Remarks on Space-time and Locality in Everett's Interpretation*

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Abstract

Interpretations that follow Everett's idea that (at some level of description) the universal wave function contains a multiplicity of coexisting realities, usually claim to give a completely local account of quantum mechanics. That is, they claim to give an account that avoids both a non-local collapse of the wave function, and the action at a distance needed in hidden variable theories in order to reproduce the quantum mechanical violation of the Bell inequalities. In this paper, I sketch how these claims can be substantiated in two renderings of Everett's ideas, namely the many-minds interpretation of Albert and Loewer, and versions of many-worlds interpretations that rely on the concepts of the theory of decoherence.

1 Measurements, Classicality and Interpretations of Quantum Mechanics

The measurement problem of quantum mechanics can be concisely formulated as follows: what we usually call measurements are examples of interactions in which the linear dynamics of quantum mechanics generally produces superpositions of classical-like wave functions.

The thought implicit in this formulation is that there are some special quantum states, which under certain circumstances behave very much like classical states, say, they spread very slowly and follow approximately classical trajectories (in position representation and under the free Hamiltonian evolution). Were it not that under other circumstances (such as measurements) this behaviour is unstable, we would presumably be happy to follow Schrödinger's [56] early intuitions and *identify* classical states with such special quantum states.

*Talk delivered at the NATO Advanced Research Workshop on ***Modality, Probability, and Bell's Theorems***, Cracow, 19–23 August 2001. Proceedings to be published by Kluwer Academic (NATO Science Series).

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However, since there appear to be interactions in which superpositions of very different such states are produced, this straightforward reinterpretation of classical physics in quantum mechanical terms is put into question, and it is unclear which elements of the quantum mechanical description (if any) correspond to elements of the 'classical world', the 'world of everyday experience', and in particular the 'classical measurement context' that Bohr considered to be a prerequisite for the formulation of quantum mechanics.

As a matter of fact, measurement-like interactions arise in nature even without the concourse of observers. More general — and very pervasive — interactions in which such superpositions of classical-like states arise, are studied in the theory of decoherence (for a general reference, see [30]). For instance, a Brownian particle becomes quantum mechanically entangled with its environment, and the quantum state of the total system develops into a superposition of components corresponding to the various classical trajectories of the particle. I say 'trajectories', because one of the crucial features of decoherence is that it provides one with a criterion for reidentifying over time suitable components of a quantum state.

Thus, on the one hand, the pervasiveness of decoherence shows that the 'measurement problem' should be thought of as a more general problem than the one of reconciling the definiteness of measurement outcomes with the linearity of the Schrödinger equation.[1] By the same token, however, a successful solution to the problem that takes into account also the phenomenon of decoherence, should be able to recover many more 'classical events' from the quantum mechanical description.[2]

Attempted solutions which bear some promise of success can be arguably classified as follows: (1) collapse theories, such as the spontaneous localisation theories by Ghirardi, Rimini and Weber [29] and Pearle [46], or arguably the primary state diffusion of Percival [47]; pilot-wave theories such as de Broglie–Bohm theory [12,13], Bell's stochastic 'beable' theory [9, Chapter 19], and arguably Nelson's mechanics [44] (but not as understood by Nelson himself); and finally theories deriving from Everett's ideas [25], which are the subject of the present paper.[3]

All of these — be they more physical or more philosophical in their approach — implicitly or explicitly take a stance about the *meaning* or *interpretation* of the quantum states. Collapse theories are compatible with the 'straightforward' identification of classical states with special quantum states, since according to collapse theories the non-classical superpositions never develop (or, rather, superpositions

[1] Indeed, not all approaches to quantum mechanics have taken due account of this more general measurement problem. In particular, modal interpretations in versions that privilege the spectral decomposition (or biorthogonal decomposition) of the state, arguably fail to recover classicality in the presence of decoherence; see [3] and references therein.

[2] The role of decoherence will be particularly emphasised in the many-worlds approach discussed below in Section 5. (For a sketch of how other approaches to quantum mechanics relate to the theory of decoherence, see [4].)

[3] Yet a different kind of approach may be one that assumes the existence of only one world among the many possible ones, or in other words that one 'history' of the world simply unfolds, and that the quantum state encodes some information about it or partially constrains it in some way. Examples of this kind of approach may be [59], as well as the original version of the modal interpretation by Van Fraassen [27].

of 'classically distinguishable' wave functions are dynamically highly unstable). According to pilot-wave theories, the superpositions do develop, but the description of the world as we observe it involves crucially some element not included in the wave function (e. g. a classical configuration).[4] Finally, Everett theories take the presence of the superpositions literally as describing at some appropriate level a multiplicity of coexisting (classical-like?) realities.

This characterisation of Everett interpretations is very crude. Indeed, any interpretation of this kind will have to specify and/or justify the choice of those components of the wave function that are taken to embody this multiplicity (this is often called the 'preferred basis' problem, although these different components need not be exactly orthogonal, or correspond to the same 'basis' at different times).

Also, each version of Everett will face the problem of giving a meaning to the quantum mechanical probabilities, since in some sense the random events that are taken as alternatives in the standard theory are now supposed to coexist. Thus, it is unclear whether classical notions of probability can be used.

In general, Everett interpretations will try to *combine* a linear and deterministic evolution at the level of the wave function of the universe with an 'apparent', 'effective' or 'subjective' non-linear and indeterministic 'collapse', emerging at the level of components within the universal wave function. This is unlike the case of collapse theories, which wholeheartedly embrace classical indeterminism; and of pilot-wave theories, which at least in the case of de Broglie and Bohm's theory totally restore classical determinism.

Everett theories also claim to follow a 'third way' between collapse theories and pilot-wave theories as far as the notion of *locality* is concerned. Indeed, while collapse theories embrace non-locality at the level of the universal wave function, and pilot-wave theories at the level of the 'hidden variables', a frequent claim about Everett theories is that they are completely local.

At first sight Everett theories seem to be so metaphysically extravagant that — together with the claim that they 'keep intact' the theory of quantum mechanics, by not requiring a modification of the Schrödinger equation, nor the introduction of a more fundamental level of description — the claim to locality is indeed the main feature that commends an Everett approach.

2 Non-Locality in Quantum Mechanics

The standard discussion of locality and non-locality in quantum mechanics derives from Bell [9, e. g. Chapters 2 and 4]. Bell considered a standard EPR setting (two entangled particles travelling in opposite directions, say two electrons in the singlet state). He then showed that a mathematically precise locality condition for (the probabilities of) measurement outcomes on the two wings of the experiment leads

[4] Such additional elements are often and quite misleadingly termed 'hidden variables'. Notice that none of the theorems that supposedly show the impossibility of hidden variables theories rules out pilot-wave theories, as emphasised by Bell [9, Chapters 1 and 17].

to certain inequalities (Bell inequalities) that are violated by the usual statistical predictions of quantum mechanics (maximally so in the singlet state). Thus, any approach that allows one to reproduce these predictions must violate Bell's locality condition.

This condition can be further analysed in terms of the conjunction of two separate conditions, called by Jarrett [38] locality and completeness, and corresponding, respectively, to Shimony's [57] conditions of parameter independence and outcome independence.[5] Roughly, violation of parameter independence means that the arrangement of a piece of apparatus at one end of an EPR experiment will affect (the probability of) the outcome on the other wing; while violation of outcome independence means that even once all possibly relevant information is included in the description, the outcomes on the two wings of the experiment are probabilistically dependent upon each other. Violation of the Bell inequalities implies violation of either or both of these conditions. Collapse theories provide the standard example of violation of outcome independence alone, and de Broglie–Bohm theory of violation of parameter independence alone.

As far as Lorentz invariance or non-invariance is concerned, the issue is less clear-cut, but it seems difficult to construct a Lorentz-invariant version of either collapse theories or pilot-wave theories.

In Everett theories, it is unclear how one should apply Bell's analysis. To be sure, in an Everett interpretation there will be *something* that corresponds to the probabilistic predictions of standard quantum mechanics, but there is a presumption that the correlations are not mysterious. Suppose I have the choice of travelling to Scotland by the high road or by the low road, and my true love has the choice of travelling (in the opposite direction!) by one of the same two routes. We may never meet again. If, however, we can make sense of the idea that we both travel both routes, we shall be sure to meet on either route.

It is also unclear how to apply an analysis in terms of Lorentz invariance. One obviously has Lorentz invariance at the level of the universal wave function (in a relativistic quantum theory). However, there are conflicting intuitions about the description in terms of worlds: are they to be described non-invariantly as in a collapse theory? (This might be true of a naive view in which universes literally multiply at each 'splitting'.) Or should they be thought of as alternative complete histories of the universe, each of which has a perfectly Lorentz-invariant description?

In the following, we shall sketch how the locality claim can be made more precise in two versions of the Everett theory, namely Albert and Loewer's [2] version of the many-minds interpretation (Sections 3 and 4), and my favourite understanding of the many-worlds theory, mainly inspired by Saunders [52, 53, 54] (Sections 5 and 6). The account of locality in many-minds merely expands on the account by Albert and Loewer and by Albert [1]. The account in the many-worlds case relates both to ideas by Saunders and to work in the theory of branching space-times, for which see e. g. Belnap [10, 11], Placek [48, 49], as well

[5] For subtle differences between the two analyses, which need not concern us here, see [39]

as McCall [42, 43]. For the purposes of this paper, I shall ignore other proposed treatments of locality in (versions of) Everett, as summarised e. g. in Vaidman [62].

3 Many-Minds

There are several versions of the many-minds interpretation, mainly due to Albert and Loewer [2], Lockwood [41] and Donald [19, 21, 22], and there are important differences between them, also with regard to issues of locality. For the purposes of this paper, I shall focus on the version by Albert and Loewer (also treated in [1]).[6]

In many-minds theory, the multiplicity of coexisting realities is not at the level of the physical realm but of the mental realm. Suppose the physical state of the universe (including an observer) develops into a superposition involving brain states of the observer that correspond to different perceptions, which we represent schematically as, say,

$$\frac{1}{\sqrt{2}}\Big(|+\rangle|\text{see }+\rangle + |-\rangle|\text{see }-\rangle\Big). \tag{1}$$

This is indeed a complete and accurate description. There is but *one* physical state. However, this observer will possess a multiplicity of *mental* states. This is expressed by saying that he or she possesses *many minds*, some of which will be in the mental state $||\text{see }+\rangle\rangle$, and some in the mental state $||\text{see }-\rangle\rangle$. (These are the mental states that would correspond to the physical brain state $|\text{see }+\rangle$ and $|\text{see }-\rangle$, respectively, if the brain were in such a state.)

The aim of a many-minds theory is to solve the measurement problem in the sense that *our mental experiences*, as predicted by standard quantum mechanics, should be recoverable. The outside world, insofar as it contains definite measurement results and tables and chairs with (more or less) definite positions and momenta, is purely 'ideal'. If one wishes to recover a 'real' classical world outside of our minds, one must look elsewhere for a solution to the measurement problem.

One might ask for a more precise formulation of the concept of a brain state corresponding to a definite perception.[7] That, however, is arguably the only ambiguity in the answer to the question of the preferred basis. Notice that — in a sense that will become clearer below — this choice should not be thought of as providing a 'global' preferred basis for the universe, but a set of 'local' preferred bases, one for every observer.

Probabilities also enter the many-minds theory only at the mental level. At the physical level the evolution of the (single) physical state is perfectly deterministic. But whenever the physical state develops into a superposition such as (1), the states of the observer's minds will evolve stochastically to either $||\text{see }+\rangle\rangle$ or $||\text{see }-\rangle\rangle$. (Individual minds are reidentifiable over time.)

[6] A quite different discussion of probability and locality in Lockwood's and in Albert and Loewer's interpretations is provided by Hemmo and Pitowsky [36]. Their discussion has partly prompted me to clarify my understanding of Albert and Loewer's position.

[7] This aspect is given a very careful treatment in [19].

The probabilities are determined by the evolution of the reduced state of each observer's brain *alone*. For instance, if the observer performs a Stern–Gerlach measurement, his or her brain will evolve, say between t and t', as

$$|\text{ready}\rangle\langle\text{ready}| \mapsto \frac{1}{2}\Big(|\text{see } +\rangle\langle\text{see } +| + |\text{see } -\rangle\langle\text{see } -|\Big) \tag{2}$$

(again schematically). Then the probability for any of the observer's minds to jump from $||\text{ready}\rangle\rangle$ to $||\text{see } +\rangle\rangle$ or to $||\text{see } -\rangle\rangle$ during that time interval is 0.5.[8] It should be noted that in order for the transition probabilities for the minds to be well-defined, also the physical brain states corresponding to the different components in the superposition ought to be reidentifiable over time. (This identity over time of wave components is precisely one of the features provided by the theory of decoherence, which is thus directly relevant to the many-minds interpretation!) Each of the observer's minds evolves independently of the others, and independently of the minds of other observers (as we shall presently discuss in more detail).

The probabilities that appear in the many-minds interpretation are thus entirely classical: they are probabilities for any mind of an observer to follow a particular random trajectory in the space of mental states.

4 Locality in Many-Minds

The claim to locality in Albert and Loewer's version of the many-minds theory is embodied in the fact that the minds of different observers evolve *independently* of each other: the stochastic evolution of the minds of each observer is determined by the form of the reduced state of that observer's brain alone. The independence of the evolution of different observers' minds is directly connected with the feature of having *many* minds for each observer, and we shall discuss the two points together.

Assume each observer had only one mind evolving according to the stochastic dynamics sketched above, and take the case of two observers observing an EPR pair in the singlet state. The overall physical state (in obvious notation) evolves as

$$\begin{aligned} &\tfrac{1}{\sqrt{2}}\Big(|\text{ready}\rangle|+\rangle|-\rangle|\text{ready}\rangle - |\text{ready}\rangle|-\rangle|+\rangle|\text{ready}\rangle\Big) \mapsto \\ &\qquad \tfrac{1}{\sqrt{2}}\Big(|\text{see } +\rangle|+\rangle|-\rangle|\text{see } -\rangle - |\text{see } -\rangle|-\rangle|+\rangle|\text{see } +\rangle\Big). \end{aligned} \tag{3}$$

The evolution of the states of the two brains is given in both cases by (2), so that each observer's (single) mind has a 0.5 probability of ending up in either mental state, $||\text{see } +\rangle\rangle$ or $||\text{see } -\rangle\rangle$. But then, the evolution of the 'combined mental state'

[8] Notice that Markov processes can be reconstructed uniquely from an 'initial' probability distribution and the 'forward' transition probabilities between any two times.

of the two observers will be

$$||\text{ready}\rangle\rangle||\text{ready}\rangle\rangle \mapsto \begin{cases} ||\text{see }+\rangle\rangle||\text{see }+\rangle\rangle \\ ||\text{see }+\rangle\rangle||\text{see }-\rangle\rangle \\ ||\text{see }-\rangle\rangle||\text{see }+\rangle\rangle \\ ||\text{see }-\rangle\rangle||\text{see }-\rangle\rangle \end{cases}, \tag{4}$$

each with probability 0.25, even if the physical state (3) is one in which the brain states of the two observers are perfectly correlated.

A similar situation will arise if the physical state does not exhibit perfect correlations between the brain states of the two observers, in particular if the observers measure different components of spin on the two particles. While all four components appear in the final quantum state, in general the state will still exhibit *some* correlation between the two observers seeing the same or different results. However, there is again *no* correlation between the observers' mental states.

The anticorrelation of the brain states in (3) is of course caused by the original preparation of the EPR pair in the singlet state. Now, on the basis of that information, each of the observers' brain state components (if the observers know their quantum mechanics!) can figure out what to expect about the other observer's findings, and this will be reflected in the corresponding mental states:

$$\begin{matrix} ||\text{see }+\text{ and expect }-\rangle\rangle||\text{see }+\text{ and expect }-\rangle\rangle \\ ||\text{see }+\text{ and expect }-\rangle\rangle||\text{see }-\text{ and expect }+\rangle\rangle \\ ||\text{see }-\text{ and expect }+\rangle\rangle||\text{see }+\text{ and expect }-\rangle\rangle \\ ||\text{see }-\text{ and expect }+\rangle\rangle||\text{see }-\text{ and expect }+\rangle\rangle \end{matrix} \tag{5}$$

(each with probability 0.25). We see that the mental states of the two observers 'harmonise' *only with probability* 0.5 (in the second and third case), while there is 'disharmony' between them in the remaining cases.

The following is an inessential but graphic illustration of this situation. If the two observers were subsequently to meet, there is probability 0.5 that they would be actually *experiencing* each other as being in states that do not correspond to their mental states. In Albert and Loewer's phrase, each would be interacting with a 'mindless hulk'.

There are two ways out of this quandary. Either we introduce 'pre-established harmony' between the mental states of the two observers, that is we introduce *correlations* into their stochastic evolution, or — indeed — we give up on the idea that each observer has a single mind!

The former solution would mean that the stochastic evolution of the minds is determined on the basis of the reduced state of the *composite* system consisting of all observers' brains, say

$$\begin{aligned} &|\text{ready}\rangle\langle\text{ready}| \otimes |\text{ready}\rangle\langle\text{ready}| \mapsto \\ &\quad \tfrac{1}{2}\Big(|\text{see }+\rangle\langle\text{see }+| \otimes |\text{see }-\rangle\langle\text{see }-| + |\text{see }-\rangle\langle\text{see }-| \otimes |\text{see }+\rangle\langle\text{see }+|\Big), \end{aligned} \tag{6}$$

so that the evolution of the mental states is indeed

$$||\text{ready}\rangle\rangle \otimes ||\text{ready}\rangle\rangle \mapsto \begin{cases} ||\text{see } +\rangle\rangle \otimes ||\text{see } -\rangle\rangle \\ ||\text{see } -\rangle\rangle \otimes ||\text{see } +\rangle\rangle \end{cases} \tag{7}$$

(each with probability 0.5). Thus, the preferred basis which defines the evolution of the mental states would be indeed a 'global' basis. This, however, amounts to explicitly introducing *non-locality* to the evolution of the minds.[9]

The alternative, as mentioned, is to allow each observer to have (infinitely) many minds. After the observation of the measurement result on his or her side of the experiment, the minds of each observer follow two different paths, so that each observer has minds of two kinds: those in the mental state $||\text{see } +\rangle\rangle$ and those in the mental state $||\text{see } -\rangle\rangle$. Each of the kinds of mental states will also include expectations about the findings of the other observer (for any measurement the other observer may decide to make). These expectations will in general be different for the minds in state $||\text{see } +\rangle\rangle$ and for the minds in state $||\text{see } -\rangle\rangle$), but they will match the findings of the other observer.[10]

This matching of expectations is a consequence of the evolution of the quantum state of the total system. This can be thought of as entirely local, and indeed it is explicitly so in relativistic formulations of quantum mechanics (although the quantum state itself is a global object).

If the two observers meet, each mind of each observer will experience only some component of the other observer's brain state. The other observer, however, will have some minds in mental states corresponding to that brain state component. Thus, each mind of each observer interacts, as it were, only with part of the other observer, but each such part is indeed a *mindful* being.

Notice that by the independence assumption, all probabilities factorise. However, the event space we are dealing with is the infinite product of all minds of all observers, so that there is no event corresponding to an 'outcome' of an experiment. Thus, Bell's analysis does not apply.

Finally, notice also that no special treatment of space-time is needed or emerges from this approach. Indeed, space-time is presupposed as a background structure in which the quantum state of the universe evolves, and which defines what we mean by local evolution of the quantum state, by (possibly spacelike) separation of the two observers when they perform their observations, by the meeting of the two brains, and indeed by the locality of the minds' evolution.

[9] Indeed, this evolution has the same form as the evolution used in beable theories such as those by Bell [9, Chapter 19], Vink [63] and Bub [14], and in certain versions of the modal interpretation (see [6]).

[10] Suppose observer A has measured spin in direction a, and a certain fraction of his or her minds are in state $||\text{see } +\rangle\rangle$ (respectively, $||\text{see } -\rangle\rangle$). These minds expect a fraction, call it β^+_+ (respectively, β^+_-), of observer B's minds to be in state $||\text{see } +\rangle\rangle$, if B has measured spin in direction b. Suppose B has indeed measured spin in direction b. Among *those* minds of B expecting that if observer A has measured spin in direction a, A's minds will be in state $||\text{see } +\rangle\rangle$ (respectively, $||\text{see } -\rangle\rangle$), a fraction β^+_+ will be, indeed, in the mental state $||\text{see } +\rangle\rangle$, and a fraction β^+_- in the mental state $||\text{see } -\rangle\rangle$.

5 Many-Worlds

There are very many versions of many-worlds theories, which differ in their treatment of the problem of the preferred basis, as well as in their treatment of the interpretation of the probabilities. The most insightful work to date is in my opinion that by Saunders [52, 53, 54] and by Wallace [64], and my favoured understanding is very much related to their approaches.[11]

Many-words theories all identify the multiplicity of coexisting realities as a multiplicity in the *physical world*, such as the coexistence of different outcomes in a measurement, or of a live cat with a dead cat. Consideration of these examples leads directly to the so-called *problem of the preferred basis*: why should the coexisting realities correspond to the states $|\text{live}\rangle$ and $|\text{dead}\rangle$, and not to the states, say,

$$\frac{1}{\sqrt{2}}(|\text{live}\rangle + |\text{dead}\rangle \qquad \text{and} \qquad \frac{1}{\sqrt{2}}(|\text{live}\rangle - |\text{dead}\rangle? \tag{8}$$

One way of understanding this is as follows. The wave function of the universe should be thought of in analogy to a 'block universe' in special relativity, and we identify components within the wave function with worlds if they exhibit appropriate structure. A natural *necessary* condition for components of the universal wave function to qualify as independent realities, is that these components be reidentifiable over time, or — which amounts essentially to the same thing — that they do not *interfere* with each other. From this point of view, what is crucial in defining the coexisting realities is the presence of *decoherence*, the complex of phenomena that involve suppression of interference in a system in interaction with some appropriate environment.

A related more abstract approach is that introduced by Griffiths [31] and Gell-Mann and Hartle [28] (for a clear and brief introduction, see [34]), and known as the *consistent histories* or the *decoherent histories* formalism.

In this approach, the primary object is a *history*, which is a time-ordered sequence of projectors (usually in the Heisenberg picture):

$$P_1(t_1), P_2(t_2), \ldots, P_n(t_n). \tag{9}$$

One normally considers sets of histories that are exhaustive and alternative, in the sense that at each of the times t_j, one takes the projection P_j to be any one from a family of mutually orthogonal projections $P^j_{i_j}(t_j)$ summing to $\mathbf{1}$ (a projection-valued resolution of the identity):

$$P^1_{i_1}(t_1), P^2_{i_2}(t_2), \ldots, P^n_{i_n}(t_n). \tag{10}$$

For each $i = (i_1, \ldots, i_n)$, this is a history from the given set.

[11] Another notable proponent of many-worlds is Vaidman [60, 61]. For discussion and review of these and other approaches to Everett, see the papers by Butterfield [16, 17, 18], the book by Barrett [7] and the encyclopedia articles by Barrett [8] and by Vaidman [62]. The version of Everett that I sketch below was originally worked out together with Meir Hemmo.

One can define the *probability* of a history using the standard formula one would use for a sequence of measurements, i. e.

$$\mathrm{Tr}\Big(P^n_{i_n}(t_n)\dots P^1_{i_1}(t_1)\rho P^1_{i_1}(t_1)\dots P^n_{i_n}(t_n)\Big), \tag{11}$$

e. g. in a two-slit experiment, we would have probabilities such as

$$\mathrm{Tr}\Big(P_i^{\mathrm{screen}}(t_2)P^{\mathrm{slits}}_{\mathrm{up/down}}(t_1)\rho P^{\mathrm{slits}}_{\mathrm{up/down}}(t_1)P_i^{\mathrm{screen}}(t_2)\Big). \tag{12}$$

In general, however, these probabilities exhibit *interference*. If the particle passes through the apparatus without interacting with its environment, interference is unavoidable, but if it does interact with some other particle just after passing the slits, the probabilities satisfy the following condition, which characterises the absence of interference:

$$\begin{aligned}\mathrm{Tr}\Big(P_i^{\mathrm{screen}}(t_2)\rho P_i^{\mathrm{screen}}(t_2)\Big) =& \\ &\mathrm{Tr}\Big(P_i^{\mathrm{screen}}(t_2)P^{\mathrm{slits}}_{\mathrm{up}}(t_1)\rho P^{\mathrm{slits}}_{\mathrm{up}}(t_1)P_i^{\mathrm{screen}}(t_2)\Big)+ \\ &\mathrm{Tr}\Big(P_i^{\mathrm{screen}}(t_2)P^{\mathrm{slits}}_{\mathrm{up}}(t_1)\rho P^{\mathrm{slits}}_{\mathrm{up}}(t_1)P_i^{\mathrm{screen}}(t_2)\Big).\end{aligned} \tag{13}$$

This is equivalent to the so-called *consistency* condition (or *weak decoherence* condition), which in its general form reads: for any two different histories i and i',

$$\mathrm{ReTr}\Big(P^n_{i_n}(t_n)\dots P^1_{i_1}(t_1)\rho P^1_{i'_1}(t_1)\dots P^n_{i'_n}(t_n)\Big) = 0. \tag{14}$$

More often, one considers the condition of *strong decoherence*, in which both the real and the imaginary part of the trace expression in (14) are required to vanish for different histories. This trace expression is called the *decoherence functional*.

The way the decoherent histories approach is often presented states that the decoherence condition identifies when *quantum* probabilities behave classically (the idea being that in that case they can indeed also be *interpreted* classically). As a matter of fact, the analogue of the consistency condition (and with the same name) appears as a standard requirement on the distribution functions of a classical stochastic process (but see footnote 15).

A different characterisation of the decoherence condition is based on the theorem about *persistance of records* (see e. g. [34]), which essentially states the equivalence (if ρ is pure) of the (strong) decoherence condition with the existence of permanent records of the past elements of the histories. This characterisation is particularly useful for understanding the significance of the decoherence condition as being a necessary condition for representing Everett worlds.

My favoured understanding of Everett's many-worlds theory (in fact, the one I find most intelligible) is thus as follows. The universal wave function is the main object of the theory. Within this wave function we need to isolate structure that will represent the visible reality, i. e. the worlds we live in. Decoherence is largely responsible for creating such structure within the wave function. (I say 'largely', because decoherence may be only a necessary condition.[12])

In such a picture, one has to make sense of the identity of 'observers' (or indeed of any other objects), which — at least on an intuitive reading — seem to have a unique past but several futures at any one instant. The simplest way of dealing with identity in this case seems to be to talk of split identity, somewhat analogously to what we do in the case of reproduction by cell division, or in the cases discussed by Parfit [45] (for a different point of view, see [58]).

The problem that is perhaps most pressing in this framework is the interpretation of the probabilities. Again, very insightful work has been carried out by Saunders, with emphasis on an analogy with *time* in the block universe picture. The block universe picture (all space-time events are equally real) seems incompatible with a view of time as fundamentally given by the A-series (objective becoming), and lends itself rather to a view of time as given by the B-series (relational view of time). Similarly, probabilities ought to be viewed as relations defined between components of the wave function (relations in Hilbert-space norm). Both Saunders [52] and Vaidman [60] have sketched ways in which such probabilities would govern a rational agent's expectations. While I believe that sketch to be correct, I also believe that a full understanding of probabilities in such a version of Everett will be gained only when a full understanding of the experienced flow of time in a block universe will be attained (the B-series alone arguably does not provide this). The most promising approach would seem to be an extension of the approach developed by Ismael [37] in the case of time.[13]

In this framework we shall now sketch a discussion of space-time and locality.

6 Space-time and Locality in Many-Worlds

In order to discuss whether such a version of Everett is indeed local, we must first reflect on what notion of space-time is appropriate to the discussion.

If we are dealing with a relativistic theory, the universal wave function can be represented on a background (Minkowski) space-time. We thus have events as points of the background space-time. We still have to identify *concrete events*,

[12] Indeed, there are many examples of mutually 'inconsistent' ways — in the sense of the consistency condition — of selecting sets of decoherent histories with respect to the same state ρ, including sets of decoherent histories with mutually inconsistent future extensions. These and other criticisms of the decoherent histories approach can be found in [23] and [24]. As far as the specific problems of non-uniqueness and of non-uniqueness of future extensions are concerned, there seem to be two avenues worth exploring: (a) the identification of some conditions stronger than decoherence (so-called quasiclassicality conditions), which would single out a unique set of histories (requiring that at least one history be quasiclassical for all times is insufficient: see [40]); (b) the idea that we happen to inhabit one of these sets of decoherent histories and that other sets are also just as real, whether or not they be inhabited, too. Given my proposals in the next sections, I would be more interested in pursuing option (b).

[13] A convincing Everettian account of the meaning of probabilities would further provide an alternative approach to the foundations of probability, as suggested by Donald [20] and discussed by Wallace [65], since one could argue that all (objective) chances have their origin in quantum mechanical processes.

however. The situation is perfectly analogous to that in non-relativistic quantum mechanics, in which the wave function can be represented in configuration space, but we still need to identify what in the description (if anything) corresponds to, say, the trajectory of an α-particle in a cloud chamber.

The natural Everettian suggestion in the framework we have sketched above is to identify concrete events at the level of components of the wave function, e. g. as elements (projections) in the decoherent histories. Intuitively, the set of concrete events should have two salient properties. First, if the decoherence interactions are appropriately 'local' with respect to the background space-time, the concrete events will inherit some of the structure of the background space-time. Second, if observers or objects in general indeed split and have several futures, the structure of the concrete events cannot be a standard (Minkowski) space-time, but in some sense, a *branching* space-time.

A notion of branching space-time which should serve this purpose was introduced by Belnap as a framework for causal structures compatible with both relativity and indeterminism, and was applied, among other things, to discuss EPR-like situations [10, 11]. It is used by Placek to discuss the Bell inequalities [48, 49]. A similar notion was introduced also by McCall, and is used in the context of his branching interpretation of quantum mechanics [42, 43].[14]

Belnap's branching space-time generalises to a space-time framework the more familiar idea of a branching time (which formalises the idea of fixed past and open future). Branching time is characterised by the presence of splitting points, at each of which one past segment of time is pasted to two or more future segments, giving time a tree-like structure. Branching space-time is analogously characterised by divergence surfaces, which are (segments of) forward light cones along which different leaves (branches) of the space-time diverge.

Branching space-times are quite more complex structures than branching times. For instance, there may be more than one branching space-time with the same set of splitting points (vertices of the light cones forming the divergence surfaces). In the simple case of two spacelike separated splitting points, one can have different kinds of branching, depending on whether and how the space-time leaves to the future of the splitting points split again at the intersection of the forward light cones of the two splitting points. If there is *no* further splitting, the divergence surface of the two leaves is a jagged piecewise lightlike surface. This case can be used to represent a perfect correlation between events at the two splitting points.

The conjecture, which is spelled out in slightly more detail below, is that under appropriate assumptions about decoherence interactions, one will be able to rigorously derive that 'decoherence events', i. e. projections in a set of decoherent histories, will have a causal structure that is realisable in terms of a branching space-time, and that we can use this structure to spell out the locality of the Everett theory.

[14] McCall's interpretation shares some features with the interpretation we are considering in this paper, in particular with regard to the concept of locality, but it incorporates a notion of 'branch attrition' which is a version of genuine (as opposed to effective) collapse.

6.1 Time

The picture of branching time or space-time, or indeed the intuitive picture of Everett worlds and of objects and observers in them splitting, is highly time-asymmetric. In order to show that a branching structure can emerge through decoherence phenomena one needs to show first of all that decoherence gives rise to some form of time asymmetry (at the level of components of the wave function). In this section I shall sketch how I believe this can be shown, already in the non-relativistic case, for the decoherent histories formalism. This claim will be discussed more fully in [5]. Once this is established, one should be able to show that this time asymmetry can be spelled out further in terms of a branching time at the level of the Everett worlds.

The claim that the decoherent histories formalism incorporates the emergence of time asymmetry in physics is not new; in fact, it is explicitly made by Gell-Mann and Hartle (see e. g. [35]). The details of their treatment are quite different, however. The main point to be noted is that the definition of the decoherence functional (and thus the very statement of the decoherence condition) is time-asymmetric: in general

$$\begin{aligned}\mathrm{Tr}\Big(P^n_{i_n}(t_n)\dots P^1_{i_1}(t_1)\rho P^1_{i'_1}(t_1)\dots P^n_{i'_n}(t_n)\Big) \neq \\ \mathrm{Tr}\Big(P^1_{i_1}(t_1)\dots P^n_{i_n}(t_n)\rho P^n_{i'_n}(t_n)\dots P^1_{i'_1}(t_1)\Big)\end{aligned} \tag{15}$$

(in particular, the two expressions need not vanish simultaneously).[15]

Let us call one of the two expressions the ‘forward’ decoherence functional, the other the ‘backward’ decoherence functional (the names being purely conventional). The choice of one of these functionals as *the* decoherence functional would seem to distinguish by *fiat* a direction of time, and Gell-Mann and Hartle suggest to restore the symmetry of the theory by introducing a second quantum state ρ_{fin} in addition to $\rho_{\mathrm{in}} := \rho$, and define the decoherence functional symmetrically as:

$$\mathrm{Tr}\Big(\rho_{\mathrm{fin}}P^n_{i_n}(t_n)\dots P^1_{i_1}(t_1)\rho_{\mathrm{in}}P^1_{i'_1}(t_1)\dots P^n_{i'_n}(t_n)\Big). \tag{16}$$

The asymmetry of the standard decoherence functional would then be due to the (contingent) asymmetry between $\rho_{\mathrm{in}} = \rho$ and $\rho_{\mathrm{fin}} \approx \mathbf{1}$.

The Everett framework sketched above suggests instead that the ‘forward’ and ‘backward’ decoherence conditions are both legitimate ways of characterising structure within the time-symmetric (indeed, in the Heisenberg picture, time-independent) wave function ρ, structures which are both time-directed, but in opposite directions. The way time asymmetry in our world(s) emerges would then be through the fact that the set of histories we inhabit satisfies one of the two conditions, but not both (and we call that condition the ‘forward’ one, or for short

[15] This shows incidentally that the way stochastic processes are defined in the decoherent histories approach is *different* from the way they are defined in the classical theory of probability (where distribution functions are symmetric under any permutation of the indices).

the decoherence condition).[16]

Thus, it seems that we can indeed have coexistence between time symmetry (at the level of the universal wave function) and time asymmetry (at the level of the Everett worlds). The step from time asymmetry to the idea of branching time (fixed past and open future) can then presumably be justified by exploiting the relation between decoherence and the permancence of records. This formal setting will then motivate giving an asymmetric interpretation to the transition probabilities one can define. Thus the coexistence between time symmetry and time asymmetry will go exactly hand in hand with the coexistence between determinism (at the level of the universal wave function) and indeterminism (at the level of the Everett worlds).[17]

6.2 Space-time

If decoherence in the non-relativistic case can lead to a picture of branching time, in a relativistic setting it should be possible — by imposing suitable locality restrictions on the decoherence interactions — to obtain a branching structure that satisfies the axioms of a branching space-time.[18] As a specific example of such 'locality restrictions', I would suggest adopting the framework of algebraic quantum field theory, or local quantum theory, initiated by Haag and Kastler [33]. (This is another point of contact with Saunders.)

Algebraic quantum field theory is characterised by certain axioms on the algebra of 'observables', which in particular is *quasi-local*, i. e. suitably generated by algebras $\mathcal{A}(O)$ associated with open regions O of (Minkowski) space-time. These axioms embody, among other things, the local and relativistic structure of the theory, e. g. by requiring the commutativity of 'observables' associated with spacelike separated regions.

In the usual 'operational' interpretation of the theory, the local algebras represent the possible operations performable on the (global) state, such operations being constrained by locality requirements.[19] In line with the intuition that in

[16] One may further want to argue that sets of histories that decohere in both directions of time are (in some appropriate sense) uninteresting.

[17] Notice that if recoherence takes place (interference of histories), it will not imply that branching time goes over to a merging time, but merely that 'our' time is future inextendible (with respect to the background linear time).

[18] This picture is quite different from suggestions in the quantum gravity literature that (classical) space-time may emerge through decoherence. Such proposals attempt to derive space-time from a quantum theory of gravity, and are not specifically concerned with branching. See [50] for review and discussion.

[19] Haag and Kastler [33, Appendix] are adamant that the (bounded and not necessarily self-adjoint!) operators in the local algebras represent operations, and *not* observables, i. e. they are operators A_i which (suitably normalised) are used to form not necessarily projection-valued resolutions of the identity $\sum_i A_i^* A_i = \mathbf{1}$, which can be associated with probability measures on the sets I of 'outcomes', of the form $\langle\Omega|\sum_{i\in I} A_i^* A_i|\Omega\rangle$, and with corresponding families of transformations of the (global) state, of the form $\sum_{i\in I} A_i|\Omega\rangle\langle\Omega|A_i^*$ (where I is some subset of indices, and $|\Omega\rangle$ is the global state of the universe). For a general treatment of the operational approach to quantum mechanics, see e. g. [15].

the theory of decoherence we are interested in interactions that are 'measurement-like' but not necessarily part of a measurement set-up, one could reinterpret the locality axioms as constraints on the *decoherence interactions* that may take place in the universe. Indeed, this reading of the axioms could be a way of implementing Haag's suggestion [32] that the theory ought to describe *observer-independent* 'events'.

The conjecture is thus the following. If the axioms characterise decoherence interactions, any decoherent histories will be made up of local projections (in fact, of local *effects* $\sum_{i\in I} A_i^* A_i$ — the formalism calls for unsharp histores, as in [51]). The 'events' in each history will be embeddable in a Minkowski space-time. Identifying histories with Everett worlds, Everett worlds will thus have a Minkowski space-time structure.

Crucially, however, it should be possible to show that the total set of events will not fit into one simple space-time, but into a *branching* space-time. The branching space-time ought to be reconstructed from the causal structure of the decoherence events.

The background space-time of the axiomatic theory is thus not the space-time we live in (the set of 'concrete events'), but only the embodiment of the constraints on the possible sets of decoherent histories (which presumably give us sufficient grounds for postulating the background space-time in the first place).

The original motivation for the axioms was that *measurement events* should be organised in a space-time framework, irrespective of whether we understand measurements operationally, or whether we assume with Bohr that the space-time description of measurements is a precondition for the formulation of quantum mechanics. Our current motivation is to consider the (branching) space-time we live in as the collection of events *created* by the process of decoherence. Issues of locality ought to be discussed in the context of this space-time.

6.3 Locality

In the framework sketched above, two elements come together to provide a local description of quantum mechanics. First, we have the branching structure of space-time, which is (largely) determined by decoherence. Thus, we can tell whether the branching structure to the future of two measurements performed on an EPR pair in the singlet state will exhibit further splitting or not (will have four leaves or two). This process takes place at the universal level, and is fully Lorentz-invariant.

Second, we have the evolution of observers (or pieces of apparatus) monitoring the outcomes, which is such that their worldlines split whenever they reach a divergence surface.

The relation between this feature and the collapse of the wave function can be described as follows. Each leaf of the space-time can be associated with a particular outcome of the measurements. For the purpose of future predictions, the branch of an observer in that leaf will need to use only the component of the wave function associated with that outcome. In the EPR case with two leaves, this will be one of the components in the decomposition

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|+\rangle|-\rangle - |-\rangle|+\rangle), \tag{17}$$

and in the case of four leaves, it will be one of the components in a decomposition of the form

$$|\psi\rangle = \alpha|+\rangle|+'\rangle + \beta|+\rangle|-'\rangle + \gamma|-\rangle|+'\rangle + \delta|-\rangle|-'\rangle \tag{18}$$

(where primed and unprimed states correspond to different directions of spin).

Thus, the collapse experienced by the observers is only an effective collapse: parts of the wave function (which are still there at the universal level) become causally inaccessible to other branches of the observer whenever the observer crosses a divergence surface.

The description of one's leaf from the point of view of a splitting worldline can be given in three ways. (1) By mimicking the standard collapse postulate, i. e. changing the description of the state along a simultaneity plane in the instantaneous inertial frame of the observer. This reflects correctly each observer's future expectations, but (unsurprisingly) these descriptions are not Lorentz invariant. (2) By mimicking a hyperplane-dependent collapse *à la* Fleming [26], i. e. including in each description the collapses along all possible planes of simultaneity. This is the way McCall [43] describes the first stage of collapse (before 'branch attrition'). (3) By using collapse along future light cones. The latter method — adopted by Saunders [55] — corresponds exactly to distributing the different components of the wave function over the different space-time leaves, and is fully Lorentz covariant (since the leaves diverge along piecewise lightlike surfaces).

In the context of a collapse theory, collapse along the future light cone is not admissible, because in the case of perfect correlations (17), the collapses on the future light cones of the two measurement events would not necessarily match up. If they did, the collapse would be along the piecewise lightlike surface connecting the two spacelike separated events. In the present context, however, (effective) collapse along the future light cone can account for the phenomena, because all leaves of the space-time are inhabited. This is indeed our central claim about locality.

Notice first of all that whether observer B performs this or that measurement on the singlet state (equivalently, whether the decomposition (17) or (18) defines decoherent histories) will affect observer A only locally (if at all). Indeed, if B measures spin along the same direction as A (case (17), two space-time leaves), observer A will be totally unaffected, since he or she enters the two branches of that space-time already at the measurement event on his or her own side. If B performs a different measurement (as in case (18), four leaves), A will be affected only locally, by splitting a *second* time when crossing the divergence surface (future light cone) centred at B's measurement event.

The question now — similarly to the many-minds case above — is whether such a local evolution can account for the correlations observable in any particular leaf of the space-time.

A full quantitative account of less-than-perfect correlations will require providing an interpretation of the weights in the decomposition of the state (say, (18)) as probabilities for the evolution along splitting worldlines. However, we can already give a complete explanation of perfect correlations. Indeed, if there is no further splitting and the space-time has just the two leaves associated with the components $|+\rangle|-\rangle$ and $|-\rangle|+\rangle$, the mere fact that all leaves of the space-time are inhabited by some branch of an observer (if any leaf is) suffices to explain why different observers have matching results if they meet in any one leaf.

More work will be needed for a complete analysis of the Bell inequalities in this framework once a full interpretation of the probabilities is given. The bottom line, however, should be that in each leaf of the space-time the probabilities exhibit outcome dependence, but that this outcome dependence is metaphysically benign, since we can explain locally how both on the high road and on the low road me and my true love will always meet again.

Acknowledgements

I am very grateful to Tomasz Placek and to Jeremy Butterfield for the invitation to contribute to the conference and proceedings, and I am particularly grateful to Tomasz Placek for his patience in waiting for this paper. I wish to thank the following for discussions, suggestions and information: Paul Busch, Jeremy Butterfield, Jenann Ismael, Adrian Kent, Storrs McCall, Lionel Mason, Thomas Müller, Paul Teller and Bill Unruh. The ideas contained in this paper would never have developed were it not for the sustained intellectual stimulus provided by my interaction with David Albert, Meir Hemmo and Simon Saunders. Finally, I would like to thank Meir Hemmo for sending me a preprint of [36], and Storrs McCall for a copy of [43].

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