Motivating Structural Realist Interpretations of Spacetime

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ABSTRACT. Motivated by examples from general relativity and Newtonian gravitation, this essay attempts to distinguish between the *dynamical* structure associated with a theory in physics, and its *kinematical* structure. This enables a distinction to be made between a structural realist interpretation of a theory based on its dynamical structure, and a structural realist interpretation of spacetime, as described by a theory, based on its kinematical structure. I offer category-theoretic formulations of dynamical and kinematical structure and indicate the extent to which such formulations deflect recent criticism of the radical ontic structural realist's conception of structure as "relations devoid of *relata*".

Keywords: structural realism, spacetime, category theory

1. Introduction

This essay is concerned with ways of motivating an ontic structural realist interpretation of spacetime. I'll begin in Section 2 by first considering a particular underdetermination argument against scientific realism due to Jones (1991). This argument claims that the existence of alternative formulations of a single theory that differ at the level of ontology effectively undermines the scientific realist's inclination to interpret theories literally. Examples of "Jones Underdetermination" from general relativity will suggest a distinction between the *dynamical* structure associated with a given theory in physics and its *kinematical* structure. My claim will be that this provides a means to distinguish between a structural realist interpretation of a theory based on its dynamical structure, versus a structural realist interpretation of spacetime as described by a given theory, based on its kinematical structure.

Section 3 indicates how the distinction between dynamical and kinematical structure addresses a recent argument raised by Pooley (2006) against using Jones Underdetermination as a means to motivate ontic structural realism. Briefly, the argument is that different formulations of the same theory not only underdetermine individuals-based ontologies, but also the structures these individuals may instantiate; hence appeals to alternative formalisms cannot motivate structural realism. I will claim that structural realist interpretations of different formulations of GR do not suffer from underdetermination of dynamical structure; and while different formulations of GR admit different structural realist interpretations of spacetime, the underdetermination involved is less severe than that associated with individuals. Whereas the individuals-based ontologies associated with alternative formulations of spacetime in GR are in-principle underdetermined, the structures they instantiate are open to empirical investigation in the form of extensions of GR to quantum gravity. Such extensions are currently research programs with little if any empirical support, but their advocates foresee a time at which they, and hence the structures they attribute to spacetime, may be distinguished by empirical evidence.

Finally, Section 5 considers how the dynamical and kinematical structure of a theory in physics might be given a category-theoretic formulation. This formulation, I will argue, goes some way in deflecting a recent criticism of a radical flavor of ontic structural realism. This radical flavor conceives of structures as existing independently of the individuals that instantiate them, and the criticism is that, to the extent that this is a conception of structure as consisting of relations devoid of *relata*, it is incoherent. I will argue that a conception of structure as consisting of relations devoid of *relata* amounts to a literal interpretation of a set-theoretic definition of structure, and that shifting to an alternative category-theoretic formulation may make the radical ontic structural realist's concept of structure less problematic.

2. Jones Underdetermination: Realism With Respect to What?

Scientific realism can be associated with the general claim that successful theories in science should be interpreted literally; in other words, we should take them at their face-value. Jones (1991) raised the following worry about this general view: Successful theories typically admit alternative mathematical formulations that disagree at the level of ontology. Thus, regardless of whether there are cogent arguments for being a scientific realist, there's a prior worry about *what* scientific realists should be realists about. After Pooley (2006, pg. 87), call this type of underdetermination of ontology by formalism, "Jones Underdetermination". In this section, I'd like to consider, as an example, General Relativity (GR, hereafter).

In the tensor formalism, models of GR are typically given by a pair (M, g_{ab}), where M is a differentiable manifold and g_{ab} is a metric field defined on M and satisfying the Einstein equations. In the following I will briefly review three alternative formulations of GR and suggest that on a literal interpretation, while they may disagree at the level of "individuals-based" ontology, they agree at the level of structure, appropriately construed. Each of these alternative formalisms has been discussed in more detail in

Bain (2006) in the case of general field theories in physics. In the following, I will only be concerned with their application to GR.

2.1. Einstein Algebra Models of GR

Models of GR in the Einstein algebra (EA) formalism consist of a triple ($\mathcal{R}^\infty$, $\mathcal{R}$, g), where $\mathcal{R}^\infty$ is a commutative ring, $\mathcal{R}$ is a subring of $\mathcal{R}^\infty$ isomorphic with the real numbers, and g is a multilinear map defined on the space of derivations of ($\mathcal{R}^\infty$, $\mathcal{R}$) and its dual space, and satisfying the Einstein equations (Geroch 1972). A 1-1 correspondence between such models and tensor models exists, based on the 1-1 correspondence between the points of a differentiable manifold and the maximal ideals of the commutative ring of smooth functions defined on M.[1] This correspondence allows all the relevant tensorial objects defined on M in tensor models of GR to be translated into appropriate algebraic objects defined on (spaces constructed from) ($\mathcal{R}^\infty$, $\mathcal{R}$). Thus the Einstein algebra formalism is as expressive as the tensor formalism in the sense that any model of GR in the latter corresponds to a model of GR in the former.

Now, arguably, on a literal construal, tensor models and EA models disagree at the level of "individuals-based" ontology. The individuals associated with tensor models are the points of M, insofar as these points are the basic objects of predication in tensor models; whereas the individuals associated with Einstein algebra models are (a bit more abstractly) maximal ideals of smooth functions. However, the isomorphism between these models suggests they agree at the level of structure. In general, tensor models of GR are invariant under the actions of *Diff*(M), the group of diffeomorphisms on M. EA models of GR share this invariance property, although in the EA formalism, it gets translated into actions of the group of homomorphisms that leave invariant ($\mathcal{R}^\infty$, $\mathcal{R}$, g). In both cases, the structure associated with these transformations may be identified as *differentiable structure*. In tensor models, this is predicated on the points of M, whereas in EA models, it is associated with the structure of a commutative ring of smooth functions on M.

2.1.1. Points vs Maximal Ideals: Local vs Global Differentiable Structure

There is a sense in which the individuals in tensor models (*i.e.*, manifold points) play a greater role in formulating GR than do the individuals in Einstein algebra models (*i.e.*, maximal ideals). In particular, one might say that manifold points *kinematically matter*, whereas maximal ideals do not. To see this consider imposing certain types of asymptotic boundary conditions on GR. Examples include solutions to the Einstein

[1] A maximal ideal of a commutative ring is the largest subset of the ring closed under the ring product. Each point of a differentiable manifold M corresponds to a maximal ideal of smooth functions on M that vanish at that point.

equations that are asymptotically flat, and solutions involving certain types of curvature singularities. In such examples, the boundary conditions can be implemented geometrically by encoding them in a boundary space ∂M (itself a differentiable manifold) and attaching it to the manifold M. The result is a manifold with boundary $M' = M \cup \partial M$. One then observes that, while tensor models (M, g_{ab}) without such boundary conditions are invariant under the group *Diff*(M), tensor models (M', g_{ab}) *with* such boundary conditions are in general invariant under the subgroup *Diff*$_c(M)$ of diffeomorphisms on M with compact support (a diffeomorphism is in *Diff*$_c(M)$ just when there is a compact region of M outside of which it is the identity). One can think of *Diff*$_c(M)$ as the group of "local" diffeomorphisms on M. Intuitively, such local diffeomorphisms are guaranteed to preserve the local structure of *any* manifold (including boundary spaces). Elements of the larger group *Diff*(M), on the other hand, are only guaranteed to preserve the structure of a given M, and may fail to preserve the structure of a boundary space distinct from M. Thus, in general, there are no morphisms (*i.e.*, transformations) that preserve both M and M' (no d on M is guaranteed to extend smoothly to a d on M'). Technically this means that manifolds and manifolds with boundaries belong to different categories.

On the other hand, asymptotic boundary conditions of this type can be imposed on Einstein algebra models of GR in two steps (*cf.*, Heller and Sasin 1995, pg. 3657). We first replace the *ring* $\mathcal{R}^\infty \cong C^\infty(M)$ of real-valued smooth functions on a given M with the *sheaf* $\mathcal{R}^\infty{}_{Asymp} \cong C^\infty(M')$ of real-valued smooth functions on the corresponding manifold with boundary $M' = M \cup \partial M$. We then replace the Einstein algebra $(\mathcal{R}^\infty, g)$ defined on M with the sheaf of Einstein algebras $(\mathcal{R}^\infty{}_{Asymp}, g)$ defined on M', where for each open region U of M', $(C^\infty(U), g)$ is an Einstein algebra. One can think of such a sheaf of Einstein algebras as the collection of Einstein algebras defined on all open regions of M'. It turns out that, as algebraic objects, $(\mathcal{R}^\infty, g)$ and $(\mathcal{R}^\infty{}_{Asymp}, g)$ belong to the same category , what Heller and Sasin (1995, pg. 3647) have dubbed the category of structured spaces. In particular, one can define morphisms that preserve the structure of *both* $(\mathcal{R}^\infty, g)$ and $(\mathcal{R}^\infty{}_{Asymp}, g)$.[2]

This suggests that the kinematical structure of Einstein algebra models of GR (both with and without asymptotic boundary conditions) can be identified as "global"

[2] A *structured space* is a pair $(M, \mathscr{C})$, where M is a topological space and $\mathscr{C}$ is the sheaf of real continuous functions on M satisfying the following condition (*closure with respect to composition with smooth Euclidean functions*): For any open set U in the topology τ on M and any functions $f_1, ..., f_n$ in $\mathscr{C}(U)$, and any smooth function ω on $\mathbb{R}^n$, the composite $\omega \circ (f_1, ..., f_n)$ is in $\mathscr{C}(U)$ (Heller & Sasin 1995, pg. 3645). Now let $(M, \mathscr{C})$ and $(N, \mathscr{D})$ be structured spaces. A continuous mapping $f : M \to N$ is said to be *smooth* if, for any cross section g in $\mathscr{D}(U)$, the composite $g \circ (f|f^{-1}(U))$ is in $\mathscr{C}(f^{-1}(U))$ (Heller & Sasin 1995, pg. 3647) <u>*Claim*</u>: The set of structured spaces as objects and smooth mappings as morphisms forms a category.

differentiable structure associated with a single type of morphism. In contrast, the kinematical structure of tensor models of GR (both with and without asymptotic boundary conditions) can be identified with "local" differentiable structure, in so far as, in general, the differentiable structure at a given point p of a tensor model will depend on whether p is in the interior space M or the boundary space ∂M.

2.2. Twistor Models of GR

Now suppose we require that the metric field in tensor models of GR be anti-self-dual and satisfy the vacuum Einstein equations. Such models are schematically of the form $(M, g_{ab}{}^{ASD})$, where $g_{ab}{}^{ASD}$ satisfies the vacuum Einstein equations with the anti-self-dual constraint $*g_{ab}{}^{ASD} = -ig_{ab}{}^{ASD}$. One can now establish a 1-1 correspondence between such tensor models and models of GR in the twistor formalism of the schematic form $(\mathcal{P}, \tau, \rho)$. Such twistor models consist of a curved twistor space $\mathcal{P}$ and two differential forms τ, ρ defined on it and satisfying certain requirements (this construction was dubbed the non-linear graviton by Penrose 1976). Briefly, the idea is to modify the correspondence that exists between Minkowski spacetime and flat twistor space in an infinitesimal way for particular curved general relativistic spacetimes. (For the correspondence between Minkowski spacetime and flat twistor space, and a brief explanation of the Penrose non-linear graviton, see the discussion in Bain 2006, pg. xx, and references therein).

One might again argue that tensor models and twistor models disagree at the level of "individuals-based" ontology: Points in the case of tensor models, as opposed to twistors in the latter case. And, again, the fact that these models are isomorphic indicates they share common structure. In this case, the relevant structure is the conformal structure associated with Ricci-flat Lorentzian metrics.

2.3. Geometric Algebra Models of GR

Finally suppose we restrict the tensor models of GR to those in which the metric field is everywhere decomposable into a tetrad field. A tetrad field $(e_\mu)^a$ consists of a set of orthonormal vector fields that serves to define an orthonormal frame in the tangent space at each point of a manifold M (where the index $\mu = 0, 1, 2, 3$ labels the vector fields of the tetrad). At each point p of M, the Lorentzian metric of a tensor model of GR can always be decomposed as $g_{ab} = (e_\mu)_a(e_\nu)_b\eta_{\mu\nu}$, where $\eta_{\mu\nu}$ is the Minkowski metric on the tangent space at p. For g_{ab} to be everywhere so-decomposable requires the existence of a global tetrad field on M. In general, tensor models of GR need not admit such global tetrad fields. Those that do may be schematically represented by $(M, g_{ab}, (e_\mu{}^a))$.

One can now establish a 1-1 correspondence between such tensor models, and a subclass of models of GR formulated in the geometric algebra (GA, hereafter) formalism. This subclass of GA models of GR takes the schematic form ($\mathcal{D}$, h, Ω), where $\mathcal{D}$ is the Dirac algebra, and h and Ω are linear functions defined on $\mathcal{D}$ interpreted as displacement and rotation gauge fields. The Dirac algebra is the real Clifford algebra $\mathcal{C}_{(1,3)}$ of Minkowski vector space.[3] Intuitively, it encodes the metrical structure of Minkowski vector space, in so far as the bilinear form that defines $\mathcal{C}_{(1,3)}$ is induced by the Minkowski metric. The correspondence between tensor models (M, g_{ab}, $(e_\mu{}^a)$) and GA models can be justified by appeal to Lansby, *et al.* (1998), who construct what they refer to as a Gauge Theory of Gravity (GTG) in the geometric algebra formalism. GTG is obtained by imposing displacement and rotation gauge invariance on a matter Lagrangian defined on $\mathcal{D}$, following the same procedure used in Poincaré gauge theory.[4] The resulting local gauge fields then define a metric field that satisfies the Einstein equations and a connection with non-vanishing torsion. Restricting such solutions to those in which the torsion vanishes then reproduces tensor models of GR of the above form (M, g_{ab}, $(e_\mu{}^a)$).

One might again argue that tensor models and GA models disagree at the level of "individuals-based" ontology: Points in the case of tensor models, as opposed to multivectors in the case of GA models (multivectors being the objects of predication of the Dirac algebra). And, again, the fact that these models are isomorphic indicates they share common structure. As indicated above, in this case this structure is *metrical*. In particular, it is encoded in the Poincaré group, which is the isometry group of Minkowski spacetime.

2.4. Dynamical vs. Kinematical Structure

The above examples suggest that models of GR in different formalisms can agree *dynamically*; provided they represent appropriate solutions to the Einstein equations. This is the case for the following models:

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[3] Let V be a real vector space equipped with a bilinear form $g: V \times V \to \mathbb{R}$ with signature (p, q). The real Clifford algebra $\mathcal{C}_{(p,\,q)}$ is the linear algebra over $\mathbb{R}$ generated by the elements of V *via* "Clifford multiplication" defined by $xy + yx = g(x, y)\mathbf{1}$, $x, y \in V$, where $\mathbf{1}$ is the unit element.

[4] See Blagojevic (2002) for a comprehensive review of the latter. Briefly, in Poincaré gauge theory, local Poincaré gauge invariance is imposed on a matter Lagrangian with matter fields defined on a manifold M, and this requires the introduction of gauge potential fields. These are then identified as the connection on a Poincaré frame (*i.e.*, tetrad) bundle over M encoding rotational gauge degrees of freedoms, and sections of this bundle (*i.e.*, global tetrad fields) encoding translational gauge degrees of freedom. The Einstein equations are then obtained by extremizing the Lagrangian with respect to the gauge potentials. This procedure also produces a non-vanishing torsion, thus, strictly speaking, Poincaré gauge theory is both a restriction and an extension of GR, in so far as models of GR need not admit global tetrad fields and require a vanishing torsion.

(1) Tensor models, with and without asymptotic boundary conditions, and Einstein algebra models.

(2) Anti-self-dual vacuum tensor models and twistor models.

(3) Tensor models with global tetrad fields and Geometric algebra models.

These examples also suggest that models of GR in different formalisms can disagree *kinematically*; in so far as they can disagree over what they take to be the structure they attribute to spacetime. Thus tensor models, arguably, take the point set as the basic structure of spacetime, which might be identified with local differentiable structure. In contrast, spacetime structure in Einstein algebra models may be identified as global differentiable structure, and in twistor models as conformal structure, and finally in geometric algebra models as metrical structure.

One can thus make a distinction between *a structural realist interpretation of a theory*; namely, an ontological commitment to the dynamical structure associated with the theory, and *a structural realist interpretation of spacetime as described by a particular formulation of a given theory*; namely, an interpretation of spacetime as given by the kinematical structure associated with that formulation of the theory. Section 4.3 will attempt to make the distinction between dynamical and kinematical structure a bit more precise, but before doing so, I'd like to look at an example of how such a distinction can be made to do work for the structural realist.

3. Is Structure Jones-Underdetermined?

Section 2 attempted to motivate structural realist interpretations of theory and spacetime by means of Jones Underdetermination. However some authors claim that Jones Underdetermination cannot motivate structural realism. This is because alternative formalisms disagree at the level of individuals, *and* at the level of structure. Thus not only are individuals-based interpretations of a single theory underdetermined; so are structural realist interpretations. For instance, Pooley (2006) argues in the following way:

> Consider a model of a theory of Newtonian gravitation formulated using an action-at-a-distance force and an *empirically equivalent* model of the Newton-Cartan formulation of the theory. There is no (primitive) element of the second model which is structurally isomorphic to the flat inertial connection of the first model, and there are no (primitive) elements of the first model which are structurally isomorphic to the gravitational potential field, or the non-flat inertial structure of the second. Clearly a

more sophisticated notion of structure is needed if it is to be something common to models of both formulations of the theory. (Pooley 2006, pp. 87-88; my italics.)

In response, I would first point out that this example is not really an example of Jones Underdetermination, as I understand it. The empirically equivalent flat-space and curved-space formulations of Newtonian gravity that Pooley refers to are better thought of as two ways of formulating the same theory in the *same* (*viz.*, tensor) formalism, at least as they're typically presented. But Pooley's example does raise the following question: Can a single theory admit distinct formulations in a single formalism that differ at the level of structure? This might indeed prove difficult for a structural realist to explain.

To consider this question, I'd like to look more closely at Pooley's example, which is a bit more complex than he presents it. It turns out that there are *many* potential candidates for theories of Newtonian gravity in both flat and curved spacetimes, and only some of these may be considered empirically equivalent to each other. Consider first theories of Newtonian Gravity that make use of a gravitational potential field Φ. Tensor models of such theories are given by a 6-tuple $(M, h^{ab}, t_{ab}, \nabla_a, \Phi, \rho)$ that consists of a manifold M, spatial and temporal metric fields h^{ab}, t_{ab}, a derivative operator ∇_a, and scalar fields on M that represent a Newtonian gravitational potential Φ and a mass density ρ. These objects are required to satisfy orthogonality and compatibility conditions, the Poisson equation, and an equation of motion:

$$h^{ab}t_{ab} = 0 = \nabla_c h^{ab} = \nabla_c t_{ab} \qquad \textit{(orthogonality and compatibility)} \qquad (1)$$

$$h^{ab}\nabla_a \nabla_b \Phi = 4\pi G\rho \qquad \textit{(Poisson equation)} \qquad (2)$$

$$\xi^a \nabla_a \xi^b = -h_{ab}\nabla_a \Phi \qquad \textit{(equation of motion)} \qquad (3)$$

where ξ^a is a tangent vector field for a timelike particle trajectory worldline that encodes the particle's four-velocity. At least three formally distinct theories of Newtonian Gravity can now be identified (for details see Bain 2004, pp. 353-355).

(i) *Neo-Newtonian Newtonian Gravity.* This theory describes Newtonian gravity in terms of a potential field and a mass density defined in spatiotemporally flat Neo-Newtonian spacetime. It does this by placing an additional constraint on the curvature tensor, requiring it to vanish, $R^a{}_{bcd} = 0$, which encodes the fact that Neo-Newtonian spacetime is spatiotemporally flat. One can then identify the *spacetime symmetries* of this theory as the symmetries of Neo-Newtonian spacetime, which are generated by the Galilei Lie algebra $\mathfrak{gal}$. If we now identify a theory's *dynamical symmetries* with the transformations that leave invariant its dynamical equations, then one can show that the dynamical symmetries of Neo-Newt NG are generated by elements of an extension $\widetilde{\mathfrak{max}}$ of the Maxwell Lie algebra.

(ii) *Island Universe Neo-Newtonian Newtonian Gravity.* A second example can be had by imposing a boundary condition on Example (i) that forces the gravitational potential to vanish at spatial infinity (we require $\phi \to 0$ as $x^i \to \infty$). The result is a concentration of mass in the center of the universe in what has been referred to as an "island universe effect". One can show that this reduces the dynamical symmetries to those generated by the Galilei Lie algebra, plus a gauge transformation on the potential; namely, $\Phi \mapsto \Phi + \varphi(t)$, where $\varphi(t)$ is an arbitrary function of time.

(iii) *Maxwellian Newtonian Gravity.* Finally, one can impose a weaker constraint on the curvature tensor, $R^{ab}{}_{cd} = 0$, and end up with Newtonian Gravity in Maxwellian spacetime. This weaker constraint effectively relativizes acceleration, but not rotation. The spacetime symmetries are now generated by the Maxwell Lie algebra $\mathfrak{max}$, while the dynamical symmetries are the same as for Neo-Newt NG.

One can also formulate theories of Newtonian gravity by incorporating the gravitational potential field into the spacetime connection. These may be called theories of Newton-Cartan gravity (NCG). Tensor models of such theories eliminate the Newtonian gravitational potential, and are given by $(M, h^{ab}, t_{ab}, \nabla_a, \rho)$. These objects are required to satisfy the same orthogonality and compatibility constraints (1) as the theories above, but they replace the Poisson equation (2) with a generalized Poisson equation, and replace the equation of motion (3) with the geodesic equation:

$$R_{ab} = 4\pi G\rho t_{ab} \qquad \text{(generalized Poisson equation)} \qquad (4)$$

$$\xi^a \nabla_a \xi^b = 0 \qquad \text{(equation of motion)} \qquad (5)$$

These changes enforce the principle of equivalence on theories of Newtonian gravity. Again, at least three distinct theories of NCG can be identified (for details see Bain 2004, pp. 356-372).

(iv) *Weak Newton-Cartan Gravity.* This theory imposes a "curl-free" condition on the curvature tensor, $R^{[a}{}_{[b}{}^{c]}{}_{d]} = 0$ that is necessary in recovering weak NCG as the non-relativistic limit of GR. It also is necessary, but not sufficient, in recovering the Poisson equation. One can show that both its spacetime symmetries and its dynamical symmetries are generated by an extension of the Leibniz Lie algebra $\widetilde{\mathfrak{leib}}$.

(v) *Asymptotically spatially flat weak Newton-Cartan Gravity.* To recover the Poisson equation, one may impose a boundary condition on weak NCG in the form of asymptotic spatial flatness. This has the result of reducing the spacetime symmetries of Weak NCG to those generated by the Galilei algebra, and the dynamical symmetries reduce to those generated by the Galilei algebra plus a particular gauge transformation $\Phi \mapsto \Phi + \varphi(t)$ on a scalar field Φ that can be identified as a Newtonian gravitational potential.

(vi) *Strong Newton-Cartan Gravity.* This theory imposes both "curl-freeness", $R^{[a}{}_{[b}{}^{c]}{}_{d]} = 0$, and the Maxwell condition, $R^{ab}{}_{cd} = 0$, on the curvature tensor. This is also sufficient to recover the Poisson equation. Both its spacetime and dynamical symmetries are generated by an extension of the Maxwell Lie algebra $\widetilde{\mathfrak{max}}$.

Theory	**Spacetime symmetries**	**Dynamical symmetries**
Neo-Newtonian NG $R^a{}_{bcd} = 0$	$\mathfrak{gal}$	$\widetilde{\mathfrak{max}}$
Island Universe Neo-Newt NG $R^a{}_{bcd} = 0$ $\Phi \to 0$ as $x^i \to \infty$	$\mathfrak{gal}$	$\mathfrak{gal}$ $\Phi \mapsto \Phi + \varphi(t)$
Maxwellian NG $R^{ab}{}_{cd} = 0$	$\mathfrak{max}$	$\widetilde{\mathfrak{max}}$
Weak NCG $R^{[a}{}_{[b}{}^{c]}{}_{d]} = 0$	$\widetilde{\mathfrak{leib}}$	$\widetilde{\mathfrak{leib}}$
Asymp. spatially flat weak NCG $R^{[a}{}_{[b}{}^{c]}{}_{d]} = 0$ $R^{abcd} = 0$ at spatial infinity	$\mathfrak{gal}$	$\mathfrak{gal}$ $\Phi \mapsto \Phi + \varphi(t)$
Strong NCG $R^{[a}{}_{[b}{}^{c]}{}_{d]} = 0$ $R^{ab}{}_{cd} = 0$	$\widetilde{\mathfrak{max}}$	$\widetilde{\mathfrak{max}}$

Table 1. Theories of Newtonian Gravity in Flat and Curved Spacetimes

In all, there are at least six distinct theories of Newtonian gravity in flat and curved spacetime (see Table 1). Pooley's example is based on *empirically equivalent* theories that exhibit different structure. Suppose two theories are empirically indistinguishable just when they share the same solution space of a set of dynamical equations. This entails that they make the same predictions, in so far as any set of admissible observables is evolved in time in exactly the same way by both theories. Now this shared solution space is reflected in the examples above in terms of shared dynamical symmetries. Again, these are the symmetries of a theory's dynamical equations, and in some of the above examples, the dynamical equations of a given theory effectively reduce to those of another. Whether this entails that such theories are really different formulations of the same theory will depend, in this context, on whether or not they share the same spacetime symmetries.

If this is right, then there are at least two cases of empirically indistinguishable theories of Newtonian gravity:

(a) Island Universe Neo-Newtonian NG, and asymptotically spatially flat weak NCG.

(b) Neo-Newtonian NG, Maxwellian NG, and Strong NCG.

Do these cases exhibit different structures? Case (a) does not. Both Island Universe Neo-Newtonian NG and asymptotically spatially flat weak NCG possess the same spacetime symmetries; hence, arguably, they make the same ontological commitments with respect to spacetime structure. They constitute an example of different formulations of the same theory.

Now consider Case (b). Here the theories do disagree on their spacetime symmetries, and hence, arguably, on what they take to be the structure of spacetime. So it might be an example of this type that drives Pooley's argument against structural realism. But on the other hand, all these theories do agree on one aspect of structure; namely, they all agree on *dynamical* structure. This suggests that a structural realist interpretation of such theories is *still* viable.

I would thus claim that structural realist interpretations of different formulations of a single theory do not suffer from underdetermination of dynamical structure, appropriately construed. And, granted, structural realist interpretations of spacetime as represented by a particular formulation of a given theory *are* underdetermined, both in cases of Jones Underdetermination that involve *different* formalisms, as well as in cases in which a theory can be formulated in different ways in a *single* formalism. But such underdetermination of spacetime structure does not affect the *current* empirical adequacy of the given theory, as determined by its dynamical structure. And moreover, arguably, spacetime structure *is* susceptible to future empirical tests. In the GR examples in Section 2, for instance, which formalism one adopts may depend on how one thinks GR can be extended to a quantum theory of gravity. Each of the formalisms in these examples is associated with a particular approach to constructing a quantum theory of gravity.[5]

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[5] See Heller and Sasin (1999) for an approach to quantum gravity motivated by the Einstein algebra formalism. The twistor program initiated by Penrose is also viewed as an approach to quantum gravity.

4. What is Structure?

The type of structural realism that is motivated by Jones Underdetermination takes structure to be more fundamental than the individuals that instantiate it. Thus it's similar to the radical ontic structural realism (ROSR, hereafter) associated with French and Ladyman (2003) that takes structure to consist of relations devoid of *relata*. This view has been criticized by many authors. Here is just a sample: Esfeld and Lam (2008, pg. 31) acknowledge that one might posit the existence of abstract relations-as-universals without reference to *relata*, but "...when it comes to the physical world, the point at issue are concrete relations that are instantiated in the physical world and that hence are particulars in contrast to universals. For the relations to be instantiated, there has to be something that instantiates them... ." With respect to the view that there are only relations without *relata*, Stachel (2006, pg. 54) states: "As applied to a particular relation, this assertion seems incoherent. It only makes sense if it is interpreted as the metaphysical claim that ultimately there are only relations; that is, in any given relation, all of its *relata* can in turn be interpreted as *relations*." Wüthrich (2008, pg. 3) agrees with Stachel's assessment: "Taken at face value... [radical ontic structural realism] is clearly incoherent...". Finally, Dorato (2008, pg. 21) states "I daresay that no ontic structural realist should be falling into the trap of accepting the view that 'relations can exist without *relata'*."

As Chakravartty (2003, pg. 871) notes, criticism of this type assumes that there is a conceptual dependence between the notions of relation and *relata*, and to the extent that ROSR recommends a revision of such concepts, it cannot be faulted simply for denying this dependence. On the other hand, as Greaves (2009, pp. 17-18) suggests, the onus is still on ROSR to make good on just how such a dependence can be denied. Here's a suggestion for how ROSR might proceed to do this. One might claim that the conceptual dependence between relations and *relata* that critics of ROSR assume is a consequence of formulating the notion of structure in a particular formalism; namely, set theory. In the spirit of this essay, one might consider alternative formalisms in which the notion of structure might be presented and in which such a conceptual dependence between relations and *relata* is not implied. To see how this might proceed, I'd like to look first at the typical set-theoretic formulation of the notion of a structure, and then compare it with a category-theoretic formulation.

4.1. Set Theory vs. Category Theory

The geometric algebra formalism isn't associated with a particular approach, but arguably is in the same family as background dependent approaches that prioritize Minkowski spacetime structure.

If one adopts a set-theoretic formalism, then radical ontic structural realism may indeed seem incoherent. Suppose, for example, that by "*structure*" we mean "*isomorphism class of structured sets*", $[\{X, R_i\}]$, where a structured set $\{X, R_i\}$ consists of a domain X of individuals together with a collection of n-ary relations R_i defined on it. The ontic structural realist's claim then is that the specification of the domain X of individuals is arbitrary to the concept of structure: what matters is the structure of the relations these arbitrary individuals enter into. Now suppose, to take the simplest example, by "*binary relation R on X*", we mean "*subset of the Cartesian product* $X \times X$ ". In so far as the latter consists of all ordered pairs (x_1, x_2), where $x_1, x_2 \in X$, this definition makes ineliminable reference to the elements of X (let the ordered pair (x_1, x_2) be the set $\{x_1, \{x_1, x_2\}\}$). Hence if the *relata* of a relation in a structure are identified with the elements of its domain, the set-theoretic definition of structure as an isomorphism class of structured sets makes ineliminable reference to *relata*. In general, one might argue that any set-theoretic definition of structure does likewise, in so far as membership "$\in$" is a primitive concept in set theory.

However, consider adopting a *category*-theoretic formalism to represent structure. In brief, a category $\mathcal{C}$ consists of objects A, B, ... and morphisms between objects $f: A \rightarrow B$, ... In addition, we require that for each object A, there be an identity morphism $1_A : A \rightarrow A$, which satisfies the Identity Laws $1_A \circ f = f$, and $f \circ 1_A = f$, for any morphism f with A as domain; and we require that there be composite morphisms $f \circ g : A \rightarrow C$ for each pair of morphisms of the form $f: A \rightarrow B$, $g : B \rightarrow C$, which satisfy the Associative Law $f \circ (g \circ h) = (f \circ g) \circ h$, for $h : C \rightarrow D$. It turns out that set theory can be formulated as a category, **Set**, in which the objects are sets and the morphisms are functions defined on sets. Moreover, for any given structured set, there is a category in which the objects are that type of structured set and the morphisms are functions that preserve the structure of the set (see Lawvere and Shanuel 1997 for elementary examples). This suggests that the intuitions of the ontic structural realist may be preserved by defining "*structure*" in this context to be "*object in a category*".

To what extent does such a category theoretic definition of structure eliminate reference to *relata*? As Bell (1988, pg. 5) observes, "[i]n category theory many concepts formulated in terms of *elements* are instead formulated in terms of *arrows* [*viz.*, morphisms]". In particular, the notion of an element of an object only makes sense in those categories with certain types of objects; namely, *terminal objects*. An object $\mathbf{1}$ in a category $\mathcal{C}$ is a terminal object of $\mathcal{C}$ if for each object X of $\mathcal{C}$, there is exactly one $\mathcal{C}$-morphism $X \rightarrow \mathbf{1}$. In categories with terminal objects, an *element* of an object A is then defined as a morphism $\mathbf{1} \rightarrow A$ from the terminal object to A. (In the category **Set**, the terminal object is the isormorphism class of singleton sets.) Thus, instead of saying "$x \in X$", one says "$x : \mathbf{1} \rightarrow X$".

As another example, consider the concept of Cartesian product which underlies the set-theoretic concept of relation. In category theory, the Cartesian product of an object X with itself is an object P together with a pair of morphisms $p_1 : P \to X$ and $p_2 : P \to X$, such that, for any object T with morphisms $f_1 : T \to X$, $f_2 : T \to X$, there is exactly one morphism $f : T \to P$ for which $f_1 = p_1 \circ f$ and $f_2 = p_2 \circ f$. Explicitly in this definition there is no reference to the "internal" members of X. One might say that category theory prohibits direct reference to "internal" elements of an object. Thus in forming the definition of Cartesian product we need to construct the right external "probe" T, f_1, f_2, f, that directly encodes what in set theory is the "internal" pair structure of P. This suggests that the definition of structure as an object in a category does not make ineliminable reference to *relata* in the set-theoretic sense. And this suggests that in category theory, the concept of structure devoid of *relata* may be placed on a firmer foundation than in set theory.

Now one might object in the following way. Category theory eliminates reference to *relata* only in name. Instead of calling the *relata* associated with a structure "elements of the structure's domain", as in set theory, category theory calls them "morphisms from the terminal object". Assumedly, or so the objection goes, any given set theoretic structure will have a category theoretic analog, and however many *relata* the former is associated with, so the latter will be associated with the same number of morphisms from the terminal object. The argument against the radical ontic structural realist might then be modified from the slogan "*no relations without relata*" to the slogan "*no objects without morphisms*"!

One way to address this objection might be to suggest an analogy between set theory and category theory on the one hand, and the tensor and Einstein algebra (EA) formulations of GR on the other. In particular, the question of the extent to which category theory does away with *relata* might be mapped onto the similar question of the extent to which the EA formalism does away with manifold points. Recall that manifold points in the tensor formalism have correlates in the EA formalism; namely, maximal ideals of smooth functions. But recall, too, that maximal ideals in the EA formalism don't do the work that points do in the tensor formalism. In particular, maximal ideals in the EA formalism are not the objects of predication of the global differentiable structure associated with EA models. This global differentiable structure is encoded directly in a sheaf of Einstein algebras. In contrast, the local differentiable structure of tensor models is predicated directly on the points of the manifold. Thus we might say that the point correlates in EA models of GR are *surplus structure*. They can be defined, but they do not play leading roles in articulating the structure associated with EA models. And, importantly, the global differentiable structure associated with EA models is more unifying than the local differentiable structure associated with tensor models: In the EA formalism, models of GR with and without asymptotic boundary conditions fall under the same structure, whereas in the tensor formalism they do not.

One might attempt to tell a similar story about set-theoretic *relata* in the context of category theory. Such *relata* are the elements of sets, and while they have correlates in category theory, they don't play the essential role there that they play in set theory. They amount to *surplus structure* in category theory. To make good on this claim, one would have to demonstrate that, just as EA models of GR are more general than tensor models, and this generality does actual work in providing a unifying description of the theory, so objects in a category are more general than structured sets, and this generality does actual work in providing a more comprehensive notion of structure. Two examples of categories described by Baez (2006, pp. 246-247) might be seen in this light. The first is the category ***n*Cob** whose objects are (n-1)-dimensional (compact oriented) topological manifolds and whose morphisms are n-dimensional topological manifolds that "go between" them (what are called cobordisms). The second example is the category **Hilb** with (finite-dimensional) Hilbert spaces as objects and bounded linear operators as morphisms. These differ from the category **Set** of sets (with functions as morphisms) in the following three respects.

(i) First, the objects of ***n*Cob** and **Hilb** cannot be considered structured sets, in so far as their morphisms are not simply functions that preserve the relevant set-theoretic notion of structure associated with them. Set-theoretically, the functions that preserve the structure of an (n-1)-dim topological manifold are homeomorphisms (*i.e.*, maps that preserve the topological properties of points). But the morphisms in ***n*Cob** are not even functions. Set-theoretically, the functions that preserve the structure of a Hilbert space are unitary operators that preserve the inner-product. The morphisms in **Hilb** in contrast are general bounded linear operators that do not necessarily have to be unitary. (Baez 2006, pg. 251, defines an inner-product on the objects in **Hilb** in terms of an adjoint operation, thus turning **Hilb** into a *-category: see (3) below.)

(ii) Second, unlike **Set**, the categories ***n*Cob** and **Hilb** are *monoidal* categories. This means they admit a tensor product but not a Cartesian product. In particular, in both of these categories, for any pair of objects H, K, there is an object $H \otimes K$ called the tensor product of H and K, but there are no morphisms $p_1 : H \otimes K \rightarrow H$ and $p_2 : H \otimes K \rightarrow K$ with the properties of a Cartesian product (Baez 2006, pg. 257).

(iii) Third, unlike **Set**, the categories ***n*Cob** and **Hilb** are *-categories. This means they admit a morphism * that sends each morphism $f : X \rightarrow Y$ to a morphism $f^* : Y \rightarrow X$ called the "adjoint" of f and satisfying $1^*_X = 1_X$, $(f \circ g)^* = g^* \circ f^*$, and $f^{**} = f$ (Baez 2006, pg. 251).

Now both ***n*Cob** and **Hilb** admit terminal objects, and hence a well-defined notion of an element of an object. (For ***n*Cob** elements are points of (*n*-1)-dim manifolds, for **Hilb**, elements are vectors.) But, in so far as the objects of these categories are not structured sets, this notion of element does not do work in articulating the relevant notion of structure associated with these categories. Again, because the objects of these categories are not structured sets, the "properties" of such elements are not what gets preserved under the morphisms. Thus the structure associated with the objects in ***n*Cob** and **Hilb** is arguably more general than that associated with their set-theoretic counterparts. In other words, the category-theoretic definitions of (*n*-1)-dim topological manifold and Hilbert space, as provided by the categories ***n*Cob** and **Hilb**, are more general than the set-theoretic definitions. Baez (2006) further argues that this generality is more than cosmetic: Baez sees the similarities between ***n*Cob** and **Hilb** -- in particular, those features above that distinguish them from **Set** -- as suggestive of how GR and quantum theory might be reconciled. Briefly, ***n*Cob** has an essential role to play in a category-theoretic formulation of topological quantum field theories, which have been viewed by some authors as attempts to reconcile the background independent nature of GR with quantum field theory. One might view this as one way that the generality associated with the notion of structure in ***n*Cob** and **Hilb** has the *potential* to do actual work in articulating a notion of structure that addresses a key issue in physics.

Thus a definition of structure as an object in a category is more general than a definition of structure as an isomorphism class of structured sets. And for categories with objects that cannot be identified as structured sets, correlates of set-theoretic *relata* (*i.e.*, morphisms from the terminal object) arguably have diminished roles in articulating the nature of the structures under consideration.

4.2. What the Category-Theoretic Radical Ontic Structural Realist Must Do

Of course if the types of structures that ROSR is (or should be) concerned with are all of the structured set type (and hence depend definitionally on the notion of *relata*), then adopting a category-theoretic definition of structure would not be all that helpful. More perniciously, one might also argue that the generality afforded by category-theoretic definitions of structure is a moot point if it turns out that category theory presupposes set theoretic concepts. If this is the case, then categories are really just sets in disguise, even those categories that do not have structured sets as objects; thus there would be no greater expressiveness to be associated with category theory. In particular, the claim would be that membership really is a primitive in category theory, examples like Baez's none withstanding; hence a category-theoretic definition of structure will not, ultimately, break free of *relata*. Thus there is still work to be done by the category-theoretic radical ontic structural realist:

(1) She will have to provide a rationale for the fundamentality of category theory over set theory. For instance, Kraus (2005, pg. 114) claims the following:

> The reason [ontic structural realists] don't use category theory is still not clear to me, but perhaps this is due to the fact that from an intuitive point of view a category is nothing more than an ordered pair (hence a set) whose elements are a collection of objects (the structures) and a collection whose elements are called morphisms (both concepts of course are subjected to adequate postulates). That is, even in category theory we are not completely free from the intuitive notion of sets.

If category theory can be shown to be more fundamental than set theory, this argument is blunted. The fact that a category can be presented as an ordered pair would reduce to the fact that a category can be presented as a category. This on-going debate in the philosophy of mathematics deserves more space than can be provided here. I will suffice to refer to a recent article by Pedroso (2008) which addresses some of the major charges against category-theoretic fundamentalism.

(2) She will have to provide category-theoretic reformulations of theories in physics that explicitly do not depend on sets. The fundamentality of category theory would be moot if it turned out that structures in the physical world are better represented by set-theoretic constructions. Döring and Isham (2008) are engaged in this project in the context of theories in quantum physics (see also Isham and Butterfield 2000), and Baez (2006) has argued against set-theoretic intuitions in formulating approaches to quantum gravity.

(3) Finally, the category-theoretic ontic radical ontic structural realist will have to identify the relevant notion of structure in category-theoretic terms. In particular, in the context of this essay, one would have to distinguish between kinematical structure and dynamical structure in category-theoretic terms. I'd now like to consider one way this last task might be approached.

4.3. How to Do Category-Theoretic Physics

Consider, first, how one might attempt to do physics in category theory. Baez (2006, pp. 256-257) suggests the following. Given a theory T, we identify its "kinematics" with objects in a particular category $\mathcal{C}$. And we identify its "dynamics" with morphisms in $\mathcal{C}$. Take, for example, classical physics. The relevant category here is the category **Symp** of symplectic manifolds. Objects in this category are symplectic manifolds, which encode the structure of classical phase spaces. The morphisms in **Symp** are symplectic transformations. These are intended to represent maps that take dynamically possible states to dynamically possible states. As another example, consider quantum physics. The relevant category here is **Hilb**, whose objects are Hilbert spaces and whose morphisms are bounded linear operators. The latter, again, are meant to represent maps that take dynamically possible states of a quantum system into dynamically possible states.

Now while this scheme does a good job in encoding the notion of the dynamical structure of a theory T, one might balk at employing it to encode the notion of kinematical structure. In particular, the "kinematics" that Baez describes is really the space of dynamically possible states of a physical system. One would like a framework underwhich a distinction can be made between *kinematically* possible states and *dynamically* possible states. Consider, then, the following framework for doing field-theoretic physics in the tensor formalism (as described by Belot 2007, pp. 155-157). Under this scheme, a field theory consists of a pair $(\mathcal{K}, \Delta)$, where $\mathcal{K}$ is the space of kinematically possible fields $\phi : M \to W$. Such fields are represented by maps from a differentiable manifold M to an appropriate space in which the fields take their values (this space W depends on the type of field under consideration). Δ is a set of differential equations consisting of independent variables that parameterized M, and dependent variables that parameterize W. Given such a pair, one can define the space $\mathcal{S}$ of dynamically possible fields as the subspace of $\mathcal{K}$ consisting of all kinematically possible fields that are solutions to Δ; *i.e.*, $\mathcal{S} = \{\phi_0 \in \mathcal{K} : \phi_0 \text{ is a solution of } \Delta\}$.

This framework is specifically for field theories in the tensor formalism, but the distinction between $\mathcal{K}$ and $\mathcal{S}$ is more general. Arguably, it is an essential aspect of *any* formulation of a theory that depends on specifying a set of differential equations. And in fact, all the examples of alternative formulations of GR considered in this talk assume such a distinction. This suggests identifying the *dynamical structure* associated with a field theory with the structure of the solution space $\mathcal{S}$. This structure is in part encoded in the theory's equations, which place constraints on the kinematically possible variables. The *kinematical structure* might then be identified with the *basis of support* for the space $\mathcal{K}$ of kinematically possible variables. This is the structure of the independent variables in the theory's equations. This is the structure overwhich the fields of the theory predicate, and is presupposed by the dynamics of the theory. In the tensor formalism, this structure is encoded in a differentiable manifold, but it may take other forms when field theories are formulated in alternative formalisms.

Consider the examples of kinematically distinct models of GR in Section 2. Each of these models is associated with a different kinematical structure, now identified as the structure into which the independent variables of the relevant set of differential equations enter. For tensor models this set consists of the Einstein equations defined in terms of tensor fields on a manifold. For EA models, this set consists of the correlates of the Einstein equations defined in terms of algebraic objects in an Einstein algebra or its generalization. For twistor models, the set of differential equations consists of the constraints on the differential forms defined on curved twistor space; and for GA models, the set of differential equations can be identified with the Euler-Lagrange equations of the Lagrangian that describes the GA gauge theory of gravity. These equations govern functions defined on the Dirac algebra.

These kinematical structures can be tentatively identified with the following categories. The local differentiable structure of tensor models can be represented by the category **Man** of smooth manifolds (for GR without asymptotic boundary conditions), or by the category of smooth manifolds with boundary, call it **Manb**, for GR with asymptotic boundary conditions. The global differentiable structure of EA models can be represented by Heller and Sasin's (1995) category of structured spaces, call it **Struc**. The conformal structure of twistor models (*i.e.*, the conformal structure of Ricci-flat Lorentzian metrics) might be represented by the category of curved twistor spaces, call it **Twist**. And the metrical structure of GA models might be represented by the category of Dirac algebras, which can be identified with the category of real Clifford algebras of signature (1,3), call it $\mathbf{Cliff}_{(1,3)}$.

Sector	Models	Spacetime Structure		Dynamical Structure	
GR *sans* b.c.'s	tensor	local differentiable	**Man**	$(M,\ g_{ab})$	$\mathbf{Symp}_1$
	EA	global differentiable	**Struc**	$\cong (\mathcal{R}^\infty,\ g)$	
GR with b.c.'s	tensor	local differentiable	**Manb**	$(M \cup \partial M,\ g_{ab})$	$\mathbf{Symp}_2$
	EA	global differentiable	**Struc**	$\cong (\mathcal{R}^\infty{}_{Asymp},\ g)$	
ASD-GR	tensor	local differentiable	**Man**	$(M,\ g_{ab}{}^{ADS})$	$\mathbf{Symp}_3$
	twistor	conformal	**Twist**	$\cong (\mathcal{P},\ \tau,\ \rho)$	
tetrad-GR	tensor	local differentiable	**Man**	$(M,\ g_{ab},\ (e_\mu{}^a))$	$\mathbf{Symp}_4$
	GA	metrical	$\mathbf{Cliff}_{(1,3)}$	$\cong (\mathcal{D},\ h,\ \Omega)$	

Table 2. Structural Relations Among Sectors of General Relativity.

Thus, to recap, consider again the examples of sectors of GR in Section 2 and their associated kinematical and dynamical structure (see Table 2). GR without boundary conditions can be formulated either using tensors or Einstein algebras. These formulations differ on what kinematical structure they attribute to this sector: the tensor formalism suggests the structure represented by the category **Man**, whereas the EA formalism suggests the category **Struc**. But both formulations agree on what dynamical structure they attribute to this sector; namely, the dynamical structure represented by a subcategory of the category of symplectic manifolds, call it $\mathbf{Symp_1}$. This subcategory consists of objects that are particular symplectic manifolds; namely, those that encode the structure of the space of solutions to the particular differential equations of this sector. It is this structure that tensor models and EA models of this sector have in common. Similar stories can be told for the remaining GR examples, where, in general $\mathbf{Symp} \supset \mathbf{Symp_i} \cong \mathcal{S}$ for a given sector's $(\mathcal{K}, \Delta)$.

5. Conclusion

Structural realism with respect to spacetime can be motivated by a distinction between the kinematical structure associated with a theory in physics and its dynamical structure. The former encodes the structure of spacetime, and can be identified with the structure of the independent variables in the theory's differential equations. This structure forms the background for the space $\mathcal{K}$ of kinematically possible states of the theory. The dynamical structure of a theory in physics encodes the dynamics associated with the theory's differential equations. It is given by the space $\mathcal{S}$ of solutions to these equations (the dynamically possible states of the theory). This distinction is possible both for sectors of general relativity, our current best-confirmed theory about the nature of spacetime, as well as for Newtonian theories of gravitation that preceded it. These examples suggested that models of a theory in different formalisms will agree on dynamical structure, but may disagree on kinematical structure. This motivated a version of structural realism that commits to the dynamical structure of a theory, and remains agnostic about the kinematical structure, allowing that the latter may depend on future extensions of the theory.

This paper also suggested that a promising approach to articulating the nature of structure is given by category theory. In particular, the dynamical structure of a theory in physics as encoded in the space $\mathcal{S}$ of solutions to its dynamical equations, can be identified with objects in a relevant category. For theories in classical physics like general relativity, the dynamical structure encoded in $\mathcal{S}$ finds its home in a subcategory of the category **Symp** of symplectic manifolds. The kinematical structure of a theory in physics, as encoded in the background structure for the space $\mathcal{K}$ of kinematically possible variables, likewise can also be identified with objects in a relevant category. And, again, this latter depends on the formalism in which the theory is presented. To the extent that category theory allows us to speak of structures (as objects in a category) without reference to their internal constituents, the intuitions of the radical ontic structural realist are preserved, and arguments against such intuitions based on the slogan "no relations without *relata*" are blunted.

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