Universal one-way light speed from a universal light speed over closed paths

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Abstract

This paper gives two complete and elementary proofs that if the speed of light over closed paths has a universal value c, then it is possible to synchronize clocks in such a way that the one-way speed of light is c. The first proof is an elementary version of a recent proof. The second provides high precision experimental evidence that it is possible to synchronize clocks in such a way that the one-way speed of light has a universal value. We also discuss an old incomplete proof by Weyl which is important from an historical perspective.

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1 INTRODUCTION

There has been much confusion about the relationship between Einstein's definition of synchronized clocks * and his postulate of the universality of the one-way speed of light. Some authors [1, 2] assume a universal one-way speed of light before discussing synchronization. We believe that this is a logical error, as a one-way speed has no meaning until clocks are synchronized. For the one-way speed of light from a point of space A to a point B is defined as $\overline{AB}/(t_B - t_A)$, where t_A is the time of departure of a light beam from A as measured by a clock at A, and t_B is the time of its arrival at B as measured by the clock at B. For this definition to be meaningful, the two clocks need to be synchronized.

*In this paper we are interested only in the standard Einstein synchronization. We do not enter into the debate over the conventionality of synchronization [8, 9].

Other authors [3, 4] state that once clocks have been synchronized according to Einstein's definition, then the speed of light is universal, that is, independent of the point of space, of time, or of the direction followed by a light beam. However, this is not true, as the example of a Newtonian spacetime with an ether frame shows [5]. Thus the assumption of a universal one-way light speed c is actually two assumptions, which together we call **1c**:

(a) Clocks can be set so that every pair of them is Einstein synchronized.

(b) The one-way speed of light with respect to the synchronized clocks is a universal constant c.

Let us denote by **L/c** the assumption of a universal light speed c around closed paths. Note that this is a synchronization independent concept since for the measure of such an average speed only one clock is required.

Our purpose here is to discuss this theorem, expressed in the title of this paper:

Theorem. L/c ⇒ 1c.

We shall give two separate proofs of the theorem. The first is the most direct. It is a considerable simplification of a proof of one of the authors [6]. The second is based on work of the other author [5]. It provides high precision experimental evidence for **L/c** and thus, by the theorem, for **1c**. Finally, we discuss a proof given by Hermann Weyl in his book "Raum, Zeit, Materie" [7]. We point out a tacit assumption in the proof, which makes it incomplete, and then show that the assumption follows from Weyl's other assumptions.

All this provides new logical and experimental insights into the foundations of special relativity, since it justifies the fundamental assumption of a universal one-way speed of light.

2 THE PROPERTIES

We consider observers at rest with respect to each other, and assume that their space is Euclidean. Spatial points are denoted with letters $A, B, C \ldots$. Next, we assume that light propagates on straight lines and that if a beam leaves a point A at time t_A, with respect to A's clock, directed toward B, it reaches B at a finite time t_B with respect to B's clock.

Einstein defined "clocks at A and B are synchronized" as follows. Emit a flash of light from A to B at time t_A. Let it arrive at B at time t_B. Similarly, let a flash emitted from B at time t'_B arrive at A at time t'_A. Say the clocks are *synchronized* if

$$t_B - t_A = t'_A - t'_B \tag{1}$$

for all times t_A and t'_B. If clocks can be set so that this equation holds for any pair of them, then we say that Einstein synchronization can be applied consistently. When this is the case, we can define a global time t: the time t of an event at P is the time of the event according to the clock at P. We shall call this *Einstein time.*

Notice that with respect to Einstein time the one-way speed of light between two points is equal to the two-way speed between the points. Indeed Eq. (1) states that the time needed by light to go in direction AB is the same as that needed to go in the opposite direction BA.

In what follows we shall relate a number of properties which we list here, adding mnemonics to the left.

z=0. Emit flashes of light from B at times t_{1B} and t_{2B} according to a clock at B. Let them arrive at A at times t_{1A} and t_{2A} according to a clock at A. Then

$$t_{2A} - t_{1A} = t_{2B} - t_{1B}. \tag{2}$$

This property is another way of saying that there is no redshift.

△. The time it takes light to traverse a triangle (through reflections over suitable mirrors) is independent of the direction taken around the triangle.

2c. The two-way speed of light has a constant value c.

L/c. The time it takes light to traverse a closed polygonal path (through reflections over suitable mirrors) of length L is L/c, where c is a constant.

syn. Einstein synchronization can be applied consistently.

1c. Einstein synchronization can be applied consistently and the one-way speed of light with respect to the synchronized clocks has a constant value c.

A constant c appears in the definitions of **2c**, **L/c**, and **1c**. As we shall see in the proofs below, the implications among these properties refer to the same value of c. This will justify our notation.

Notice that the first four properties do not depend on clock synchronization.

Figure 1 summarizes the implications we will establish between these properties.

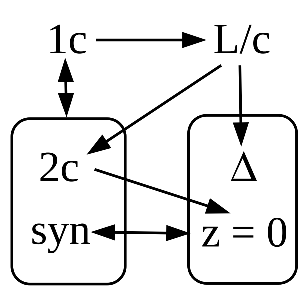

Figure 1: Implications proved in the text.

We can immediately establish several trivial implications:

1c ⇒ **L/c**. This requires no comment.

$\mathbf{L/c} \Rightarrow \mathbf{2c}$. A light beam that goes from A to B and back traverses a closed path. According to $\mathbf{L/c}$, the two way speed is c.

$\mathbf{L/c} \Rightarrow \triangle$. According to $\mathbf{L/c}$, the time to traverse a triangle is L/c, which is independent of the direction taken around the triangle.

$\{\mathbf{syn}$ and $\mathbf{2c}\} \Leftrightarrow \mathbf{1c}$. We have noted that if **syn** holds, then the two-way speed is equal to the one-way speed. Since from **2c** the two-way speed is c, the one-way speed is c. The equivalence now follows easily.

The only implications remaining are $\mathbf{2c} \Rightarrow (\mathbf{z} = \mathbf{0})$ and $\mathbf{syn} \Leftrightarrow \{(\mathbf{z} = \mathbf{0})$ and $\triangle\}$. They will be established as part of the proof of $\mathbf{L/c} \Rightarrow \mathbf{1c}$ in Section 4.

3 THE MOST DIRECT PROOF

A proof of the theorem $\mathbf{L/c} \Rightarrow \mathbf{1c}$ was given by one of the authors in [6]. The elementary nature of the proof was however hidden by an infinitesimal approach. The strategy was to assign a label, an "Einstein time" to any event and, in the end, to synchronize clocks. We give here an elementary version of that proof.

Consider a point O and a clock at rest at O. We define a time $t(e)$ of a generic event e. Emit a light beam from O to e, where it is reflected back to O. † Let the departure and arrival times of the light beam be $t^i(e)$ and $t^f(e)$ according to the clock at O. Define

$$t(e) = \frac{t^i(e) + t^f(e)}{2}. \tag{3}$$

This procedure assigns a time $t(e)$ to every event e. The time t at O is measured by the clock at O. The procedure also defines times $t^i(e)$ and $t^f(e)$ for every event e. From $\mathbf{L/c}$, if e is at point E, then $t^f(e) - t^i(e) = 2\overline{OE}/c$.

Let us prove that the one way speed of light with respect to t is c. Emit a light beam from point A at event a. Let it arrive at point B at event b. We can imagine that the beam starts at O, arrives at a, is reflected to b, and is reflected back to O. From $\mathbf{L/c}$ we have

$$t^f(b) - t^i(a) = \overline{OABO}/c\,, \tag{4}$$
$$t^f(a) - t^i(a) = 2\overline{OA}/c\,, \tag{5}$$
$$t^f(b) - t^i(b) = 2\overline{OB}/c\,. \tag{6}$$

Subtracting the second and third equations from twice the first gives

$$t(b) - t(a) = \overline{AB}/c\,; \tag{7}$$

the one way speed of light with respect to t is c.

Now consider another "global time" $\tilde{t}$ established by a clock at point $\tilde{O}$. Since light moves at speed c with respect to both t and $\tilde{t}$, one has that if a and b are events on the world line of a light beam, then

$$t(b) - \tilde{t}(b) = t(a) - \tilde{t}(a). \tag{8}$$

Since light moves at constant speed with respect to t, given any two events whatsoever e and f, there is a third event g in the intersection of their light cones. Therefore

$$t(e) - \tilde{t}(e) = t(g) - \tilde{t}(g) = t(f) - \tilde{t}(f). \tag{9}$$

Now if we fix f and let e vary, then we see that $t(e) = \tilde{t}(e) + const$. Thus we can take $\tilde{t} = t$ by resetting the clock at $\tilde{O}$ which defines $\tilde{t}$. The definition of global time is therefore independent of the initial point O chosen.

† We show that given two points A and B, and a time t_{1B}, a light beam can be sent from A that reaches B at t_{1B}. Define t_{0B} by $t_{1B} - t_{0B} = 2\overline{AB}/c$. Emit a light beam from B at time t_{0B} toward A. At A reflect it back to B. Then from $\mathbf{L/c}$, after the reflection at A the beam returns to B at t_{1B}.

The time $\tilde{t}$ at $\tilde{O}$ is measured by the clock at $\tilde{O}$. Since $\tilde{t} = t$ at $\tilde{O}$, the time t at $\tilde{O}$ is also measured by the clock at $\tilde{O}$.

Thus **L/c** implies that clocks can be synchronized using Einstein's method without bothering about an origin O. Moreover the one way speed of light with respect to the synchronized clocks is c.

4 A PROOF PROVIDING EXPERIMENTAL EVIDENCE FOR L/c

The proof in this section has two advantages. First, it passes through **syn**, thus making clear its relation to **L/c**. Second, it provides high precision experimental evidence for **L/c**.

We noted at the end of Section 2 that of the implications in Figure 1, only **syn** $\Leftrightarrow \{(\mathbf{z} = \mathbf{0})$ and $\Delta\}$ and **2c** $\Rightarrow (\mathbf{z} = \mathbf{0})$ remain to be established. We shall do this momentarily. Then following the arrows in Figure 1 gives our second proof of **L/c** $\Rightarrow$ **1c**.

syn $\Leftrightarrow \{(\mathbf{z} = \mathbf{0})$ and $\Delta\}$. This characterization of **syn** has been proved by one of the authors in [5].

2c $\Rightarrow (\mathbf{z} = \mathbf{0})$. This result appears to be new. Consider two points A and B and arbitrary times t_{1B} and $t_{2B} > t_{1B}$ according to a clock at B. We must show that if light beams are sent from B to A at times t_{1B} and t_{2B}, arriving at A at times t_{1A} and t_{2A} according to a clock at A, then Eq. (2) holds.

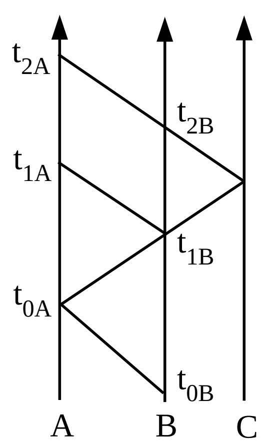

Figure 2: The reflections considered in **2c** $\Rightarrow (\mathbf{z} = \mathbf{0})$.

Refer to Figure 2. Emit a light beam from B at time t_{0B} toward A. At A reflect it back to B. Define t_{0B} by $t_{1B} - t_{0B} = 2\overline{AB}/c$. Then from **2c**, the beam will return to B at t_{1B}. At B the beam is split by a semi-transparent mirror. The reflected part of the beam arrives back at A at time t_{1A}. The transmitted part arrives at a point C, where it is reflected back to B. Choose the point C

such that $t_{2B} - t_{1B} = 2\overline{BC}/c$. Then from **2c** the beam will return to B at t_{2B}. At B the beam again encounters the semi-transparent mirror. The transmitted part arrives at A at t_{2A}. We consider the paths ABA, ACA, and BCB. Since **2c** holds:

$$ABA \quad \rightarrow \quad t_{1A} - t_{0A} = 2\overline{AB}/c, \tag{10}$$

$$ACA \quad \rightarrow \quad t_{2A} - t_{0A} = 2\overline{AC}/c, \tag{11}$$

$$BCB \quad \rightarrow \quad t_{2B} - t_{1B} = 2\overline{BC}/c. \tag{12}$$

Summing the first and third equations and subtracting the second gives Eq. (2). This completes our proof.

4.1 EXPERIMENTAL EVIDENCE

Following the implications in Figure 1, we see that

$$\mathbf{L/c} \Leftrightarrow \{\mathbf{2c} \text{ and } \triangle\} \Leftrightarrow \mathbf{1c}\,. \tag{13}$$

The implications ultimately rest upon our result $\mathbf{2c} \Rightarrow (\,\mathbf{z} = \mathbf{0})$. They show that evidence for **2c** or $\triangle$ is evidence for **L/c** and **1c**. A direct test of **L/c** requires a ruler-measurement of L for several paths. We know of no high precision test. On the other hand, tests of **2c** and $\triangle$, which we are about to discuss, are interferometric, and thus of high precision.

2c. There are well known tests of **2c**. The Michelson-Morley experiment shows that the two-way speed of light is the same in perpendicular directions from a point. This result is consistent with **2c**. But it is not conclusive: the equal-in-perpendicular-directions speed could still be different at different times, places, or speeds, violating **2c**. The Kennedy-Thorndike experiment was performed to eliminate this possibility. Together the experiments test **2c**. A modern version of both experiments with an impressive accuracy has recently been performed [10].

$\triangle$. We know of no direct test of $\triangle$. But an experiment of Macek and Davis using a ring laser in the shape of a square shows that the time it takes light to traverse the square is independent of the direction taken around it to one part in 10^{12} [11]. There is no reason that the experiment could not be performed with a ring laser in the shape of a triangle. For further discussion, see [12, 13].

The ring laser experiment support for $\triangle$ is consistent with **L/c**. But it is not conclusive: our counterexamples in the appendix show that $\triangle$ does not imply **L/c**. The equal-in-opposite-directions speed from $\triangle$ could still be different at different times, places, and speeds, violating **L/c**. Our work eliminates this possibility. It shows that $\triangle$ and **2c** together imply **L/c** to high precision.

We note that while **2c** is a consequence of **L/c**, our counterexamples show that **2c** does not imply **L/c**. Without evidence, one should be hesitant to extrapolate from **2c** to **L/c**, as light traversing paths which enclose an area behaves differently from light traversing paths which do not. An example is provided by the Michelson-Morley and Macek-Davis experiments. If the apparatus of the Michelson-Morley experiment is rotating, the interference fringes do not shift. Indeed, the null result of the experiment is just this fact. If the ring laser in the Macek-Davis experiment is rotating, then the interference fringes do shift. Indeed, the experiment was performed to test just this effect.

5 WEYL'S INCOMPLETE PROOF

The fifth edition of "Raum, Zeit, Materie" by Hermann Weyl contains, among the other modifications, a new Section 23 devoted to an analysis of the postulates of special relativity. Here Weyl gives a proof of the theorem $\mathbf{L/c} \Rightarrow \mathbf{1c}$.

To the best of our knowledge, Weyl's proof has not been published in English. (Henry Brose's English translation [14] is of the fourth edition.) We thus provide a translation:

> Section 23. Analysis of the Relativity Principle. The division of the world into space and time as projection.
>
> We want to as carefully as possible determine upon which conditions and facts the validity of Einstein's relativity is based. The Michelson experiment shows that in any uniformly moving reference frame K (which we imagine as a flat structure) the proportionality Equation (20) [our **2c**] is valid with a constant c. If A and B are two points in K, then a clock at B runs synchronized with a clock at A, whenever a light signal, which at any given time t is emitted at A, arrives at B at time $t + \overline{AB}/c$; that is the definition of "synchronized". Conversely, as a matter of fact, the clock at A is synchronized with B (or, in short, A runs in the same way as B). Moreover, we need to consider another circumstance, that if B runs as A and C as B, then C runs as A; what does that mean? Let us take two clocks set up at A; we synchronize the clock at B, by means of light signals, with the first clock at A; we synchronize the clock at C with the clock at B; and the second clock at A with the clock at C; then our claim is that the two clocks at A constantly show the same time. The second clock at A shows the time $t + L/c$ when a light signal, which is emitted from A at the time t at A of the first clock at A and has traversed the path $ABCA$ of length L, arrives back at A. Our claim thus says that the time which passes between the departure and arrival of the light signal equals L/c. It is reasonable to establish this fact not only for the "two sided triangle" ABA, and the triangle $ABCA$, but also for any polygonal path. We express it here as a first experimental fact:
>
> *If one lets the light, in a reference frame K in uniform translation, traverse a closed polygonal path of length L, then between the departure and arrival of the light signal a time τ elapses which is proportional to L: $\tau = L/c$.*
>
> Accordingly, it is possible to introduce a time $t = x_0$ anywhere in the reference frame; the clocks which show it all run synchronously. They can be synchronized by means of light signals from a central clock; this synchronizing is independent of the center that is selected. We expressly note that in these cases sources of light are always to be used which are at rest in K.

Weyl starts his analysis from the experimental evidence for **2c** and uses the constant c to give a definition of synchronization that differs from Einstein's definition. According to Weyl's definition, a clock at B is synchronized with a clock at A if a light beam that leaves A at time t with respect to A's clock reaches B at time $t + \overline{AB}/c$ with respect to B's clock. From **2c**, this definition of synchronization is symmetric. Note that with **2c** Einstein's and Weyl's definition are equivalent.

Weyl next proves the transitivity of synchronization. As his proof is perhaps not completely transparent, we offer this explication. Suppose that B's clock is synchronized with A's and C's clock is synchronized with B's. We need to prove that A's clock is synchronized with C's. Synchronize a second clock at A with C's clock. Consider a light beam that leaves A at time t_A according to A's first clock and traverses the path $ABCA$, returning to A at time $t^r_{A'}$ according to A's second clock. From the given synchronizations,

$$t^r_{A'} - t_A = L/c, \tag{14}$$

where L is the length of the path $ABCA$. Let t^r_A be the time of the return of the light beam at A according to A's first clock. Then from L/c

$$t^r_A - t_A = L/c. \tag{15}$$

Thus

$$t^r_A = t^r_{A'}, \tag{16}$$

that is, the two clocks at A are synchronized (in the end they are the same clock), and therefore A's first clock is synchronized with C's clock.

Clocks over space can be synchronized with respect to a central clock. Since synchronization is transitive, the result does not depend on the central clock chosen (up to a global resetting of clocks).

In order to be meaningful, Weyl's definition of synchronization must be independent of the time the light beam is emitted, i.e., the property $\mathbf{z} = \mathbf{0}$ must hold. Weyl apparently assumes this tacitly. We have shown above that $\mathbf{2c} \Rightarrow (\,\mathbf{z} = \mathbf{0})$, a fact of which Weyl was probably not aware.

Weyl, in the translation above, takes $\mathbf{L/c}$ as an "experimental fact". However, as noted in Section 4, we are unaware of any precision test of $\mathbf{L/c}$.

C. Møller has given a proof of **1c** from two assumptions [15]. He proves his Eq. (2.1), which is **1c**. The two assumptions are:

L/c. This is Møller's Eq. (2.3). He justifies it by appeal to Fizeau's experiment. But this experiment is a test of $\boldsymbol{\Delta}$, not **L/c**, as we argued with respect to the ring laser experiment at the end of Section 4.

$\mathbf{z} = \mathbf{0}$. This is Møller's condition 1. Unlike Weyl, Møller realizes the relevance of $\mathbf{z} = \mathbf{0}$. He justifies it with these words:

> [Condition 1] is no doubt fulfilled, since all points in an inertial system are equivalent, so that two standard clocks which have the same rate when placed together at O will also have the same rate when they are installed at different points O and P.

The phrase "have the same rate when they are installed at different points" expresses $z = 0$. His justification is spatial homogeneity: "all points in an inertial system are equivalent". Thus according to Møller, $\mathbf{z} = \mathbf{0}$ in any spatially homogeneous space. The following counterexample shows that this is wrong. Consider a Newtonian spacetime with an absolute space and an absolute time t. Suppose that the speed of light in the spacetime depends on $t:\ c = c(t)$. This spacetime is spatially homogeneous and $z \neq 0$. It thus appears to us that Møller provides no valid justification for $\mathbf{z} = \mathbf{0}$. We obtain $\mathbf{z} = \mathbf{0}$ from $\mathbf{2c} \Rightarrow (\mathbf{z} = \mathbf{0})$.

We conclude that we provide better evidence for **L/c** and a more complete proof of $\mathbf{L/c} \Rightarrow \mathbf{1c}$ than do Weyl or Møller.

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APPENDIX: COUNTEREXAMPLES

One can wonder whether there are, between the properties in Figure 1, relations that are not obtainable from the figure by simply following the arrows. The answer is negative since the counterexamples, listed below, show that $\mathbf{2c} \not\Rightarrow \boldsymbol{\Delta}$, $(\mathbf{z} = \mathbf{0}) \not\Rightarrow \boldsymbol{\Delta}$, $\mathbf{syn} \not\Rightarrow \mathbf{2c}$, and $\boldsymbol{\Delta} \not\Rightarrow (\mathbf{z} = \mathbf{0})$. It can be seen from the counterexamples and the figure that there are no other implications among individual properties.

$\mathbf{2c} \not\Rightarrow \boldsymbol{\Delta}$. Consider a Newtonian spacetime with an absolute space and time. Suppose that the velocity of light at the point $\mathbf{x}$ in the direction $\hat{\mathbf{v}}$ is

$$\frac{c\hat{\mathbf{v}}}{1 + c\hat{\mathbf{v}} \cdot \mathbf{A}(\mathbf{x})}\ , \tag{A1}$$

where $\mathbf{A}(\mathbf{x})$ is a field such that $\nabla \times \mathbf{A} \neq 0$ and $|\mathbf{A}| < 1/\,2c$. Then the time it takes light to traverse a closed path γ of length L is [6]

$$\Delta t = \frac{L}{c} + \oint_\gamma \mathbf{A} \cdot \mathrm{d}\mathbf{l}\,. \tag{A2}$$

Given $\mathbf{x}$, let γ be an equilateral triangle centered at $\mathbf{x}$ and orthogonal to $\nabla \times \mathbf{A}(\mathbf{x})$. If γ is small enough, then the integral in Eq. (A2) is not zero. Thus the round trip time Δt depends on the direction followed by the light, and hence $\triangle$ does not hold. However, **2c** holds because the integral vanishes for the path that goes from A to B and back.

$(\mathbf{z} = \mathbf{0}) \not\Rightarrow \triangle$. Consider again the setting of the last counterexample. We have already noticed that with the speed Eq. (A1), $\triangle$ does not hold. Moreover, the speed of light at a given point does not change in time. Thus the worldlines of light that goes from A to B are time translations of each other, and so $z = 0$.

syn $\not\Rightarrow$ **2c**. Notice that the definition of **syn** does not involve any metric over space. Start from Minkowski spacetime. Taking unaltered its light cone structure in the coordinates t, x, y, z redefine the the space metric to be $\mathrm{d}l^2 = k^2 \mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2$. You have obtained a space where **syn** still holds but the two-way speed of light is anisotropic since that in the x direction is kc whereas that in the y direction is c.

Another counterexample [5] comes from the old ether theory of the propagation of light. In such a theory the light propagates at a constant speed with respect to the ether. Inertial observers in motion with respect to the ether can apply consistently the Einstein synchronization method but the one-way speed turns out to be anisotropic since the two-way speed itself, in those frames, is anisotropic.

$\triangle \not\Rightarrow (\mathbf{z} = \mathbf{0})$. Consider Minkowski spacetime with the usual coordinates $\{x^i, t\}$. Suppose, however, that the clocks at rest do not measure t but $t' = t(1 + r^2/a^2)$ where r is the radial distance from the origin, and a is a constant. In this space $\triangle$ still holds since it involves only one clock. On the contrary, $z \neq 0$.

References

[1] R. Resnik, *Introduction To Special Relativity* (John Wiley & Sons, New York, 1968).

[2] G. F. R. Ellis and R. M. Williams, *Flat and curved space-times* (Oxford University Press, Oxford, 2000), 2nd ed.

[3] A. P. French, *Special Relativity* (Chapman and Hall, London, 1968).

[4] R. W. Brehme, Am. J. Phys. **56**, 811 (1988).

[5] A. L. Macdonald, Am. J. Phys. **51**, 795 (1983).

[6] E. Minguzzi, Found. Phys. Lett. **15**, 153 (2002).

[7] H. Weyl, *Raum, Zeit, Materie* (Springer-Verlag, New York, 1988), seventh edition based on the fifth German edition (1923).

[8] R. Anderson, I. Vetharaniam, and G. E. Stedman, Phys. Rep. **295**, 93 (1998).

[9] A. Janis, *Conventionality of Simultaneity*, The Stanford Encyclopedia of Philosophy, (Fall 2002 Edition) Edward N. Zalta (ed.), http://plato.stanford.edu/archives/fall2002/entries/spacetime-convensimul/.

[10] P. Wolf, S. Bize, A. Clairon, A. N. Luiten, G. Santarelli, and M. E. Tobar, Phys. Rev. Lett. **90**, 060402 (2003).

[11] W. M. Macek and D. T. M. Davis, Jr., Appl. Phys. Lett. **2**, 67 (1963).

[12] H. Erlichson, Am. J. Phys. **41**, 1298 (1973).

[13] G. E. Stedman, Am. J. Phys. **41**, 1300 (1973).

[14] H. Weyl, *Space Time Matter* (Dover Publications, Inc., New York, 1952), based on the fourth German edition (1921) translation by H. L. Brose.

[15] C. Møller, *The Theory of Relativity,* (Clarendon Press, Oxford, 1972), 2nd ed, Sec. 2.2.