A Two-Sided Ontological Solution to the Sleeping Beauty Problem

Paul Franceschi
University of Corsica

p.franceschi@univ-corse.fr
http://www.univ-corse.fr/~franceschi

ABSTRACT. I describe in this paper an ontological solution to the Sleeping Beauty problem. I begin with describing the Entanglement urn experiment. I restate first the Sleeping Beauty problem from a wider perspective than the usual opposition between halfers and thirders. I also argue that the Sleeping Beauty experiment is best modelled with the Entanglement urn. I draw then the consequences of considering that some balls in the Entanglement urn have ontologically different properties form normal ones. In this context, considering a Monday-waking (drawing a red ball) leads to two different situations that are assigned each a different probability. This leads to a two-sided account of the Sleeping Beauty problem. On the one hand, the first situation is handled by the argument for 1/3. On the other hand, the second situation corresponds to a reasoning that echoes the argument for 1/2 but that leads however, to different conclusions.

1. The Entanglement urn

Let us consider the following experiment. In front of you is an urn. The experimenter asks you to study very carefully the properties of the balls that are in the urn. You go up then to the urn and begin to examine its content carefully. You note first that the urn contains only red or green balls. By curiosity, you decide to take a sample of a red ball in the urn. Surprisingly, you notice that while you catch this red ball, another ball, but of green colour, also moves simultaneously. You decide then to replace the red ball in the urn and you notice that immediately, the latter green ball also springs back in the urn. Intrigued, you decide then to catch this green ball. You note then that the red ball also goes out of the urn at the same time. Furthermore, while you replace the green ball in the urn, the red ball also springs back at the same time at its initial position in the urn. You decide then to withdraw another red ball from the urn. But while it goes out of the urn, nothing else occurs. Taken aback, you decide then to undertake a systematic and rigorous study of all the balls in the urn.

At the end of several hours of a meticulous examination, you are now capable of describing precisely the properties of the balls present in the urn. The latter contains in total 1000 red balls and 500 green balls. Among the red balls, 500 are completely normal balls. But 500 other red balls have completely astonishing properties. Indeed, each of them is linked to a different green ball. When you take away one of these red balls, the green ball which is associated with it also goes out at the same time of the urn, as though it was linked to the red ball by a magnetic force. The red ball and the green ball which is linked to it behave then as one single object. Indeed, if you take away the red ball from the urn, the linked green ball is also extracted instantly. And conversely, if you withdraw from the urn one of the green balls, the red ball which is linked to it goes out immediately of the urn. You even tried to destroy one of the balls of a linked pair of balls, and you noticed that in such case, the ball of the other colour which is indissociably linked to it was also destroyed instantaneously. Indeed, it appears to you that these pairs of balls behave as one single object.

The functioning of this urn leaves you somewhat perplexed. In particular, your are intrigued by the properties of the pairs of correlated balls. After reflection, you tell yourself that the properties of the pairs of correlated balls are finally in all respects identical to those of two entangled quantum objects. The entanglement (Aspect & al. 1982) is indeed the phenomenon which links up two photons, for example, so that when one modifies the quantum state of one of the entangled photons, the quantum state of the other one is instantly modified accordingly, whatever the distance where it is situated. Indeed, the pair of entangled photons really behave as one and the same object. You decide to call "Entanglement urn" this urn with its astonishing properties. After reflection, what appears peculiar with this urn, is that it includes at the same time some normal and some entangled balls. The normal red balls have nothing different with our familiar balls. But entangled balls behave in a completely different way. What is amazing, you think, is that nothing seemingly differentiates the normal red balls from the red entangled ones. You tell yourself finally that it could be confusing.

Your reflection on the pairs of entangled balls and their properties also leads you to question the way the balls which compose the pairs of entangled balls are to be counted. Are they counted as normal balls? Or do specific rules govern the way these pairs of entangled balls are counted? You add a normal red ball in an Entanglement urn. It is then necessary to increment the number of red balls present in the urn. On the other hand, the total number of green balls is unaffected. But what when you add in the Entanglement urn the red ball of a pair of entangled balls? In that case, the linked green ball of the same pair of entangled balls is also added instantly in the urn. Hence, when you add a red ball of a pair of entangled balls in the urn, it also occurs that you add at the same time its linked green ball. So, in that case, you must not only increment the total number of red balls, but also the total number of green balls present in the urn. In the same way, if you withdraw a normal red ball from the urn, you simply decrement the total number of red balls of the urn, and the number of green balls in the urn is unaffected. But if you remove the red ball (resp. green) of a pair of entangled balls, you must decrement the total number of red balls (resp. green) present in the urn as well as the total number of green balls (resp. red).

At this very moment, the experimenter happens again and withdraws all balls from the urn. He announces that you are going to participate in the following experiment:

> *The Entanglement urn** A fair coin will be randomly tossed. If the coin lands Heads, the experimenter will put a normal red ball in the urn. On the other hand, if the coin lands Tails, he will put a pair of entangled balls in the urn, composed of a red ball and a green ball, both indissociably linked. The experimenter also adds that the room will be put in absolute darkness, and that you will therefore be completely unable to detect the colour of the balls, no more that you will be able to know, when you will have withdrawn a ball from the urn, whether it is a normal ball, or a ball which is part of a pair of entangled balls. The experimenter tosses the coin and while you catch a ball from the urn, he asks you to assess the likelihood that the coin felt Heads.

2. The Sleeping Beauty problem

Consider now the well-known *Sleeping Beauty problem* (Elga 2000, Lewis 2001). Sleeping Beauty learns that she will be put into sleep on Sunday by some researchers. A fair coin will be tossed and if the coin lands Heads, Beauty will be awaken once on Monday. On the other hand, if the coin lands Tails, Beauty will be awaken twice: on Monday and on Tuesday. After each waking, she will be put into sleep again and she will forget that waking. Furthermore, once awakened, Beauty will have no idea of whether it is Monday or Tuesday. On awakening on Monday, what should then be Beauty's credence that the coin did land Heads?

At this step, one obvious first answer (I) goes as follows: since the coin is fair, the initial probability that the coin lands Head is 1/2. But during the course of the experiment, Sleeping Beauty does not get any novel information. Hence, the probability of Heads still remains 1/2.

By contrast, an alternative reasoning (II) runs as follows. Suppose the experiment is repeated many times, say, to fix ideas, 1000 times. Then there will be approximately 500 Heads-wakings on Monday, 500 Tails-wakings on Monday and 500 Tails-wakings on Tuesday. Hence, the reasoning goes, the probability of Heads equals 500/1500 = 1/3.

The argument for 1/2 and the argument for 1/3 yield conflicting conclusions. And the Sleeping Beauty problem is usually presented as a problem arising from conflicting conclusions resulting from the two above-mentioned competing lines of reasoning aiming at assigning the probability of Heads once Beauty is awaken. I shall argue, however, that this statement of the Sleeping Beauty problem is restrictive and that we need to envisage the issue from a wider perspective. For present purposes, the Sleeping Beauty problem is the issue of calculating properly (i) the probability of Heads (resp. Tails) once Beauty is awaken; (ii) the probability of waking on Monday (resp. Tuesday); and (iii) the probability of Heads (resp. Tails) on waking on Monday. From the halfer perspective, the probability of waking on Monday equals 3/4, and the probability of waking on Tuesday is 1/4. By contrast, from the thirder's perspective, the probability of waking on Monday equals 2/3 and the probability of waking on Tuesday is 1/3.

But the argument for 1/2 and for 1/3 also have their own account of conditional probabilities. To begin with, the probability of Heads on waking on Tuesday is not a subject of disagreement, for it equals 0 in both accounts. The same goes for the probability of Tails on waking on Tuesday, since it equals 1 from the halfer's or from the thirder's viewpoint. But agreement stops when one considers the probability of Heads on waking on Monday. For it equals 2/3 for a halfer but from a thirder's perspective, it amounts to 1/2. On the other hand, the probability of Tails on waking on Monday is 1/3 from a halfer standpoint, and 1/2 for a thirder.

3. The urn analogy

In what follows, I shall present an ontological solution to the Sleeping Beauty problem, which rests basically on the Entanglement urn experiment. A specific feature of this account is that it incorporates insights from the halfer and thirder standpoints, a line of resolution initiated by Nick Bostrom (2007) that has recently inspired some new contributions (Groisman 2008, Delabre 2008)[1].

The argument for 1/3 and the argument for 1/2 rest basically on an urn analogy. This analogy is made explicit in the argument for 1/3 but is less transparent in the argument for 1/2. The argument for 1/3, to begin with, is based on an urn analogy which associates the situation inherent to the Sleeping Beauty experiment with an urn that contains, in the long run (assuming that the experiment is repeated, say, 1000 times), 500 red balls (Heads-wakings on Monday), 500 red balls (Tails-wakings on Monday) and 500 green balls (Tails-wakings on Tuesday), i.e. 1000 red balls and 500 green balls. In this context, the probability of Heads upon awakening is determined by the ratio of the number of Heads-wakings to the total number of wakings. Hence, P(Heads) = 500/1500 =1/3. The balls in the urn are normal ones and for present purposes, it is worth calling this sort of urn a “standard urn”.

On the other hand, the argument for 1/2 is also based on an urn analogy, albeit less transparently. It is worth rendering this analogy more apparent. For this purpose, let us investigate how the calculation of the probability of drawing a red ball is handled by the argument for 1/2. If the coin lands Heads then the probability of drawing a red ball is 1, and if it lands Tails then this latter probability equals 1/2. We get then accordingly the probability of drawing a red ball (Monday-waking): P(R) = 1 x 1/2 + 1/2 x 1/2 = 3/4. By contrast, if the coin land Tails, we calculate as follows the probability of drawing a green ball (Tuesday-waking): P(G) = 0 x 1/2 + 1/2 x 1/2 = 1/4. To sum up, according to the argument for 1/3: P(R) = 3/4 and P(G) = 1/4. Suppose the Sleeping beauty experiment is iterated. Now it appears that the argument for 1/2 is based on an analogy with a standard urn that contains 3/4 of red balls and 1/4 of green ones. These balls are also normal ones and the analogy underlying the argument for 1/2 is also with a “standard urn”.

Both above-mentioned analogies are based on a standard urn. But at this step, a question arises: is the analogy with the standard urn well-suited to the Sleeping Beauty experiment? In other terms, isn't another urn model best suited? In the present context, this alternative can be formulated more accurately as follows: isn't the situation inherent to the Sleeping Beauty experiment better put in analogy with the *Entanglement urn**, rather than with the *standard urn*? I shall argue, however, that the analogy with the standard urn is mistaken, for it fails to incorporate an essential feature of the experiment, namely the fact that Monday-Tails wakings are indissociable from Tuesday-Tails wakings. For in the Tails case, Beauty cannot wake up on Monday without also waking up on Tuesday and reciprocally, she cannot wake up on Tuesday without also waking up on Monday. The argument for 1/3 and the argument for 1/2 are thus based on an analogy with a standard urn.

In the argument for 1/3 and the argument for 1/2, one feels intuitively entitled to add red-Heads (Heads-wakings on Monday), red-Tails (Tails-wakings on Monday) and green-Tails (Tails-wakings on Tuesday) balls to compute frequencies. But red-Heads and red-Tails balls appear to be objects of a fundamentally different nature in the present context. In effect, red-Heads balls are in all respects similar to our familiar objects, and can be considered properly as single objects. By contrast, it appears that red-Tails balls are quite indissociable from green-Tails balls. For we cannot draw a red-Tails ball

[1] Bostrom opens the path to a third way out to the Sleeping Beauty problem: “At any rate, one might hope that having a third contender for how Beauty should reason will help stimulate new ideas in the study of self-location”. In his account, Bostrom sides with the halfer on P(Heads) and with the thirder on conditional probabilities, but his treatment has some counter-intuitive consequences on conditional probabilities.

without picking the associated green-Tails ball. And conversely, we cannot draw a green-Tails ball without picking the associated red-Tails ball. In this sense, red-Tails balls and the associated green-Tails balls do not behave as our familiar objects, but are much similar to entangled quantum objects. For Monday-Tails wakings are indissociable from Tuesday-Tails wakings. And Beauty cannot be awaken on Monday (resp. Tuesday) without being also awaken on Tuesday (resp. Monday). From this viewpoint, it is mistaken to consider red-Tails and green-Tails balls as separate objects. The correct intuition is that the red-Tails and the associated green-Tails ball are a pair of entangled balls and constitute but one single object. In this context, red-Tails and green-Tails balls are best seen intuitively as constituents and mere parts of one single object. In other words, red-Heads balls and, on the other hand, red-Tails and green-Tails balls, cannot be considered as objects of the same type for probability purposes. And this situation justifies the fact that one is not entitled to add unrestrictedly red-Heads, red-Tails and green-Tails balls to compute probability frequencies. For in this case, one adds objects of intrinsically different types, i.e. one single object with the mere part of another single object.

Given what precedes, the correct analogy, I contend, is with an Entanglement urn that contains 2/3 of red balls and 1/3 of green balls. And among the red balls, 1/2 are normal balls, but 1/2 are entangled ones, each being associated with a different green ball. As will become clearer later, this new analogy incorporates the strengths of both above-mentioned analogies with the standard urn.

4. Consequences of the analogy with the Entanglement urn

At this step, it is worth drawing the consequences of the analogy with the Entanglement urn, that notably result from the ontological properties of the balls. Now the key point appears to be the following one. Consider the Entanglement urn. Recall that there are in total 2/3 of red balls and 1/3 of green balls in the Entanglement urn, and that nothing seemingly distinguishes the normal balls from the entangled ones. Among the red balls, half are normal ones, but the other half is composed of balls that are each entangled with a different green ball. If one considers the behaviour of the balls, it appears that normal red balls behave as usual. But entangled ones do behave differently, with regard to statistics. Suppose I add the red ball of an entangled pair in the Entanglement urn. Then I also add instantly in the urn the associated green ball of the entangled pair. Suppose, conversely, that I remove the red ball of an entangled pair from the urn. Then I also remove instantly the associated green ball. Now the same goes for Sleeping Beauty, as the analogy suggests. And the consequences are not so that innocuous. What is the probability of a Monday-waking? This is tantamount to calculating the probability P(R→) of *drawing a red ball* from the Entanglement urn? On Heads, the probability of drawing a red ball is 1. On Tails, we can either draw the red or the green ball of an entangled pair. But it should be pointed out that if we pick on Tails the green ball of an entangled pair, we also draw instantly the associated red ball. Hence, the probability of drawing a red ball on Tails is also 1. Thus, P(R→) = 1 x 1/2 + 1 x 1/2 = 1. Conversely, what is the probability of a waking on Tuesday? This is tantamount to the probability P(G→) of drawing a green ball. The probability of drawing a green ball is 0 in the Heads case, and 1 in the Tails case. For in the latter case, we either draw the green or the red ball of an entangled pair. But even if we draw the red ball of the entangled pair, we draw then instantly the associated green ball. Hence, P(G→) = 0 x 1/2 + 1 x 1/2 = 1/2. To sum up: P(R→) = 1 and P(G→) = 1/2. The probability of a waking on Monday is then 1, and the probability of a waking on Tuesday is 1/2. Now it appears that P(R→) + P(G→) = 1 + 1/2 = 1,5. In the present account, this results from the fact that drawing a red ball and drawing a green ball – in general – are *not exclusive events*. And – in particular – drawing a red ball and drawing a green ball from an entangled pair are not exclusive events for probability purposes. For we cannot draw the red-Tails (resp. green-Tails) ball without drawing the associated green-Tails (resp. red-Tails) ball.

On the other hand, as mentioned above, there are unambiguously 2/3 of red balls and 1/3 of green balls in total in the urn. This casts light on the fact that we need to distinguish two different situations with regard to the Entanglement urn: the probability P(R↑) of a red ball *being* in the urn; and the probability P(R→) of *drawing* a red ball. For as we did see it, the former equals 2/3 and the latter equals 1. And the same goes for green balls: the probability P(G↑) of a green ball being in the urn is 1/3 and the probability P(G→) of drawing a green ball equals 1/2. In sum, from an *internal* viewpoint, the probability of a red (resp. green) ball being in the urn is 2/3 (resp. 1/3) in the Entanglement urn. By contrast, from an *external* viewpoint, the probability of drawing a red (resp. green) ball from the the urn is 1 (resp. 1/2). The same goes analogously for Sleeping Beauty: we need to distinguish (i) from an *internal* point of view, the probability of being on Monday (to say it otherwise, the probability that this waking is a Monday-waking); and (ii) from an *external* standpoint, the probability of a waking on Monday.

Let us forget for a moment the fact that, according to its classical formulation, the Sleeping Beauty problem arises from conflicting conclusions resulting from the argument for 1/3 and the argument for 1/2 on calculating the probability of Heads once Beauty is awaken. This could be a red herring. For as we did see it before, the problem also arises on the calculation of the probability of waking on Monday (drawing a red ball), where conflicting conclusions also result from concurrent lines of reasoning: Elga argues for 2/3 and Lewis for 3/4. Hence, the Sleeping Beauty problem could then have been formulated alternatively as follows: once awaken, what probability should Beauty assign to waking on Monday (drawing a red ball)?

What is now the response of the present account, based on the Entanglement urn, to the latter question? In the present context, we need then to distinguish between two different questions: (i) what is the probability of drawing a red ball (a Monday-waking)? And (ii) what is the probability that this ball is a red one (this waking is a Monday-waking)? This distinction makes sense in the present context, since it results from the properties of the entangled balls. In particular, this richer semantics results from the case where you draw the green ball of an entangled pair from the urn. For in this case, this ball is not a red one, but it occurs that you also draw a red ball, since the associated red ball will be drawn simultaneously. Now it appears that the response to the first question equals 1, since it corresponds to the probability $P(R\rightarrow)$ of drawing a red ball (a Monday-waking). And the probability $P(G\rightarrow)$ of drawing a green ball (a Tuesday-waking) also equals 1/2. On the other hand, the response to the second question turns out to be different. For the probability $P(R\uparrow)$ that this ball is a red one (this waking is a Monday-waking), as we did see it, equals 1/3. And the probability $P(G\uparrow)$ that this ball is a green one (this waking is a Tuesday-waking), equals 2/3.

5. A two-sided account

Grounded as they are on an unsuited analogy with the standard urn, both arguments do have, however, their own strengths. In particular, the analogy with the urn in the argument for 1/3 does justice to the fact that the Sleeping Beauty experiment leads to a choice between three wakings (Heads-waking, Tails-waking on Monday, Tails-waking on Tuesday), each corresponding to a different ball in the urn. On the other hand, the analogy with the urn in the argument for 1/2 handles adequately the fact that the Heads-waking is put on a par with the two Tails-wakings. Nevertheless, these two analogies are one-sided and fail to handle the whole notion of the probability of drawing a red ball (waking on Monday). In the present account, the analogy with the Entanglement urn proves to be two-sided and encapsulates both insights. For on the one hand, there are 2/3 of red balls and 1/3 of green balls in the urn, in the same way as with the thirder's urn. It appears then that the probability $P(R\uparrow)$ that this drawn ball is red, in the present account, corresponds to the thirder's insight. On the other hand, the halfer's insight is also taken into account, since the normal red ball is put on a par with a pair of entangled balls, which behave as one single object. This casts light on the fact that the probability $P(R\rightarrow)$ of drawing a red ball, in the present account, is the mere transposition of the halfer's intuition. Recall then the halfer's calculation: $P(R) = 1 \times 1/2 + 1/2 \times 1/2 = 3/4$ and $P(G) = 0 \times 1/2 + 1/2 \times 1/2 = 1/4$. Now the present standpoint echoes this reasoning, by only pondering the calculation with the properties of the entangled balls: $P(R\rightarrow) = 1 \times 1/2 + 1 \times 1/2 = 1$ and $P(G\rightarrow) = 0 \times 1/2 + 1 \times 1/2 = 1/2$. In sum, it appears that the probability $P(R\uparrow)$ that this drawn ball is red does justice to the thirder's intuition, and that the probability $P(R\rightarrow)$ of drawing a red ball vindicates the halfer's insight. At this step, it appears that the present account is two-sided, since it incorporates insights from the argument for 1/3 and from the argument for 1/2.

Now what precedes casts new light on the halfer and the thirder's accounts. For given that the Sleeping Beauty experiment, is modelled with a standard urn, both accounts lack the ability to express the difference between the probability $P(R\rightarrow)$ of drawing a red ball (a Monday-waking) and the probability $P(R\uparrow)$ that this drawn ball is red (this waking is a Monday-waking). For it does not make sense in the standard urn, since these probabilities are equal in the latter model. Consequently, there is a failure to express this difference in the analogy with the standard urn. But such distinction makes sense in the Sleeping Beauty experiment and in the analogy with the Entanglement urn. For in the richer ontology that results from this two-sided model, the distinction between P(R→) and P(R↑) yields two different results: P(R→) = 1 and P(R↑) = 2/3.

At this step, it is worth recalling the diagnosis of the Sleeping Beauty problem made by Benoit Groisman (2008). Groisman attributes the two conflicting responses to the probability of Heads to an ambiguity in the protocol of the Sleeping Beauty experiment. He argues that the argument for 1/2 is an adequate response to the probability of Heads on awakening, under the setup of coin tossing. On the other hand, he considers that the argument for 1/3 is an accurate answer to the latter probability, under the setup of picking up a ball from the urn. Groisman also considers that putting a ball in the box and picking out a ball out from the box are two different events, that lead therefore to two different probabilities. In the present account, though, putting a ball in the urn is no different from drawing a ball from the urn. For if we put a red ball of an entangled pair in the urn, we also put immediately the associated green ball. Rather, from the present standpoint, being in the urn is probabilistically different from withdrawing (or putting) a ball in the urn. The present account and Groisman's analysis share the same overall direction, although the details of our motivations are significantly different.

As we did see it, the calculation of the probability of drawing a red ball (waking on Monday) is the core issue in the Sleeping Beauty problem. But what is now the response of the present account on conditional probabilities and on the probability of Heads upon awakening? Let us begin with the conditional probability of Heads on a Monday-waking. Let us recall first how it is calculated on the two concurrent standard lines of reasoning. To begin with, the probability P(Heads|G) of Heads on drawing a green ball is not a subject of disagreement for halfers and thirders, since it equals 0 in both accounts. The same goes for the probability P(Tails|G) of Tails on drawing a green ball, since it equals 1 from a halfer or thirder viewpoint. But agreement stops when one considers the probability P(Heads|R) of Heads on drawing a red ball. For P(Heads|R) = 2/3 for a halfer and P(Heads|R) = 1/2 from a thirder's perspective. On the other hand, the probability P(Tails|R) of Tails on drawing a red ball is 1/3 from a halfer standpoint, and 1/2 for a thirder. Now the response of the present account to the calculation of the conditional probability of Heads on a Monday-waking parallels the answer made to the issue of determining the probability of waking on Monday (drawing a red ball). In the present account, P(Heads|G) = 0 and P(Tails|G) = 1, as usual. But we need to distinguish between P(Heads|R→) and P(Heads|R↑), to go any further. For P(Heads|R→) is the probability of Heads on drawing of a red ball. And P(Heads|R↑) is the probability of Heads on this ball being a red one. Now we get accordingly: P(Heads|R→) = [P(Heads) x P(R→|Heads)] / P(R→) = [1/2 x 1] / 1 = 1/2. The reasoning parallels the halfer's standpoint, but takes additionally into account the property of entanglement. And we also get: P(Tails|R→) = [P(Tails) x P(R→|Tails)] / P(R→) = [1/2 x 1] / 1 = 1/2. On the other hand, P(Heads|R↑) is calculated in the same way as in the thirder's account. And we get then accordingly: P(Heads|R↑) = 1/2 and P(Tails|R↑) = 1/2.

Now the same goes for the probability of Heads upon awakening. For there are two different responses in the present account, depending on whether one considers P(R→) or P(R↑). If one envisages things from the viewpoint of P(R→), the probability of drawing a red ball (a Monday-waking), then it ensues, in the same way as in the halfer's account, that there is no shift in the prior probability of Heads. It ensues, that the probability of Heads (resp. Tails) on awakening still remains 1/2. On the other hand, if one considers P(R↑), the probability of Heads upon awakening is calculated in the same way as in the argument for 1/3, and we get accordingly: P(Heads↑) = 1/3 and P(Tails↑) = 2/3.

Finally, the above results are summarised in the following table:

	halfer	*thirder*	*present account*
P(Heads→)	1/2		1/2
P(Tails→)	1/2		1/2
P(Heads↑)		1/3	1/3
P(Tails↑)		2/3	2/3
P(a Monday-waking) ≡ P(R→)	3/4		1
P(a Tuesday-waking) ≡ P(G→)	1/4		1/2
P(this waking is a Monday-waking) ≡ P(R↑)		2/3	2/3
P(this waking is a Monday-waking) ≡ P(G↑)		1/3	1/3
P(Heads| a Monday-waking) ≡ P(Heads|R→)	2/3		1/2
P(Tails| a Monday-waking) ≡ P(Tails|R→)	1/3		1/2
P(Heads| this waking is Monday-waking) ≡ P(Heads|R↑)		1/2	1/2
P(Tails| this waking is a Monday-waking) ≡ P(Tails|R↑)		1/2	1/2

6. Handling the variations of the Sleeping Beauty problem

From the above, it follows that the present treatment of the Sleeping Beauty problem, is capable of handling several variations of the original problem which have recently flourished in the literature. For the above solution to the Sleeping Beauty problem applies straightforwardly, I shall argue, to these variations of the original experiment. Let us consider, to begin with, a variation were on Heads, Sleeping Beauty is not awaken on Monday but instead on Tuesday. This is modelled with an Entanglement urn* that receives one normal green ball (instead of a red one in the original experiment) in the Heads case.

Let us suppose, second, that Sleeping Beauty is awaken two times on Monday in the Tails case (instead of being awaken on both Monday and Tuesday). This is then modelled with an Entanglement urn* that receives one pair of entangled balls which are composed of two red balls in the Tails case. (instead of a pair of entangled balls composed of a red and a green ball in the original experiment).

Let us imagine, third, that Beauty is awaken two times – on Monday and Tuesday – in the Heads case, and three times – on Monday, Tuesday and Wednesday – in the Tails case. This is then modelled with an Entanglement urn* that receives one pair of entangled balls composed of one red ball and one green ball in the Heads case; in the Tails case, the Entanglement urn* is filled with one triplet of entangled balls, composed of a red, a green and a blue ball.

Finally, the lesson of the Sleeping Beauty Problem appear to be the following: our current and familiar objects or concepts such as balls, wakings, etc. should not be considered as the sole relevant classes of objects for probability purposes. We should bear in mind that according to an unformalised axiom of probability theory, a given situation is standardly modelled with the help of urns, dices, balls, etc. But the rules that allow for these simplifications lack an explicit formulation. However in certain situations, in order to reason properly, it is also necessary to take into account somewhat unfamiliar objects whose constituents are pairs of indissociable balls or of mutually inseparable wakings, etc. This lesson was anticipated by Nelson Goodman, who pointed out in *Ways of Worldmaking* that some objects which are prima facie completely different from our familiar objects also deserve consideration: ‘we do not welcome molecules or concreta as elements of our everyday world, or combine tomatoes and triangles and typewriters and tyrants and tornadoes into a single kind’.[2] As we did see it, in some cases, we cannot add unrestrictedly an object of the Heads-world with an object of the Tails-world. For despite the appearances, objects of the Heads-world may have ontologically different properties from objects of the Tails-world. And the status of our probabilistic paradigm

[2]Goodman (1978, p. 21).

object, namely a ball, appears to be world-relative, since it can be a whole in the Heads-world and a part in the Tails-world. Once this goodmanian step accomplished, we should be less vulnerable to certain subtle cognitive traps in probabilistic reasoning.

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