have come from their own experience, and that means they were frequencies [...]” (p. 347)<sup>4</sup>.

This paper is a critical examination of his evolutionary argument. I shall argue, first, that, even if Gigerenzer’s responses to the criticisms are right, there are some reasons to believe that using the frequency format was not more adaptive than using the standard format and, second, that Gigerenzer’s evolutionary argument and his historical description of the notion of probability —the percentage representation is a fairly recent development— are in tension with each other and he does not give any additional argument to ease the tension. In the next section, I will briefly review the evolutionary argument for the frequency hypothesis given by Gigerenzer and his followers. In the following section, I will point out two problems with these arguments. The first problem is that there are several reasons such as burden on memory to believe that the frequency format was not more adaptive than what Gigerenzer calls the standard probability format (i.e., the percentage representation). Second, the evolutionary arguments conflict with Gigerenzer’s interpretation of the history of probability and his account of computational simplicity in the frequency format. Moreover, I also argue that Gigerenzer’s hypothesis is in a less favorable position in this respect than the other hypothesis of evolutionary psychology, that is, the cheater-detection hypothesis.

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<sup>4</sup>Robert Nozick (1993) cites Gigerenzer’s paper as an important result.

## 2 Evolutionary arguments

In a number of experiments, Gigerenzer and other researchers showed that subjects are better able to solve certain probability problems, such as the Linda problem and various Bayesian inference problems (see appendices for details) if the probabilities involved are expressed as frequencies (e.g., ‘1 out of 5’) rather than percentages (‘20%’). Gigerenzer argues that humans have built-in abilities (mental algorithms) to solve problems using a frequency representation of probability while they lack corresponding algorithms for percentages.

Gigerenzer tries to explain this from an evolutionary vantage point. In the early evolution of humans, reasoning in terms of frequencies was more adaptive than reasoning in terms of percentages. And thus the ability to reason in terms of frequencies was selected.

[...]If evolution has selected some kind of [probabilistic] algorithm in the mind, then it will be tuned to frequencies as representation (Gigerenzer 1993, p. 291).

Gigerenzer offers two rationales for his evolutionary argument. First, percentages and single-event probabilities “took millennia of literacy and numeracy to evolve as tools for communication” (op.cit., p.290).

Probabilities and percentages are quite recent forms of representations of uncertainty. [...] Percentages became common notations only during the French Revolution (mainly, though, for interest and taxes rather than for representing uncertainty). (Gigerenzer 2000*a*, p. 62)

Second, a frequency representation (like “7 out of 30”) carries more information than a percentage (like “13.3 %”), because the former tells us the sample

size, and thus tells us more about the reliability of a prediction based on this datum:

[I]n an illiterate world, the input representation would be frequencies of events, sequentially encoded, such as 3 out of 20 (as opposed to 15 per cent or  $p = 0.15$ ). [...] Moreover, frequencies such as 3 out of 20 contain more information than percentages such as 15 per cent. These frequencies contain information about the sample size (here: 20), which allows one to compute the ambiguity or precision of the estimate. (ibid.)

Leda Cosmides and John Tooby (1996), who followed up Gigerenzer's study by doing a series of elaborate experiments, also added two other rationales. First they pointed out that it is easier to update one's database by using a frequency representation. If George the hunter has so far killed 5 game in 20 past hunting expeditions when he went to the north mountain but fails today, the original frequency is easily updated to "5 out of 21," which is by no means easy if you use percentages.

Second, under the frequency representation it is easier to reconstruct reference classes to compute another frequency. Suppose George has killed 5 game in 20 attempts when he has gone to the north mountain. But suppose it turns out that he has killed 4 games in 10 attempts when he carried a spear, whereas he got only one kill out of 10 times when he used a club. Reorganization of statistical information like this seems to be very advantageous for prediction and only the frequency representation allows it.

### 3 Problems with the evolutionary arguments

These evolutionary arguments by Gigerenzer and Cosmides & Tooby may seem to provide sufficient support for the frequency hypothesis<sup>5</sup>. However, there are problems with their arguments. First, analysis of the frequency and percentage representations seems to show that the frequency representation was not always more adaptive than the standard probability format in the environment in which early human beings encountered their statistic and probabilistic tasks (3.1.). Second, the evolutionary argument and Gigerenzer's other arguments on the history of probability and computational simplicity conflict with each other (3.2. and 3.3.).

#### 3.1 Comparison and burden on memory

In this section, we will see a couple of advantages which the percentage representation has over the frequency representation. The first such advantage is ease of comparison. To use the example of hunters again, suppose that you want know whether Bob (having 130 successes out of 402 attempts) or George (159 out of 530) is a better hunter. If you only use the frequency representation, how would you compare them? In some cases, we can easily make comparisons using the frequency representation; for example, when both of Bob and George have 10 attempts, all you have to do is just to compare the number of successes each has. But this is not the case all the time. However, with the percentage representation (under which Bob's record is.32 and George's is.30), it is relatively easy to make a comparison which we need to answer the question.

Another advantage of the percentage representation is that it places less of a burden on memory than the frequency representation. As Tooby & Cosmides say, it is easy under the frequency representation to update the data as we get

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<sup>5</sup>For criticisms, see, for example, Girotto & Gonzalez (2001).

a new sample, and reorganize the data by changing reference classes. But this means that you have to memorize a new frequency whenever you get another sample. This increases the load memory is asked to bear. If Fred, a hunter, whose record is 5 successes out of 16 attempts, succeeds today, you have to update two numbers (“6 out of 17”). As long as we use the frequency representation, we have to do this updating whenever Bob goes out for hunting<sup>6</sup>. One may object that even under the percentage representation, you have to memorize the frequency to make a percentage representation. But this does not mean that one should use always the frequency representation. Taking the hunting example, maybe one has to memorize frequency of success in only recent 10 hunting attempts to make the percentage representation. Or maybe one memorizes initial ten hunting attempts to make the initial percentage representation and ignores further attempts (and keep only the percentage in your mind); one still uses *some* memory to memorize the frequency, but needs much less memory than when one keeps all the record in one’s mind. Notice that Gigerenzer only considers two options: using *only* the frequency or the percentage representation and ignores a mixed strategy like the ones I have just suggested.

I do not mean that the percentage representation is generally *more* adaptive than the frequency representation. Instead, I argue that the three advantages which Gigerenzer and Cosmides & Tooby mention (section 2) — showing a sample size, the ease to update the data, and the possibility of reorganizing the reference classes— are possible only when one memorizes all the cases. In other words, those advantages *trade off* against an increased burden on memory. So the frequency format for probabilistic reasoning could have as many adaptive disadvantages as advantages when it is compared to the percentage

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<sup>6</sup> Sloman & Over (2003) make a similar point: we have to memorize all relevant experiences in order to keep frequency. But since they do not explicitly compare the frequency and the percentage representations, it is not clear whether this difficulty is avoided in percentage representation.

representation.

### 3.2 Was selection possible?

The second general problem for Gigerenzer's account concerns the history of the notion of probability. He seems to hold at least one time that the notions of single-case probability and of its percentage representation are a fairly recent development<sup>7</sup>. It is not clear whether such claims are historically correct (for a different view, see Girotto & Gonzalez (2002)). But if they are literally true, it is puzzling how this supports his evolutionary argument. The reason is that in order for a trait to be evolutionarily advantageous, there must be some alternative trait(s) to be selected against, or some phenotypic (but transmittable) variation (see, for example, Sober 1984, 2000, Sterelny & Griffith 1999). Thus, if we see Gigerenzer as claiming that nobody had the percentage representation in the Pleiocene, then there was no selection of the psychological mechanism for the frequency representation over the one for the percentage representation. Thus it is because there was some (developmental or other type of) constraint on the mode of probability interpretation and representation which prevented other modes from occurring, not because a particular mode was adaptive, that we are better at probabilistic reasoning in the frequency representation. When there is only one trait in a population, there *cannot* be recent selection. In this case, Gigerenzer can claim that human beings have the psychological mechanism specific for the frequency representation, but it is not because of selection but some form of constraint.

But as I have said, this straightforward interpretation may not be the only interpretation. There are at least two ways to reconcile Gigerenzer's historical description and his selection scenario concerning the frequency representation. One way is something like this: suppose that percentage representation emerged

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<sup>7</sup>He cites Daston (1988), Hacking (1975).

(as a result of mutation) in the human population from time to time (or only once), but it was selected against and disappeared in it whenever it emerged until percentage representation and a theory of single-event probability finally fixed themselves in human intellectual (nonevolutional) history. This is compatible with the history of probability (as Gigerenzer sees it) and the unfavorable selection of the percentage mode of representation. But Gigerenzer and others have not provided any evidence that there were such representations and notions of probability in the evolutionary history of human beings (and as we saw above, there are reasons to think that the frequency is no more adaptive than the percentage representation).

Alternately, early human beings might have used only the frequency format, but some of them might have gotten better at using the frequency format and those might have been selected. For example, once most human beings might have used only the last case to make predictions about future events<sup>8</sup>. But some human beings might have used the last two or more cases to make a prediction and have succeeded in terms of survival and reproduction. Then this way of using the frequency representation was selected over other ways. In this scenario, the selection was on different ways to use the frequency representation, not the frequency representation itself. I have two responses: first, again, Gigerenzer does not offer anything to support this reading. On the contrary, Gigerenzer contrasts the frequency representation and the percentage representation rather than different ways of using the frequency representation. Gigerenzer says:

What is the format of the numerical information humans encountered during their evolution? [...] Probabilities and percentages were not the way organisms encountered information. [...]

I propose that the original format was *natural frequencies*, acquired

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<sup>8</sup>I am indebted for this interpretation to John Beatty.

by *natural sampling*. (Gigerenzer 1998, p.62, original italics)

Gigerenzer does not say here explicitly that the percentage representation was selected against. However, it is clear that he pits the frequency representation and the percentage representation against each other, not various ways of utilizing the frequency representation. Second, even if it is a possible scenario, it is not incompatible with my criticism against his evolutionary argument: recall I argue that it is difficult to provide enough reasons to show that the frequency representation is *more adaptive than* the percentage representation. But this interpretation provides only a reason to show that one way of using the frequency representation is more adaptive than the other. Even under this interpretation, it may be fine to say that having an ability specific for the frequency representation was adaptive, but it is only a very loose way of saying. Even if the subjects are better at handling probabilistic reasoning in the frequency format than the percentage format, this is not the result of selection for the mechanism *as a whole*.

### 3.3 Computational simplicity and evolutionary advantage

The last problem facing the frequency hypothesis concerns the computational simplicity of the frequency format. Let us recall the mammography problem (See Eddy 1982, Gigerenzer & Hoffrage 1995, Gigerenzer 2000*c*, see also Appendix for details.). In this problem, subjects are asked to calculate the probability that a woman has breast cancer (H) given that she has a positive mammography result (D). In a quiz, base rate ( $\Pr(H)$ ), true positive rate ( $\Pr(D|H)$ ), and false positive rate ( $\Pr(D|\sim H)$ ) are given. If you use the Bayesian equation, the calculation is

like this:

$$\begin{aligned} Pr(H|D) &= \frac{Pr(D|H)Pr(H)}{Pr(D|H)Pr(H) + Pr(D|\neg H)Pr(\neg H)} \\ &= \frac{(0.80)(0.01)}{(0.80)(0.01) + (0.096)(0.99)} \approx 0.078 \end{aligned}$$

However, according to Gigerenzer, with the frequency representation, “[a]ll one needs is the number of cases that had both the symptom and the disease (here, 8) and the number of symptom cases (here, 8+95)” (Gigerenzer 2000*a*, p. 98). Therefore, in this case, the Bayesian equation is like this:

$$Pr(H|D) = \frac{(\text{symptom \& disease cases})}{(\text{symptom cases})} = \frac{8}{8 + 95} = \frac{8}{103}$$

From comparing these two equations, Gigerenzer concludes,

Bayesian algorithms are computationally simpler when information is encoded in a frequency format rather than a standard probability format. (Gigerenzer & Hoffrage 1995, p. 687), (Gigerenzer 2000*a*, p. 99)

Gigerenzer also notes that this computational simplicity involves the number of operations (in the case of frequency representation, we have to do fewer computations); furthermore, one deals with natural numbers in the second case but with percentages in the first case (*ibid.*).

However, if this analysis is right, then it undermines Gigerenzer’s hypothesis concerning the content-specific psychological mechanism for the frequency representation. For the sake of argument, suppose his simplicity analysis is true. It follows that we do not have to assume the selection of the psychological mechanism specific for the frequency representation to explain Gigerenzer’s (and his followers’) experimental results, because the subjects’ improved performance under the frequency representation may be only due to the computational sim-

plicity. After all, it is by no means surprising that human beings are better at computing simpler operations. Let me put it another way. Suppose we have two hypotheses to explain the phenomena:

**(H1)** Human beings have psychological mechanisms *specific* for the frequency representation. [Gigerenzer's hypothesis]

**(H2)** Human beings have *general* abilities for mathematical calculation and the difference in subjects' performance comes from how computationally easy the tasks are.

(H1) is Gigerenzer's hypothesis, and (H2) is an alternative hypothesis we may have. We also have one observation (to be explained) and two background assumptions:

**(O1)** Subjects solve the quizzes under the frequency representation more correctly than under the percentage representation. [From the experiments]

**(A1)** Under the frequency representation, computational load is lighter than under the percentage representation. [Gigerenzer's argument on the computational simplicity]

**(A2)** Human beings compute with fewer mistakes computationally easier tasks than otherwise [A general fact].

Without (A1)-(A2), (H1) seems to explain (O1) better than (H2). But once we have (A1)-(A2), then (H2) seems to explain (O1) as well. But (H1) ontologically postulates one more object than (H2): psychological mechanism for the frequency representation. Therefore, if (H1) and (H2) explain (O1) equally well, (H1) is unfavorable in terms of *ontological parsimony*. Although it is likely that natural selection works on *more general* computational abilities, there is

no need to assume *specific* selection pressure to the modes of representation in order to explain the improvement of subjects' performance as Gigerenzer does.

## 4 Summary

In this paper, we have examined Gigerenzer's and his followers' evolutionary arguments regarding the human problem solving abilities measured by Bayesian inference quiz. From a series of experiments in which subjects solve Bayesian inference quizzes much better under the frequency representation, Gigerenzer, and Cosmides and Tooby argue that it is because human beings have acquired the mathematical algorithms specific for the frequency representation as an adaptation by which they can deal with probability properly and smoothly.

However, it turns out that their rationales for this interpretation are not convincing enough. Firstly, even if Gigerenzer and Cosmides and Tooby focus only on the frequency representation, it is still unclear that the frequency representation is more evolutionarily advantageous than the percentage representation. The frequency representation puts more burdens on our memory and makes comparisons more difficult. Rather, it is more likely that some advantages of the frequency representation which Gigerenzer points out *trade off* disadvantages which this paper shows. Therefore, their points on them are not sufficient to claim that the frequency representation was more adaptive than the percentage representation.

Second, if, as Gigerenzer claims, the percentage representation emerged very recently, then there may have been no natural selection between the percentage and frequency representations, because there need to be at least two different traits for selection to work. So far as I know, Gigerenzer does not deny that this is what he means.

Finally, Gigerenzer himself proposes an explanation of the performance in

Bayesian Inference which seems to make no appeal to the existence of a *specific* adaptation for it. In this explanation the difference of performance between the frequency and percentage representations are explained by computational simplicity. It is just because the frequency representation is computationally simpler that subjects perform better under this representation and it is ontologically parsimonious to postulate the psychological mechanism specific for the frequency representation.

## APPENDIX

### A The Bayesian inference quizzes

The Bayesian inference is one of Gigerenzer's favorite examples. This inference is used when one calculates the probability of a hypothesis (e.g., that of one's having HIV) given certain information (e.g., that one's HIV test is positive). In an example of the relationship between breast cancer and mammography test, a subject is asked to solve a problem like this (Gigerenzer 2000*a*, p. 97, he says this is from Eddy (1982)):

The probability of breast cancer is 1% for a woman at age forty who participates in routine screening.

If a woman has breast cancer, the probability is 80% that she will get a positive mammogram.

If a woman does not have breast cancer, the probability is 9.6% that she will also get a positive mammogram.

A woman in this age group had a positive mammogram in a routine screening. What is the probability that she actually has breast cancer?

To calculate the probability, subjects can use the following Bayesian equation.

$$Pr(H|D) = \frac{Pr(D|H)Pr(H)}{Pr(D|H)Pr(H) + Pr(D|\neg H)Pr(\neg H)}$$

Where H means a hypothesis (in this case it is "a woman has breast cancer"), and D means data ("a woman gets a positive result in a mammography test"). In this case, Pr(H) is 1% (.01), Pr(D|H) is 80% (.80), and Pr(D|¬ H) is 9.6% (.096), so the answer is

$$Pr(H|D) = \frac{(0.80)(0.01)}{(0.80)(0.01) + (0.096)(0.99)} \approx 0.078$$

that is, only 7.8%. But when Eddy made this experiment on 100 physicians, the average of 95 physicians' estimation is about 75% (not 7.5%). When college students and staff at the Harvard Medical School were subjects on this and similar experiments, the experimenters get the similar results, i.e., their average estimations are from 70 to 80%. It seems that this error is caused by a phenomenon called *base rate neglect*. In this case, many subjects seem not to take into account the probability of breast cancer in population (1%) when they estimate the posterior probability ( $Pr(H|D)$ ). Thus, many researchers like Kahneman and Tversky (1972) conclude that what these results show is that human beings do not always do Bayesian reasoning correctly.

For the solution of the Bayesian inferences, Gigerenzer emphasizes the importance of the way those problems are represented to the subjects. He argues that those “irrational” results are caused when they are represented in terms of probability, and when those quizzes are described in terms of frequency, subjects show remarkable improvement in their performances. Under the frequency representation, the same problem of breast cancer (see the previous section) may be described as follows (Gigerenzer 2000*a*, p.97):

10 out of every 1,000 women at age forty who participate in routine screening have breast cancer.

8 out of every 10 women with breast cancer will get a positive mammogram.

95 out of every 990 women without breast cancer will also get a positive mammogram.

Here is a new representative sample of women at age forty who got

a positive mammogram in routine screening. How many of these women do you expect to actually have breast cancer? \_\_\_\_\_ out of \_\_\_\_\_.

When Gigerenzer and his fellow researchers made this experiment in probability and frequentist representations, about a half of physicians (11 out of 24 physicians) gave the Bayesian answer under the frequentist representation, while only two of (other) 24 physicians gave the correct answer under the probability representation (see Hoffrage & Gigerenzer 1998).

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