Relational Spacetime Ontology

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Abstract

It is argued that, contrary to the widespread impression, a relational account of spacetime is plausible.

1 Introduction

The ontology of physics is a difficult soil to seed. Leibniz and Newton planted the arena of discussion. They were concerned in the debate over the ontological status of space and time. In the post-relativistic era the debate is still persuasive, facing relationalists against substantivalists in this issue. Einstein shifted the battlefield from the ontology of space (plus time) to the ontology of spacetime. It is worth to mention that Einstein’s original intention was to cast GR’s spacetime as a relational entity à la Leibniz-Mach. However, despite early relational interpretations of GR -such as Reichenbach’s (1928)- most philosophers of science feel comfortable with the now standard sophisticated substantivalist (SS) account of spacetime (Mundy:1992, Brighouse:1994, Di Salle:1994, Hoefer:1996, Pooley: 2002). Furthermore, most philosophers share the impression that although relational accounts of certain highly restricted models of GR are viable, at a deep down level, they require substantival spacetime structures. I have reasons to disagree, but before bringing them out into the light I will briefly recall some history in order to describe what sophisticated substantivalism is about and why it is troublesome.

Back in the 70s the increased interest in relativistic cosmology in addition to the expanding scientific realism made people confident in the literal reality of theoretical GR structures. Now, a model of GR, broadly speaking, consists of a manifold and some (matter and metric) fields spread over the points (events) of the spacetime manifold.

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Consequently, within the realm of scientific realism it seemed natural to understand the point manifold as an independent background substantival spacetime containing physical fields. The manifold had become the successor of Newton’s absolute space container (Earman 1970, Stein 1970, Friedman 1983). Furthermore, it is a basic structure of our best theories and the corresponding field equations suggest that physical fields should be considered as integral properties of the points of the spacetime manifold.

However, this conviction in the independent existence of spacetime in the form of the manifold was broken in the late 80s. Spacetime philosophers went on to reconsider manifold substantivalism when they were forced to struggle against the interpretative complications brought out by the hole argument. The hole argument was the rediscovery -in differential geometry language- by Earman and Norton (1987) of Einstein’s own hole argument dating back from 1913. At the time, Einstein thought that general covariance -the key to the hole- failed to conform consistently to the ‘law of causality’. Likewise, in its modern version, the hole argument was intended to put forward an interpretative obstacle: If interpreted along realistic lines GR should be accounted as a nondeterministic theory. This proposition is, evidently, contrary to the usual understanding of the theory inherited from its current scientific practice. As a result different responses to the hole argument have populated the literature. Some of them opted to reject the type of indeterminism within the hole construction since it seemed physically irrelevant, others went on to modify spacetime substantivalism. This is the case of spacetime sophisticated substantivalism (SS). This latter substantivalist doctrine (SS) is usually presented as the natural escape from the hole argument. In Hoefer’s words, as ‘the best way of cashing out substantivalism’ (1996,p.6). Since the hole argument is well known nowadays I will offer a brief overview.

2 The Hole Argument

The possibility of the hole construction comes from the active general covariance of GR. By definition, this means that if any tensor field $X$ on the manifold $M$ is a solution of the corresponding field equations, then the so called pushed-forward tensor $\phi \ast X$ of $X$ is also a solution of the same field equations for any active diffeomorphism $\phi$. As stated previously a model of GR -taken usually to represent a possible world- consists on a differential manifold $(M)$, a metric tensor field $(g)$ defined over the whole manifold, and a stress-energy tensor field $(T)$ representing the material contents of spacetime. Two models of GR, $U = \langle M, g, T \rangle (M, g, T)$ and $U^* = \langle M, \phi \ast g, \phi \ast T \rangle$, related by a diffeomorphism $\phi$ share the same background point manifold, but the action of the diffeomorphism pushes the fields $(g, T)$ onto different manifold points. To build the hole argument, it is convenient to choose a spacetime $M$ admitting a $(3+1)$ foliation consisting of 3-spacelike slices labelled by a time parameter $t$. After doing so, one can pick the diffeomorphism to be the identity up to $t$ (say $t = 0$) and differ thereafter. As a result, one has two models of GR identical up $t = 0$ but smoothly different afterwards.
This should be taken as a failure of determinism within GR since the field equations plus the past cannot single out a unique evolution of fields within the manifold. However one should be careful when judging this type of determinism breakdown. The standpoint is that the hole argument becomes effective if one grants primitive identity to the manifold points; if one is willing to be a manifold substantivalist.

In this case each diffeomorphic model would represent an ontologically distinct state of affairs. Things would be, loosely speaking, somewhere else after \( t = 0 \). The problem comes when one notices that the field equations plus the past cannot pick between \( U = \langle M, g, T \rangle \) or \( U^* = \langle M, \phi \ast g, \phi \ast T \rangle \) to determine which of the two (or more) models represent the factual history. All diffeomorphic models of GR are empirically equivalent; they share the same background manifold but the field equations (plus the past) cannot tell, for example, if a star \( W \) collapses at point \( p \) of \( M \), or at point \( r \) of \( M \). The hole construction clearly resembles the Leibniz shift argument against Newtonian substantival space, where, by means of the Principle of Identity of Indiscernibles (PII), Leibniz was able to conclude that a hypothetically shifted world and a non-shifted world represented the same state of affairs since they were observably (dynamically) identical given the invisibility of absolute space and the principle of relativity. The argument involved and implicit rejection of primitive identity of space points, otherwise the shifted and the non-shifted world would represent different states of affairs. In the language and context of GR, PII is known as Leibniz Equivalence (LE).

LE can be stated as follows: two field distributions related by an active diffeomorphism represent the same physical situation, this is, \( U = \langle M, g, T \rangle \) and \( U^* = \langle M, \phi \ast g, \phi \ast T \rangle \) are equivalent for any \( \phi \in \text{Diff}(M) \). Where \( \text{Diff}(M) \) is the set of all possible diffeomorphisms defined on the same manifold \( M \).

There have been several reactions to the hole argument, but the general conclusion most philosophers have shared is that, as stated by Earman and Norton, ‘determinism is worth a fighting chance’. Consequently, most philosophers endorse LE, thereby they reject primitive identity of spacetime manifold points. I agree.

3 Points unidentified: ¿Relational or Substantival Spacetime?

The hole argument is constructed under the strong belief that substantivalism is a realistic doctrine about the independent existence of individual manifold spacetime points. This way of presenting substantivalism makes it sound as if relationalism consisted primarily in the denial of the statement that ‘spacetime points enjoy this robust sort of existence’ (Belot and Earman 2001, p. 227). I have to disagree. Recall the situation in Newtonian dynamics. Despite the impossibility to single out space points, almost

\(^1\)In Einstein’s original version the diverging manifold diffeomorphism was not necessarily in the future, it was placed anywhere within the manifold; hence the hole terminology.
everyone granted physical reality to Newtonian absolute space. Why was it so? The real issue at stake was that Newtonian space encompassed important spatiotemporal structures that played an important explanatory role in dynamics. Mach (1883), for example, suggested that an alternative relational dynamics would have to, somehow, put together a novel inertial structure depending explicitly on material sources. It was clear that the reality of space was linked to the explanatory power of its inertial structure and not to the robust existence of its individual parts (points). For that reason, I find more accurate and healthier for the current controversy to consider substantivalism as realistic doctrine about the independent existence of spatiotemporal structures (not points). I quote Hoefer’s definition along this line of thought (Hoefer 1996, p. 5):

A modern-day substantivalist thinks that spacetime is a kind of thing which can, in consistency with the laws of nature, exist independently of material things (ordinary matter, light, and so on) and which is properly described as having its own properties, over and above the properties of any material things that may occupy parts of it.

Hoefer is speaking as a sophisticated substantivalist. Evidently, I find his general definition convincing. Nevertheless, I will shortly show my discomfort as to the classification of ‘material things’. As stated earlier, SS is advertised as the best escape from the hole argument. Thus, a Sophisticated Substantivalist subscribes LE, is someone who is realist about the independent existence of spacetime but, contrary to the manifold substantivalist, she does not grant the bare manifold the ontological status of a complete independent spacetime. One of the compelling reasons to do so, aside from the hole argument, is the fact that the bare manifold can work as a collection of points with certain differential and topological structures, but lacks practically of every paradigmatic property required to properly understand it as an independent spacetime in its own right.

Of course, to look for the paradigmatic properties of an independent spacetime it is convenient to recall -once more- Newtonian dynamics. Newtonian absolute space (or, alternatively, neo-Newtonian spacetime) is widely considered as an independent entity in its own right mainly because it had all the structures that made theoretically intelligible the idea of one single body moving without relation to anything external (matter). On the other hand, the bare manifold structure is incapable of defining spatial -or temporal- distances, it does not contain any inertial (affine) structure, and furthermore, it does not allow any distinction between past and future. In brief, the manifold is far from filling the paradigmatic role provided to classical dynamics by Newtonian absolute time and space or, alternatively, by neo-Newtonian spacetime.

\[2\] The alternative neo-Newtonian spacetime, removes absolute space (rest), keeping the full inertial (affine) structure of Newtonian space without any need of reference to identified points.

\[3\] Relationism would then be the denial of this thesis, i.e the non-independence of spacetime structures from matter objects (particles or fields).
It sounds paradoxical enough to envisage the manifold as a spacetime without spacetime structures. The lacking spatiotemporal structure, is essentially metrical structure. Therefore, the manifold requires additional structures to be cast as substantival spacetime. This is why sophisticated substantivalists claim that the couple manifold + metric \((M, g)\) plays the role of a realistically constructed substantival spacetime in GR. In GR, the metric tensor field \((g)\) plays the explanatory role provided by Newtonian absolute space to classical dynamics. From this perspective it might seem suggestive to cast the metric tensor field \((g)\) as a substantival entity. But things don’t fit so smoothly. SS has done well in revealing the weakness of manifold substantivalism. However when the g-field is judged as a pure substantival structure, even from the standpoint of the Newtonian inherited perspective, things start to get twisted.

The metric field carries energy-momentum. This is a distinctive feature of physical matter fields. Relationalists can therefore question -as the few remaining seldom do- the alleged substantival ontological status of the metric field.

How do we interpret the gravitational field? Should it be understood as a matter field \textit{a la par} with every other physical field or as a spacetime structure? Why candidly raise the metric field to the status of an independent spacetime? What about gravitational waves and the energy carried (and released in stellar collapse) by gravitational fields with no sources?

The conflicting issue is that GR incorporates fundamentally geometry, gravity and inertia in the same field: the metric tensor field. In Newtonian physics (in field theoretic presentation) geometry and inertia are structural qualities of space, whereas the gravitational field is a physical matter field. Under these circumstances, Sophisticated Substantivalists (most philosophers) pick geometry and inertia -the spatiotemporal or chronogeometrical structures- to support their ontological doctrine, whereas Relationalists, such as Stachel (1993) and Rovelli, pick the gravitational structure of the metric field to support their own doctrine. This is how Rovelli pictures the whole thing (Rovelli 1997, p. 193):

Einstein’s identification between gravitational field and geometry can be read in two alternative ways:

i. as the discovery that the gravitational field is nothing but a local distortion of spacetime geometry; or

ii. as the discovery that spacetime geometry is nothing but a manifestation of a particular field, the gravitational field.

The choice between these two points of view is a matter of taste, at least as long as we remain within the realm of nonquantistic and nonthermal general relativity. I believe, however, that the first view, which is perhaps more traditional, tends to obscure, rather than enlighten, the profound shift in the view of spacetime produced by general relativity.
Like Rovelli, I find convincing the relational interpretation of the metric field (ii), but at this point one can imagine why Rynasiewicz (1996) has argued that the debate is outmoded, or why Saunders (2002) delineates some kind of relationism neutral as to the matter/space distinction. Giving up -I believe- is not the right thing to do, and to avoid the matter/space distinction in the relationalist/substantivalist debate seems far away from the core of the original Leibniz-Newton controversy.

As in the case of Newtonian dynamics, the Special Theory of Relativity (SR) incorporates basically geometry and inertia -the same structures- to the substantival qualities of space (or spacetime). Likewise, the gravitational field is presented as a physical field. In this sense, SR perpetuates Newtonian tradition. Things are quite different in GR. Saunders and Rynaciewicks’ uneasiness is understandable given that the theoretical structure and the explanatory role of the metric field in GR are usually judged taking as only reference the Newtonian dynamics tradition. And we have seen that from the Newtonian perspective there seems to be no possible clear cut distinction in order to settle a fair triumphant side in the debate. ‘It seems like a mere choice of taste’.

The reason, I believe, why the substantival interpretation of spacetime has become more traditional is, tautologically speaking, the power of tradition. Newtonian dynamics -paradigmatic theory of our ideas about matter, space and time- is unanimously understood as a substantivalist theory of space (and time). Concerning space and time, GR is taken as the accepted inheritor or replacement of Newtonian dynamics. Accordingly, most physicists, astrophysicists and cosmologists have aligned, following the Newtonian tradition -the only tradition-, along the substantival spacetime interpretation of GR. Most philosophers of science have also taken this interpretative path but, when facing the interpretative difficulties and ambiguities of GR, there ought to be more persuasive arguments than mere historical inertia.

Certainly, when turning back to the original Leibniz-Newton dispute one sees that substantivalism turns out prima facie triumphant since Newton was able to successfully formulate dynamics. But condemning relationalism based upon Newton’s success seems like atavistic obstinacy. Therefore, to give relationalism a fair chance, one can also put forward the following hypothetical questions: What if Leibniz -or some leibnizian like Mach- had had a good relational theory? What role would geometry play in this type of theory? Would it be natural to view geometry and inertia as intrinsic properties of substantival space -if not spacetime? Would it still seem natural to interpret the metric field of GR along substantival lines regardless of the fact that it also encodes important material properties such as energy-momentum? Would Rovelli’s -and almost everyone’s- choice between geometry (i.) and gravitational field (ii.) be rightly formulated?

I believe these sort of counterfactuals are adequate to spread important doubts over the SS account of spacetime. According to the SS account of spacetime, one should view the metric field of GR as the modern version of a realistically constructed spacetime since it has the properties -or contains the structures- that Newtonian space had. But then again, if we imagine that Leibniz had had a good relational theory of classical
dynamics we can readily envisage the alternative explanatory role of certain alleged substantival structures. Leibniz considered space as nothing else than the full set of relations amongst coexisting material objects. Geometry in this type of leibnizian theory should function as a structure that codifies the set of metrical relations amongst material coexisting bodies. If the leibnizian theory does the job, would it be sound then to come up with the conclusion that geometry *per se* is a pure relational structure and that, consequently, GR is to be interpreted as a genuine relational theory? I think everyone will answer in the negative to this question. But, this type of hidden reasoning is employed by several philosophers who find it natural to emphasize the geometrical character of the $g$-field to defend substantivalism. If we put side by side Newtonian dynamics and a Leibnizian relational alternative, geometry seems rather neutral in the relationalist/substantivalist dispute. It works as a structure that codifies certain matter relations in one case, and it works as a structure that describes substantival space or spacetime in the other. A similar argument can be applied to topological and inertial structure. Thus, using the geometrical (or metrical) character of the $g$-field *per se* in support of a substantival interpretation of spacetime puts substantivalism standing on very fragile grounds. Geometry seems to be proving nothing against the material nature of the $g$-field. Further arguments should be provided. On the other hand, the relational account of spacetime seems to have important reasons to consider the $g$-field as a physical matter field. It generates the gravitational field structures; it carries energy-momentum and, equally important, it is a dynamical object field. All these are distinctive qualities of material objects, even when judged from the Newtonian perspective.

I have argued that the popularity of the SS interpretation of spacetime is broadly accepted, largely, due to the atavistic burden of Newtonian dynamics. With the purpose of giving relationalism a fair chance, I have proposed the exercise of imagining that Leibniz, Mach, or any Leibnizian, had been able to successfully formulate a relational dynamics. This hypothetical theory would allow us to remove part of the historical inertia within the current controversy. Turns out that this need not be a mere hypothetical exercise of imagination in view of the fact that Barbour and Bertotti (BB2, 1982) have provided and alternative relational formulation of classical dynamics.

4 Relational Dynamics

BB2 is a relational theory in the sense that it is formulated in the relative configuration space satisfying Poincare’s criterion. In Newtonian dynamics spatial coordinates $(r_i, \dot{r}_i)$ are expressed relative to some inertial frame (i.e. in the absolute configuration space). Poincare (1905) noticed that if, in the equations of motion, these coordinates were replaced by purely relative quantities $(r_{ij}, \dot{r}_{ij})$, then by means of the mere knowledge of the relative configuration of a Newtonian system of coexisting particles, and of the relative velocity at which they move, it is not possible to unequivocally determine the future evolution of the system.
One can picture the problem as follows: Let us suppose that two successive snapshots of a world made up by \( N \) point particles \((m_i)\) are taken. These particles move in Euclidean space according to Newton’s laws. The two snapshots slightly differ intrinsically, and reveal the relative distances between material bodies only. Temporal separation between the snapshots is also not known. (The snapshots were printed in transparent paper) ¿Would it be possible, in this case, to univocally determine the future evolution of the particle system? The answer is, plainly, NO. What is happening is that part of the background spacetime structures, such as the whole family of inertial frames and the absolute temporal metric, is lost.

Recovery of the temporal metric would amount to full knowledge of the elapsed time between snapshots. Recovery of the inertial system structure would amount, on the other hand, to fixing the snapshot camera. This way we would be able to know what points of space were successively occupied by material bodies. This recovery procedure is equivalent to setting and a-priori equilocality relation. Now, for relationalists the idea of fixing a camera in order to identify the points of space is an anathema. This was, in my opinion, for long time Newton’s substantial advantage. O. Pooley and H. Brown clearly state this issue as follows (Pooley y Brown 2001, p. 185):

Newton effectively postulated a preferred equilocality relation between the points of space at different times and a primitive measure of the temporal distance between them in order to associate with every body an unambiguous measure of its motion. His equilocality was defined by the simple persistence of the points of space.

Now, the obligatory problem, according to a relational ontology, would be how to define an alternative equilocality relation from the mere knowledge of the relative configuration of material point particles. Barbour and Bertotti proposed a procedure called Best Matching (B-M). Informally, it can be described as follows:

Let us suppose that the two snapshots of the system are printed in a transparent paper that reveals only points at different locations that have different color intensities proportional to each particle’s mass. The first snapshot is held fixed. The second snapshot is then moved around it until is brought into the closest fit (B-M). Formally this is achieved by a variational principle. Both snapshots are labeled by arbitrary Cartesian coordinate systems and when best-matched two points having the same particle location, in both snapshots, are declared equilocal. This amounts to a suitable definition of relational coordinate system, and of a metric in the relative configuration space that allows the determination of the distance \( \delta x_i \) between the positions of a particle \( m_i \) at different times (the two snapshots). The new coordinates emerging are said to be horizontally stacked. This new equilocality relation defines a new set of relational coordinates that dispenses with Newtonian dynamics use of the independent inertial frame structure. They are relational in the sense that the B-M framework is set to give a coordinate independent objective measure of the intrinsic difference between relative material con-
configurations. When two relative configurations are horizontally stacked, coordinates do not represent positions in some inertial system. Rather, they represent particle positions in some arbitrary Cartesian system. The resulting Equilocality Relation is a measure, so to speak, of the maximum congruence between relative material configurations. It has not been defined by the ‘simple persistance the points of space’ in time.

After solving the problem of equilocality (defining relational intrinsic coordinates) a new geodesic-variational principle is set to describe the evolution of the whole system in the relative configuration space. BB2 uses a Jacobi-Type principle that formally dispenses also with Newtonian absolute time metric. The resulting BB2 theory meets, as said before, Poincare’s criterion in the sense that the dynamical evolution of the a particle system is univocally determined from the mere knowledge of the relative configuration (relative distances and velocities) of the bodies. Neither absolute time, or absolute space are used.

It should be mentioned, that the best matching machinery was developed to be properly extended to the field theoretic framework. This allows one to better understand physical fields as extended material objects, given that two relative field configurations can, similarly, be brought up to closest fit and, again, a Jacobi-type variational principle can be set up to determine the evolution of the physical system dispensing with the background substantival structure. This is readily achieved within the geometrodynamical representation of GR (Barbour et al., 2002, Anderson et al., 2003).

In the Best-Matching framework, the geometrical structure of the planes of simultaneity -in classical dynamics- and the whole differential and topological structure of the manifold -in both classical an generally relativistic dynamics- are used as important structural qualities of the whole set of relations amongst material bodies. But these structures acquire only a pure relational meaning given that only the intrinsic variation of relative material configurations is physically meaningful within this formalism. The intrinsic theories make use of relational coordinates that, at no level, presuppose the existence of a containing physical space inhabited by particles or/and fields. Particles do not occupy absolute space, nor fields fill a topological space ($M$). The background substantival space -or spacetime- is completely eliminated via B-M.

Turning back to the main issue, the important point I want to draw attention to is the fact that Barbour and Bertotti (BB2, 1982) have provided an alternative formulation of classical dynamics. They provide a ‘genuinely relational interpretation of dynamics’ (Pooley and Brown 2001). Geometry and inertia become -contra SS- relational structures in BB2. There is no real sense in which geometry and inertia can be said to be intrinsic properties of classical space, so the usually alleged interpretation of the $g$-field along substantival lines is strongly undermined.

As stated before, on the other hand, the relational account of spacetime has important reasons to consider the $g$-field as a physical matter field. It generates the gravitational field structures; it carries energy-momentum and, equally important, it is a dynamical object field. All these are distinctive qualities of material objects in
both Newtonian dynamics and the recently introduced alternative intrinsic relational dynamics (BB2).

I want to briefly stress out the latter point: Matter is linked to dynamical objects in physical theories. I believe this general consensus should not be broken just to save an ontological doctrine. The Sophisticated Substantivalist also ignores the material dynamical nature of the g-field. In doing so, she wishes to have a clear cut distinction between spatiotemporal structures, and material object structures. We have seen that this is not the case in GR when judged from the Newtonian perspective.

However this alleged distinction works out pretty well in the case of Newtonian, neo-Newtonian, and Minkowski models. The separation between matter and space-time seems to be clearly defined in these cases. To illustrate it, let us suppose we have an ensemble of physical fields (electromagnetic, gravitational, dust fluid fields, etc.) living in their corresponding Newtonian, neo-Newtonian or Minkowski space-times. In this cases a model can be represented as follows: $\mathcal{M} = (M, h, \Gamma, \phi, \rho, E, \ldots)$. Where $M$ is the manifold, $h$ is the metric structure, $\Gamma$ is the affine connection or inertial structure, $\phi$ is the gravitational field, $\rho$ is a fluid density that can represent any conventional matter distribution, $E$ is the electromagnetic field, and the suspension points correspond to any other kind of matter we wish to include in the model.

In all these pre-GR models $h$ and $\Gamma$ are understood as substantival spacetime structures, whereas $\phi, \rho, E$ and others, are material objects. Now, it is well known that in these models, $h$ and $\Gamma$ are absolute objects representing spacetime. On the other hand, matter is represented by dynamical object fields ($\phi, \rho, E, \ldots$). An absolute object is a theoretical object that retains its structure in all models of the theory. An absolute objects partially determines the evolution of matter without it been affected back. These absolute objects are usually characterized by their symmetry group and the effect of the corresponding symmetry transformations is to move or drag along the physical fields preserving the background absolute structure. On the other hand the dynamical objects, such as $\phi, \rho, E, \ldots$, change from model to model according to their respective field equations\(^4\).

In GR models $\langle (M, g, T) \rangle$ $g$ and $T$ are dynamical object fields. Then again, it seems quite stubborn to ignore the material nature of dynamical object fields -common to all models of pre-GR theories- just to save an ontological doctrine. Thus, it seems obstinate enough to ignore the dynamical material nature of the g-field. I believe, then, that GR is more naturally interpreted as a relational theory where a material dynamical object field $g$ has finally absorbed the spatiotemporal structures. We have also seen that this spatiotemporal structures receive a genuine relational treatment in the alternative intrinsic dynamics.

Everything is matter according to GR. There is no such thing as an independent spacetime. This was Einstein’s own late conclusion (Einstein 1954 Apendix V, P.155):

\(^4\)The distinction between absolute and dynamical objects is originally due to Anderson (1967 p. 83-84) It has been well developed by Friedman (1984 p. 47-60)
There is no such thing as empty space, i.e. a space without a field. Space-time does not claim existence on its own, but only as a structural quality of the field.

References


