UNSHARP QUANTUM REALITY

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ABSTRACT. The positive operator (valued) measures (POMs) allow one to generalize the notion of observable beyond the traditional one based on projection valued measures (PVMs). Here, we argue that this generalized conception of observable enables a consistent notion of *unsharp reality* and with it a concept of joint properties. A property manifests itself as an element of unsharp reality by its power, or tendency, of becoming actual or actualizing a specific measurement outcome, where this tendency of actualization is quantified by the associated quantum probability. The resulting single-case interpretation of probability as a degree of reality will be explained in detail and its role in addressing the tensions between quantum and classical accounts of the physical world will be elucidated. It will be shown that potentiality can be viewed as a causal agency that evolves in a well-defined way.

1. INTRODUCTION

The advent of quantum mechanics has created a puzzling and in fact unique situation in physics. On the one hand this theory turned out enormously successful as a framework for the explanation of the behavior of matter at all scales—from the level of elementary particles, atoms and molecules to the world of macroscopic and even cosmic phenomena; on the other hand, researchers have not been able, in more than eight decades, to reach a consensus on the interpretation of quantum mechanics. There is not even an agreement over the status and significance of such strange quantum features as incommensurability, indeterminacy, indeterminism, and non-locality. Nor is it considered established whether the so-called quantum measurement problem or the classical limit problem are actually *conceptual problems* or artifacts of some particular choices of interpretation.

That quantum mechanics bears profound consequences for our physical world picture became evident from the early beginnings, as witnessed, for example, by Albert Einstein, who expressed his reservations against the irreducibly indeterministic character of this theory in a letter to Max Born from 1926:

> "Quantum mechanics is very worthy of regard. But an inner voice tells me that this is not yet the right track. The theory yields much, but it hardly brings us closer to the Old One's secrets. I, in any case, am convinced that He does not play dice."

By contrast, with his discovery of the uncertainty relations in 1927 [1], Werner Heisenberg felt forced to conclude that he had invalidated the principle of causality.

Nearly four decades later, at a time when Foundations of Quantum Mechanics was on its way to becoming a research field in its own right, Richard Feynman espoused his views towards it with these words (in his book of 1965, *The Character of Physical Law*):

> "...I think I can safely say that nobody understands quantum mechanics.
> ...Do not keep saying to yourself, if you can possibly avoid it, 'But how

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> can it be like that?' because you will get 'down the drain', into a blind alley from which nobody has yet escaped. Nobody knows how it can be like that."

These quotations capture the tension between two opposing philosophical positions: scientific realism versus instrumentalist empiricism. Einstein's concern was to hold up a world view based on "local realism," in which probability plays a primarily epistemic role. Heisenberg was prepared to accept quantum indeterminacy and probability as primarily ontic, that is, as essential features of the physical world.

Many physicists adopt a pragmatic double approach: they practice a realist outlook for the purposes of heuristic explorations of new models and the discussion of experiments, using intuitive pictures of individual (sub)atomic objects; but when challenged, they only admit to the minimal probabilistic or statistical interpretation, according to which quantum mechanics is not more than a bookkeeping tool to process data obtained from physical experiments. This has similarly been noted by D'Espagnat [2].

It seems to us that a more coherent and productive approach would be to systematically investigate all possible variants of realistic interpretations of Quantum Mechanics. This would not only recognize the possibility that such an interpretation could in the end enable the best description of the physical world; it also has the potential benefit of providing us with guidance in developing new, appropriately adapted intuitions about microphysical objects. Here we do not venture to embark on a comprehensive survey and critique of all known realistic interpretations; instead we will identify a key element of such interpretations and explore a particular way of formalizing it.

On a realist interpretation of Quantum Mechanics as a complete theory, the referent of quantum mechanical propositions is an individual system. A primary role of any interpretation is to provide a rule that determines, for every state, which physical quantities have definite values in that state, thus representing *actual properties*, or *"elements of reality,"* pertaining to the quantum system under investigation.

In the present paper we put forth an interpretational point of view that has not yet been much considered but that seems to us worthy of further exploration—the concept of *unsharp quantum reality*. This notion requires us to review the issue of quantum indeterminacy and its implications. This will constitute the start of Sec. 2, where we then proceed to make precise the notion of *degrees of reality* with reference to the general representation of observables as positive operator measures (POMs). The attribution of unsharp properties to a physical system, and the associated assignment of approximate truth values, is discussed in Subsec. 2.2. We will show that the language of elements of unsharp reality has the potential of reconciling some if not all contrasting aspects of the quantum and classical descriptions of physical systems (Sec. 3). However, along the way it will be seen that due to the notorious quantum measurement problem, such reconciliation will require, under the proposed interpretation, a stochastic modification of the unitary time evolution postulate for quantum dynamics. Finally (Sec. 4) we summarize the individual interpretation of quantum mechanics resulting from the notion of unsharply real properties, showing how it brings more aspects of a quantum system into the realm of the "speakable" in the sense of John Bell's notion.

Many of the ideas laid out in this work are inspired by discussions with Peter Mittelstaedt as well as his lecture courses and philosophical work. One of us (PB) owes Peter Mittelstaedt a great debt of gratitude for his generous guidance and mentorship over many years. It is a pleasure to have the opportunity to dedicate this work to him in honor of his 80^{th} birthday.

2. POTENTIALITY: FROM QUANTUM INDETERMINACY TO INDETERMINISM

2.1. **Elements of reality.** We adopt the standard Hilbert space formalism of quantum mechanics. States are represented by *density operators* ρ (positive operators of unit trace), the pure states among them being given by the rank one projections $P = P_\psi$, or one of their representative (unit) vectors, ψ. Observables are associated with positive operator measures (POMs). The traditional notion of an observable as a self-adjoint operator is included in the form of projection valued measures (spectral measures of the self-adjoint operators); such observables will be referred to as *sharp observables*, and all other observables will be called *unsharp observables*.

On a minimal probabilistic interpretation, quantum mechanics is understood as a formalism for the calculation of probabilities that correspond to predicted frequencies of the outcomes of measurements performed on identically prepared systems. Thus if the state resulting from the preparation is given by operator ρ and the observable being measured is a POM E on a σ-algebra Σ of subsets of the value space Ω, the associated probability measure is

$$\Sigma \ni X \mapsto p^E_\rho(X) := \mathrm{tr}[\rho E(X)] \in [0,1]. \tag{1}$$

In a realistic interpretation, one is first of all concerned with identifying the formal elements representing the properties or associated propositions that may or may not pertain to the system. These elements are commonly taken to be the eigenvalues or more generally the spectral projections of the sharp observables. The orthocomplemented lattice of orthogonal projections (or the associated closed subspaces, with the order being defined as inclusion) forms the basic proposition structure for an individual quantum system. Maximal properties are given by the atoms of the lattice, that is, the rank-one projections (or one-dimensional subspaces). There is a far-reaching analogy between the set Γ_q of one-dimensional projections and the phase space Γ_c of a classical mechanical system: first, both sets represent maximal-information states, and second, they carry a natural symplectic structure. From this perspective a closed quantum mechanical system with its unitary Schrödinger dynamics can be viewed as a (generally infinite-dimensional) Hamiltonian dynamical system. As will be reviewed in Subsection 3.2 the analogy between Γ_c and Γ_q extends also to the treatment of probabilities in that all quantum states are representable as probability distributions on Γ_q.

The generalized notion of a quantum observable as a POM entails an extension of the ordered set (lattice) of projections to the ordered (convex) set of *effects* (the positive operators that may occur in the range of a POM). We will explain below that effects may be taken to represent *unsharp properties*, in analogy to the notions of fuzzy sets and vague propositions.

We provide next an argument for the orthodox property attribution rule, also known as the *eigenstate–eigenvalue link*. A *sufficient* condition for a physical quantity to have a definite value, or for a property to be actual, is that the value or property can be ascertained without changing the system. This is the famous Einstein–Podolsky–Rosen criterion for a physical quantity to be an *element of reality*. Using the apparatus of the quantum theory of measurement, one can show [3] that there is a distinguished class of measurement operations that do allow just such procedures of determining the value of an observable without changing the state of the system. These operations are the so-called *Lüders operations*, which are restrictions of linear, positive, trace-non-increasing maps on the linear space of self-adjoint trace-class operators. In fact, let P be a projection, then the associated Lüders operation is defined as

$$\rho \mapsto \phi_L^P(\rho) := P\rho P. \tag{2}$$

A state ρ remains unchanged exactly when it is an eigenstate of P, in the sense that $P\rho = \rho (= \rho P)$. Thus, if the system's state is an eigenstate of a given observable, that observable can be said to correspond to an element of reality. This shows that the EPR condition is naturally implemented in quantum mechanics if one posits the rule that a property is actual if the system's state is an eigenstate of the corresponding projection or observable.

In considering the question of what might constitute a *necessary* condition for a property to be actual, one may arrive at an answer by reflecting on the usual content of the terms "real" or "actual." A *thing* (Latin: *res*) has the power to make itself felt, it *acts* on its environment. Hence the reality or actuality of a property of an object entails that it has the capability of influencing other objects—in particular, measurement devices—in a way that is characteristic of that property.[1]

Here, the properties of concern are physical magnitudes of a quantum system. If a property is absent, the system's action or behavior will be different from that when it is present. Applied to the context of a measurement, in which an observer induces an interaction between the system and part of its environment (a measurement apparatus), this means that a property is actual—that is, an observable has a definite value—if its measurement exhibits this value or property unambiguously and (hence) with certainty. This condition, which has been called *calibration condition* in [3], is taken as the defining requirement for a measurement scheme to qualify as a measurement of a given observable. Again, its implementation within the quantum theory of measurement is possible if a property's being actual is associated with the system being in an associated eigenstate.

In summary, the structure of the quantum theory of measurement suggests the adoption of the eigenvalue–eigenstate link as a necessary and sufficient criterion of reality in quantum mechanics. However, the application of quantum mechanics to the description of measurement processes also leads directly into the *quantum measurement problem*, or *objectification problem*, to which we will return shortly.

This problem is one of the main reasons for the continued debate about the interpretation of quantum mechanics. Insofar as an interpretation entails its own specific rule for the attribution of actual properties, it is interesting to note that effectively all existing interpretations can be formally characterized in a unified way by a construction in which this property assignment is uniquely determined by the quantum state and a particular observable that is always definite; this "preferred" observable is specific to the interpretation at hand. This result, which we will not spell out here, is the content of the Bub–Clifton uniqueness theorem for (modal) interpretations of quantum mechanics [7]. It highlights an important fact: any interpretation other than the orthodox one described above faces the problem of explaining the definiteness of its particular "preferred" observable. The eigenstate–eigenvalue rule is based on the preferred observable being the identity operator, which could be trivially regarded as having a definite value.

2.2. **Vagueness and quantum properties.** We now provide a general consideration of the third possibility, that is, of a property being neither actual nor absent. The next subsection

[1] The sufficient and necessary conditions for the reality of physical objects or properties proposed here are in accordance with an account of conditions of objective experience in the spirit of Kant that is based on the categories of substance, causality, and interaction; as Peter Mittelstaedt and his collaborator Ingeborg Strohmeyer have shown, this account can be cogently phrased in a way that encompasses objects of quantum-physical experience [4, 5, 6].

proceeds to a more formal discussion. A property of an entity is typically thought of as *indeterminate* when there is no way, even in principle, of attributing it in pertinent contexts. The corresponding vagueness of propositions regarding the properties of objects can arise, for example, when the concepts or definitions involved are either insufficiently clear or overly restrictive.

In classical physics, which is known to correctly describe macroscopic objects (to a hitherto practically unlimited accuracy) when quantum effects are imperceptible, all magnitudes have a definite value at any given time that can in principle be simultaneously known with certainty at that moment by their measurement, even though they may not always then be predicted with certainty; for example, when the classical evolution is chaotic, or may be difficult to measure. Hence, vagueness is considered alien to classical physics.

By contrast, in geographical descriptions of macroscopic systems vagueness readily arises, in that the boundaries of geographical entities—for example, a mountain or a cloud—are often vague in some respects such as their precise extent in square meters, but independently not in others, such as extent in relation to greater scales, say continental location or atmospheric regions. That sort of vagueness is less a question regarding objective reality than one of a choice of concepts; in the above instance it arises when an overly restrictive concept corresponding to the property of geographical location is imposed.

In the case of microscopic entities to which quantum mechanics pertains, there is vagueness the origin of which is substantially different. Vagueness in quantum mechanics appears endemic, arising directly from the indeterminate nature of quantum entities themselves rather than a choice of concepts within a flexible theoretical framework. Indeed, in quantum physics spatial location can be almost *entirely* indeterminate, such as when the momentum is specified with extremely high precision, say in the case of a free electron.

Unlike the situation of macroscopic entities with vague geographical characteristics, it is not the case that the concept *electron* is overdetermined in the sense that the criteria for it to have a unique spatial location—or for that matter, for it to be spatially dispersed—cannot be satisfied in principle; nor is the concept of electron underdetermined in the sense that the definitions of its physical properties are insufficiently precise [8]. Quantum mechanical position is typically indeterminate but can, in principle, be measured precisely (i.e., with arbitrary accuracy) and be determinate (to a high degree) immediately after measurement (but with the result that momentum is disturbed and indefinite, and vice-versa). However, at least one of the two quantities, position and momentum (and typically each of these), is also limited in its determination.

In the case of quantum systems, properties can be considered objectively indefinite and sets of propositions regarding them complementary to specific other sets of propositions, so that it becomes impossible to jointly attribute them. Thus, quantum mechanics involves a unique form of vagueness distinct from those considered before.

If a traditional quantum observable A has an eigenvalue a, *i.e.* $A\psi = a\psi$, then when the state *is* an eigenstate of A a measurement of that observable will yield the value a with certainty; the physical magnitude corresponding to the observable A has a definite value if the state of the system is in one of its eigenstates, under the standard interpretation. However, there are always some other observables that have no definite value. It is the linear structure of the Hilbert-space formulation of quantum mechanics that is responsible for the indeterminateness of properties: Every pure state is a non-trivial linear combination of eigenvectors of observables with which the associated density operator (one-dimensional projection) does not commute, so that the values of those observables are indefinite. The indeterminacy relation for state preparations governs the (in)definiteness of joint properties of quantum systems.

2.3. **Indeterminacy.** The novel feature in the description of physical reality brought about with the advent of quantum mechanics is thus the fact that physical properties will not in general be either actual or absent but that there is an additional modality of being that is called *indefiniteness* or *indeterminacy*.[2] Thus it is impossible, according to Quantum Mechanics, to ascribe definite truth values to all propositions regarding all properties of a physical system at any one time, for any of its quantum states. This quantum feature is acknowledged in any realistic interpretation of Quantum Mechanics as a complete theory.

There has been a variety of approaches and attempts to demonstrate the inevitability of quantum indeterminacy, starting from different premises.

(I1) *Interference:* We adopt as the basis for our discussions an interpretation according to which an observable is assigned a definite value if, and only if, the system's state is an eigenstate of the observable (the eigenvalue-eigenstate link). On that interpretation there are obviously many observables for every given state that do not have definite values—they are *indeterminate*. This indeterminacy is exhibited in the presence of *interference terms* in the probability distribution of a suitable test observable. For example, if ϕ_A, ϕ_B are the two orthogonal states representing passage along disjoint paths A and B in a split-beam experiment, and the system's state is $\psi = (\phi_A + \phi_B)/\sqrt{2}$, the indeterminateness of the path observable is made manifest by analyzing the probability distribution of an *interference observable* with eigenstates $\psi_\pm := (\phi_A \pm \phi_B)/\sqrt{2}$. Indeed, in a situation in which the system's path observable was definite – which is represented by either path eigenstate ϕ_A or ϕ_B, the probabilities for both outcomes $\pm$ of the interference observable would be 1/2 if there was maximal ignorance about the actual path; whereas when the system's state is $\psi = \psi_+$, these probabilities are 1 and 0, respectively.

(I2) *Bell–Kochen–Specker theorem:* In a system represented by a Hilbert space of dimension greater than 2, it is impossible to consistently assign *truth values* (1 for "true", 0 for "false") to all elements of all complete orthogonal families of rank-1 projections in such a way that for each basis the value 1 is assigned just once. (An orthogonal family of projections is complete if the sum of all the projections is equal to the unit operator.) This result entails the impossibility of *non-contextual* hidden-variable supplementations of quantum mechanics.

(I3) *Gleason's theorem:* Any (generalized) probability measure on the lattice of orthogonal projections of a complex Hilbert space of dimension greater than 2 is of the well-known form $P \mapsto p_\rho(P) = \mathrm{tr}[\rho P]$, where ρ is some positive operator of trace equal to 1 (a density operator). This entails that there are no dispersion-free probability measures on the complete lattice of all projections. (Hence the Bell–Kochen–Specker theorem follows.)

This last statement can be extended to include fuzzy quantum events represented by effects: any generalized probability measure on the set of effects is of the usual quantum mechanical form, $E \mapsto p_\rho(E) = \mathrm{tr}[\rho E]$, with ρ being some density operator. Owing to the stronger premise, this result is found to hold also for Hilbert spaces of dimension two.

[2] In [3] the term *objective* is introduced to characterize a property that is either actual or absent in a general quantum state represented by a density operator. In the present paper we consider mostly pure states and in this context use both *determinate* or *definite* as synonym to *objective*, and likewise for the negations.

(I4) *Bell's theorem:* In the description of EPR experiments involving entangled, spatially separated systems, Bell's inequalities follow from the conjunction of assumptions of "realism" and "locality." As Bell's inequalities are violated in the quantum mechanical description of these experiments, this shows that local realistic hidden-variable models and their ensuing value attributions are not compatible with quantum mechanics. EPR–Bell experiments have confirmed the quantum mechanical predictions.

The generally accepted upshot is that Quantum Mechanics does not admit global, *non-contextual* value attributions to *all* physical quantities at once. It has been shown (for example, by Bell) that global value assignments are possible provided they depend on the measurement, i.e., they are *measurement-contextual*; that is, the same projection may take a different value depending on the observable in whose range it occurs. The best elaborated and known example of a viable hidden-variables approach to quantum mechanics is Bohm's theory, which gives the position variable of a quantum system the special status of a definite quantity. The Schrödinger representation of the state vector is considered to take the role of a guidance field. On that approach, in a two-slit experiment, each quantum system is ascribed a definite path that passes either through slit A or B, but it remains unknowable as a matter of principle through which path the system passes. In fact, the quantum system comprises the particle aspect, represented by the position and path, *and* the guidance field, and the latter propagates through both slits and acts non-locally.

2.4. **Degree of reality, actualization, and measurement.** We noted above that what is actual has the power to act. Similarly, when a property is absent there is no power to act. If this idea is extended to apply to the intermediate mode of existence, potentiality, one may say that an indeterminate property has a quantifiable, limited *degree of reality* that manifests itself in a limited capacity—*potentiality*[3]—to induce the associated measurement outcome. A quantitative measure of the degree of reality and associated potentiality is given by the quantum mechanical probability, which provides the likelihood for an individual outcome to occur in the event of measurement. We show now how this interpretation of the quantum state ψ in terms of single-case probabilities, or propensities, is supported by the quantum theory of measurement.

Measurement is commonly understood as a process in which a physical system (the *object* system) is made to interact with a *probe* or *apparatus* system in such a way that the apparatus ends up indicating the value of a particular physical quantity pertaining to the system under investigation. This indication is modeled by means of a *pointer observable* assuming a definite value that corresponds to a value of the system quantity of interest.

As also noted above, a minimal condition for a measurement process to qualify as a measurement of a given observable is the *calibration condition*: if the system is in an eigenstate of that observable when measurement begins, then the corresponding eigenvalue will be indicated with certainty. This means that the evolution of system plus apparatus is such that the apparatus state at the end of the joint evolution is the associated pointer eigenstate. If the state is a superposition of eigenstates of the observable being measured, then according to the minimal interpretation, the probability of the occurrence of a particular pointer value is given by the squared modulus of the amplitude of the corresponding object eigenstate.

[3]This term was introduced by W. Heisenberg to express the tendency of quantum events to actualize in a measurement [9, 10]. A similar idea was expressed by Popper [11, 12], who used the term *propensity* to refer to the objective single-case probabilities of indeterminate quantum events. In the present context we will consider the terms (degrees of) potentiality and propensity as synonym.

In the framework of the quantum theory of measurement, this result is in fact a consequence of the calibration condition [3]. Moreover, adapting arguments initially developed in the context of the relative-state interpretation of Quantum Mechanics, Peter Mittelstaedt has shown that the quantum mechanical probability emerges as an approximately definite property of a large ensemble of identically prepared systems represented by the same state [13]. The quantum mechanical probability postulate is thus found to be deducible from the probability-free eigenvalue–eigenstate rule, which specifies the definite properties of a quantum system in any given state.

However, the entity that carries the probability value as a definite property is a (strictly speaking, infinite) *ensemble* of equally prepared quantum systems. In a finite ensemble, this property can only be considered as approximately actual. Since this system is composed of independent subsystems, the question remains how one could then explain that such independent objects act together to cause the emergence of an approximately actual eigenvalue of the frequency operator of the ensemble that these systems constitute. After all, the description of the ensemble as a compound system is hardly more than a conceptual construct. The answer is therefore to be found in the fact that each constituent has been treated—*prepared*—in an identical way. One may phrase an explanation as follows: through its preparation, each individual system is constrained to "respond" to a measurement, so as to induce a specific outcome in accordance with the probability law specified by its quantum state. Accordingly, the individual has a limited capacity to actualize each possible property, and this actualization tendency, or potentiality, is what justifies attributing the physical magnitude's limited degree of (or unsharp) reality, as quantified by the probability value associated with it.

This interpretation accounts for such phenomena as the intriguing emergence of an interference pattern in a two-slit experiment in which quantons are sent through the slitted diaphragm one by one, each giving a light spot in an apparently random location on the capture screen: what appears as random behavior at the level of an individual member of an ensemble is found to be guided by the (approximate) actuality of the path and interference properties of the ensemble. This account can be supplemented with one that reduces this ensemble behavior to that of the individual systems: The remarkable collective behavior of many, equally prepared, quantons can be explained by noting that the interference pattern produced by the ensemble is a manifestation of the potentiality inherent in the state of the individual quanton.

In summary, it is a consequence of quantum measurement theory that quantum *indeterminacy* entails the *indeterminism* of measurement outcomes: if a property has no definite value, all a measurement can do is induce the random occurrence of one of the possible outcomes; that is, the individual outcome is not necessitated by any identifiable cause, yet this individual event is governed by *probability causality*—the so-called Born rule, which attributes to it a well-defined, quantified potentiality.

This result rests on the premise that measurements and the occurrence of their outcomes, being physical processes, are correctly described and explained by Quantum Mechanics itself. What is at stake here is the *semantic consistency* of the theory—an issue of great concern in Mittelstaedt's work [13] which is called into question by the problem of objectification (see Sec. 3).

2.5. **Sharp vs. unsharp properties.** The idea of unsharp quantum reality was conceived originally with the aim of interpreting *unsharp observables*, that is, POMs that are not projection valued. It is therefore important (though perhaps at first surprising) to observe that the notion of degrees of reality is also applicable to sharp observables (projection valued POMs). In fact, up to this point our consideration of these concepts mostly referred to sharp observables.

It was in the context of a discussion of the EPR–Bell experiment that the Einstein–Podolsky–Rosen condition of elements of reality was relaxed to a condition regarding elements of *approximate* reality, represented accordingly by *approximately true* propositions [14]. As the spin or polarization observables of the entangled particles in an EPR experiment are measured with progressively more limited accuracy, there is a corresponding progressive degradation of the degree of Bell inequality violation due to the appearance of additional terms reflecting the inaccuracy; the violation of the Bell inequality becomes unobservable above a certain degree of inaccuracy.

This consideration illustrates two facts: (1) the notion of elements of reality is robust in the sense that reality in the sharp sense can be continuously well approximated by measurements and associated unsharp observables with decreasing degrees of unsharpness; (2) quantum observables with sufficient degrees of unsharpness may display classical features, such as the satisfaction of Bell inequalities.

The sufficient condition for elements of reality given by Einstein, Podolsky and Rosen can be adapted to the realm of effects, by way of a relaxation of its formalization for a sharp proposition given above. A good candidate of a measurement operation generalizing the Lüders operation (2) for an effect $E(X)$ (called E for short) is the following:

$$\phi_L^E : \rho \mapsto \phi_L^E(\rho) = E^{1/2}\rho E^{1/2}.$$

It has been shown [15] that if $\mathrm{tr}[\rho E] \geq 1-\varepsilon$, then ϕ_L^E does not decrease this probability and the state change, measured by the trace-norm distance, is small to the order of $\sqrt{\varepsilon}$. This demonstrates the stability of the EPR reality criterion against small deviations from actualization, and one may say that approximately real properties can be ascertained almost with certainty and without significantly altering the system. It is in this sense that effects that are not projections are still capable of being elements of approximate reality, and therefore it is justified to regard them as *unsharp properties*, as opposed to the *sharp properties* which are represented as projections. We note that the weakened EPR condition for elements of unsharp reality can be applied to sharp properties as well, which will thus correspond to elements of approximate reality if the system's state is a near-eigenstate.

In general, an effect may only allow limited degrees of reality, and the degree of reality may not be enhanced by a measurement (as would be the case with a generalized Lüders operation). In such cases the above *sufficient* EPR reality condition will be of little use. But one may refer to the *necessary* condition for elements of reality introduced above: any effect has a potentiality or power to actualize an indicative measurement outcome, quantified by its degree of reality, or probability, in the given state. The set of all effects of a quantum system then comprises the totality of the ways in which the system may act on its environment, specifically in measurement-like interactions. At this interactional level, and in the sense of our necessary reality criterion, an effect can thus be described as a sort of *relational* property, or disposition. If, in addition, these interactions are of such a nature that the influence on the system is described by a (generalized) Lüders operation, this can be used to ascertain the actuality of a *property* of the system whenever this operation does not (significantly) alter the system state: in the spirit of the EPR criterion, it can be said that what can be observed without disturbance must already have been present or actual.

3. Unsharp Quantum Reality and the Quantum–Classical Contrast

We now address the question of the extent to which the notion of unsharp quantum reality could contribute to a resolution of some of the most notorious conceptual problems of quantum mechanics. Here we focus our attention on the following aspects of the quantum–classical contrast: quantization; dequantization; the classical limit problem; and the problem of objectification. We will consider each of these issues in turn and then show how they seem to be interrelated.

3.1. **Quantization.** Quantization is a procedure of formulating the quantum theory of a type of physical system using a form of *correspondence* with an analogous classical system. In the original heuristic version introduced by the pioneers of quantum mechanics, quantization consisted of associating selfadjoint operators on a complex Hilbert as representations of quantum observables with certain functions on the phase space of a classical system corresponding to classical observables; this correspondence was to translate the Poisson bracket relations of canonically conjugate variables such as position and momentum into commutator relations of the operator counterparts. In a more refined fashion quantum observables are defined through their covariance properties under the fundamental spacetime symmetry group, the Galilei or Poincaré group. The group parameters are labels that describe the—classically characterized—locations and orientations of macroscopic preparation and measuring devices.

The problem faced with quantization consists of the fact that on the one hand, quantum mechanics is considered to supersede classical mechanics, while on the other hand it is not fully emancipated from the latter in that classically described macroscopic apparatuses located in a classical background spacetime are presupposed in any quantization procedure.

There are various strategies for solving or eliminating this problem. One approach is to take the dualistic view (and hope) that physical theories of both objects and spacetime structures come in quantum and classical versions, each with their own domains of validity separated by different scales. Then one need not require that classical physics be reduced to quantum physics (or vice-versa). However, advocates of this position have (so far) failed to provide an explanation of any hierarchic or emergence relationship between the theories for the small and those for the large that could account for the fact that the constituents of macro-objects are systems obeying the laws of quantum theory—and thus provide an explanation of the border between the supposedly separate domains of validity.

An alternative approach is pursued in the search for a theory of *quantum gravity*, which takes quantum theory and the quantum structure of spacetime to be fundamental (so that classical objects and classical spacetime would be emergent, large-scale structures). One recent version of this approach has grown out of the C^*-algebraic formulation of physical theories by taking seriously the notion that spacetime coordinates should be described as quantum variables, thus giving rise to a concept of *quantum spacetime* [16]. First promising steps towards formulations of quantum field theories on quantum spacetime have been reported in [17], for example. Insofar as spacetime coordinate observables turn out to be noncommuting, the natural framework for a description of spacetime measurements is that of the theory of unsharp observables, as this allows one to account for the simultaneous measurability of these noncommuting coordinates.

3.2. **Dequantization.** The question whether quantum mechanics can be recast in terms of classical physical theories has been asked ever since the advent of quantum mechanics. This is the dequantization problem, also known as the problem of finding a supplementation of Quantum Mechanics by means of *hidden variables*, that is, classical quantities that have definite values in all states. In Subsection 2.3 we surveyed arguments showing that quantum mechanics cannot be recast in terms of a non-contextual hidden variables theory. We have also recalled Bub's uniqueness theorem that elucidates the possible partial value attributions given one distinguished, definite observable.

Perhaps surprisingly at first, it turns out that there is a formulation of quantum mechanics as a classical probability theory which yields a universal attribution of unsharp values, or elements of unsharp reality. This formulation, which is based on the set Γ_q of pure quantum states as the "classical" phase space, was first studied rigorously in 1974 by B. Misra [18] and elaborated fully as a probabilistic model of quantum mechanics by S. Bugajski in the 1990s [19]. The space Γ_q is naturally equipped with a Borel structure via the metric induced by the trace norm. The convex set of all probability measures on Γ_q forms the classical state space S_c. There is an affine map R from S_c to the convex set S_q of all quantum states (density operators) given by the association

$$S_c \ni \mu \mapsto R(\mu) = \int_{\Gamma_q} P\mu(dP) \equiv \rho_\mu \in S_q. \tag{3}$$

The integral is defined in the sense that for every probability measure μ there is a density operator ρ_μ such that for any effect E,

$$\mathrm{tr}[\rho_\mu E] = \int_{\Gamma_q} \mathrm{tr}[PE]\mu(dP) \equiv \int_{\Gamma_q} f_E(P)\mu(dP). \tag{4}$$

This is the probability of the occurrence of the effect E in the state ρ_μ.

We note that the map R thus defined is many-to-one and associates with any density operator all probability distributions with which it can be decomposed. Further, the correspondence $E \mapsto f_E =: R'(E)$ defines the map R' dual to R, which sends all quantum effects to classical effects.

We find that quantum mechanics is thus presented as a classical theory with the set of effects restricted to a certain subset of fuzzy sets or vague propositions. In fact, even if E is a projection other than O or I, the corresponding classical effect f_E is a fuzzy set function assuming all values in the interval $[0,1]$. Correspondingly, the set of observables is restricted to certain *fuzzy random variables*. This observation has led to the development of a classical fuzzy probability theory, as documented in [19] and works cited there.

The classical presentation of quantum mechanics via the maps R, R' is comprehensive in that all quantum states and quantum effects have their classical counterparts. Moreover, it has been shown to be essentially unique by W. Stulpe and one of the authors [20]. This result provides formal support to the interpretation of indefinite quantum propositions in terms of degrees of reality defined by the quantum probabilities. Indeed, the formal probability expressions $\mathrm{tr}[EP]$ are recovered here as measures of the fuzziness of the vague proposition represented by E.

Moreover, quantum fuzziness is seen to be a reflection of a geometric peculiarity of the quantum phase space Γ_q: if E is taken to be a rank one projection, $E = P'$, then $f_E(P) = \mathrm{tr}[PP']$; this quantity can assume any value between 0 and 1 for any given P, by varying P'. In particular, $f_{P'}(P) = 0$ only when the two projections are mutually orthogonal. This shows that two maximal information states, or quantum phase space points, are "overlapping" rather than disjoint, and it is well known that this is the reason for the fact that two non-orthogonal states cannot be unambiguously separated by any single-shot measurement. In other words, two nonorthogonal pure quantum states always coexist in that the maximal properties they represent are fuzzy, giving rise to nonzero degrees of reality, $f_{P'}(P) = f_P(P')$.

This phenomenon is geometric in nature insofar as the overlap $f_{P'}(P) = \mathrm{tr}[PP']$ is related to the operator norm distance between P and P' via

$$\|P - P'\|^2 = \left[1 - \mathrm{tr}[PP']\right]. \tag{5}$$

By contrast, two distinct points in a classical physical phase space are always disjoint, the associated probability norm generates a discrete topology where two points have distance 1 if they are identical and 0 if they are different. Seen from this perspective, quantum mechanics appears *more continuous* than classical mechanics.

With this consideration the 'canonical classical presentation' of quantum mechanics due to to Misra and Bugajski explains the impossibility of a universal non-contextual hidden-variable formulation of quantum mechanics: insofar as this presentation is essentially unique, it shows that the only possible classical formulation is one whose global value ascription is intrinsically fuzzy. We note that this presentation displays a form of *preparation contextuality*, in that a change of preparation is not always reflected in a change of state—one and the same mixed quantum state ρ admits an infinity of different preparations, as reflected by the multitude of mixtures μ of point measures mapped to the same ρ [21].

3.3. **Classical limit of quantum mechanics.** The classical limit problem arises if Quantum Mechanics is taken to be a universal theory, encompassing classical physics. Intuitively, one would expect that the typically classical behavior of a macrosystem should emerge within a quantum mechanical description if relevant quantities are large compared to Planck's constant. The classical limit is commonly thought to be associated with the formal limit of Planck's constant approaching zero, $\hbar \to 0$. However, it takes considerable effort to carry out this limit in a mathematically satisfactory way [22]. Moreover it is not clear whether this simple process is capable of capturing the emergence of classical behavior *in operational terms*. In fact, there is more involved than taking the limit of a single parameter, as will be explained in the following example.

It is often said that coherent states constitute the best approximation to the states of a classical particle or a classical electromagnetic field. Yet it seems that this statement in isolation makes little sense. In order to see, for example, whether the centre of mass degree of freedom of a large system could be regarded as approximately classical, it is necessary to consider the conditions under which this variable could be said to have (relatively) well-defined values that can be measured without disturbing this degree of freedom. As a classical variable one would expect the centre of mass to have a definite position at all times, which would, according to classical deterministic reasoning, require its momentum to be equally definite.

The only way these requirements can be approximately reconciled with quantum mechanics is by way of representing the position and momentum of the centre of mass simultaneously as marginals of a quantum mechanical phase space observable. The approximate definiteness of the values of these quantities is given if they can be considered as relatively sharply defined on a macroscopic scale, while their noninvasive measurability requires their degrees of unsharpness to be large in comparison to Planck's constant. It then turns out that minimum uncertainty states are those that are least disturbed in such macroscopic, coarse-grained phase space measurements. Thus the underlying phase space observable must be equipped with macroscopically large inaccuracies. At the same time, in order for the values of these macroscopic pointer observables to be considered 'point-like', their unsharpness has to be small on the readout scale. A more detailed technical account of these classicality conditions for the observation of a macroscopic trajectory can be found in [23] together with further references to relevant studies.

We see that the theory of unsharp observables and their interpretation as elements of unsharp reality provides necessary key elements in the modeling of approximately classical behavior within quantum mechanics. More generally, within the set of general observables, pairs of observables may be jointly measurable without having to commute with each other. A sufficient degree of unsharpness will allow two POMs to be jointly measurable. Two POMs are called jointly measurable if they are marginals of a third POM. Joint measurability of a triple or quadruple of spin-1/2 observables entails the validity of Bell's inequalities for the pair distributions among these observables. This explains the fact, mentioned earlier, that Bell's inequalities are satisfied in the case of sufficiently unsharp variables. If this happens in the context of an EPR experiment, the usual argument against "local realism" no longer applies. One may say that in the case of sufficiently unsharp measurements, Bell-locality can be maintained as a classical fiction.

So far, we have discussed operational aspects of the classical limit problem. The universality claim of quantum mechanics also entails that there should be an account of the time evolution of macroscopic objects and, moreover, of hybrid microscopic-macroscopic systems, including measurement processes. Given this requirement, we are faced with the quantum measurement problem.

3.4. **The problem of objectification.** In the context of the quantum mechanical description of a measurement, quantum indefiniteness in conjunction with the linearity of (the unitary) time evolutions gives rise to the *measurement problem*, or *objectification problem*: if the system's initial state is a superposition of different eigenstates, then by the linearity of the unitary coupling map, the total system state will evolve into an *entangled* state, each component of which corresponds to a different pointer value. Thus the conclusion that a definite pointer value has been realized is not warranted. This is the problem of *pointer objectification*.

In a realist interpretation it is noteworthy that discrete observables (for example, sharp yes-no measurements) admit *repeatable* measurements, in which the coupling interaction leads to the measured property being strictly correlated with the pointer in the final total state. Thus, if a definite pointer value is indicated, one can conclude that the measured system observable possesses the definite value associated with this reading. However, the objectification problem extends to those cases in which the initial state is a superposition of eigenstates. Then, the post-measurement state of the total system is not an eigenstate of the measured observable, and hence the objectification of that object observable is not warranted according to the quantum mechanical description of a repeatable measurement.

These negative results have been made precise in a succession of *insolubility theorems* initiated by Wigner. The presently most general version and a brief history of these theorems is given in [24] and [25]. The assumptions of the insolubility theorem are (UD) the linear, unitary dynamics of quantum states, (R) the reality criterion given by the eigenvalue–eigenstate link, and (DO) the stipulation that a physical measurement process terminates with a definite outcome. These three requirements lead to a contradiction. One can eliminate the objectification problem by adopting some modified form of the the reality criterion (R); in each there will be a specific way of accounting for definite outcomes (DO). This has the advantage of leaving the formalism of quantum mechanics intact and untouched but—as made evident by the Bub–Clifton uniqueness theorem—it raises the problem of explaining the special status of one distinguished observable. As with the “many-worlds” interpretation, it appears in these interpretations as though quantum mechanics describes the world as if watched “from outside” by some non-physical entity.

By contrast, if one adopts the point of view that the reality of physical magnitudes is generally indefinite (or *unsharp*) and seeks an explanation for objective actualization of properties during measurement, the insolubility theorem is most naturally read as indicating the necessity of a modification of the dynamical law of quantum mechanics, given that in any measurement-like interaction there is the potential that an indicator outcome is actualized when in contact with the object system. It suggests that there is room for a modification of the Schrödinger equation along the lines proposed by Pearle [26] and Ghirardi, Rimini, and Weber [27], that is for the *spontaneous collapse* or *dynamical reduction* of quantum states.

3.5. **Unsharp objectification.** The question has been raised whether the generalized notion of unsharp reality underwrites a process of *unsharp objectification* that would allow one to avoid the conclusion of the insolubility theorem without the need for a modified dynamical law. A partial result has been obtained to the negative [25], but this is still based on the requirement of the—possibly unsharp—pointer observable admitting definite values. The idea of unsharp objectification is based on the description of a pointer observable as an *intrinsically unsharp* observable. Such observables, unlike sharp observables, do not admit outcomes that can occur with certainty. Technically speaking, the positive operators (other than the identity I) in the range of an intrinsically unsharp observable do not have eigenvalue 1.

If a pointer observable Z (defined as a POM on the (Borel) subsets of $\mathbb{R}$, say) is intrinsically unsharp, so that its effects ($\neq I$) do not admit probabilities equal to one, then the measured observable E defined according to the probability reproducibility condition is *also* an intrinsically unsharp observable given as a POM on $\mathbb{R}$. Thus, although a general proof is still outstanding, we would expect that a unitary measurement coupling cannot turn an element of unsharp reality with a potentiality (probability) of (say) 1/2 into an element of approximate reality with probability close to 1.

It is interesting to note here Peter Mittelstaedt’s comment on unsharp objectification. In [28] he writes:

> “It is well known, that within the framework of quantum mechanics it is not possible after a unitary premeasurement to attribute sharp or unsharp values to the pointer observable (Busch/Lahti/Mittelstaedt 1996; Mittelstaedt 1998 and Busch 1998). Even if merely unsharp values are attributed to the pointer, these values turn out to be not strictly reliable.[1] For macroscopic quantities, however, this unreliability is practically negligible, and hence it was never observed. On the other side, we should keep in mind that in quantum physics the requirement of objective and reliable pointer values - the pointer objectification postulate - is merely a reminiscence to classical physics. However, classical physics is based at least partially on ontological hypotheses without any rational or empirical justification. Hence, there is in principle no reason to maintain the requirement of objectification...
>
> [1] This means that ‘...one cannot claim with certainty that the reading one means to have taken is reproducible on a second look at the pointer’ (Busch 1998, p. 246).”

This argument seems to suggest that the notion of objective properties is based on ontological premises made tacitly in classical physics that were found empirically untenable and invalidated in the realm of quantum mechanics. If one assumes that quantum mechanics supersedes classical mechanics, it would then follow that it is the requirement of objectification that is untenable. This begs the question of what measurements are about if they cannot be regarded as providing definite outcomes in some sense.

However, there is an alternative possibility to the universality of quantum mechanics that has been put forth by a number of physicists, prominently including Günther Ludwig: it may eventually turn out to be the case that both quantum mechanics and classical mechanics are limiting cases of a more general and more comprehensive physical theory that assigns different domains of validity to these two theories. Such an extended framework would leave room for a notion of objective reality as it underlies the objectification requirement.

We mention two examples of theory or model constructions wherein this scenario has been schematized. The first example is given by the C^*-algebraic approach to the formulation of physical theories, which provides a framework in which the structure of the algebra of observables can either be Abelian (representing the classical case), or irreducible (as in standard quantum mechanics), or intermediate. In the last case the center of the algebra is neither trivial nor does it comprise the whole algebra. This corresponds to a quantum system with superselection rules, which may arise in relativistic quantum field theories or in quantum theories of macroscopic systems. In this approach there is thus a rigid distinction between cases with or without superselection rules, and the emergence of classical properties in the modeling of larger and larger quantum systems is somewhat discontinuous, as it requires a limit to a genuinely infinite number of degrees of freedom.

The second example that allows hybrid classical-and-quantum behavior are alterations of quantum mechanics with modified Schrödinger equations, such that spontaneous collapse happens at a rate that is related to the size of the system. In this approach the transition from quantum to classical features seems more continuous than in the latter approach. The criticism has been put forth that despite the fact that spontaneous collapse happens practically instantaneously, it is never actually complete, in the sense that it would leave the system in a single component state of its initial superposition. The distribution over the different component states is rather sharply peaked but never strictly concentrated on a single state. This is the so-called *tail problem* [29, 30, 31].

It seems that the feature just described as the tail problem can be accounted for naturally as an instance of the notion of approximate reality. Furthermore, it appears that the discontinuity problem for classical properties in the algebraic approach can at least be alleviated by giving due account of the nature of a supposedly classical pointer observable. In fact, prototypical pointer observables in a quantum measurement are the locations of pointer needles, and thus of systems with simultaneously rather well-defined (on a macroscopic scale) positions and momenta. As pointed out in Subsection 3.3, the natural representation of such observables within the framework of quantum mechanics is in terms of (covariant) phase space observables with intrinsic inaccuracies that are macroscopically large compared to Planck's constant but that can still be considered small on the scale of macroscopic measurement accuracies [23].

3.6. **Summary: emergence of classicality.** From the perspective of a position that takes quantum mechanics to be universal, a resolution of the quantum–classical contrast as reflected in the above four problems should be envisaged as follows. First, *quantization* should be recast as a procedure to formulate the quantum theory of a type of object *intrinsically*, without recourse to a classical physical background. This will presumably have to comprise a quantum account of spacetime. The *dequantization* (hidden variables) problem just highlights the fundamentally nonclassical nature of quantum mechanics and thus demonstrates that the *classical limit* can only be expected to account for an *approximate* emergent classical behavior of macroscopic objects and large-scale spacetime structure.

Finally, if in addition to the universality of Quantum Mechanics the (unsharp) objectification of measurement outcomes is maintained, the insolubility theorem suggests that this can only be realized by accepting a change of the dynamical postulate so as to allow for spontaneous collapse of the quantum state.

If the notion of state collapse as a real, stochastic physical process is considered in the context of localization experiments, one is forced to address the nonlocal nature of the collapse: the fact that a quantum object has been localized in one space region entails that the probability of its being localizable in another region has instantly become zero; the wavefunction (state in the position representation) will have collapsed to zero everywhere except in the region of localization. In the language of potentialities or their quantification as degrees of reality, this means that the system's power to trigger a localization event has instantaneously changed to zero outside the localization region. As is known from the no-signaling theorem, this collapse does not give rise to superluminal causal influences that could be used as signals. Still this abrupt change of the spatial distribution of the potentiality of localization appears to call for a causal account. Such an account could conceivably be obtained in a quantum theory that provides a description of the system *and* spacetime as quantum dynamical entities. Then the so-called spontaneous collapse could be construed as induced by dynamical features of spacetime.

4. Elements of unsharp quantum reality and their evolution

In this section we put the realist interpretation of Quantum Mechanics in terms of elements of (generally) unsharp reality to work in order to show how it affords a richer description of quantum systems than the standard account in terms of sharp realities alone. We begin with a historical commentary.

Bell believed that the idea that quantum systems might be "unspeakable," in that one cannot properly make statements about the properties of quantum systems but only about apparatus that measure them, was an error of the "founding fathers" of quantum theory [32, p. 171]. Instead of accepting such a restriction, realist interpretations of quantum states (in the physical sense of the term *realist*) consider the values precisely ascribed by pure quantum states as actual, and these actual properties constitute the *ontic state* of the system; any change in a pure state (be it through free evolution or measurement) is associated with a corresponding change in the ontic state.

Contrary to Bell's contention, such a realist view was in fact held by one of the founding fathers of quantum theory, W. Heisenberg, as laid down in his later work on the foundations of the theory [9]. He considers the 'probability function' (pure or mixed quantum state) as comprising objective and subjective elements. The former are "statements about possibilities or better tendencies ('potentia' in Aristotelian philosophy)" [9, p. 53], whereas the latter represent the observers' limited knowledge; this subjective element (lack of knowledge) is said to be negligible in the pure case. In particular, Heisenberg puts great emphasis on the "notion that a course of events in itself is not determined by necessity but that the possibility or rather the 'tendency' towards a course of events possesses itself a kind of reality—a certain intermediate level of reality midway between the massive reality of matter and the mental reality of an idea or picture..." [10, p. 140] (our translation). He notes that this concept of possibility is given in quantum theory in the form of probability.

Heisenberg thus goes beyond the usual interpretation of the pure quantum state ψ as the catalogue of all actual properties—those with probability equal to one—of an individual system in that he considers ψ as the catalogue of the potentialities of all possible (sharp) properties Q of the system, quantified by the probabilities $p_\psi(Q) = \langle\psi|Q\psi\rangle$. The notion of actualization of potentialities then makes it possible for one to "say that the transition from the 'possible' to the 'actual' takes place as soon as the interaction between the object and the measuring device, and thereby with the rest of the world, has come into play; it is not connected with the act of registration of the result in the mind of the observer." [9, pp. 54-55].

In the more recent work in the philosophy of physics, potentiality has been further articulated by Shimony as a modality of existence of physical systems that confers an intermediate status to properties between that of bare logical possibility and full actuality [33]. As Shimony points out, the existence of such an intermediate modality is in fact *required* by the non-local correlations violating Bell's inequality [33, Vol. II p. 179]. This connection is in accordance with the fact that, formally, entanglement is an instance of the superposition of mutually orthogonal product states, and thus gives rise to quantum indeterminacy.

Here we propose to add the idea that potentiality—initially conceived as a tendency or propensity of the property or effect in question to actualize a value or an indication of it in a measurement device—can be understood as a causal agency and therefore constitutes a limited degree of reality, rendering the property or effect as an element of unsharp reality.[4]

To further explicate this extended single-case interpretation of quantum mechanical probabilities, we take up the observation, noted earlier, that the pure state ψ of an isolated quantum system, represented as a projection $P = P_\psi$, can be viewed as a point in this system's phase space Γ_q that evolves deterministically along a trajectory given by the Schrödinger equation. This time development gives rise to a continuous evolution of the potentialities, that is, degrees of reality $f_E(P) = \mathrm{tr}[PE] = \langle\psi|E\psi\rangle$ of all the effects E of the system, which thereby are all simultaneously real to a degree (actualization tendency) given by $f_E(P)$.

In addition to such attribution of degrees of reality, the use of POMs enables the simultaneous measurement of approximate values of noncommuting properties. In fact, the joint measurement of properties for which sharp observables are incompatible is possible: Although non-commuting sharp (standard) observables are never jointly measurable, they are capable of approximation by a pair of related unsharp observables, represented by suitable POMs, which *will be* jointly measurable. Moreover, similar to Lüders measurements, which are perfectly repeatable, it is possible to perform joint approximate measurements that have approximate repeatability properties, leading to increased degrees of reality of the measured properties.

Thus, let us take quantum probability as characterizing the tendency to actualize in the ontic, as opposed to the merely epistemic sense. That is, let us take the expression $p^E_\psi(X) = \langle\psi|E(X)\psi\rangle$, for effect $E(X)$ associated with the value set X, as providing the likelihood of the actualization of the potential property, whether sharp or unsharp, when

[4] In current research in the philosophy of physics a case is being made for the causal power of dispositions, or potentialities; this is laid out in the context of the quantum measurement problem in a contribution to this Festschrift by M. Esfeld [34].

measured on a system prepared in pure state ψ. In the standard account, the measured system interacts with a probe system, resulting in an entangled state for the total system. We can assume a similar scheme for approximate joint measurement of both position and momentum since the existence of such schemes is warranted by the existence of POMs that represent joint observables for momentum and position (as reviewed in [35]).

If the couplings between the object system and two appropriate measurement probes are activated simultaneously, the resulting measurement scheme constitutes a joint approximate measurement of both position, q, and momentum, p. No matter how small the position and momentum imprecisions if measurements are made of them separately, the joint coupling of both probes results in a re-adjustment of the individual measurement imprecisions in such a way that they satisfy an uncertainty relation. It is possible to choose measurement couplings such that the joint measurement is approximately repeatable [23]. Thus, upon obtaining a joint reading of q and p, the quantum particle will be found afterwards in a state in which position and momentum are unsharply localised at that point: the centers and widths of the associated position and momentum distributions are equal to the values q, p and the measurement imprecisions $\delta q, \delta p$, respectively.

The example of approximately repeatable phase space measurement provides a theoretical underpinning of the reality of bubble chamber or cloud chamber tracks of elementary particles. It also corroborates Heisenberg's famous dictum, "I believe that the existence of the classical 'path' can be pregnantly formulated as follows: The 'path' comes into existence only when we observe it." [1, p. 185]. We are interested in observing these tracks because they are probes for what is going on in the high energy collisions or decays in which these particles are produced. One may even say that the approximate repeatability is necessary because without it one cannot reconstruct the energy-momentum balance of the processes.

These considerations shows that the possibility of simultaneously attributing unsharp(ly defined) values to noncommuting quantities is underwritten by the quantum theory of measurement, according to which these values are likelihoods of actualization (thus degrees of reality) that can be tested in appropriate measurements.

Thus, while the theory of approximate joint measurements of position and momentum delineates the possibilities of joint measurements of noncommuting effects, the 'phase-space' presentation of quantum mechanics provided by the Misra-Bugajski map describes the limited possibilities of preparation. An indeterminacy interpretation of quantum uncertainties thus establishes consistency between the possibilities of state definition, that is, the preparation of noncommuting sets of effects, and the possibilities of determination, that is, the joint measurement of these quantities.

When a measurement is performed, there is a stochastic change of the states of both the measured system and the measuring system, whether either a sharp or an unsharp observable is measured. When a *repeatable* measurement is made of a sharp observable of the measured system, the properties associated with sharp observables compatible with that observable become actual and definite and those associated with incompatible sharp observables become indefinite and potential. When a measurement is made of an *unsharp* observable, it remains fuzzy but can become less indeterminate and properties associated with compatible unsharp observables remain fuzzy and can become more indeterminate.

To summarize, given the POM formalism, the ontic potentiality approach gains further reach than is the case given only the PVMs; one is able to provide further explanations of the behavior of quantum systems, because one can make use of unsharp observables to explain the results of joint measurements of physical properties traditionally thought of as incompatible and unspeakable. In particular, better predictions of subsequent measurements can be provided by the updated probabilities corresponding to the observed outcomes, whether the measured observable is sharp or unsharp, so long as the corresponding uncertainty product is reduced.

In addition to accommodating the reality of restrictions on the joint determination of properties of quantum systems, the use of the POM formalism under the above ontic potentiality interpretation allows one conceptually to ground quantum probability and to provide explanations for the statistics of outcomes of both sharp and unsharp measurements, rather than leaving many situations "unspeakable."

Although, when joint approximate measurements are made of both of a pair of physical magnitudes, say position and momentum, there are only actually fuzzy and potentially sharp properties, both properties do exist and can be spoken of. Depending on an experimenter's interests, he can choose to make either a sharp measurement to determine a single property of a complementary pair or he can make an optimal joint unsharp measurement to *jointly* specify a pair of corresponding indeterminate properties; sharp observables that are incompatible with the one that is measured are real to a limited degree.

Quantum fuzziness is not a reflection of some theoretical deficiency but can be naturally related to probability. Although the sharp physical magnitudes are not (fully) actualized during approximate, unsharp measurements, the likelihoods for the value ranges corresponding to the outcomes that are found can increase. In this way, a clear and consistent meaning is given to (generalized) quantum observables, namely, as fuzzy properties always consistently jointly attributable to individual quantum systems, where these properties have an evolution that is explicable in a well defined manner and can be used to provide physical explanations.

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