Miraculous Success? Inconsistency and Untruth in Kirchhoff's Diffraction Theory[1]

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Kirchhoff's diffraction theory is introduced as a new case study in the realism debate. The theory is extremely successful despite being both inconsistent and not even approximately true. Some habitual realist proclamations simply cannot be maintained in the face of Kirchhoff's theory, as the realist is forced to acknowledge that theoretical success can in some circumstances be explained in terms other than truth. The idiosyncrasy (or otherwise) of Kirchhoff's case is considered.

> The sole virtue of Kirchhoff's theory of diffraction lies in its correct predictions and not in its false assumptions.
>
> (Andrews 1947, 784)

1. Introduction. Scientific realists seek to establish a link between theoretical truth and predictive success, suitably understood. Different realist strands can be discerned by asking how theoretical truth on the one hand, and predictive success, on the other, are to be understood. What sort of link holds between the two? This paper introduces a new case-study pertinent to the above questions: we adduce some facets of Gustav Kirchhoff's diffraction theory that call for a more nuanced treatment of the central connection between predictive success and truth.

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Much realist ink has been spilt over the anti-realist challenge regarding the high level of predictive success of (what appear to be) grossly false theories. Ether theories of light have played a justifiably central role in the debate: their predictive triumphs are prima facie rather surprising by realist lights. We believe that realists got the upper-hand in this dispute some time ago; in the ensuing discussion the devil has been purely in the detail. For example, it has become apparent that surprisingly subtle manoeuvrings are required to properly draw out the correspondence that holds between Fresnel's theorising and our current understanding of light, and there are still open questions about the most appropriate level of realist commitment in some other cases. But all this is relatively minor tinkering and disagreement within the realist camp, as most realists agree that there is a sense in which Fresnel's success can be fully accounted for in terms of what Fresnel actually got right (by the present lights). Whichever streak of "selective realism" one prefers, arguably our best current theory says the very same things about the world in those *relevant respects* that are explanatory of the past theory's success in the realist sense.[2] For example—just to illustrate by mentioning one option—one might argue that the ether scenario on which Fresnel's success is built instantiates *exactly the same* critical (higher-level) properties as the corresponding Maxwellian scenario (Saatsi 2005). Although we cannot take at face value Fresnel's description of this scenario, ether and all, we can nevertheless take seriously the equations employed in his derivation. For those very same equations, minimally interpreted, describe the relevant properties of the electromagnetic field.

[2] Selective realism comes in many variants: *structural realism* (Worrall 1989), *semi-realism* (Chakravartty 1998), *divide-et-impera* (Psillos 1999), *eclectic realism* (Saatsi 2005). The whole stratagem has been accused of relying too much on hindsight (e.g. Stanford 2006). Although we believe that this accusation can be rebutted (cf. Saatsi forthcoming), we shall not tackle this contentious issue here.

Regardless of which brand of selective realism one settles on, the case-study introduced here is problematic since it doesn't fit the required mould. The fact that Gustav Kirchhoff (1824–1887), like Fresnel, was operating in the ether paradigm is not problematic *per se*. By employing the ether-wave conception of light Kirchhoff (1882) derived a celebrated equation in the scalar diffraction theory of optics, describing the behaviour of light with remarkable accuracy. The predictive accuracy achieved is *prima facie* amazing for two reasons: it turns out that Kirchhoff's derivation turns on crucial assumptions regarding the *amplitude* of light waves that (i) *differ considerably from the actual situation* (as described by Maxwell's equations, for example) in various respects, and (ii) as a matter of fact are *inconsistent*. The selective realist cannot explain this success by pointing out the fact that 'ether' in Kirchhoff's theorising referred to an idle metaphysical posit that didn't play a role in the actual derivation. The problem simply is that there is no appropriate correspondence between Kirchhoff's theory and our best current understanding of diffraction at the level of success-fuelling properties, "structure", or whatever the selective realist might attempt to capitalise on.

Kirchhoff's feat has not gone unnoticed in the physics literature; quite the contrary—physicists and mathematicians have carefully analysed Kirchhoff's theory in order to understand what makes it tick. Examining the theory more closely yields an understanding of its success, but the case is fundamentally different from many other successful ether theories: the selective realist is led intolerably astray if she optimistically commits to those premises of Kirchhoff's derivation that are responsible for its success. It is impossible to view Kirchhoff's theory as approximately true in any reasonable sense, *even if* the derivation is construed in *contemporary realist terms* that ignore the assumption that the ether is the bearer of light waves.

Kirchhoff's case demonstrates how it is possible to derive highly accurate predictions from misguided and even inconsistent premises. This undermines a certain (implausibly) strong form of realism, which we outline in the next section under the label "Naïve Optimism". Thereafter in §3 we introduce Kirchhoff's theory, before we highlight the specific details of the theory which act to undermine naïve optimism in §4. In §5 we consider the wider consequences for realism, and in particular for less naïve forms of realism. Any such form of realism must allow that sometimes the best explanation of success is not in terms of truth. We will argue that this need not entail the demise of realism, as long as Kirchhoff's case can be viewed as having idiosyncratic features that do not generalise across the rest of science. §6 is the conclusion.

2. The Naïve Optimist. Part of the debate between realists and anti-realists turns on how best to explain the success of a particular theory. Realists adhere to their intuition about the low likelihood ('miracle') of a (duly) successful theory that is not even approximately true. There are difficult questions to be answered about how the relevant likelihood should be conceived (Magnus and Callender 2004, Psillos 2006). We do not wish to engage in this debate here. Rather, we are concerned with the more specific question whether the realist should expect *every* instance of novel success to be explained by the truth content of the key theoretical assumptions. Let's call those who answer this question in the affirmative *naïve optimists*.

It is not clear why any realist would *a priori* deny the possibility of some actual predictive successes being explainable in terms other than truth. After all, the possibility of there being *some* such successes is not inconsistent with the realist credo that the best explanation of the success of science *on the whole* is that theories latch onto unobservable reality *by and large*. Although it is difficult to make this precise in probabilistic terms, the general thought is clear enough: there is no clear motivation for any realist to insist *a priori* on a connection between success and truth that allows *no leeway whatsoever*. Most realists, we believe, would concur.

Nevertheless, the Naïve Optimist position is not a complete strawman. For example, Leplin's classic characterisation of realism included the clause 'The (approximate) truth of a scientific theory is *the only possible explanation* of its predictive success' (1984, 1, our emphasis). And Psillos and Ladyman carry this spirit on by effectively talking about the possibility of a *counter-example* to the realist's no-miracles argument:

> [A]t least *some* past theories which pass both realist tests of maturity and success are nevertheless considered false. […] If these theories are false, despite their being distinctly successful and mature, then the intended explanatory connection between empirical success and truthlikeness is still undermined. (Psillos 1999, 108)

> Even if there are only one or two [problematic] cases, the realist's claim that approximate truth explains empirical success will no longer serve to establish realism. This is because, where a theory is empirically successful, but is not approximately true, we will need some other explanation. If this will do for some theories then it ought to do for all, and then we do not need the realist's preferred explanation that such theories are true. (Ladyman 2002, 244)

Surely matters are not so clear-cut, however. It is quite conceivable that we might be able to explain the success of some particular theory T_1 in such a way that we would not expect that kind of explanation to generally apply across the board. Understanding Kirchhoff's success from the present-day perspective will provide an example of such explanation, and exploring this case will allow us to elaborate on this preliminary, purely conceptual point.

In order to be more precise, let us characterise the Naïve Optimist position as adhering to the following assumption:

(NO) *Any* significant novel predictive success is explainable by the truth content of the assumptions (equations, models, structure, etc.) which play an *essential* role in the derivation yielding that success.

Different realists have different ideas about how to best capture the essential truth content that can explain success in a 'non-miraculous' way. Naive optimism is clearly compatible with any realist position with respect to this separate issue. We can talk about a derivation being (explanatorily) approximately true, using the term 'approximately true' broadly so as to include structural realism, say, which would only commit to truths about structure as the essential explanatory truth content.[3]

There are some respects in which (NO) certainly *isn't* naïve, but captures a relatively careful realist position. To begin with, such a realist is only willing to make her inference when there is evidence of *novel predictive* success, rather than mere explanatory success, or mere ad hoc accommodation of data, say. Furthermore, such a realist is focusing on the *derivations* that generate successes, rather than simply "theories". The reason for this is twofold. First of all, the realist needs to recognise that a general theoretical framework often has excess baggage that in closer analysis plays no

[3] There are two informal senses to 'approximate truth' in the literature. First of all, we can talk about *theories* being approximately true in the (broad) sense that can incorporate the various selective realist manoeuvres (Saatsi, 2005). Secondly, we can talk about various individual *assumptions* of a theory being approximately true in the (narrow) sense that pertains to the numerical values of various quantitative properties, say.

essential part in generating a predictive success of interest.[4] Secondly, the realist needs to recognise that it can be very difficult to say what a theory is, and what its role is in generating a success—as opposed to a set of specific modelling assumptions, say, that drive a derivation. It is better to focus directly on the derivation of a successful prediction and on the assumptions that go into that derivation; after all, the realist needs to explain how the outcome of that derivation does not indispensably depend on radically false assumptions about the world. Given such provisos, there is no need to avoid using the word 'theory', understood broadly as a set of assumptions about the world, required by a derivation, such that these assumptions are purportedly true about the system in question (and hence excluding manifest idealisations).[5]

Although (NO) captures a careful realist position in some respects, the position is nevertheless naïve in the supposition that novel predictive success could not possibly be born of a derivation based on radically false assumptions. All derivations of novel predictive success are lumped together, without allowing that distinctions might need to be drawn between different cases. Included is the implicit assumption that we can infer approximate truth from predictive success without taking into account any particular features of the theory in question. That is, the connection between success and truth is not qualified by the domain of theorising in question, by the mathematics used in the derivation, by the nature of the system under theorising, or by anything of that ilk that might conceivably power success-production under some particular circumstances. Hence, according to Ladyman, for example, one can undermine the realist's gambit by

[4] For example, only by studying the actual derivation of Fresnel's equations will one find that, surprisingly, the wave aspect of the ether-wave theory of light played no essential role in the derivation. (Cf. Saatsi 2005)

[5] We will use 'Kirchhoff's theory' and 'Kirchhoff's derivation' (including the assumptions required for it) interchangeably, as is customary in the physics literature.

producing a single *counterexample* to the success-to-truth inference, with no regard to the potential idiosyncrasies of such an example.

The case study that follows forces the Naïve Optimist to abandon her realism—unless she is willing to be less naïve, of course. We will argue that attending to the details of this case should lead the realist to qualify the connection between success and truth.

3. Kirchhoff's Theory. The naïve optimist's position is to be tested in the face of Kirchhoff's diffraction theory. Kirchhoff's derivation of the *Fresnel–Kirchhoff diffraction formula* for the amplitude of light waves, $U(P)$[6]

$$U(P) = -\frac{iA}{2\lambda}\iint_A \frac{\exp(ik(r+s))}{rs}\left[\cos(n,r) - \cos(n,s)\right]dS \qquad (1)$$

is relatively simple, yet mathematically rich in interesting ways (see Figure 1). Connection with observations is made by calculating from (1) the intensity of light as the amplitude squared ($I = |U(P)|^2$). This predicts how light originating from a point source P_0, and passing through a small aperture in a flat, thin screen, will give rise to some intensity I at a point P on the other side of the screen.

There is an essential bit of *purely mathematical* background to Kirchhoff's success. Kirchhoff, a leading figure in the study of Green's functions in connection with the wave equation, employed Green's theorem and the time-independent (i.e. Helmholtz) wave equation to prove the central *Helmholtz–Kirchhoff integral theorem*:

[6] We follow Born & Wolf 1999, chapter 8.

$$U(P) = \iint_S \left[U \frac{\partial}{\partial n}\left(\frac{\exp(iks)}{s}\right) - \frac{\exp(iks)}{s}\frac{\partial U}{\partial n}\right] dS \qquad (2)$$

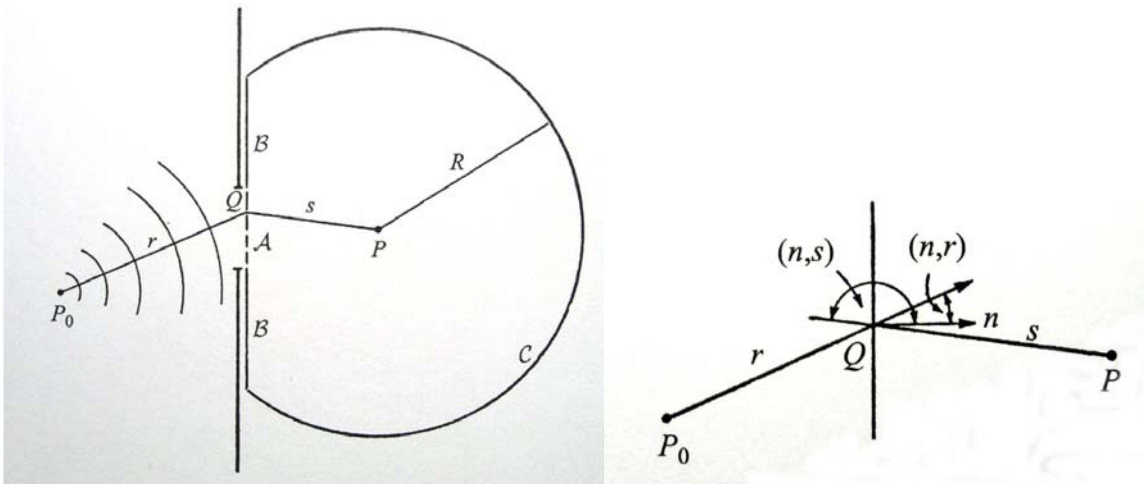

Figure 1. Kirchhoff's method of determining diffraction at an aperture. Obviously P_0 is the source of the light, and *P* is the point beyond the screen at which we want to know the light intensity. In addition *Q* is a point in the aperture whose contribution we are considering at a given time, *r* is the distance from P_0 to *Q*, *s* is the distance from *Q* to *P*. An imaginary surface of integration *S* is comprised of *A* (the aperture), *B* (part of the screen), and *C* (part of a circle of radius *R* which has *P* at its centre). *n* is a normal to the aperture, (*n*, *r*) is the angle between this normal and the line joining P_0 to *Q*, and (*n*, *s*) is the angle between this normal and the line joining *Q* to *P*. (Figure taken from Born and Wolf 1999, p.421.)

This formula expresses the scalar amplitude U at a point P in terms of the values of U and $\partial U/\partial n$ on *any* closed surface of integration S drawn around P.[7] This allows one to calculate the wave amplitude at a given point as a boundary value problem.

[7] Here s is the distance from P to the surface of integration, and $\partial/\partial n$ denotes differentiation along the inward normal to S (cf. Born & Wolf 1999, 417–419).

For the diffraction problem at hand this is not very useful by itself, of course, since the values of U and $\partial U/\partial n$ are not known behind the screen. The Helmhotz–Kirchhoff theorem can be usefully applied to a variety of special cases, however. Our central point of interest, *Kirchhoff's diffraction theory*, applies the mathematical theorem to the scenario depicted in figure 1, with a point-source P_0 sending out a monochromatic spherical wave. The closed surface of integration S is chosen to comprise (a) the aperture A, (b) the non-illuminated side of the screen B, and (c) a portion C of a large sphere of radius R, centred at P (cf. figure 1). At the heart of Kirchhoff's theory are the following three assumptions about the system:

(A1) The field at the aperture A is as if the screen did not exist; i.e. the screen does not perturb waves at the aperture.

(A2) The field and its normal derivative vanish immediately behind the screen, i.e.

a. $U = 0$ on *B*

b. $\partial U/\partial n = 0$ on *B*.

(A3) The contribution of the integral around C vanishes as $R \to \infty$ ('Sommerfeld radiation condition').

As one can readily see from the form of equation (2), it follows immediately from these assumptions that the only contribution to the integral comes from the field at the aperture A. Assuming furthermore that the point source P_0 emits a spherical field

$$U = \frac{A\exp(ikr)}{r}, \qquad \frac{\partial U}{\partial n} = \frac{A\exp(ikr)}{r}\left[ik - \frac{1}{r}\right]\cos(n,r)$$

one straightforwardly derives the Fresnel–Kirchhoff diffraction formula (1). This formula is supremely accurate in predicting diffraction effects.[8] It is still widely used in practice and discussed in the optics literature (Cf. e.g. Mielenz 2002, Li 2005, Bruce 2007).

What should the naïve optimist's attitude be in the face of Kirchhoff's novel predictive success? The assumptions (A1)-(A3) above are absolutely vital to Kirchhoff's derivation (they are certainly 'working posits') and they are also quite plausible intuitively speaking if one thinks of light as waves in the ether. The realist happily infers from all this that the key assumptions behind Kirchhoff's derivation are most probably at least approximately true. In other words, Kirchhoff's assumptions (A1), (A2) and (A3) at least closely approximate the *actual* wave-amplitudes at the aperture and behind the opaque screen. For wouldn't it be quite 'miraculous' if Kirchhoff's assumptions were not even nearly true? The Naïve Optimist, in particular, is willing to stick her realist neck out as she proclaims that the approximate truth of (A1)–(A3) is the *only* explanation of Kirchhoff's success that is consistent with realism. Were that really the case, realism would presently flop.

4. Inconsistency and Untruth in Kirchhoff's Theory. Like any non-fundamental theory, Kirchhoff's incorporates *various levels* of approximation and idealisation, well documented in the literature. For example, as a scalar diffraction theory it ignores the vectorial nature of the electromagnetic field to begin with. But our claim that the theory is not even approximately true is not based on such small-scale idealisations. These are uninteresting compared with the ways in which the assumptions (A1) and (A2)—

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[8] In typical circumstances, that is: the aperture needs to be several wave lengths in width, and the inspection-point P and the source-point P_0 need to be several wave lengths from the aperture, for example.

collectively known as *Kirchhoff's boundary conditions*—misrepresent the behaviour of light. One would expect these not to be *literally* true, of course, on the basis that the edges of the aperture, however thin, are bound to perturb the field to *some* degree.[9] Such minor simplifications are unproblematic and insignificant, however, relative to the startling fact that Kirchhoff's boundary conditions are (a) inconsistent altogether, and (b) *wide of the mark* regarding how light actually behaves at the aperture.

Thus two puzzles present themselves: the 'inconsistency puzzle' and the 'error-tolerance puzzle'. It will turn out that only the latter presents a genuine challenge for the realist, but we will give a reasonably detailed account of the inconsistency of Kirchhoff's assumptions since it is widely discussed in the optics literature, intrinsically interesting, and naturally leads to the puzzle about error-tolerance, as we will now see.

4.1. The Inconsistency Puzzle. The inconsistency of Kirchhoff's theory is much discussed in the earlier optics literature. The opening sentences of Heurtley (1973) capture the inconsistency puzzle:

> A problem of continuing interest in scalar diffraction theory is why the mathematically inconsistent theory of Kirchhoff predicts results that are in substantial agreement with experiment. (1003)

The inconsistency manifests itself in various ways. Mathematically it is rooted in incorrectly over-determining the boundary conditions by fixing both U and $\partial U/\partial n$, when fixing either would uniquely determine the solution of the elliptic wave equation (see e.g.

[9] From the modern perspective this follows from the continuity conditions for Maxwell's equations, but plausibility arguments can also be given in the elastic ether framework.

Barton 1989). Furthermore, it can be shown that fixing U and $\partial U/\partial n$ to vanish over a finite line segment entails $U = 0$ across the whole plane (Sommerfeld 1954, 198). Another way to demonstrate the inconsistency is by using (1) to calculate U and $\partial U/\partial n$ as the observation point P approaches the screen or the aperture: the boundary values assumed in the derivation of (1) are not recovered at the boundary (as already noted by Poincaré 1892, 187).

One possible way scientists have attempted to solve this 'puzzle' is to find a theory which is a close relative of Kirchhoff's but which is consistent. Obvious candidates arise in the form of the so-called Rayleigh–Sommerfeld (RS) diffraction theories.[10] These theories are consistent by virtue of adhering to a proper *subset* of Kirchhoff's over-specified boundary conditions: regarding the field behind the screen only *either* (ii *a*) *or* (ii *b*) is assumed to hold, but not both simultaneously. These two alternative sets of boundary conditions yield two different equations that correspond to the Fresnel–Kirchhoff diffraction formula (1)—the RS diffraction integrals:

$$U_{RS}^{(I)}(P) = \frac{1}{2\pi}\iint_A \frac{A\exp(ikr)}{r}\left(ik + \frac{1}{s}\right)\frac{\exp(iks)}{s}\cos(n,s)ds$$

$$U_{RS}^{(II)}(P) = -\frac{1}{2\pi}\iint_A \frac{A\exp(ikr)}{r}\left(ik - \frac{1}{r}\right)\frac{\exp(iks)}{s}\cos(n,r)ds\ .$$

[10] In the optics literature it is customary to talk about *the* Rayleigh–Sommerfeld theory, which then has two solutions corresponding to two alternative boundary conditions. We prefer to speak of *two* RS theories, unified by a common mathematical framework, because one can identify two different (although partly overlapping) sets of physical assumptions.

Comparing these with the Fresnel–Kirchhoff diffraction formula at the limit $r >> \lambda$, $s >> \lambda$ shows that the field amplitude U according to Kirchhoff's theory is essentially equivalent to the two different predictions that come out from the consistent RS theories. For small angles of incidence and diffraction $(\cos(n,r) \approx 1,\ \cos(n,s) \approx -1)$ we have

$$U \approx U_{RS}^{I} \approx U_{RS}^{II}.$$ [11]

Since the assumptions made by these two theories are proper subsets of the assumptions of Kirchhoff's theory, one might hope that one or another of them keeps what is right about Kirchhoff's theory and does away with what is false. At first it might even be supposed that this situation follows the pattern identified by Norton (1987, 2000, 2002), where the success of an inconsistent theory is seen to be due to it incorporating a consistent "sub-theory". But as a matter of fact Kirchhoff's derivation does not get off the ground without *both* assumptions (ii *a*) and (ii *b*). There is no clear sense in which the RS theories would constitute sub-theories of Kirchhoff's theory; the RS theories only work by giving derivations fundamentally different to Kirchhoff's. At any rate, it isn't clear that the realist would be happy with turning to these two theories, since as a matter of experimental fact Kirchhoff's theory outperforms both Rayleigh–Sommerfeld theories in many circumstances. That is, in many circumstances the inconsistent theory is more accurate than the theories which are in a sense its nearest consistent alternatives, although the two mutually incompatible alternatives each hold sway over certain ranges of parameter values (Totzeck 1990).

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[11] In fact, curiously enough, U is an arithmetic average of the two RS theories:

$$U = \tfrac{1}{2}\left(U_{RS}^{I} + U_{RS}^{II}\right).$$

It is no wonder that physicists have been puzzled by these fascinating facts about Kirchhoff's theory and its relationship to RS-theories. However, nothing yet said need overly perturb the realist. Even if she can't straightforwardly turn to either of the RS theories as representing consistently what makes Kirchhoff's theory tick, no reason has yet been given why she can't maintain that the assumptions (A1)-(A3) are each approximately true. But things do become more awkward for the realist in the face of another theory which is a somewhat more distant relative: the Marchand-Wolf (MW) theory.

Marchand and Wolf (1966) show that Kirchhoff's diffraction formula (1) can be derived—exactly—from a consistent set of assumptions. Kirchhoff's boundary conditions are modified by introducing an effect due to the "scattering" of light off the edges of the aperture. This systematically changes the amplitude and its normal derivative both at the aperture and behind the screen, and renders them quite different from what Kirchhoff assumed. Instead of an undisturbed "flat" wave across the aperture one finds a number of peaks and troughs of intensity as one moves across it (see Marchand and Wolf 1966, 1716, fig.3(b)). It turns out that Marchand and Wolf come close to matching what we find when we work directly with Maxwell's equations (see figure 2, below).

Marchand and Wolf suggest that their theory explains the success of Kirchhoff's theory in the face of its inconsistency:

> In the present paper we show that the inconsistency in Kirchhoff's diffraction theory is only apparent (Marchand and Wolf 1966, 1713).

And this attitude is found throughout the literature; for example Stamnes (1986) writes,

> The consistency of the Kirchhoff diffraction theory was first demonstrated by Marchand and Wolf (1966). (27)

Prima facie, these remarks are rather puzzling: the derivation of Kirchhoff's formula makes essential use of boundary assumptions which are simply not recovered when the formula is evaluated at the boundary! But a more careful reading of the aforementioned authors indicates that the above claims can be put down to careless use of language and a different perspective on the inconsistency puzzle. These authors are happy to explain why Kirchhoff's *diffraction formula* is so successful—namely, exactly the same formula springs from a consistent, well-motivated theory the assumptions of which may well represent the reality quite accurately. The realist, on the other hand, wants to explain why Kirchhoff's *theory* is so successful and, if anything, MW theory presents a challenge for the realist by demonstrating how underdetermination can be realised in actual science. We have here two theories which make radically *different* assumptions about how light behaves in the aperture, but both of which can be used to derive the *same* diffraction formula!

It should be made clear that the MW theory is strikingly different from the Kirchhoff theory. Marchand and Wolf's assumptions about the scattering effects are nothing like Kirchhoff's boundary assumptions (A1) and (A2). Thus the realist is faced with two quite different theories which are both equally successful. From that success she cannot possibly infer that the relevant assumptions in *both* theories are approximately true, because the respective sets of assumptions are simply too different. Furthermore, Marchand and Wolf's assumptions about the scattering wave are *independently* *motivated* by the contemporary understanding of diffraction, so they cannot be dismissed simply as post hoc.[12]

Our best contemporary theory favours the MW boundary conditions over Kirchhoff's, revealing (A1) and (A2) not to be even approximately true, and thus challenging the realist. But note that this challenge has nothing to do with consistency *per se*. The issue is no longer how an inconsistent theory can generate such success, but how a theory with seriously false assumptions can generate such success. The gravity of this problem will become clearer in the next section.

4.2. The Error-Tolerance Puzzle. The central challenge for the realist is that the success-generating assumptions of Kirchhoff's theory are wide of the mark. Here we provide further evidence for this claim. Since Maxwell's identification of light as an electromagnetic phenomenon, correctly described by Maxwell's equations, we have had the wherewithal to study diffraction from the first principles by imposing the correct electromagnetic boundary conditions over the edges of an aperture.[13] The most serious problem for the realist is the discrepancy between what Maxwell's equations tell us, on the one hand, and Kirchhoff's assumption (A1) that the presence of the screen does not affect the field at the aperture, on the other.

Brooker (2001) illustrates this by a model of an infinitely long slit of width a in a screen of zero thickness and infinite conductivity. For such a system it can be shown that the amplitude of the $\mathbf{E}$-field across the aperture varies as a function of the state of

[12] Marchand and Wolf draw on the work of Rubinowicz and others which started to attract serious attention from 1917 onwards. The assumption is made that diffraction is the combined effect of an incident wave and a scattered boundary wave (see Born and Wolf 1999, 499ff.).

[13] It should be noted, however, that computational intractability in the present scenario prevented scientists from working directly with Maxwell's equations until advances in computing in the later twentieth century.

polarisation, and that there is a major departure from Kirchhoff's assumption (A1) for light so polarized that its $\mathbf{E}$-field is oriented along the slit. In this case the amplitude has a significantly fluctuating sinusoidal shape instead of being "flat" as assumed by Kirchhoff (cf. Figure 2).

So from the perspective of Maxwell's equations some of Kirchhoff's key assumptions are not even approximately true. At the same time, however, diffraction effects calculated directly from Maxwell's equations coincide with almost perfect accuracy with Kirchhoff's predictions, regardless of the state of polarisation!

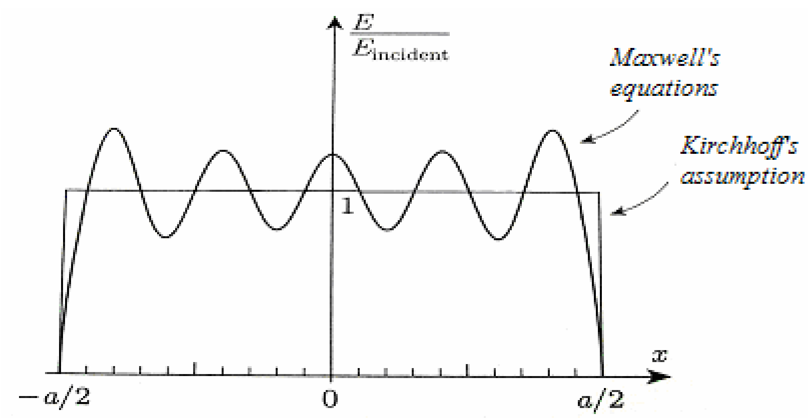

Figure 2. Comparison of Kirchhoff's assumption of a "flat" amplitude function across an aperture of width *a* with the amplitude function derived from Maxwell's equations. (Adapted from Brooker 2001, 71).

This creates a genuine puzzle: as far as predictive accuracy is concerned, why does our wave-theoretic modelling of diffraction tolerate such significant errors with respect to the amplitude U at the aperture? As Brooker remarks,

> Kirchhoff's assumptions give a poor representation of the field in the plane of the slit, yet give a remarkably good approximation to the diffraction pattern. But—again we ask—why? (2003, 72)

With some ingenuity Brooker is able to provide an answer. Of all the possible ways in which Kirchhoff's assumption (A1) differs from the truth (according to Maxwell's equations), it just so happens that the difference has negligible effect. This has to do with the fact that as one moves across the aperture the difference between Kirchhoff's assumption and the real amplitude is a close approximation to a sine wave with a period equal to the wavelength of the incident light. Brooker goes on to show that due to the nature of diffraction this particular error won't show up in a final diffraction formula. Thus he concludes,

> [T]here are good reasons why we can get away with using Kirchhoff's boundary conditions at a diffracting aperture. Nature has been unusually kind to us. (Ibid.)

Of course Kirchhoff had no idea that an error of this kind would not show up in the final diffraction formula. If he were a realist, he would no doubt have taken the success of his derivation as strongly indicating that the difference between his boundary conditions and reality is negligible. But in fact the difference between his boundary conditions and reality is large, and the reason they can be used without engendering large error is something that was discovered only much later on. Note that it is not the case that Kirchhoff was playing with mathematics, trying many different ideas without any good physical reason, and that he hit upon a very successful formula by trial and error. This would perhaps be a natural way to explain why Kirchhoff's "luck" was to be expected after all. But in fact he had a very plausible physical explanation behind the specific assumptions he made, an explanation which, in the end, turned out to be quite mistaken. This makes Kirchhoff's success all the more remarkable.

5. Ramifications for Realism. How should the realist respond to Kirchhoff's case? To begin with, it is clear that the Naïve Optimist must either renounce her realism, or become less optimistic about predictive success as an indicator of approximate truth. Kirchhoff's theory functions as a "counter-example" par excellence against realism as construed by the Naïve Optimist, regardless of which of the contemporary selective realist positions she adopts. As a selective realist she may easily side-step the fact that Kirchhoff operated in the ether paradigm, and that Kirchhoff achieved success even though "ether" is a non-referring term. But such a realist cannot get around the fact that Kirchhoff's theory makes wildly wrong assumptions about the amplitude attributed to light waves at the aperture and behind the screen (regardless of what those waves are waves *of*). Even the structural realist---perhaps the most lightly committed breed of selective realism---can be challenged by Kirchhoff's theory. Given the nature of Kirchhoff's wildly wrong assumptions there simply cannot be a natural structural correspondence between Kirchhoff's theory and Maxwell's theory, like the one we find in the Fresnel case. Neither is there a mathematical correspondence *at some natural limit*, like in the case of relativity and classical mechanics, say, that would allow the structural realist to explain Kirchhoff's success in a realist fashion.

But what about less naïve forms of realism that wish to allow for such "exceptions which prove the rule"? How can they accommodate the above explanation of Kirchhoff's success and yet retain their optimism for science in general? Let's consider first the quick suggestion that the realist need not be bothered by this singular case-study, simply because this sort of underdetermination doesn't occur with any significant frequency in science. One might assume that, given the vast literature on the realism debate, there cannot possibly be many other historical examples like Kirchhoff's hiding in the woodwork. Hence, the suggestion is that perhaps in this case nature really has been *unusually* kind to us, as Brooker puts it, and Kirchhoff's success from falsity is just a one-off.

We are reluctant to make such an assumption, however. Philosophers of science operating in the realism debate have a tendency to write a great deal about a handful of case studies, leaving many stones of science unturned. So far philosophers have mainly focused on instances of radical theory-shift accompanied by ontological and referential change. It is worth stressing again that the ether aspect of Kirchhoff's theorising does not play any role in our analysis. What matters is that Kirchhoff's assumptions about the *wave-amplitudes* were so badly mistaken. To find examples of successful theories which are not even approximately true one does not need to look for cases of ontological and referential change. This increases dramatically the scope for finding potentially problematic cases.

Simply appealing to the apparent rarity of such cases also fails to directly respond to the challenge posed to the Naïve Optimist. Since Kirchhoff's success can be explained in this way, why not assume that a similar explanation may be available (although it may remain unknown to us) for many, or most cases?[14] The appropriate response to this challenge turns on the details of the present case-study. It is telling that scientists themselves have explored and gained an understanding of Kirchhoff's success from a firmly realist stance. By studying the nature of waves and diffraction further they have

[14] It seems that the realist can have some room for manoeuvre here, by accusing the challenger of having shifted the focus away from where it should be. That is, the realist can take her explanandum to be the success of science *on the whole*, and she may claim on various grounds that her explanans—"successful scientific theories are by and large approximately true"—still provides the best explanation of *this* explanandum, given our best understanding of science. Such a response would require much elaboration, of course, and some have worried that the whole debate is reduced to a fruitless disagreement about inaccessible statistical factors. (Magnus and Callender 2004). Although we remain unconvinced by this worry, here we want to focus on a different response.

discovered that the world of waves just is such that it creates a certain limited underdetermination that turned out to be rather auspicious for a scientist operating with a set of assumptions much simpler than the reality. Due to the nature of diffraction there is an interesting many-to-one mapping, so to speak, from amplitude-distribution-at-the-aperture to diffraction patterns. We have discovered that *the world in this specific respect* is very kind to a human scientist who works her way upwards from the simplest assumptions.

However, the fact that we have such underdetermination in the case of Kirchhoff's theory need not mean that it is to be found everywhere. There is nothing in this case to indicate that the world gives rise to *similar* underdeterminations in the physics involved in analysing Brownian motion in terms of the atomic nature of matter or in our best theories involved in making inferences about the genetic nature of inheritance, say. A realist can insist that, in the present case study, the nature of diffraction and the mathematics employed are idiosyncratically fecund to this kind of underdetermination. In other words, the historical lesson learnt from Kirchhoff need not be widely generalisable. It is admittedly a difficult question how exactly to spell out this idiosyncrasy in general terms, and further work is needed here. But it should not be implausible to anyone that given the enormous variation in the nature and methods of scientific theories across the whole spectrum of "successful science", some domains of enquiry can be more prone to this kind of underdetermination than others. And, witnessing Kirchhoff's case, there is every reason to expect that from a realist stance we can grasp the features of physics and mathematics that contribute to such differences. The Naïve Optimist goes wrong in simply assuming that there are no such differences between theories and domains of theorising, as one "counter-example" from whichever field can stand as a proxy for the whole edifice of successful science.

6. Conclusion. Kirchhoff's theory is the strongest single historical example against realism yet, and certainly seems to undermine Naïve Optimism. It is also the first case which gives realists a concrete reason *not* to be naïve: up until now *every* case put forward (e.g. those on Laudan's list) can be and has been handled in ways compatible with Naïve Optimism. Now the realist must accept that sometimes a novel predictive success is explainable in terms other than underlying truth. But quite what is meant by "sometimes" is an open question.

Still, the anti-realist cannot simply claim that because we have one such case we should infer that such cases abound. This is an example particular to one field, and it is not at all clear that it gives cause for wide-spread pessimism about realism more generally. The anti-realist may claim that Kirchhoff's predictive success is *just like* any other success the realist views as indicative of underlying theoretical truth. And since Kirchhoff's success is not explainable in realist terms—the argument continues—we should not expect other predictive successes to be thus explainable either. We have argued that the realist should try to resist this line of thought by showing how the field of theorising in question is idiosyncratic in relevant respects, so that Kirchhoff's curious case remains isolated and doesn't provide the anti-realist with grounds for projectable pessimism. Whether or not this response can be made precise enough to convince the opponent is an open question that calls for further research. Here we have merely argued for the prima facie plausibility of such a response.

On the other hand, the anti-realist might make something of the fact that this new case makes manifest a type of underdetermination which has not been given much attention in the literature: namely the possibility of equally successful theories which are different qualitatively and quantitatively, but not ontologically or referentially. But whether we should expect the type of underdetermination in question to crop up elsewhere in science is not at all clear. Kirchhoff's theory *may* be the start of a newly invigorated case against the realist, but by itself it does very little damage to a realist who is not naïve, in our sense.

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