Conventional and Objective Invariance: Debs and Redhead on Symmetry

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Abstract

This review is a critical discussion of three main claims in Debs and Redhead's thought-provoking book *Objectivity, Invariance, and Convention*. These claims are: (i) Social acts impinge upon formal aspects of scientific representation; (ii) symmetries introduce the need for conventional choice; (iii) perspectival symmetry is a necessary and sufficient condition for objectivity, while symmetry *simpliciter* fails to be necessary.

Much work in the philosophy of science falls into one of two camps. On the one hand, we have the system builders. Often with the help of formal methods, these philosophers aim at a general account of theories and models, explanation and confirmation, scientific theory change, and so on. Here, examples from the sciences are at best illustrations of the philosophical claims in question. On the other hand, we find those philosophers who, dissatisfied by general accounts and often prompted by a close study of the history of science, present detailed accounts of very specific parts of science, without aiming at the development of a general account. If a general account of science is referred to at all, it is refuted (by a case study) or only used to describe a certain episode of scientific research.

Although both of these approaches have their merits and limitations, very few research projects span the whole range of philosophy of science, such that results from general accounts are non-trivially applied to case studies of specific sciences. It is a virtue of Debs and Redhead's book (2007, page numbers refer to this work) that they

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have set out to do exactly that. The authors give a general model of scientific representation, analyse the relations of symmetry, objectivity, and convention in this model, and connect their results with three elaborate examples from physics.

This is a formidable task, especially given the relevance and difficulty of the topic. In our contribution, we try to identify some of the inevitable weaknesses in the first part of the book. This foundational section makes three main philosophical claims: (i) Scientific representation has both a formal and an intertwined social aspect; (ii) symmetries introduce the need for conventional choice; (iii) perspectival symmetry is a necessary and sufficient condition for objectivity, while symmetry *simpliciter* fails to be necessary.

The contribution is organised as follows. First, we discuss Debs and Redhead's analysis of scientific representation, albeit with a different focus: While Debs and Redhead stress the interaction of the formal and social aspects of representation, we first focus on the formal aspect (section 1) and then critically discuss Debs and Redhead's arguments for the dependence of the formal aspect on the social (section 2). This discussion also allows us to elucidate their claim that symmetry entails the need for convention. In section 3, we analyse and discuss Debs and Redhead's argument for symmetry as a sufficient criterion for objectivity, their argument against its necessity, and their introduction of perspectival invariantism.

1 The Formal Chain of Representation

Debs and Redhead formalise representation by relations between structures, tuples $S = \langle |S|, P_1, P_2, \ldots, f_1, f_2, \ldots, c_1, c_2, \ldots \rangle$, whose first element is their domain, followed by predicates (subsets of Cartesian products of $|S|$), functions on $|S|$, and constants (elements of $|S|$). The structures' domains determine their position in the formal chain of representation. Structures W whose domains contain objects of the (not further explicated) "world" are mapped to structures O, which are idealised models of W, by a partial isomorphism $p : W \longrightarrow O$ (p. 22). Here, for example, W could be X-radiation and O could be electromagnetic waves (p. 21). A partial isomorphism maps one partial structure isomorphically to another. A partial structure is like a structure, except that each predicate P tripartitions the (Cartesian product of the) domain into one set where it applies (say, P_+), one where it does not apply (P_-), and one where it is undetermined whether it applies ($P_\circ$). A partial structure with $P_\circ = \emptyset$ for all P is a normal structure (cf. Bueno 1997).

The formal relation between O and a mathematical model M that represents O is given by a homomorphism $h : O \longrightarrow M$, where in an ideal representation, the homomorphism is also an isomorphism. When there are objects in the mathematical representation that do not correspond to any object in O, M is a substructure of a larger structure M', and whatever is in M' but not in M is surplus structure. Debs and Redhead carefully point out that the existence of a homomorphism h is not a sufficient, but only a necessary condition for M to represent O (p. 23). We assume that an analogous claim can be made for p.

Debs and Redhead leave the formal relation between the world and the mind of the scientist unspecified because they consider it to be related to the problem of perception, while they wish to focus on the problem of scientific representation. In their view this also excludes the relation between world and idealised model (p. 9), so that representation "in its greatest generality includes O, M, and M' [...]. In practice, analysis will usually be limited to the features of idealised conceptual model O, mathematical model M, and the relations between them" (p. 24).

This seems to us like a severe limitation of the analysis. At best, a discussion wholly restricted to the representation of O by M leads to an analysis of the relation between different representations of the world. This limitation is all the more severe because at least two of Debs and Redhead's definitions of objectivity, Obj_2 and Obj_3 (p. 57, see below), involve the existence of an observer independent world, and thus seem to demand that the representation of W by O be taken into account. It may be that there is a reason why W is not relevant to the analysis, but as far as we can tell, Debs and Redhead do not provide an argument for this.

On the other hand, restricting the discussion to representations of an idealised model O invites the assumption that the represented object is already described in a certain way, and thus carries a structure. But this is not the case for the world. Take, as an example, a graduate student's office furniture. A possible structure for the furniture is $\langle \{\text{chair}, \text{desk}\}, \{\langle \text{chair}, \text{desk} \rangle\} \rangle$. The first set is the domain, the second the extension of 'is lighter than'. Another structure that fits the furniture is one whose domain contains all furniture legs, vertical surfaces, and backs, and has extensions for the predicates 'belongs to a chair' and 'belongs to a desk'. The choice of either structure is, further arguments pending, a clear case of convention. For the discussion of convention below, it is important to note that this conventional choice is preanalytic: While the formal relations rely on this choice, the implications of these formal relations can be determined without reference to the conventionality of the choice.

On a more technical note, we are unsure about $p : W \to O$. If W is a proper partial structure, that is, not also a normal structure, O is a proper partial structure as well, by the definition of partial isomorphism. Hence, unless the introduction of partial structures is superfluous, O is a proper partial structure and, via h, so is M. This result does not seem to be intended by Debs and Redhead. Maybe what is meant is that O is isomorphic to a normal structure W' that contains a predicate P' for each predicate P of W such that

$P_+ \subseteq P'$ and $P_- \subseteq \complement P'$.

As we hope to have demonstrated, the formal relations developed by Debs and Redhead allow for a discussion of the problems of representation, but they do not lead to obvious solutions. It is a virtue of Debs and Redhead's work that they are so explicit about their assumptions regarding the connection of W and O and their exclusion of W from the discussion.

2 The Social Influence on Formal Representation

The formal relations described above constitute what Debs and Redhead call the formal chain of representation. There is also a second, social chain of representation, and Debs and Redhead argue that the formal chain is dependent on the social chain for a variety of reasons. We will not discuss the more traditional arguments based, for example, on the theory-ladenness of data, but will rather focus on the role of symmetry.

Debs and Redhead's new argument for the reliance of the formal chain of representation on social acts is based on ambiguities in the formal chain. "[A]mbiguity exists", Debs and Redhead write, "where formal analysis is underdetermined. Furthermore, in order for the social dimension of representation to function, this ambiguity must be resolved by choice, often known as convention" (p. 26). Since "the interaction [between formal and social representation is] especially [...] evidenced by the resolution of formal ambiguity via convention" (p. 50), we take it that Debs and Redhead assume the choice of a convention to be a fundamentally social act.

If the resolution of ambiguity is only needed for the *social* dimension of representation to function, it may seem that the formal chain of representation can go without social acts altogether. But Debs and Redhead give two arguments why any kind of representation needs the social dimension. The first argument is, following Wittgenstein, based on the impossibility of private language. "[R]epresenting reality to oneself", Debs and Redhead write, "might require the use of language that can only take its meaning from a wider social context" (p. 14). The other argument is a "more direct response" (p. 26):

> If the social mediation of scientific knowledge is to involve only a single person, then there is no reason to choose which structure, of a number of similar structures, to designate as [representing which other structure] unless this act is a rehearsal for an attempt to transfer information to others.

Having so established a need to resolve ambiguities in order for the formal chain of representation to function, Debs and Redhead identify an ambiguity and hence a need for convention in the ideal case of representation: If $h : O \to M$ is an isomorphism, the direction of the representation relation is a property not captured in the formalism, since the inverse of h (i.e. h^{-1}) is also an isomorphism (pp. 23, 50).

Debs and Redhead identify another source of formal ambiguity in symmetries of O (p. 42):

> If a symmetry transformation is understood as mapping elements of one mathematical representation to another, then one may just as well choose either representation. In this way symmetries introduce ambiguity that must be resolved by conventional choice in the way scientific models are used.

This quotation needs some unpacking, as Debs and Redhead use the word 'symmetry' in two different ways. In one sense, "the word 'symmetry' takes on the meaning of invariance under a group of structure-preserving transformations" (p. 34), but in another, "symmetry transformations, or automorphisms [are] often simply [called] 'symmetries'" (p. 34, emphasis removed). An automorphism of a structure O is a bijection $f : |O| \to |O|$ such that for any predicate $P \in O$, $f(P) = P$, where f is assumed to apply to the elements of the tuples of P. An analogous relation holds for functions; constant elements are mapped to themselves, as if they were singleton predicates. In the following, we will usually limit our discussion to predicates. In this notation, Debs and Redhead use 'symmetry' to refer both to the fact that $f(P) = P$ and to f itself. Now the automorphisms of a structure O are determined by its domain and its predicates, and the more bijections f there are that leave its predicates invariant, that is, the more symmetries O has, the more functions there are to choose from. In this way, "symmetries introduce ambiguity that must be resolved by conventional choice".

Debs and Redhead (p. 42) give the example of

> the ambiguity existing between inertial reference frames within special relativity. [...] [O]ne may think of special relativity as proposing an idealised model O, of actual events in space and time. This is represented by a mathematical model, M, [...] in the form of a manifold. It is central to special relativity that certain relations on this manifold are invariant under the Poincaré group.

The admissible transformations of M, resulting from h and the automorphisms of O, are the Poincaré transformations, and a choice of representation cannot rely on Poincaré invariant predicates, since they are the same in all representations.

Of course, once the set of automorphisms (and thus the set of representations) is fixed by O, any newly introduced predicate over $|O|$ (that is, some subset Q of a Cartesian product of $|O|$ with $Q \notin O$) may not be invariant under all admissible transformations. The greater the number of automorphisms of O, the lower the number of predicates over $|O|$ that are invariant under all of them.

This interpretation of Debs and Redhead's position explains their initially surprising claim that "there is in any given circumstance a distinctive inverse relationship between [symmetries and invariants]: the greater the number of symmetries, then the smaller the number of invariants and vice versa" (p. 38). This statement is false if one understands symmetry to be invariance under a transformation. It is true, however, if one takes symmetry to be a transformation, determined by a structure O; more transformations mean fewer invariant predicates.

To discuss the social influence on the formal relations that constitute the formal chain of representation, we must first determine what is at issue. In some true and unsurprising ways, the formal chain is socially influenced: That something is being represented at all will depend on society. Which W is being represented can depend on the researcher's interest, which is influenced by society. What O is chosen as representation can depend on what has been used before as representation, the researcher's socially influenced predilections, and so on. What M is chosen as representation can depend on the mathematics known to the researcher, the available computing power for simulations, and many other socially influenced factors. Most importantly, the structure W that is given to the world depends on the researcher's concepts, which are probably influenced by society.

Debs and Redhead's claim seems to be that beyond these preanalytic, socially influenced choices, there is a need for conventional choice that *results* from the formal relations, and thus they cannot be analysed without taking the social chain of representation into account. If correct, this would indeed be an important result, since it would mean that results that are usually considered derivable within the formal chain are actually dependent on social acts.

We see several problems with Debs and Redhead's attempt to establish this claim. Their first, private language argument could be seen as a defence of our point that the world does not carry a structure, and that W is the result of applying learned concepts. But then their argument would not show that the formal relations, once determined, must be analysed by recourse to the social chain. We are not sure that the private language argument can be used to establish this for several reasons. First, the formal chain can rely on an already established language, but then proceed without there being any social influence on that language. Second, a private language is not simply one that only one person understands, but a language whose terms are in principle unintelligible to anyone but its one user, which is not typically the case when the terms refer to objects of the outside world. Third, the exact content and soundness of the private language argument are themselves under debate (cf. Candlish 2007). Debs and Redhead's second, "more direct” argument seems to be invalid: Because the researcher does not have a reason to represent anything to herself, the argument goes, she cannot represent anything to herself; thus representation is a social act. This is an inference from unwillingness to inability.

The next step in Debs and Redhead’s argument is to identify formal ambiguities. But the ambiguity of the direction of the representational relation that they identify occurs, if at all, only in the ideal case of isomorphic representation. For a homomorphism, it is generally clear which set is the domain and which the range. Even an ideal representation is not obviously ambiguous: Although an isomorphism’s inverse is indeed again an isomorphism, isomorphisms are directed, and not generally their own inverse. The formal ambiguity can at best exist for statements of isomorphy, because if S is said to be isomorphic to S', it is not obvious which isomorphism between S and S' is meant to be a representation. But this formal ambiguity can be resolved by formal means. For example, the formal relation between a structure S and its representation S' can be given by $\langle 0, S\rangle$, $\langle 1, S'\rangle$ and the claim that S and S' are isomorphic. While it is true that we are not forced to preanalytically associate 1 with the representing structure, neither must we choose S or S' in the first place, as already noted above.

This also points to a solution to the problem of ambiguity that results from symmetry. If one really wants to fix a specific frame of reference, one can formally single it out, that is, one can expand the structure O whose automorphisms are the allowed transformations, and thereby exclude unwanted ambiguities. Of course, this is what is typically done in special relativity. If a specific frame of reference is used to describe a set of events, some of its distinguishing properties are explicitly mentioned.

A predicate Q over $|O|$ is symmetric in O if and only if it is the same in any of the structures to which O’s symmetry transformations can map. Accordingly, Debs and Redhead state that in the theory of special relativity, symmetries are used “as an invariance criterion for objectivity”. They continue that “the objectivity of the synchrony relation depends on this choice of invariance criterion”, in our words, it depends on the choice of the structure O that determines allowed transformations. When Debs and Redhead then infer that “invariance has as much to do with convention as it does with objectivity” (all quotations from p. 97), it has to be kept in mind that they use ‘invariance’ in two distinct senses, just as ‘symmetry’ before: The connection of invariance to convention stems from the choice of O, while its connection to objectivity stems from the invariance of Q. The core question then seems to be whether there exists some distinguished O, and thus a distinguished group of automorphisms, that is determined by the world.

3 Against Invariantism and for Perspectival Invariantism

To evaluate the connection between symmetry and objectivity, Debs and Redhead define three kinds of objectivity: Agreement between some observers or "multi-subjective agreement", Obj_1 (p. 57), the "existence of a physical system in the world [...] over and above the experience of any number of human observers", Obj_2 (p. 57), and a combination of the two, multi-subjective agreement about an observer-independent physical system, Obj_3 (p. 59). Debs and Redhead then identify three tenets of a position they dub "invariantism", the first of which is that "[i]nvariance with respect to the group of automorphisms is both necessary and sufficient for objectivity" (p. 63). We will discuss this tenet.

Debs and Redhead provide an argument for invariantism (pp. 64, 65) starting from an "intuition [that is] common to the different senses of objectivity [Obj_1, Obj_2, Obj_3]" (p. 63):

> "(A) Objective facts are those that are the same from any perspective".

Here, "[b]y 'perspective' one might mean a literal vantage point or simply the way a given observer is situated in some figurative sense" (p. 63). Since "physicists typically view phenomena by representing them", Debs and Redhead suggest

> "(B) If a fact appears the same under any representation, then it is objective"

as a sufficient condition for objectivity. After some further transformations, they infer

> "(E) If a feature of O is invariant under its automorphism group, then it is objective".

For (E) to be non-trivial, a 'feature of O' cannot simply be a predicate, function, or constant in O, since otherwise every feature would be invariant. Debs and Redhead are not explicit about what counts as a fact or feature, but we figure they mean at least any predicates, functions, and constants over $|O|$ as well as the relations between them. For ease of discussion, we will focus on predicates in the following.

(E) is not a statement of invariantism, but only of one half of the first tenet, and Debs and Redhead claim that "there may be objective features of O that are not shown to be so by application of (E). These could be the identity of the very elements of structure O between which relations are specified", so that invariance provides only a sufficient condition for objectivity. This, then, is "a vital clue that invariantism [...] is not a supportable view" (p. 66).

A "pragmatic response" to this problem "is to find a way to *define* objectivity so that invariance can be seen as necessary as well as sufficient" (p. 73, our emphasis), which Debs and Redhead do by "restrict[ing] objectivity to the relational features of O" (p. 73) along the lines of structural realism. This, then, leads to a criterion of "perspectival objectivity (Obj_P)" (p. 74):

> [T]he objectivity of a feature of O is equivalent to its invariance under the automorphisms of O, when these automorphisms may be interpreted perspectivally and are generalisable (and therefore heuristically fruitful).

In other words: Assume that the automorphisms of O can be interpreted perspectivally and are generalisable; then a predicate Q over $|O|$ is Obj_P if and only if $f(Q) = Q$ for all automorphisms f of O. The condition on the automorphisms of O accommodates the other two tenets of invariantism, which we have not discussed. Explicating the claim of objectivity in invariantism by Obj_P then results in perspectival invariantism.

To discuss the relevance of perspectival invariantism, we first analyse Debs and Redhead's argument for invariantism itself. We will not discuss the explication of (A) by (B), which seems to turn an equivalence into an implication, but will instead focus on the inference from (B) to (E). Debs and Redhead's argument works by successive paraphrase. To gauge the validity of the inference, we reconstruct the argument as follows: Assume a feature F of a structure O is invariant under O's automorphisms. With the additional premise that a feature is so invariant only if it is a fact that appears the same under any representation, we can infer the antecedent of (B). By (B), it then follows that F is objective.

It may seem problematic to consider features of O as facts, but this assumption can be avoided by rephrasing (A) and (B), so that (B) is about features that appear the same under any representation and (A) is about objective features. We do not think that either (A) or (B) suffer from this rephrasing.

One real cause of concern with Debs and Redhead's argument is the status of (B) and its metaphorical paraphrase (A). (A) is presented as an intuition that is common to Obj_1, Obj_2, and Obj_3, so Debs and Redhead might assume that (A) follows from intersubjectivity, from observer-independent existence, and from a combination thereof. But without further argument, we would only tentatively accept the claim that a fact that appears the same from any perspective is agreed upon by all observers and hence is intersubjective, that is, Obj_1. This claim is not enough to solve the core problem, observer independence, and Debs and Redhead have not provided an argument for (A) or (B) based on Obj_2 or Obj_3.

It also seems that the additional premise in the reconstructed argument needs further support, because a feature of O that is invariant under O's automorphism group may still appear different under different representations. There may be two different mathematical models M and M', and even with an isomorphism between the two (if, for example, both ideally represent O), it is not clear that a predicate that is symmetric in O has to be represented in the same way by M and M'.

Most importantly, the source of O is not clear. O can always be chosen so that some predicate over $|O|$ is invariant, just by letting that predicate be part of O. Debs and Redhead acknowledge this (p. 66), and point out that the methods that are typically used to identify O, heuristic fruitfulness and generalisability, are fallible (pp. 67–70). Until these three concerns are resolved, we do not think that Debs and Redhead's argument establishes invariance under O's automorphisms as a sufficient condition for observer-independent existence.

As long as the source of O is not clear, invariance under O's automorphisms is also not a good candidate for a necessary condition of objectivity. This remains true even if one can somehow establish that O correctly represents the world, because O may not be complete: Q may be an objective property of the world, but if it is not in O, it may not be invariant.

In their argument against invariance as a necessary condition of objectivity, Debs and Redhead suggest that the "identity of the very elements" of $|O|$ may be objective, but not invariant under O's automorphisms. Of course, many structures have elements in their domain that are mapped to themselves under automorphisms: constant elements, singular sets, and in general singular sets that are definable by the predicates, functions, and constants in O. There are even structures whose only automorphism is the identity function, for example structures where every element of the domain is also a constant.

We therefore venture the guess that, in their argument against invariance as a necessary condition, Debs and Redhead assume that in the domains of many relevant structures, there are some elements that are not mapped to themselves by some of the automorphisms, and these mappings constitute an objective change. If this argument is to go beyond the argument from the incompleteness of O that we gave above, then even automorphisms of complete structures constitute an objective change, and an object can change its identity while all its properties remain the same. But this reeks of an extreme kind of essentialism. Automorphisms, in this view, map one essence to another.

In summary, invariance as a criterion of observer-independent objectivity seems to suffer from two main problems. First, Debs and Redhead have not established that invariance under O's automorphisms is a criterion of observer-independent objectivity. Second, they have not established that the usual methods for determining O ensure objectivity. Debs and Redhead's introduction of $\mathrm{Obj}_{\mathrm{P}}$ captures both problems by making them part of the definition of objectivity. A predicate Q is not objective *simpliciter*, but only objective relative to a set of automorphisms, determined by a structure O. The definition is also conditional, that is, it holds for any O whose automorphisms can be interpreted perspectivally and are generalisable.

If a structure does not fulfil this condition, $\mathrm{Obj}_{\mathrm{P}}$ is not defined. For a structure O that fulfils this condition, a predicate Q over $|O|$ is now indeed invariant if and only if Q is $\mathrm{Obj}_{\mathrm{P}}$, and thus perspectival invariantism is true in these structures, albeit by definition. While this result does not solve the initial problem of observer-independent objectivity, it puts the two new conceptions, perspectival objectivity and perspectival invariantism, to good use. It is a virtue of Debs and Redhead's result that its limitations are made explicit from the beginning.[1]

References

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