# Nonseparability, Classical and Quantum

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**Abstract**

This paper examines the implications of the holonomy interpretation of classical electromagnetism. As has been argued by Richard Healey and Gordon Belot, classical electromagnetism on this interpretation evinces a form of nonseparability, something that otherwise might have been thought of as confined to non-classical physics. Consideration of the differences between this classical nonseparability and quantum nonseparability shows that the nonseparability exhibited by the classical electromagnetism on the holonomy interpretation is closer to separability than might at first appear.

## 1 Introduction

How are we to think of classical electromagnetic fields? There is a view, which has its roots in the work of Wu and Yang (1975), and which has been advanced in the philosophical literature by Richard Healey (1997, 2004, 2007) and Gordon Belot (1998), according to which the traditional representation of classical electromagnetism, in terms of electric and magnetic fields (or, in Lorentz covariant formulation, in terms of the Faraday tensor), is not the most perspicuous representation of the physical content of the theory. In its place is offered what Healey (2007) calls the *holonomy interpretation*. This rests on a representation of electromagnetism that takes as its basic elements assignments of unit complex numbers to loops in spacetime. As both Healey and Belot have pointed out, the holonomy interpretation has consequences for the metaphysics of classical electromagnetism; electromagnetism on the holonomy interpretation evinces a form of nonseparability, which one might otherwise have thought was a hallmark of non-classical physics. This, in turn has consequences for the view that Lewis calls *Humean Supervenience* (Healey, 2007, 124–27).

These points are well taken. Nevertheless, there remain important differences between quantum nonseparability and the nonseparability exhibited by the holonomy interpretation of classical electromagnetism. An examination of these differences, I shall argue below, shows that the nonseparability exhibited by the classical electromagnetism on the holonomy interpretation is closer to separability than might at first appear. Furthermore, the consequences for Humean Supervenience are not as dire as would seem.

## 2 Interpretations of Classical Electromagnetism

In his book, Healey (2007) distinguishes three main lines of approach to classical gauge theories, such as electromagnetism. One is the "no new gauge potential properties view." On this view, the electromagnetic potentials represent no new properties; an electromagnetic potential represents physical reality only insofar as it encodes the electromagnetic field, represented by the Faraday tensor. This runs into trouble when we consider the manner in which electromagnetic fields couple to a quantum wave function. As the Aharonov-Bohm effect dramatically illustrates (though the issue is, of course, not confined to that particular set-up), the coupling between electric and magnetic fields and the wave-function of a charged particle cannot be a local coupling. We *can* achieve local coupling by expressing the electromagnetic field in terms of the electromagnetic potential $A_\mu$. This has been taken as an argument for the reality of the potential (notably, by Feynman (1964, §15-5)). A representation of electromagnetism in terms of the electromagnetic potential, however, has the disadvantage that it is not gauge-invariant. Any two electromagnetic potentials that are related by a gauge transformation are regarded as representing the same physical state of affairs.

The integral of the potential around a closed curve *is* gauge invariant, as is the Dirac phase factor

$$\exp[-\frac{ie}{\hbar}\oint_C A_\mu dx^\mu].$$

Wu and Yang (1975) argued that the set of all such phase factors—that is, a mapping from closed curves in spacetime to complex numbers of unit modulus—yields a representation that captures all of the physical content of the theory without superfluous structure. Such a representation, according to Wu and Yang, "constitutes *an intrinsic and complete description* of electromagnetism" (p. 3845). Healey (1997, 2004, 2007) and Belot (1998) have examined the consequences of taking this representation of classical electromagnetism, the holonomy representation, as basic, and in particular, its consequences for locality and separability.

Locality and separability are conditions on physical theories, both found in the writings of Einstein on quantum theory (though not always clearly distinguished by Einstein; it is due to the work of Don Howard (1985) that these have widely come to be recognized as distinct principles). Separability, as Einstein puts it, is the condition that "things claim an existence independent of one another, insofar as these things 'lie in different parts of space'" (Einstein 1948, 321; tr. Howard 1985, 187). Locality, or the *Prinzip der Nahewirkung*, is the condition that, if $A$ and $B$ are spatially separated systems, "an external influence on $A$ has no *immediate* effect on $B$." (Einstein 1948, 321–22; tr. Howard 1985, 188).[1] For Einstein, (classical) field theories were the epitomes of theories satisfying these conditions. Speaking of the separability condition, he wrote,

> Field theory has carried out this principle to the extreme, in that it localizes within infinitely small (four-dimensional) space-elements the elementary things existing independently of one another that it takes as basic, as well as the elementary laws it postulates for them. (Einstein 1948, 321; tr. Howard 1985, 188.)

And the locality condition, according to Einstein, "is applied consistently only in field theory" (Einstein 1948, 322; tr Howard 1985, 188).

It has become commonplace to say that classical state descriptions are separable, quantum state descriptions, nonseparable. The holonomy view of classical electromagnetism has introduced a complication into this neat dichotomy; on this view the state descriptions of classical electromagnetism exhibit a form of nonseparability (see §5, below). The argument for the holonomy view, to be sure, rests on quantum considerations, but it is a classical field, nonetheless, that is at issue. If we take it as a principle that we should let our best physical theories be our guides in our views on matters ontological, then we should take Belot's and Healey's arguments seriously in considering how to think about classical electromagnetic fields.[2]

If we accept the holonomy view of the classical electromagnetic field, and accept that, on this view, classical electromagnetism yields nonseparable state descriptions, does this obliterate any distinction between quantum nonseparability and the sort of nonseparability exhibited by classical electromagnetism? It would

[1] I take Belot's condition of synchronic locality to be essentially equivalent to separability, and his diachronic locality, to what we are here calling locality. See Belot (1998, 540).

[2] Belot (1998, 532) has expressed the point dramatically: "*until the discovery of the Aharonov-Bohm effect, we misunderstood what electromagnetism was telling us about the world.*"

be rash to say so, and neither Healey nor Belot do.[3] It is, therefore useful to examine the differences between quantum nonseparability and the nonseparability of classical electromagnetism.

## 3 Quantum nonseparability

As a warm-up, it is useful to rehearse some familiar facts about quantum nonseparability.

If two quantum systems $A$, $B$, are in an entangled state, the reduced states of $A$ and $B$—that is, restriction of attention to what the state says about measurements performed on $A$, and what it says about measurements performed on $B$—do not suffice to determine the state of the combined system, as the reduced states underdetermine the correlations between measurement results on the two systems. This is a form of holism that does not exist in classical mechanics, as, in classical mechanics, a complete specification of the states of the parts that make up a whole determines the state of the whole.

Comparison of the classical and quantum case is facilitated by the use of a framework that subsumes both theories. Both classical and quantum systems can be represented as theories whose observables are the self-adjoint part of a $C^*$-algebra (see Clifton et al. (2003)). The key difference is that classical observables can be thought of as functions on an underlying phase space; this entails that the algebra of observables is abelian. A consequence of this is that pure states are dispersion-free (that is, they assign definite values to all observables), and this in turn entails that the state space of a composite system contains no entangled states.

A little more formally: let $A$, $B$ be two physical systems, with associated $C^*$-algebras $\mathcal{A}$, $\mathcal{B}$. We associate with the composite system consisting of $A$ and $B$ the algebra $\mathcal{C} = \mathcal{A} \vee \mathcal{B}$, that is, the smallest $C^*$-algebra containing $\mathcal{A}$ and $\mathcal{B}$.[4] Let $\omega$ be a state[5] of $\mathcal{C}$, and let $\omega_A$, $\omega_B$ be the restrictions of $\omega$ to $\mathcal{A}$, $\mathcal{B}$, respectively. If $\omega$ is a pure state, and either $\mathcal{A}$ or $\mathcal{B}$ is abelian, then $\omega_A$ and $\omega_B$ are also pure, and $\omega$ is uniquely determined by specification of $\omega_A$ and $\omega_B$ (that is: there is only one state of $\mathcal{C}$ that is compatible with $\omega_A$ and $\omega_B$ being the restrictions of $\omega$ to their respective subalgebras). If, on the other hand, both $\mathcal{A}$ and $\mathcal{B}$ are non-abelian, as will be the case when both systems are treated as quantum systems, there will be

[3] In his reply (Healey, 1999) to Maudlin (1998), who emphasized the disanalogies between the case of classical electromagnetism and quantum nonseparability, Healey made it clear that he was not denying the reality of these disanalogies.

[4] We assume that $AB = BA$ for all $A \in \mathcal{A}$, $B \in \mathcal{B}$.

[5] That is, a normalized, positive linear functional.

pure states $\omega$ that are not product states. Such a state is not uniquely determined by the reduced states $\omega_A$ and $\omega_B$. This is true even if $A$ and $B$ are systems located some spatial distance apart. Thus, quantum states violate what Healey (1997, p. 26) has called *Spatial Separability.*

> *Spatial Separability.* The qualitative intrinsic physical properties of a compound system are supervenient on those of its spatially separated component systems together with the spatial relations among these component systems.

# 4 The holonomy representation of the classical electromagnetic field.

To set up the holonomy representation, some terminology is needed. We begin with a spacetime manifold $M$. A *curve* in $M$ is a continuous, piecewise smooth mapping $C : [0, 1] \rightarrow M$. A *closed curve* is a curve whose beginning point and endpoints are the same point, called its *base point.* If the endpoint of curve $C_1$ is the beginning point of curve $C_2$, we define the composition $C_2 \circ C_1$ in the obvious way. The *image* of a curve is the set of points in $M$ that it traces out. A closed curve is *thin* if it is possible to shrink it down to a point while remaining within its image. A *loop* is the oriented image of a non-self-intersecting closed curve. Closed curves $C_1, C_2$ are *thinly equivalent* iff $C_1 \circ C_2^{-1}$ is thin. A *hoop* is an equivalence class of thinly equivalent closed curves.[6] We will denote by $[C]$ the hoop containing a closed curve $C$.

Pick some point $o \in M$, and consider the set of closed curves with base-point $o$. The associated set of hoops can be given a natural group structure. For any two hoops $\alpha$, $\beta$, take curves $C_1 \in \alpha$, $C_2 \in \beta$, with common base point $o$. Define the composition $\beta \circ \alpha$ as $[C_2 \circ C_1]$. The identity element is the class of curves equivalent to the trivial curve that begins at $o$ and remains there. For any hoop $\alpha$, we can define its inverse $\alpha^{-1}$ as $[C^{-1}]$, where $C$ is any element of $\alpha$. Clearly, $\alpha^{-1} \circ \alpha$ is $[C^{-1} \circ C]$, which is the identity element. Let $L_o$ be this group of hoops. We can define a holonomy as a smooth[7] homomorphism from $L_o$ into a suitable Lie group; this gives rise to a representation of these as holonomies of a connection on a principal fiber bundle with that Lie group as structure group.

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[6]The terminology adopted here is that used in Healey (2007). In Healey (2004), hoops are called *loops* (following Gambini and Pullin 1996) and loops are called *rings.*

[7]For the meaning of "smooth" that is relevant here, see condition H3 of Barrett (1991).

In the case of electromagnetism, things are particularly simple. The structure group is just $U(1)$, the set of rotations of a circle. This group has a natural representation as complex numbers of unit modulus; thus, in electromagnetism, a holonomy assigns to each hoop in $L_o$ a complex number of unit modulus. Written in terms of the electromagnetic potential, the value of a holonomy $H$ assigns to a hoop $\gamma$ is,

$$H(\gamma) = \exp[\frac{ie}{\hbar} \oint_C A_\mu dx^\mu],$$

where $C$ is any closed curve in the equivalence-class $\gamma$. Since the group $U(1)$ is abelian, holonomies are independent of base point.

Any closed curve is composed of non-intersecting curves, whose image points are loops. To specify an electromagnetic holonomy on the group of hoops, therefore, it suffices to specify a value for each loop (since these values are independent of base point). At the price of some abuse of notation, we will write $H(l)$ for the value of $H$ on a hoop $\gamma_l$ containing a curve whose image is the loop $l$.

The holonomy interpretation of electromagnetism takes as its basic description of the electromagnetic state of the world an assignment of holonomy values to loops in spacetime. The electromagnetic state of a region $R$ of spacetime is an assignment of holonomy values to loops in $R$. These values will not be independent of each other. Suppose loops $l_1$ and $l_2$ are the images of closed curves $C_1$, $C_2$, with common base point. It may happen that there is a curve $C_3$, thinly equivalent to $C_2 \circ C_1$, whose oriented image is a loop, which we will denote $l_1 \oplus l_2$. Because a holonomy must respect the group structure of the group of hoops, we must have, for any holonomy $H$,

$$H(l_1 \oplus l_2) = H(l_2)H(l_1).$$

Call this the *loop composition condition*.

Though doing so is a bit more complicated than in the traditional representation, it is possible to write the classical action for electromagnetism in terms of loop-dependent quantities (see Ugon et al. 1994, and Gambini and Pullin 1996, §4.6). It is also possible to write Maxwell's equations in terms of loop-dependent quantities. These preserve the expected relations of dependence. Let $\Sigma_2$, $\Sigma_2$ are two maximal spacelike hypersurfaces in Minkowski spacetime, with $\Sigma_2$ to the future of $\Sigma_1$. Let $R_2$ be some subset of $\Sigma_2$, and let $R_1$ be the past domain of dependence of $R_2$ in $\Sigma_1$. Then the holonomy-state of $R_2$ is determined by the holonomy-state of $R_1$. Thus, as Healey (2007, 127) notes, there is no threat to relativistic locality from the holonomy interpretation.

We can also construct a holonomy interpretation of a classical non-abelian Yang-Mills theory. The holonomy is now a homomorphism from the group of hoops into a non-abelian Lie group. These holonomies will not, in the non-abelian case, be independent of the base point $o$, and will not be gauge-invariant. What *will* be gauge-invariant and independent of base point will be the Wilson loops.

$$W_A(\gamma) = \mathrm{Tr}\left[\mathcal{P}\exp\left(i\oint_C A_\mu dx^\mu\right)\right],$$

where $\mathcal{P}$ denotes the path-ordered product. It can be shown (see Gambini and Pullin (1996, §3.4.2)) that all the gauge-invariant content of the theory can be reconstructed from the these Wilson loops.

# 5 Holonomy and nonseparability: the classical electromagnetic field

Suppose we represent a classical electromagnetic field in this way: as a smooth assignment of elements of $U(1)$ to loops in spacetime. Any particular holonomy $H$ will represent a state of the electromagnetic field, and the restriction of a state to the set of observables associated with a spacetime region $R$ will be given by $\{H(l)\}$, where $l$ ranges over loops in $R$.

This immediately eliminates any version of separability that requires physical states to supervene on properties assigned to spacetime points. Call this *Pointilliste Separability*.[8]

> *Pointilliste Separability.* All physical processes occupying a region $R$ of space-time supervene on an assignment of qualitative intrinsic physical properties at spacetime points in $R$.

But perhaps we have reasons to reject *pointillisme*, independent of the issues we're discussing here.[9] On the traditional view, electromagnetic fields are specified by specifying fields $\mathbf{B}$, $\mathbf{E}$, or, in covariant form, by specifying the electromagnetic field tensor field $F_{\mu\nu}$. At first glance, we have here a perfect instance of *pointilliste* separability: the electromagnetic tensor field is an assignment of a tensor to each point of spacetime. The question is whether these can be regarded as properties *intrinsic* to the spacetime points. Jeremy Butterfield (2006) has argued

[8]The negation of this is called Non-separability in Healey (1991, 1994, 2007).

[9]*Pointillisme* is defined by Butterfield as "the doctrine that a physical theory's fundamental quantities are defined at points of space or of spacetime, and represent intrinsic properties of such points or point-sized objects located there" (Butterfield, 2006, 709). He credits Lewis with appropriating the art movement's name for this doctrine.

that instantaneous velocities of a classical point particle are to be regarded as *extrinsic* but *local* to the spacetime points at which they obtain. Extrinsic because possession of a velocity at an instant has implications for matters of fact at other instants, and *local* because the velocity of a particle at time $t$ is determined by the particle's trajectory in any neighborhood of $t$, no matter how small.

A similar argument can be given for the vectors assigned to points by an electromagnetic field, or, for that matter, by any field. Let us note, first of all, that a vector (or tensor) field is usually taken to be a *smooth* assignment of a vector (or tensor) to each point of the spacetime manifold. The requirement of smoothness means that assignment of a field value to a point $x$ has implications for assignments at points other than $x$. Field values are extrinsic but local to points. This meshes well with the physical interpretation of the field. The electromagnetic field at the space time point determines the acceleration of a test charge at that point, as a function of its velocity; anti-pointillisme about velocities and accelerations, therefore, would seem to dictate anti-pointillisme about electromagnetic fields.

Let us, therefore, take the electromagnetic field value assigned to a point to be extrinsic but local to that point. Note that this also meshes well with the quantum treatment of fields; in standard quantum field theory, quantum fields do not associate operators with spacetime points, but rather *operator-valued distributions*, which yield operators (which represent observables) when smeared with suitable test-functions.

The vector assigned to a point by a vector field is extrinsic but local to a point. But what of extended regions? Suppose $R$ is a subset of some manifold $M$, to each point $p$ of which is assigned a vector $V_p$ by a vector field $V$. Each of these assignments is local to the point $p$, meaning that, for any neighbourhood $n_p$ of $p$, no matter how small, the vector assigned to $p$ carries no implications for vectors assigned to points outside of $n_p$. Does it follow that the totality of such assignments is local to $R$?

Oddly enough, no. If $M$ is ordinary 3-space, then, for some regions $R$, an assignment of vectors to all points in $R$ carries implications for assignments outside of $R$, and these implications are not confined to an arbitrarily close neighbourhood of $R$.[10] This is because Stoke's theorem applies, which says that, for any smooth vector field $\mathbf{F}$, any closed curve $C$, and any capping surface $S$ of $C$,

$$\oint \mathbf{F} \cdot d\mathbf{l} = \int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$$

In words: the line integral of $\mathbf{F}$ around the curve $C$ is equal to the surface integral over a surface bounded by $C$ of the component of the curl of $\mathbf{F}$ normal to the

[10] I take a neighborhood of a region $R$ to be any open set that contains $R$.

surface. An assignment of field values to points in the image of $C$ that yields a nonzero line integral around $C$ cannot be extended smoothly to a field that is curl-free everywhere on $S$.

If we abandon *pointillisme*, what suggests itself first, as a non-pointilliste version of separability, is

> *Naive Patchy Separability.* For any spacetime region $R$, and any open covering $\{n_\alpha\}$ of $R$, all physical processes occupying a region $R$ of space-time supervene on an assignment of qualitative intrinsic physical properties to elements of $\{n_\alpha\}$.

This condition is too strong, as it is not satisfied by vector fields, the paradigm case of separability. This is because some of the open sets in the covering may be ring-shaped, and a vector field's assignment of vectors to all points in such a set is an extrinsic property of such a set.

To better capture the intuition behind a non-pointilliste version of separability, we restrict the patches by requiring the covering to consist of open balls. For any point $x$, the *open ball* of radius $\epsilon$ centered at $x$ is the set of points of distance less than $\epsilon$ from $x$.

> *Patchy Separability.* For any spacetime region $R$, all physical processes occupying a region $R$ of space-time supervene on an assignment of qualitative intrinsic physical properties to elements of any covering consisting of open balls centred at points in $R$.

This is a version of Healey's *Weak Separability* (Healey, 2007, 46), which is equivalent to Belot's *Synchronic locality* (Belot, 1998, 540). As Belot (1998, 544) and Healey (2007, 125) point out, this also fails, on the holonomy view. Consider the set-up of the Aharonov-Bohm effect, and let $R$ be a ring that circles the solenoid. If we cover this ring with small patches, the intrinsic properties assigned to those patches will be holonomies on loops within the patches. If we choose our cover so that none of the patches contain loops that circle the solenoid, the holonomy assigned to any loop contained in a patch will be 1. This fact, that all loops in elements of our covering are assigned the value 1, leaves undetermined the value assigned to a loop in $R$ that circles the solenoid.

To tie this in with the considerations raised in §3, we can associate a $C^*-$algebra with a spacetime region $R$ region by forming a suitable set of bounded functions of the holonomy values in $R$; since holonomy values are just complex numbers, this will be an abelian algebra. A holonomy-state of $R$ will assign holonomy-values to all loops in $R$, which will determine values for all functions in the algebra we associate with $R$. Take regions $A$, $B$, such that $R = A \cup B$, and consider a holonomy-state of $R$. This will not be uniquely determined by specification of values of the holonomy on all loops in $A$ and all loops in $B$. Thus, there might seem to be a tension here with what was said in §3, about absence of nonseparability in the state of a composite system whose observable-algebra is abelian. A crucial assumption there, however, was that, if algebras $\mathcal{A}$, $\mathcal{B}$ are associated with systems $A$, $B$, the algebra associated with the composite of $A$ and $B$ is $\mathcal{A} \vee \mathcal{B}$, the smallest algebra containing both $\mathcal{A}$ and $\mathcal{B}$.[11] Since there will be loops associated with a composite system that are not composed of loops belonging to the parts, this assumption fails for the holonomy interpretation of electromagnetism. Classical electromagnetism on the holonomy interpretation achieves nonseparability, not via entanglement, but because there are observables associated with a composite system whose values are underdetermined even by an assignment of dispersion-free states to the component systems.

Patchy Separability fails because it contains a quantification over all spacetime regions $R$. However, if we consider a simply connected region $R$, then it *is* true that, for any open covering $\{n_\alpha\}$ of $R$, the value of a holonomy $H$ will be determined by its values on loops contained in elements of $\{n_\alpha\}$. The idea is this. Given a loop $\sigma$, take two points $x, y$ on $\sigma$, and join them by a path $g$. Let $c_1$ be a loop that starts at $x$, traverses $\sigma$ until it reaches $y$, and then takes $g^{-1}$ back to $x$. Let $c_2$ be a loop that starts at $x$, traverses $g$ to $y$, and then traverses $\sigma$ back to $x$. Let $\sigma_1$, $\sigma_2$ be the segments of $\sigma$ traversed by $c_1$, $c_2$, respectively. Then we have

$$\begin{aligned} c_1 &= g^{-1} \circ \sigma_1 \\ c_2 &= \sigma_2 \circ g \\ \sigma &= \sigma_2 \circ \sigma_1. \end{aligned}$$

Then, for any holonomy $H$

$$H(c_2 \circ c_1) = H(\sigma_2 \circ g \circ g^{-1} \circ \sigma_1) = H(\sigma_2 \circ \sigma_1) = H(\sigma),$$

and so the value of $H$ on $\sigma$ is determined by its values on $c_1$ and $c_2$.

$$H(\sigma) = H(c_1 \circ c_2) = H(c_1)H(c_2).$$

We can further subdivide $c_1$ and $c_2$, and so on, and, as long as $R$ is simply connected, we will be able to continue this process until each of the component loops lies within an element of our open cover $\{n_\alpha\}$.

Thus, the following condition, weaker than *Patchy Separability*, will be satisfied.

[11] This assumption is introduced without comment by Clifton et al. (2003). It is interesting to find a setting in which it fails in a natural way.

> *Patchy Separability for Simply-Connected Regions.* For any simply-connected spacetime region $R$, all physical processes occupying a region $R$ of space-time supervene on an assignment of qualitative intrinsic physical properties to elements of any covering consisting of open balls centred at points in $R$.

In particular the state of the whole world (that is, either the state of $M$, or the state of some maximal spacelike surface in $M$, depending on whether one is in a 4-dimensionalist or 3-dimensionalist mood) will supervene on assignments of states to arbitrarily small regions, if spacetime (or, for the 3-dimensionalist, space) is simply connected. Separability fails only when we consider subregions of the world, and ones with special properties, at that. We have, when $M$ is simply connected,

> *Global Patchy Separability.* All physical processes supervene on an assignment of qualitative intrinsic physical properties to elements of any covering of $M$ consisting of open balls centred at points in $M$.

Let us consider again spatial separability.

> *Spatial Separability.* The qualitative intrinsic physical properties of a compound system are supervenient on those of its spatially separated component systems together with the spatial relations among these component systems.

If $A$ and $B$ are spatially separated regions, then there are no loops in $A \cup B$ that are not in either $A$ or $B$. It follows that a holonomy assigning elements of $U(1)$ to loops in $A \cup B$ supervenes on assignments to loops in $A$ and $B$. Classical electromagnetism, on the holonomy view, satisfies spatial separability.

If we ask what the holonomy view requires a conservative classical metaphysician (ccm) to abandon, then certainly, *pointillisme* is something that has to go. But *pointillisme* should be rejected even on the traditional field interpretation of electromagnetism. Patchy Separability, with its quantification over all spacetime regions whatsoever, though satisfied by the field interpretation, is violated by the holonomy interpretation of electromagnetism.[12] But a ccm who rejects *pointillisme* will be able to retain Spatial Separability and, provided her spacetime is simply connected, Global Patchy Separability. Thus, there is a marked difference between the form of non-separability exhibited by classical electromagnetism on the holonomy interpretation, and quantum non-separability, as the latter requires rejection of both Spatial Separability and Global Patchy Separability.

[12]Unless the ccm is willing to make a move to a claim that subsets of spacetime that are not simply connected don't count as *bona fide* regions.

# 6 On Humean Supervenience

In the introduction to Volume II of his *Philosophical Papers*, Lewis formulates the thesis of Humean Supervenience (HS).

> Humean supervenience ... is the doctrine that all there is to the world is a vast mosaic of local matters of particular fact, just one little thing and then another ... We have geometry: a system of external relations of spatiotemporal points. Maybe points of spacetime itself, maybe point-sized bits of matter or aether or fields, maybe both. And at those points we have local quantities: perfectly natural intrinsic properties which need nothing bigger than a point at which to be instantiated (Lewis 1986, ix–x).

Later, in "Humean Supervenience Debugged," we have,

> Humean Supervenience ... says that in a world like ours, the fundamental relations are exactly the spacetime relations: distance relations, both spacelike and timelike, and perhaps also occupancy relations between point-sized things and spacetime points. And it says that in a world like ours, the fundamental properties are local quantities: perfectly natural intrinsic properties of points, or of point-sized occupants of points. Therefore it says that all else supervenes on the spatiotemporal arrangement of local quantities throughout all of history, past and present and future.
>
> The picture is inspired by classical physics (Lewis 1994, 474).

The primary motivation behind Humean Supervenience is the idea that laws of nature are compact summaries of patterns in non-nomic facts. Lewis' statement of HS commits him to *pointillisme*, but it is not clear that this is essential to the core idea. A Lewisian metaphysician might be satisfied with properties that are extrinsic but local: note that in the first quotation Lewis first refers to "local matters of particular fact," and then proceeds to put a *pointilliste* slant on this, and in the second quotation "local quantities" is glossed as assignments of intrinsic properties to points. It is not clear that Lewis is distinguishing between local matters of particular fact and matters of particular fact intrinsic to points. One could, perhaps, without doing too much violence to Lewis' intentions, advance a version of Humean supervenience that has all else supervene on *local* matters of fact, construed as assignments of intrinsic properties to arbitrarily small neighborhoods of points.[13] This version of Humean Supervenience would be committed only to some version of Patchy Separability.

Some version, but which? Patchy Separability, which requires the state of *any* part of the world to supervene of state-assignments to coverings by arbitrarily small patches, or Global Patchy Separability, which only requires this to be true of the whole world? The language "all there is to the world" in the first quotation suggests the global version. If we take it this way, and replace talk of properties intrinsic to points with talk of local properties, then, in spite of the nonseparability evinced by classical electromagnetism in the holonomy representation, this nonseparability is compatible with Humean supervenience—as long as spacetime is simply connected. That it matters which of these two versions of separability we consider is perhaps surprising. There is a temptation to think that, if the state of the whole world supervenes on assignments of intrinsic properties to arbitrarily small patches of it, the same must be true of any subregion of the world. One lesson from the holonomy interpretation of electromagnetism is that this need not be the case. Global Patchy Separability does not entail Patchy Separability.

It seems that Global Patchy Separability ought to be enough to satisfy the intuitions that underwrite the thesis of Humean supervenience. If Global Patchy Separability is satisfied, then the state of the world can be specified by an assignment of local properties to arbitrarily small patches, and laws can be taken to be patterns in such assignments. Thus, it seems that Healey overstates matters when he says that *any* kind of non-separability violates Humean supervenience (Healey, 2007, 124).[14]

The underlying idea of Humean Supervenience is that laws are nothing more than compact summaries of patterns in the Humean mosaic. A Humean could think of a holonomy on a spatial hypersurface, assigning holonomy values to loops within that hypersuface, as a specification of the Humean mosaic at an instant of time, and the law of evolution by which holonomies on one spacelike hypersurface determine holonomies on other hypersurfaces as laws of nature. However, as

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[13]It is worth noting in connection with this that Lewis intends a vector field to be an arrangement of local quantities (Lewis, 1994, 474) . And so it is, but, as we have discussed, the vector assigned to a point by a vector field should be taken to be a local but extrinsic property of that point.

[14]One might also consider a version of Humean Supervenience (or, rather, a thesis like Humean Supervenience) that captures the basic intuition that laws are no more than patterns in non-nomic facts, without Lewis' commitment to separability, of any sort. This has been suggested by Chris Smeenk (2009) in a review of Healey's book, citing Earman and Roberts (2005) for the idea.

already mentioned, the holonomy-values assigned to spatial loops, even within a single hypersurface of simultaneity, are not independent of each other, as they must satisfy the loop composition condition,

$$H(l_1 \oplus l_2) = H(l_1)H(l_2).$$

Are these dependencies also to be regarded as laws of nature? If so, then these laws of nature dictate relations *within* a spacelike slice of spacetime, and cannot be regarded as laws of evolution, specifying a relation between the state of a part of a spacetime and the state of its causal past. The sorts of dependencies encoded in the loop composition condition are not of the familiar sort that we expect to be expressed in physical laws. Is there an alternative?

A consequence of the loop composition condition is what Healey (2007, 122) calls *loop supervenience*, the requirement that the holonomy properties of any loop be determined by the holonomy properties of the loops that compose it, on any decomposition. One way of thinking about this is to regard loop composition as something analogous to fusion of parts into a whole; loop supervenience is, on this way of thinking, analogous to the requirement that properties of a whole be determined by the properties of its parts, on any decomposition.

One might be tempted to say that the relation of a composite loop to its components just *is* an instance of the part-whole relation (this is suggested by Healey (2007, 125)). On this view, $l_1$ is to be thought of as a proper part of $l_3$ if there exists a loop $l_2$ such that $l_3 = l_1 \oplus l_2$. If we accept this mereology, then loop supervenience is simply the natural requirement that the properties of a whole be determined by the properties of its parts.

Things are not quite as simple as this way of thinking about it may make it appear. That is because the loop composition relation does not respect the rules that mereology would lead us to expect it to. We would expect that if $l_1$ is a part of $l_3$, $l_3$ is a part of $l_1$ only if $l_1 = l_3$, and that, if $l_1$ is a proper part of $l_3$, then $l_3$ is not a proper part of $l_1$. These conditions do not hold for loop composition; if

$$l_3 = l_1 \oplus l_2,$$

then

$$l_1 = l_3 \oplus \bar{l}_2,$$

where $\bar{l}_2$ is $l_2$ with orientation reversed. Construing loop composition as a part-whole relation would require us to say that, whenever a loop $l_1$ is a proper part of a loop $l_3$, it is *always* the case that $l_3$ is a proper part of $l_1$.

Accepting this would do too much violence to our mereology. The Humean would be better off to avoid this mereological havoc, and take loop composition as *analogous* to fusion, with the crucial difference being that for every loop there is a loop of the opposite sense, such that composition of a loop and its inverse cancel each other out as far as holonomies are concerned. It still seems a viable option to regard the loop composition condition as no more metaphysically suspect than the requirement that, say, the mass of an object be the sum of the masses of the parts that compose it.

There remains another complication.[15] Note that, in the passage quoted from "Humean Supervenience Debugged," Humean supervenience is a thesis about "worlds like ours," not just about our world.[16] If we take this to be integral to the thesis, then, if Humean supervenience is to hold at a world $w$, then it must also hold in some "neighbourhood" of $w$, consisting of the worlds that are like $w$. Suppose we take a classical world $w$ with a simply connected spacetime, and consider the class of worlds that are just like $w$, except that an infinite open cylinder has been removed from space, and holonomy values other than 1 are permitted for loops that circle the missing cylinder. Do these count as worlds like $w$, or not?

Lewis' "inner sphere" of possibility consists of those worlds that contain no natural properties or relations that are alien to this world (Lewis, 1986, x). A change in the topology of spacetime, therefore, does not entail a move out of the inner sphere. But not all worlds in the inner sphere are "worlds like ours" (Lewis, 1994, 474–75). The question, then, is: should a Lewisian metaphysician count worlds with distinct spacetime topologies as unlike each other? *Prima facie*, the answer would seem to be no; removal of a chunk of $w$'s spacetime produces a world that is still like $w$. If that is right, and if HS is taken to hold at a world $w$ only if it also holds in worlds like $w$, then classical electromagnetism, on the holonomy interpretation, violates Humean supervenience, whether or not spacetime is simply connected.

But perhaps a classical world with a multiply-connected spacetime should be counted as relevantly *unlike* a classical world with a simply connected spacetime. When it comes to doing physics—and, in particular, when it comes to doing electromagnetism— the difference is not a minor one. In a simply-connected space-

[15]This point was raised by Richard Healey in his remarks at the Author Meets Critics Session at the Meeting of the Pacific APA, April 9, 2009.

[16]It might sound odd to some readers—I *hope* it sounds odd to some readers—that a thesis "inspired by classical physics" might be thought to hold of worlds like ours, in spite of evidence that classical physics is not fundamental. Lewis in 1986 declared himself "not ready to take lessons in ontology from quantum physics as it now is." This doesn't affect the project of this paper, whose concern is how we should think of the classical theories that apply, in some approximation, to our world.

time, specifying the electromagnetic field tensor field $F_{\mu\nu}$ suffices to determine the holonomy associated with any loop; this is not the case for a multiply-connected spacetime. This also has consequences for the quantum theory of a charged particle; there are inequivalent quantizations corresponding to different holonomies on loops that circle the missing cylinder (see Belot (1998, §4.1) for discussion). . So perhaps a world that differs from $w$ in having a cylinder removed is not a world like $w$, after all.

# 7 Conclusion

There can be nonseparability in classical physics. This is the surprising lesson of the holonomy interpretation of classical electromagnetism. However, this nonseparability has nothing to do with entanglement, which remains a non-classical phenomenon, and is decidedly weaker than quantum nonseparability. The classical nonseparability we have found in electromagnetism is compatible with suitably restricted separability theses, such as Patchy Separability for Simply Connected Regions, and, in a simply connected world, Global Patchy Separability. Furthermore, though Lewis included a commitment to separability, and, indeed, to *Pointilliste* Separability in his statement of Humean Supervenience, it seems that the holonomy interpretation of classical electromagnetism can still respect the basic intuitions underlying HS. I conclude that, though the nonseparability exhibited by classical electromagnetism on the holonomy interpretation is something that should give the conservative classical metaphysician pause, it's nothing that the ccm should lose sleep over.

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