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Scientific Explanation and Scientific Structuralism

Mauro Dorato
Department of Philosophy
University of Rome 3
Viale Ostiense 234, 00144, Rome, Italy
dorato@uniroma3.it

Laura Felline
Department of Pedagogical and Philosophical Sciences
University of Cagliari
Via Is Mirrionis 8, 09123 Cagliari, Italy
felline@uniroma3.it

Abstract:

In this paper we argue that quantum mechanics provides a genuine kind of structural explanations of quantum phenomena. Since structural explanations only rely on the formal properties of the theory, they have the advantage of being independent of interpretative questions. As such, they can be used to claim that, even in the current absence of one agreed-upon interpretation, quantum mechanics is capable of providing satisfactory explanations of physical phenomena. While our proposal clearly cannot be taken to solve all interpretive issues raised by quantum theory, we will argue that it can be successfully applied to some of its most puzzling phenomena, such Heisenberg's uncertainty relations and quantum non-locality. The discussion of these two case studies will also serve to illustrate the main properties of structural explanations and compare them to the DN and the unificationist models. Finally, we briefly discuss how structural explanations might relate to structural realism.

§1 Introduction

An interpretation of the formalism of quantum mechanics that can be regarded as uncontroversial is currently not available. Consequently, *philosophers* have often contrasted the *poor explanatory power* of quantum theory to its *unparalleled predictive capacity*. However, the admission that our best theory of the fundamental constituents of matter cannot explain the phenomena it describes represent a strong argument against the view that explanation is a legitimate aim of science, and this conclusion is regarded by the vast majority of philosophers as unacceptable.

On the other hand, it is well-known that for a consistent part of the community of "working *physicists*" the question of the explanatory power of quantum mechanics does not even arise, and quantum theory is regarded as explicative (or as non-explicative) with respect to quantum phenomena as any other physical theory with respect to its own domain of application.

Granting that there is such a chasm between the attitude of the “working physicists” and the philosophers of quantum mechanics, how can we explain it? One possible answer is that physicists are instrumentalists on Mondays, Wednesdays and Fridays, and scientific realists on the rest of the days, depending on the theory they are using. However, rather than attributing physicists such an opportunistic pragmatism, could we not partially make sense of their attitude by hypothesizing that they implicitly use a different criterion for individuating what counts as an “explanation”?

In this paper we try to answer in the positive this crucial question by defending the claim that quantum theory provides a kind of *mathematical* explanation of the *physical* phenomena it is about. Following the available literature, we will refer to such explanations as *structural explanations*. In order to illustrate our main claim, we will present two case studies, involving two of the most typical and puzzling quantum phenomena, namely Heisenberg’s Uncertainty Relations and quantum non-locality.

To the extent that structural explanations rely only on the formal properties of a theory, they are obviously independent of interpretive questions concerning its ontological posits. Consequently, they justify the claim that, even in the current absence of an uncontroversial interpretation of the formalism, quantum theory, regarded as the family of its mathematical models, can provide effective explanations of physical phenomena. While structural explanations should by no means be regarded as a replacement for a sound interpretation of quantum mechanics, they can nevertheless give some philosophical support to a well-established scientific practice, one that so far has received very little attention.

Our program is therefore to reassess the traditional claim that scientific explanations are necessarily based on *physical* or *causal* models of phenomena, and to stress the explanatory force of the *mathematical models* used by quantum mechanics: as one of the precursors of this idea put it: «if we are to understand quantum theory… we will have to take seriously the idea that locating phenomena within a coherent and unified mathematical model is explanatory in itself» (Clifton 1998, p.6).

The paper is structured as follows. Sections 2 and 4 will present, respectively, two case studies, aimed first of all at illustrating the existence of *genuine* structural explanations of puzzling quantum phenomena. Secondly, these sections will flesh out Hughes' and Clifton's seminal but thin intuitions about the nature of structural explanation (see the beginning of next section). While in §3 we will defend the claim that structural explanations are not mere *redescription* of the relevant physical phenomena, in §5 we will compare the main features of structural explanations with the Deductive-Nomological (D-N) and the unificationist models. By inquiring into the possible link between structural explanations and structural realism, in §6 we will offer some sketchy suggestions for future research.

§2 Structural explanations and Heisenberg's uncertainty relations

Unfortunately, the previous literature is not very generous in offering detailed characterizations of structural explanations. Despite the highly interesting claim that in the quantum case structural explanations provide a decisive alternative to other types of explanation, in his 1989a Hughes did not offer more than a metaphor to characterize it explicitly:

> a structural explanation displays the elements of the models the theory uses and shows how they fit together. More picturesquely, it disassembles the black box, shows the working parts, and puts it together again. "Brute facts" about the theory are explained by showing their connections with other facts, possibly less brutish. (Hughes, 1989a, p.198).

To be honest, by discussing the case study of the EPR correlations, Hughes tried to give a concrete and clear illustration of what it means "to disassemble" a black box and put it together again. However, in order to characterize a structural explanation, it seems to us more useful to draw attention to the already quoted, unpublished paper by Robert Clifton[1], in which we find the following sketch of a definition:

> We explain some feature B of the physical world by displaying a mathematical model of part of the world and demonstrating that there is a feature A of the model that corresponds to B, and is not explicit in the definition of the model. (Clifton 1998, p.7).

Consider as a first case study Heisenberg's famous Uncertainty Relations between position (x) and momentum (p) as our *explanandum* B. These relations are usually taken to entail that the values of the corresponding magnitudes are not simultaneously sharp, independently of limitations of our knowledge about the system. In a less metaphysically committed language, we could simply say that there is a limit to the simultaneous predictability of position and momentum. Why?

We will take as our *prima facie* formal representative A of the non-simultaneous definiteness of the two incompatible observables B the well-known equation:

$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2} \tag{1}$$

The explanation of such a relation usually invokes the description of a typical 'experimental setting' (a thought experiment, really), namely, the measurement of the position of an electron by the so-called Heisenberg's microscope. Heisenberg's account of this thought experiment[2] made use of a qualitative argument, according to which, due to its impact with a gamma ray generated by the microscope "[a]t the instant of time when the position is

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[1] Clifton appropriates, with some modifications, Hughes' (1993) definition of *theoretical* explanation.

[2] The inequality derived by Heisenberg was not exactly the one given in (1). Heisenberg originally derived the formula:

$$p_1 q_1 \approx \hbar \tag{2}$$

The standard formulation was for first proved by Kennard, in 1927.

determined, that is, at the instant when the photon is scattered by the electron, the electron undergoes a discontinuous change in momentum" (Heisenberg, 1927, pp. 174-175).

However, Heisenberg's 1927's derivation was quite confusing. For instance, if the meaning of an observable is determined, as he claimed, by the measurement apparatus, the crucial notion of 'imprecision' becomes quite *vague and contextual*, as it receives different meanings in different experimental situations. For measures of position, such imprecision corresponds to the limited resolving power of the microscope, while the imprecision of the observable momentum is due to the unpredictability of the behaviour of a particle *after* a measurement of its position, causing an unknown recoil due to highly energetic light. Since in Heisenberg's philosophy physical concepts are defined operationally, it follows that the 'imprecision' of position and momentum receive two different meanings.[3]

The confusion in dealing with the notion of uncertainty continued also after Heisenberg's first formulation, so that the Uncertainty Principle is nowadays typically defined as a bound to the degree to which the results of the two measurements are predictable. Moreover, the oversimplified character of this quasi-classical picture of a collision, and well-known complications of the standard account of measurement, renders a clear physical account of these relations quite difficult.

Given this state of affairs, if the availability of a clear physical interpretation were a necessary condition for having some insight into the relevant physical phenomena, the Heisenberg's Uncertainty Principle ought to be considered a mystery. However "working physicists" (and not just them) do not regard such a Principle as unintelligible.

In the reading we propose, this attitude of the working physicists is justifiable by the fact that the mathematics of quantum theory provides a solid *structural explanation* of the relations in question. Among physicists the better-known explanation of the position-

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[3] For a more detailed reconstruction of Heisenberg's relations, see Uffink (1990, p. 96).

momentum relation is *analytic*, insofar as it appeals to Fourier's analysis, and shows how the formal representative $\Psi(p_x, p_y, p_z)$ of the momentum of the electron is the Fourier transform of the function $\Psi(x, y, z)$, which formally represents the coordinates specifying the position of the particle. In this functional-analysis kind of approach, the structural explanation of Heisenberg's relations exploits a well-known *mathematical property* of the Fourier transform, on the basis of which the narrower the interval in which one of the two functions differs significantly from zero, the larger is the interval in which its Fourier transform differs from zero, in such a way that equation (1) must be satisfied.

The Uncertainty Relation between position and momentum, therefore, is understood as a direct consequence of the *mathematical*, formal properties of the Fourier transform. In other words the existence of a minimum for the product of the uncertainties of these two measurements, or the *physical*, non-simultaneous sharpness of the two observables, is explained first of all by showing how quantum systems are represented within quantum theory (i.e. by the model M of Hilbert spaces of square summable functions) and secondly by showing how in such a model $\Psi(p_x, p_y, p_z)$ is the Fourier transform of the function $\Psi(x, y, z)$. From these two assumptions, and in virtue of the mathematical properties of the Fourier transform, not only does it follow that $\Delta x \cdot \Delta p \geq \frac{\hbar}{2}$ (the formal representative of the physical explanandum) is also an element of M, but also that the *explanandum* must possess the required properties.

In the reading we propose, therefore, the properties of *the explanandum* are constrained by the general properties of the Hilbert model *M*. In this sense the *explanandum* is made intelligible *via* its *structural similarities* with its formal representative, the *explanans*. Given the typical axioms of quantum mechanics (for instance, the typical correspondence between observables and Hermitian operators, physical states and rays of the Hilbert space, a probability and a scalar product, etc.), any quantum system exemplifies, or is an instance of, the formal structure of the Hilbert space of square summable functions.

To be sure, there are many features of the Uncertainty Principle that are still object of dispute, and by claiming that there is a structural explanation of the position/momentum Uncertainty Relations we don't mean to imply that these problems have all been solved.[4] However, physicists hold that, despite the lack of a unanimous interpretation of the physical processes leading to the limits in the predictability of the results of the measurements of position and momentum on the same particle, *such unpredictability is indeed explained by quantum mechanics*.

It follows that structural explanations provide a common ground for understanding the *explanandum* in question, independently of the various different ontologies underlying the different interpretations of quantum theory. Even under the pessimistic assumption that the *explanandum* could not be given an interpretive account in terms of a future, precise ontology, we can still claim that quantum theory is capable of explaining Heisenberg's Uncertainty Relations.

§3 A crucial objection: are structural explanations *genuine* explanations?

However, how can a *mathematical* model be explicative of a *physical* phenomenon in the first place? According to what one could call the Principle of the Explanatory Closure of the physical world, *only* a physical fact or a physical law should be allowed to function as *explanans* of another physical fact or event. In order to understand how such a Principle can be violated, it is necessary to consider that what enables us to say that the structural explanation provided *via* a mathematical model *M* is about a *physical* fact is the existence of a

[4] For some of these problems, we refer the reader to the work already cited in note 3.

relation of representation between *M* and the physical explanandum. Such a representational aspect of *M* is obviously *necessary*, since otherwise the structural account provided by the model would be completely "unanchored" to the physical world. Without a representational or referential relation of some kind holding between *the formal explanans and the physical explanandum*, no explanation of a physical, concrete phenomenon could be forthcoming, since we would simply explain an abstract fact *via* another abstract fact.

Consequently, and more in general, at the basis of the possibility to transfer knowledge from the abstract mathematical model to a physical target, or to perform a so-called *surrogate reasoning* (Swoyer 1991), we claim there is the existence of a representation relation between the model and its target system.

In order for such a representational relation to be also sufficient for a structural explanation, however, we have to accept the idea that *we understand the physical phenomenon in terms of the properties of its formal representative, by locating the latter in the appropriate mathematical model*. This sufficiency, we take it, has been illustrated in the previous case study. The explanatory character of the mathematical/formal features of the Fourier functions *vis à vis* Heisenberg's Uncertainty Relations has to do with the fact that such features are exactly those that are required to make intelligible the relevant, represented physical phenomena. As such, they are an answer to the following "why question": *why do position and momentum not assume simultaneously sharp magnitudes?* Answer: *because their formal representatives in the mathematical model have a property that makes this impossible*. Consequently, the physical fact that the greater is the precision with which we measure one magnitude, the more undefined is the value of the other, being structurally equivalent to formal features of the Fourier functions in the mathematical model of the physical system, is made intelligible by this very model. Our claim is therefore that the existence of structure-preserving morphisms from the Hilbert space associated to the physical system to the structure of physical "relations" (observables) characterizing the system ensures that properties of the physical world can be made intelligible by properties of the model.

Clearly, the specification of what is required for a successful representation by scientific models is subject to a great deal of current philosophical research, which here can simply be mentioned.[5] Under the structural realist view of scientific representation, however, one typically requires that *the physical system be a concrete example,* or *a concrete instance of*, *the abstract mathematical model.* For example, the use of Minkowski's spacetime model to explain geometrically the phenomenon of length contraction is justified by the fact that bits of the physical world (say, electromagnetic phenomena) exemplify the abstract spatiotemporal structure postulated by the model. Accordingly, it is the assumption that quantum systems exemplify (relevant parts of) the structure of Hilbert spaces of square summable functions that allows us to use properties of the latter in order to explain properties of the former.

This claim, nevertheless, could still not clarify all possible doubts. What we have described so far as a structural *explanation*, someone could note, is really a mere *translation* or redescription in the convenient language of mathematics of the truly explanatory account, to be given in terms of those entities or processes which constitute the physical or the *categorial framework* of the theory.[6] To be concrete, think of a balance with eight identical apples, three on one pan and five on the other. If someone explained the dropping of the pan with five apples (or the raising of the side with three) by simply saying "5>3", he/she would not have provided a genuine explanation.[7] The side with five apples drops because it is heavier and because of the role that its gravitational mass has *vis-à-vis* the earth, not because 5 > 3!

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[5] For two recent essays, see Debs and Redhead 2007 and Van Fraassen 2008.

[6] A *categorial framework* is the set of fundamental *metaphysical* assumptions about what sorts of entities and processes lie within a theory's domain (see Hughes, 1989b, pp.17-176).

[7] This objection is due to Jim Brown, and was addressed to one of us during a presentation of a previous version of this paper at the first European Philosophy of Science Association (EPSA) conference, held in Madrid in 2007.

In one sense, of course, the above point is correct. In this example, it is the physical property weight that does the explanatory job. Consequently, one *could* explain the tipping of the scale by simply saying: "look, there is *more* weight here than there": consequently, it would not be the structure of the natural numbers that works as the *explanans*, but the fact that there is a force that causes one pan to drop. But even in this example, if we have eight identical apples, a *quantitative* rather than a merely *comparative* explanation of the tilting of the scale must *also* rely on the fact that, out of eight identical apples, *5 apples weigh more than 3 because 5 > 3*. This simple example shows that arithmetic enters the explanation, depending on the context, and depending on the kind of information we want. Typically, in physics we are interested in *quantitative* descriptions of phenomena, so that often the descriptions afforded by mathematical models are explanatory.

More in general, we claim that structural explanations are not so easily translatable into non-mathematical terms without loss of explicative power. It is to be noted, in fact, that the use of mathematical models as *explanans* offers various advantages that in a non-mathematical explanation would be lacking. On the one hand, and in virtue of the postulated Representation Relation, it enables us to exploit the more solid knowledge that we have of the model *as if it were knowledge of the structural relations of the target.* On the other hand, the abstraction of the mathematical representation lets us carry on our reasoning independently of the unknown properties of the target (see Pincock, 2007), in this case the categorial, intrinsic nature of the quanta.

Finally, it is important to clarify that the usefulness of mathematical explanations is not just a *faute de mieux* due to our ignorance of the exact ontology of the quantum world. In order to discard also this objection, consider again the case of the Uncertainty Principle. One, more general algebraic explanation, *valid for all pairs of non-commuting observables* and not just for the position-momentum relations, typically involves the non-simultaneous diagonalizability of the matrices representing non-commuting Hermitian operators, and therefore relies on *the non-commutativity of the algebra of observables related to a quantum system*.[8] Due to its greater generality, this algebraic explanation is not only shared by the different interpretations of the formalism, but is also common to the different Uncertainty Relations holding between the various non-commuting observables (time and energy, spin in different directions).

What kind of non-mathematical explanation could be given in place of the non-commutativity of the algebra of the quantum observables? Even granting the possibility of a physical or causal model of the position/momentum uncertainty relation, an all-purpose physical explanation (presumably in terms of a common mechanism), common to all non-commuting observables, seems hardly plausible. Just to give an example, think of the possibility of a common mechanism or physical process explaining equally well both the position/momentum and the spin-x/spin-y relations. It seems reasonable to believe that, in view of the difference in the relevant phenomena, the only possibility to meaningfully translate a general structural explanation into a physical one would be to provide different accounts for each Uncertainty Relation holding between two non-commuting observables. Needless to say, such different causal accounts would lose much of their explanatory power, in contrast to the single unifying universal mathematical feature offered by non-commutativity. We will come back to this unificationist feature of structural explanations in due course, when analyzing the difference between the latter and other theories of explanations, in particular the D-N and the unificationist model.

For now, we will close this section by noting that structural explanations seem important especially in those areas of physics that are very remote from the world of our experience. And the quantum world is so distant from the manifest world in which we evolved during

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[8] Within the account we propose, this plurality of explanations only apparent, since the *explanans* is the same modulo isomorphisms.

millennia that there is no a priori reason that it should obey the same principles. Consequently, attempts to apply to it "classical" categories like "causation" (or "property" or "substance" and the like) might simply fail for ever, a claim that will be further illustrated by our next case study.

§4 Entangled states and non-locality

In the introduction we have argued that one of the aims of a theory of structural explanation is to reduce the chasm between the physicists' explanatory practice and the philosophers' gloomy analysis of the explanatory virtues of quantum mechanics. The next case study we shall present is aimed exactly to achieve this goal, by showing how the hypothesis that physicists make more or less tacit use of structural explanations accounts quite well for their unproblematic attitude towards one of the most typical quantum phenomena, namely non-locality or non-separability.

Let instances of non-local phenomena be the *physical explanandum* B, and let non-factorizable, entangled states be their *formal* counterpart A in the mathematical model M. In virtue of what we already specified in the previous section, the non-factorizable states *represent* physical systems exhibiting non-local behaviour, say, correlated measurement outcomes in EPR-Bohm types of experiments, regarded as concrete physical events.

In order to realize how quantum theory explains non-local correlations *structurally*, one must consider once again the models with which the theory represents physical systems. By representing quantum systems *via* the well-known formalism, quantum theory 'invites us' (Hughes, 1997) *to see* the former *as* Hilbert spaces, and this allows us to perform surrogative reasoning about the physical systems themselves. In particular, since *composite physical systems* are formally represented by the *tensor product of the Hilbert spaces* representing each separate quantum system, such surrogative reasoning invites us to look at measurement outcomes of correlated systems as joint elements of such tensor products.

Now, it can be proved that a model M exemplifying the structural properties typical of the quantum (mathematical) description of the microscopic world must be such that some of its states are entangled. Entangled states can explain non-local behaviour structurally in virtue of the following two facts:

i) *M* − which, together with its structural properties, constitutes the *explanans* of the nonlocal behaviour of quantum systems − obeys the *principle of superposition*, a crucial formal feature that ensures that the sum of vectors of the Hilbert space (physical states) is also a vector of the space (a physically possible state);

ii) some such superpositions of state vectors in the tensor products of the Hilbert spaces associated to subsystems of a composite system *cannot* be written as a tensor product of any of the state vectors belonging to the component Hilbert spaces. This crucial feature, known as entanglement or non-factorizability, is the key to explain non-locality, since it is responsible for the peculiar holism of the quantum world.

It could be objected that these two formal features, taken by themselves, do not suffice to provide a full explanation of the correlations of measurement outcomes in experiments with, say, particles emitted in the singlet state:

$$\frac{1}{\sqrt{2}}(|\uparrow\rangle_1|\downarrow\rangle_2 - |\downarrow\rangle_1|\uparrow\rangle_2) \quad (4)$$

For this purpose, so the objection continues, we also need

iii) the law of conservation of intrinsic angular momentum, and additional information on the initial state of the emitter of particles.

However, the crucial claim that we want to put forward is that whatever goes on in iii) is necessary but not sufficient to explain non-locality,[9] and that *reference to the formal structure of the theory*, i.e., to i) and ii), *is needed essentially*. The physical *explanandum*, corresponding to the existence of non-local correlations across *spacelike* separated regions, is understood in terms of the properties of its formal representatives in the model by realizing that non-factorizable states in the Hilbert space M, *qua* formal counterpart of non-local measurement outcomes, *share* some essential properties of the latter. In particular the tensor product formalism implies that any possible (definite) outcome on one side of an EPR-Bohm experiment is inseparably linked to another (definite) outcome on the other side.

This is clear in singlet states for a spin ½-particle as expressed in (4). The mere formal structure of the singlet state makes it clear that, in the hypothesis of completeness, before measurement the fermion does not have any definite spin in any direction. If it is the act of measurement that creates a definite "element of reality" in one of the two spacelike-separated wings of the experiment, the spin pertaining to other side must, in virtue of the mere existence of anticorrelated states of this sort, be instantaneously "determined" in a non-local fashion, independently of whether such "non-locality", or such "determination", admits or does not admit a causal interpretation.[10] Since, after measurement, one of the two possible outcomes in an entangled state like (4) must be observed, the properties of the formal state (4) constrain those of the physical systems that they represent and, in cases of measurements apparatuses that are spacelike separated, imply by themselves non-local behaviour.

In sum, the non-separability or non-locality of the physical outcomes is understood in terms of the fact that that before measurement the joint state has no definite property, that each separate tensor product in (4) represents the two possible outcomes, and that the two

[9] As we will see in more details in the next section, this also marks the difference between structural explanations and the D-N model of explanation.

[10] For instance, according to the Bohmian interpretation, non-locality has a causal reading, corresponding to action at a distance.

elements of each tensor product cannot be separated, and *have* to occur always together, given the way that composite systems are formally represented. The fact that entangled states are independent of distance is then sufficient for non-locality.

A couple of striking analogies with the previous case study should be noticed. First of all, the explanation just offered *is* valid across all the various interpretations of quantum theory, and it is therefore wholly independent of any of them.[11] Furthermore, *if* one rejects (along with Fine 1989) the possibility of a *causal* explanation of non-local behaviours in quantum theory, then any interpretation of the formalism of the theory can hardly add any significant explanatory information to that already provided by the mathematical model of the phenomenon.[12]

Can we justify the latter antecedent? We claim that the possibility of regarding quantum non-locality as not needing a causal explanation is grounded in the conceptual and explanatory switch required by all major scientific revolutions. Exactly as, before Galileo, we thought that inertial motion required a causal account and then we discovered that it didn't require one; and exactly as, before Einstein's special relativity, we thought that the fact that light seemed to have the same speed in all inertial frame needed a dynamical/causal explanation, and then we discovered that it didn't need one; and exactly as before Einstein's general relativity, we thought that free fall required a force and therefore a cause, and then we discovered that it didn't require any cause; also after the quantum revolution, we may have to regard quantum non-locality as explanatory primitive or fundamental and therefore non-caused, instead of trying to use old, classical causal categories in order to understand it. What

[11] A possible exception is Everettian interpretations, where non-locality could be doubted in virtue of the fact that all outcomes are simultaneously realized. However, relative to a single branch or world, there must be a non-local correlation.

[12] If one adds the physical law and the initial conditions mentioned in (iii).

needs to be explained causally is rather the loss of coherence, or the non-entangled nature of the macroscopic world.[13]

Secondly, also in this case study there exists another, more abstract and general way to explain non-locality structurally or formally *vis-à-vis* classical separability, one that has been illustrated by John Baez (2006), and that hinges on the formal difference between the properties of the tensor product in the category "HILBERT SPACES" and the Cartesian product in the category "SET". In this explanatory approach, involving category theory, the classical intuition that a joint system can be accurately described by specifying the states of its parts corresponds to, or is denoted by, the mathematical properties of the Cartesian product: if the set of states of the first system is S and that of the second is T, the joint system has the Cartesian product $S \times T$ as its formal counterpart, where $S \times T$ is the set of all ordered pairs (s, t) such the first member s is in S and t is in T.

In order to make the idea of Cartesian product applicable in category theory and generalize it, Baez introduces projection functions (morphisms) from the set $S \times T$ to its components:

$$p_1 : S \times T \to S \qquad\qquad p_2 : S \times T \to T \tag{5}$$

such that «for any set X and any function (morphism) f_1 and f_2, with $f_1 : X \to S$ and $f_2 : X \to T$, there exists a unique function $f : X \to S \times T$ such that $f_1 = p_1 f$ and $f_2 = p_2 f$» (Baez 2006, p. 256). Figure 1 illustrates the definition by showing how, by composing f with the two projections, we get, respectively, f_1 and f_2.

[13] For a full justification of this claim, we refer the reader to Dorato (2009).

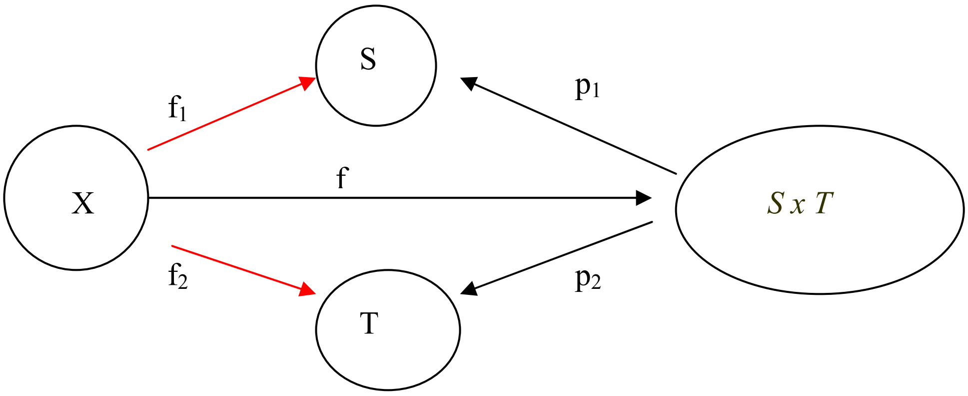

Figure 1

With such a generalized definition of (Cartesian) product, one can propose a comparison with the category given by Hilbert spaces, and give a somewhat *more general* structural explanation of non-locality. In the category of Hilbert spaces (HILBERT), the formal representative *A* of the joint state of a composite physical system (our explanandum *B*) is still the tensor product of the component Hilbert spaces, which does *not* obey the condition stated above for the product in the category SET. In particular, given two Hilbert spaces *H* and *K*, if $H\otimes K$ is their tensor product, then there are *no* morphisms (linear operators) p_1 and p_2 that project pure states in $H\otimes K$ onto pure states in the component states:

$$p_1 : H\otimes K \rightarrow H \qquad p_2 : H\otimes K \rightarrow K \quad (6)$$ [14]

It could be noted that the formal language employed in this example is not a mere redescription of the non-factorizability of the states in $H\otimes K$ that was alluded to before, but yields a *different* way of understanding it, even though the formal representative A of nonlocal states is still the tensor product. *The difference in understanding depends exclusively on the fact that the same formal representative is embedded in different models, and therefore*

[14] See Baez 2006, pp. 259.

possesses different structural properties, in such a way that in order to produce an explanation, different procedures are required.

However, even the existence of two different structural explanations, if it were to be conceded, would not represent an embarrassment for our account of structural explanation. The fact that the language of n-category theory can provide a framework to unify – formally at least − Hilbert spaces with n-cobordisms and therefore with the mathematical language of general relativity, may shed new future light on non-locality regarded as a physical phenomenon.

§5 Structural explanations, the DN model, and explanations by unification.

The illustration of the above case studies naturally leads to a further question: it could be argued that *locating* the formal representative of the explanandum within the mathematical model of the theory M is always equivalent to a *deduction* of the sentence expressing the *explanandum* from a set of sentences expressing the initial conditions. This deduction would involve laws of nature, in our two cases, the conservation of angular momentum for non-locality or Heisenberg's Principle itself for the Uncertainty Principle. If what we call "structural explanations" ended up being a mere relabeling of the D-N model, then a theory of structural explanation would in effect be superfluous.

However, as already implicitly shown in our former discussion, the D-N model cannot actually cover the presented case studies: when a phenomenon is explained structurally, the purely *mathematical* features of the model become essential, while laws of nature and initial conditions might be necessary but insufficient for the explanation. Consider the latter example discussed above. We claimed that the non-separable character of the measurement outcomes is a consequence of the existence of non-factorizable states in the Hilbert space model of the theory, but these formal properties do not also express, strictly speaking, a law of nature. In a word, even though the physical system *exemplifies* some structural features of the mathematical model, the formal representative A of the physical *explanandum* B, a mathematical property like the tensor product, or the non-existence of morphisms that project pure states in $H \otimes K$ onto pure states in the component states H and K does *not* denote physical laws, even though it carries the explanatory weight.

The same remark applies to the former case study. We have seen how, in order to explain the position/momentum uncertainty relation, we have used the Fourier transform's rule, according to which the narrower is the interval in which $\Psi(p_x, p_y, p_z)$ significantly differs from zero, the larger is the interval in which its Fourier transform $\Psi(x, y, z)$ differs from zero, and conversely. Obviously this "mathematical law", analogously to the more general non-commutativity of an algebra of observables, represents a physical fact. But even if the represented fact were a physical regularity, in the structural account of Heisenberg relations, as well as in the non-locality case, *it is the physical regularity that is explained/understood in terms of the mathematical fact*, *and not the other way around.*

A second argument separating the D-N account from the theory provided here is implicit in the following quotation:

> "if one believes (as I do) that scientific theories […] provide explanations, then one's account of explanations will be tied to one's account of scientific theories" (Hughes, 1989a, p.257)

While structural explanations are a natural by-product of the *semantic* view of theories, the natural environment for the D-N model is the 'received', *syntactic* view of theories, and it is not evident how the latter view can be consistently adapted into the framework of the semantic view.

In the previous section we have stressed the importance of unification as evidence that the structural accounts of Heisenberg's Relations in terms of non-commutativity or n-category theory are genuinely explanatory. To be sure, generality and scope are important virtues of structural explanations (remember that one of the crucial advantages of structural explanations is that they are independent of the non-structural properties of a system, and therefore of the specific interpretation of quantum theory one advocates); however, the thesis that structural explanation is merely explanation by unification, or that unification is a necessary feature of every structural explanation, need not follow, and is refuted by the first of our case studies. The explanation in terms of the Fourier's transform, in particular, is not achieved via a unification *per se*, and yet it is explanatory in virtue of the commonality of relations that the represented physical systems and the model exemplify.[15]

On the other hand, the unificationist models of explanation suffer from the well-known difficulty of defining what counts for unification, a difficulty that is solved by structural explanations: the unification of diverse physical phenomena sharing the same mathematical structure is a unification enabled by the possibility of regarding all of these phenomena as different exemplification of the same structural features characterizing the mathematical model.

Finally, notice that to the extent that traditional unificationist models are committed to the possibility of logically deducing the explananda from the unifying laws of nature or from those of the reducing theory, the structuralist view is not likewise committed. Infact, the previous examples should have convinced the reader that a representation relation between the explanans and the explanandum may suffice (Fourier transform's explanation of the position-momentum relations, or the tensor product explanation of non-locality), and that in structural explanations the unification is not achieved in a syntactic fashion.

[15] As a fact of sociology, it could be guessed that while the philosophically educated person tends to prefer the most unifying algebraic approach as the most explanatory, this attitude is far from being typical among physicists, who, in explaining the momentum/position Uncertainty Relation, more often than not rely on the less general analytic explanation.

To conclude, what said above is not meant to deny the existence of deep analogies between unificationist theories of scientific explanation and structural explanations. Unification is surely a virtue of any explanation (whether structural or causal); what we deny is simply that the received unificationist accounts can be regarded as sufficient to explain quantum phenomena.

§6 Structural explanations and structural realism

Is there any connection between the effectiveness of structural explanations and the current discussions on structural realism? In this final section we will lay down our cards on the table for further inquiry into this issue.

An often heard complaint about the blossoming literature on structural realism is that, so far, no clear definition of what a *physical structure* is available. Besides the clear distinction between *epistemic* and *ontic structural realism* − where the main divide is, respectively, whether all we can know is structure, or rather that we can know only the structure of physical entities since structure is all there is − there remains the crucial problem of clarifying once and for all what is this physical structure that theories regarded as mathematical models are meant to capture.

However, if the key idea to make sense of a physical structure is to think of it as *a net of physical relations*, the kind of net and the type of relations instantiated by physical systems must necessarily depend on the particular mathematical model one is working with. So there might simply be no question of *the* physical structure, or of what a physical structure is, since these questions are contextually dependent on particular mathematical models. This "contextualism" fits in well with the claim, illustrated by our two case studies, that one could have different structural explanations of the same physical phenomenon simply by locating its formal representative (the uncertainty relations between position and momentum for the first case study, the tensor product for the second) in *prima facie different* mathematical models (the space of L^2 functions and a non-commutative algebra of observables on the one hand, an Hilbert space and n-category theory on the other hand). It is clearly always possible to discover that the two explanations are equivalent, either because the two models are isomorphic to each other, or because the more abstract model contains the less abstract as a particular case.[16]

If this is correct, a particularly clear formulation of structural realism advocates the *primacy* of *relational* properties over *intrinsic* properties of physical entities. Now, there are various ways to analyze the crucial notion of "primacy". One involves the *identity* of objects: relational properties are more important than intrinsic properties in *defining* what an entity is. A second, more *ontological* and quite radical way of understanding the idea of primacy is to deny the existence of entities *tout court* (Ladyman 1998). A more moderate form of ontic structural realism consists instead in trying to reconceptualize all physical entities as bundle of relations which instantiate second-order relations with other "entities". This less radical form of ontic structural realism claims that there *are* indeed relata or entities, but that there are no intrinsic or monadic properties of physical entities, since all entities have extrinsic or relational properties (Esfeld and Lam 2008).[17]

A *third,* so-far unexplored way to capture this idea of *primacy*, one that is much more natural in the context of this paper, involves our *explanatory practices*: the idea is that structural properties of entities are *explanatory more important than their intrinsic properties. Is a claim about the explanatory primacy of relations sufficient to endorse some form of structural realism?*

[16] In this paper, we have not further studied these possibilities.
[17] This latter conception is committed to regard mass, charge and spin as extrinsic rather than intrinsic properties of particles, a claim that, on the face of it, looks quite implausible.

To the extent that explanatory power is an epistemic virtue, one could argue that this third way is close to a form of epistemic structural realism. However, we don't think that the effectiveness of structural explanations of physical phenomena can be used to defend *epistemic* structural realism or *ontic* structural realism of various sorts (radical or moderate). From our thesis that relational properties are explanatorily central in contemporary physical theories it need not follow, in fact, that we can only know relational properties of objects, or that the nature of things will always be hidden to us (epistemic structural realism). Nor does it follow that we can use (epistemic or) ontic structural realism in order to *justify* or explain the centrality of structural explanations in our explanatory practices: inferences to best explanations, i.e., attempts at explaining structural explanations, are a risky business.

Finally, it seems to us that the importance of structural explanations cannot used to decide between epistemic and ontic structural realisms, since both camps acknowledge the existence of physical webs of relations, and this recognition alone is sufficient to account for the genuine explanatory character of structural explanations. Since the advantage of structural explanations lies in stressing the explanatory information that is common to various interpretations of quantum theory, it would be surprising if it could contribute to solve metaphysical issues pertaining to question of scientific realism.

In conclusion, there are only two aspects of structural explanations that might be relevant for structural realism: (i) the claim that the *physical* world and the *mathematical* world share the same structure, in the precise sense that data models of the former are isomorphic to theoretical models belonging to the latter,[18] and (ii) the claim that such structure is explanatory central. As stressed in the previous part of the paper, the requirement *Representation* is the key to ensure the fact that mathematical structures refer to the physical world and can explain it. However, while the existence of a Representation relation between

[18] For an articulation of these claims, see Suppes 2002 and Dorato 2005.

model and world is required to guarantee the effectiveness of structural explanations, it cannot be regarded as committing us to any of the known forms of structural realism, neither is it sufficient to exclude other forms of scientific realism (entity realism or theory realism).[19]

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[19] Entity realism commits us to the existence of entities endowed with intrinsic properties, while theory realism commits us to the (approximate) truth of empirically successful laws or theories.