

Lorentzian theories vs. Einsteinian special relativity – a logico-empiricist reconstruction

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Introduction

1. It is widely believed that the principal difference between Einstein’s special relativity and its contemporary rival Lorentz-type theories was that while the Lorentz-type theories were also capable of “explaining away” the null result of the Michelson–Morley experiment and other experimental findings by means of the distortions of moving measuring-rods and moving clocks, special relativity revealed more fundamental new facts about the geometry of space-time behind these phenomena. For the sake of brevity, I shall use the term “Lorentz theory” as classification to refer to the similar approaches of Lorentz, FitzGerald, and Poincaré, that save the classical Galilei covariant conceptions of space and time by explaining the experimental findings through the physical distortions of moving objects—first of all of moving measuring equipments—no matter whether these physical distortions are simply hypothesized in the theory, or prescribed by some “principle” like Lorentz’s principle, or they are constructively derived from the behavior of the molecular forces. From the point of view of my recent concerns what is important is the *logical possibility* of such an alternative theory. Although, Lorentz’s 1904 paper or Chapter V. of his *The theory of electrons* (1909) are good historic examples.

According to this widespread view, special relativity was, first of all, a radically new theory about space and time. A theory about space and time describes *a certain group of objective features* of physical reality, which we call (the structure of) space-time. Consider the claims like these:

- According to classical physics, the geometry of space-time is $\mathbb{E}^3 \times \mathbb{E}^1$, where \mathbb{E}^3 is a three-dimensional Euclidean space for space and \mathbb{E}^1 is a one-dimensional Euclidean space for time, with two independent invariant metrics corresponding to the space and time intervals.

- In contrast, special relativity claims that the geometry of space-time is different: it is a Minkowski geometry \mathbb{M}^4 .

The two statements are usually understood as telling *different things about the same* objective features of physical reality. One can express this change by the following logical schema: Earlier we believed in $G_1(\hat{M})$, where \hat{M} stands for (the objective features of physical reality called) space-time and G_1 denotes some predicate (like “of type- $\mathbb{E}^3 \times \mathbb{E}^1$ ”). Then we discovered that $\neg G_1(\hat{M})$ but $G_2(\hat{M})$, where G_2 denotes a predicate different from G_1 (something like “of type- \mathbb{M}^4 ”).

This is however not the case. Our analysis will show that the correct logical schema is this: Earlier we believed in $G_1(\hat{M})$. Then we discovered for some *other* features of physical reality $\tilde{M} \neq \hat{M}$ that $\neg G_1(\tilde{M})$ but $G_2(\tilde{M})$. Consequently, it still may (and actually does) hold that $G_1(\hat{M})$. In other words, in comparison with the pre-relativistic Galileo-invariant conceptions, special relativity tells us nothing new about the geometry of space-time. It simply calls something else “space-time”, and this something else has different properties. We will also show that all statements of special relativity about those features of reality that correspond to the original meaning of the terms “space” and “time” are identical with the corresponding traditional pre-relativistic statements. Thus the only new factor in the special relativistic account of space-time is the terminological decision to designate something else “space-time”.

2. So the real novelty in special relativity is some $G_2(\tilde{M})$. It will be also argued, however, that $G_2(\tilde{M})$ does not contradict to what the Lorentz theory claims. Both, the Lorentz theory and special relativity claim that $G_1(\hat{M}) \& G_2(\tilde{M})$. In other words: *Special relativity and the Lorentz theory are identical theories about space and time in all sense of the words.*

Moreover, I shall show that the two theories provide identical description of the behavior of moving physical objects. Thus, we will arrive at the following final conclusion: *Special relativity and the Lorentz theory are completely identical in both senses, as theories about space and time and as theories about the behavior of moving physical objects.*

Empirical definitions of space and time tags

3. Physics describes objective features of reality by means of physical quantities. Our scrutiny will therefore start by clarifying how classical physics and relativity theory define the space and time tags assigned to an arbitrary event. When I say “definition”, I mean *empirical* definition, somewhat similar to Reichenbach’s “coordinative definitions”, Carnap’s “rules of correspondence”, or Bridgman’s “operational definitions”; which give an empirical interpretation of the theory. Einstein too, at least in his early writings, strongly emphasizes that all spatiotemporal terms he uses are based on operations applying measuring rods, clocks and light signals.

The empirical definition of a physical quantity requires an *etalon* measuring equipment and a precise description of the operation how the quantity to be defined is measured. For example, assume we choose, as the *etalon* measuring-rod, the meter stick that is lying in the International Bureau of Weights and Measures (BIPM) in Paris. Also assume—this is another convention—that “time” is defined as a physical quantity measured by the standard clock, say, also sitting in the BIPM. When I use the word

“convention” here, I mean the *semantical freedom* we have in the use of the uncommitted signs “distance” and “time”—a freedom what Grünbaum (1974, p. 27) calls “trivial semantical conventionalism”. Of course, these semantical conventions include not only the choices of the standard measuring-rod and the standard clock, but also includes the operational definitions we are giving below.

4. Consider two frames of reference: K , in which the *etalons* are at rest, and K' which is moving at constant velocity relative to K . As usual, we assume that the coordinate axes are parallel, and K' moves relative to K at the velocity v in the positive x -direction.

We are going to define the space and time tags of an arbitrary event A . I call them space and time “tags” rather than space and time “coordinates”. By this terminology I would like to distinguish a particular kind of space and time coordinates which are provided with *direct* empirical meaning from space and time coordinates in general, the empirical meaning of which can be *deduced* from the empirical meaning of the space and time tags. Once we have space and time tags defined, we can introduce arbitrary other *coordinates* on the manifold of space-time tags. The physical/empirical meaning of a *point* of the manifold is however determined by the space-time tags of physical/empirical meaning. Only in this way we can confirm or falsify, empirically, a spatiotemporal physical statement.

Preliminary facts

5. In order to explain the intuitions behind the definitions below, let us recall a few well known facts, to the detailed discussion of which we will return later.

(a) Both the Lorentz theory and Einstein’s special relativity “know” about the distortions of measuring-rods and clocks when they are in motion relative to the “stationary” reference frame (of the BIPM, say). Einstein writes:

A rigid body which, measured in a state of rest, has the form of a sphere, therefore has in a state of motion—viewed from the stationary system—the form of an ellipsoid of revolution with the axes.

$$R\sqrt{1 - \frac{v^2}{c^2}}, R, R.$$

Thus, whereas the Y and Z dimensions of the sphere (and therefore of every rigid body of no matter what form) do not appear modified by the motion, the X dimension appears shortened in the ratio $1 : \sqrt{1 - \frac{v^2}{c^2}}$ [...]

Further, we imagine one of the clocks which are qualified to mark the time t when at rest relatively to the stationary system, and the time τ when at rest relatively to the moving system, to be located at the origin of the co-ordinates of k , and so adjusted that it marks the time τ . What is the rate of this clock, when viewed from the stationary system?

Between the quantities x , t , and τ , which refer to the position of the clock, we have, evidently, $x = vt$ and

$$\tau = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \left(t - \frac{vx}{c^2} \right).$$

Therefore,

$$\tau = t \sqrt{1 - \frac{v^2}{c^2}}$$

[...] whence it follows that the time marked by the clock (viewed in the stationary system) is slow [...]

From this there ensues the following peculiar consequence. If at the points A and B of K there are stationary clocks which, viewed in the stationary system, are synchronous; and if the clock at A is moved with the velocity v along the line AB to B , then on its arrival at B the two clocks no longer synchronize, but the clock moved from A to B lags behind the other which has remained at B by $\frac{tv^2}{2c^2}$ (up to magnitudes of fourth and higher order), t being the time occupied in the journey from A to B . (Einstein 1905)

Probably, this last phenomenon is why Einstein suggested the light signal method of synchronization, instead of the even more obvious transportation of clocks from one place to the other.

- (b) However, the loss of phase accumulated by the clock during its transportation tends to zero if the transportation is very slow relative to the stationary reference frame:

$$\lim_{v \rightarrow 0} \frac{tv^2}{2c^2} = \lim_{v \rightarrow 0} \frac{xv}{2c^2} = 0$$

where x is the distance of the transportation. Therefore, in the stationary frame of reference, the synchronization by means of slow transportation of clocks is equivalent with Einstein's synchronization by means of light signals.

- (c) Moreover, this equivalence is true in an arbitrary reference frame moving relative to the stationary frame. Imagine a clock at rest relative to the moving frame of reference K' at a point A' . Then, we slowly (relative to K') transport the clock from A' to another point of the moving frame B' . As it will be shown in details (Point 14), the loss of phase accumulated by the clock during its slow transportation does not tend to zero but to a finite value, which is exactly equal to the one corresponding to Einstein's light signal synchronization.

6. Thus, when we will define distance by means of measuring rod, we will follow Einstein's empirical definitions in his 1905 paper; we will do the same when we will define time at the origin of a reference frame by means of a co-moving copy of the standard clock; and when we will define the time tags of events distant from the origin of the frame by means of the readings of slowly transported copies of the standard clock, we will use an operation which is equivalent to Einstein's light signal definition of simultaneity.

The reader may wonder, why do we choose this old-fashioned way of defining time and space tags, by means of measuring rods and slowly transported clocks; instead of the more modern "one clock + light signals" method offered in many textbooks—not to mention the obvious operational circularities related with the conceptions like "rigid body", "reference frame"? We have three main reasons for this choice: First, our aim in this paper is to reconstruct the definitions of the spatiotemporal terms as they are understood in Einstein's special relativity and in the Lorentz-type theories (that is, in classical physics), without questioning whether these definitions are tenable or not.

Second, by applying measuring rods and slowly transported clocks, it becomes more explicit that the Lorentz transformations express the *physical behavior* of measuring rods and clocks. In Einstein's words:

A Priori it is quite clear that we must be able to learn something about the physical behavior of measuring-rods and clocks from the equations of transformation, for the magnitudes z, y, x, t are nothing more nor less than the results of measurements obtainable by means of measuring-rods and clocks. (Einstein 1920, p. 35)

Third, the "one clock + light signals" method is actually much more complex than suggested in the textbook literature (see Szabó 2009).

Definitions

7. For the sake of simplicity consider only one space dimension and assume that the origin of both K and K' is at the BIPM at the initial moment of time.

There will be no difference between the Einsteinian and Lorentzian definitions in case of space and time tags in the stationary frame of reference, that is, in the frame of the *etalons*.

(D1) Time tag in K according to special relativity

Take a synchronized copy of the standard clock at rest in the BIPM, and slowly¹ move it to the locus of event A . The time tag $\tilde{t}^K(A)$ is the reading of the transferred clock when A occurs.

(D2) Space tag in K according to special relativity

The space tag $\tilde{x}^K(A)$ of event A is the distance from the origin of K of the locus of A along the x -axis² measured by superposing the standard measuring-rod, being always at rest relative to K .

(D3) Time tag in K according to classical physics

The same as (D1): $\hat{t}(A) = \tilde{t}(A)$.

(D4) Space tag in K according to classical physics

The same as (D2): $\hat{x}(A) = \tilde{x}(A)$.

8. The difference between the Einsteinian and Lorentzian understandings starts with the definition of the space and time tags in the *moving* reference frame K' . The two approaches agree that measuring-rods and clocks suffer distortions when transferred from the BIPM to the moving (relative to the BIPM) reference frame K' . In the special relativistic definitions, however, *we will ignore* this fact and define the space and time tags just as they are measured by means of the distorted equipments:

(D5) Time tag in K' according to special relativity

Take a synchronized copy of the standard clock at rest in the BIPM, gently accelerate it from K to K' and set it to show 0 when the origins of K and K' coincide. Then slowly (relative to K') move it to the locus of event A . The time tag $\tilde{t}^{K'}(A)$ is the reading of the transferred clock when A occurs.

(D6) Space tag in K' according to special relativity

The space tag $\tilde{x}^{K'}(A)$ of event A is the distance from the origin of K' of the locus of A along the x -axis measured by superposing the standard measuring-rod, being always at rest relative to K' , in just the same way as if all were at rest.

In contrast, in the Lorentzian approach, following the tradition of classical physics³ one has to take into account that the moving clock is slower, therefore the clock-reading $\tilde{t}^{K'}(A)$ in (D5) differs from the proper time defined by the *etalon* clock. Similarly, the “distance” $\tilde{x}^{K'}(A)$ obtained by means of the co-moving, therefore contracted, measuring-rod is not the proper distance from the origin of K' . Therefore, in the Lorentzian approach we define the space and time tags in the moving reference frame K' by means of the original *etalon* clock and *etalon* measuring-rod, according to the classical Galilean kinematics:

(D8) Time tag of an event in K' according to classical physics

The time tag of event A relative to the frame K' is

$$\hat{t}^{K'}(A) := \hat{t}^K(A) \tag{1}$$

(D7) Space tag of an event in K' according to classical physics

The space tag of event A relative to the frame K' is

$$\hat{x}^{K'}(A) := \hat{x}^K(A) - v\hat{t}^K(A) \tag{2}$$

9. Yet one might raise the historical question whether our reconstruction is correct. We shall not go into these philological details. Concerning definitions (D1)–(D2) and (D5)–(D6) I only refer to the following passage from Einstein’s 1905 paper:

Let there be given a stationary rigid rod; and let its length be l as measured by a measuring-rod which is also stationary. We now imagine the axis of the rod lying along the axis of x of the stationary system of co-ordinates, and that a uniform motion of parallel translation with velocity v along the axis of x in the direction of increasing x is then imparted to the rod. We now inquire as to the length of the moving rod, and imagine its length to be ascertained by the following two operations:

- (a) The observer moves together with the given measuring-rod and the rod to be measured, and measures the length of the rod directly by superposing the measuring-rod, in just the same way as if all three were at rest.

- (b) By means of stationary clocks set up in the stationary system and synchronizing in accordance with [the light-signal synchronization], the observer ascertains at what points of the stationary system the two ends of the rod to be measured are located at a definite time. The distance between these two points, measured by the measuring-rod already employed, which in this case is at rest, is also a length which may be designated “the length of the rod.”

In accordance with the principle of relativity the length to be discovered by the operation (a)—we will call it “the length of the rod in the moving system”—must be equal to the length l of the stationary rod.

The length to be discovered by the operation (b) we will call “the length of the (moving) rod in the stationary system.” This we shall determine on the basis of our two principles, and we shall find that it differs from l .

Concerning (D3)–(D4) and (D7)–(D8) I refer to Einstein’s own reading of the Lorentzian approach, quoted in Point **24** below. In addition, I refer to the calculation we will make in Point **14**, from which one can see that the slow-transport-of-clock and the light-signal synchronizations are equivalent.

10. With empirical definitions (D1)–(D8), in every inertial frame we defined four different quantities for each event; such that:

$$\hat{x}^K(A) \equiv \tilde{x}^K(A) \quad (3)$$

$$\hat{t}^K(A) \equiv \tilde{t}^K(A) \quad (4)$$

for the reference frame of the *etalons*, and

$$\hat{x}^{K'}(A) \not\equiv \tilde{x}^{K'}(A) \quad (5)$$

$$\hat{t}^{K'}(A) \not\equiv \tilde{t}^{K'}(A) \quad (6)$$

for any other inertial frame of reference. (\equiv denotes the identical empirical definition.)

In spite of the different empirical definitions, it could be a *contingent* fact of nature that $\hat{x}^{K'}(A) = \tilde{x}^{K'}(A)$ and/or $\hat{t}^{K'}(A) = \tilde{t}^{K'}(A)$ for every event A . However, that is obviously not the case here.

Thus, our first partial conclusion is that *different* physical quantities are called “space” tag, and similarly, *different* physical quantities are called “time” tag in special relativity and in the Lorentz theory.⁴ In order to avoid further confusion, from now on $\widehat{\text{space}}$ and $\widehat{\text{time}}$ tags will mean the physical quantities defined in (D3), (D4), (D7), and (D8)—according to the usage of the terms in classical physics—, and “space” and “time” in the sense of the relativistic definitions (D1), (D2), (D5) and (D6) will be called $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$.

Special relativity theory makes different assertions about somethings which are *different* from $\widehat{\text{space}}$ and $\widehat{\text{time}}$. In our symbolic notations, classical physics claims $G_1(\hat{M})$ about \hat{M} and relativity theory claims $G_2(\tilde{M})$ about some other features of reality, \tilde{M} . As we will see, both the Lorentz theory and Einstein’s special relativity are sufficiently complete accounts of physical reality to describe \hat{M} as well as \tilde{M} . The question is: What do the two theories say when they are making assertions about the same things?

Special relativity does not tell us anything new about space and time

11. Classical physics—from this point of view, Lorentz-type theories included—calls “space” and “time” what we denoted by $\widehat{\text{space}}$ and $\widehat{\text{time}}$. Let A be an arbitrary event and let K^* be an arbitrary inertial frame of reference. Denote $[\hat{x}^{K^*}(A)]_{relativity}$ and $[\hat{t}^{K^*}(A)]_{relativity}$ the $\widehat{\text{space}}$ and $\widehat{\text{time}}$ tags of A predicted by relativity theory and denote $[\hat{x}^{K^*}(A)]_{classical}$ and $[\hat{t}^{K^*}(A)]_{classical}$ the similar tags predicted by classical physics. Relativity theory would tell us something new if it accounted for physical quantities \hat{x} and \hat{t} differently. If there were any event A and any inertial frame of reference K^* such that $[\hat{x}^{K^*}(A)]_{relativity}$ and $[\hat{t}^{K^*}(A)]_{relativity}$ were different from $[\hat{x}^{K^*}(A)]_{classical}$ and $[\hat{t}^{K^*}(A)]_{classical}$. If, for example, there were any two events simultaneous in relativity theory which were not simultaneous according to classical physics, or vice versa—to touch on a sore point. But a little reflection shows that this is not the case. Taking into account empirical identities (3)–(4), one can calculate the relativity theoretic prediction for the outcomes of the measurements described in (D3), (D4), (D7), and (D8). The relativity theoretic prediction for $\hat{x}^{K'}(A)$:

$$[\hat{x}^{K'}(A)]_{relativity} = \tilde{x}^K(A) - \tilde{v}^K(K')\tilde{t}^K(A) \quad (7)$$

the value of which is equal to

$$\hat{x}^K(A) - \tilde{v}^K(K')\hat{t}^K(A) = [\hat{x}^{K'}(A)]_{classical} \quad (8)$$

Similarly,

$$[\hat{t}^{K'}(A)]_{relativity} = \tilde{t}^K(A) = \hat{t}^K(A) = [\hat{t}^{K'}(A)]_{classical} \quad (9)$$

This completes the proof of our first thesis that special relativity does not tell us anything new about those features of reality that correspond to the original meaning of the terms “space” and “time”.

The Lorentz theory and special relativity are identical theories of space and time

12. It is now in order to specify in a more formal way what I exactly mean by the Lorentz theory. I shall give the final and most general formulation in Point 17. At this stage, we will use the following definition:

(L1) The classical Galilean kinematics in terms of the classical conceptions of $\widehat{\text{space}}$ and $\widehat{\text{time}}$.

(L2) The assumption that

(L2a) the dimension parallel to \mathbf{v} of a solid suffers contraction by factor $\sqrt{1 - \frac{v^2}{c^2}}$ and

(L2b) a physical process in a physical system slows down by factor $\sqrt{1 - \frac{v^2}{c^2}}$

when they are gently accelerated from the reference frame of the *etalons*, K , to the frame K' moving at velocity \mathbf{v} relative to K .

This is nothing but what Grünbaum (1974, p. 723) calls the “double amended variant of the aether theory”, with the only difference, of course, that we have not even mentioned the aether yet. Note that it is sometimes mistakenly claimed that we cannot derive the Lorentz transformation equations from the doubly amended theory alone. As we will see it below, the truth is what Grünbaum correctly asserts that the “theory permits the deduction of the Lorentz transformation equations no less than does Einstein’s special theory of relativity” (*Ibid.*).

13. Since the Lorentz theory adopts the classical conceptions of $\widehat{\text{space}}$ and $\widehat{\text{time}}$, it does not differ from special relativity in its assertions about $\widehat{\text{space}}$ and $\widehat{\text{time}}$. What about the other claim— $G_2(\tilde{M})$ —about $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$? In order to prove that the Lorentz theory and special relativity are identical theories about space and time in all sense of the words, we also have to show that the two theories have identical assertions about \tilde{x} and \tilde{t} , that is,

$$\begin{aligned} [\tilde{x}^{K'}(A)]_{\text{relativity}} &= [\tilde{x}^{K'}(A)]_{LT} \\ [\tilde{t}^{K'}(A)]_{\text{relativity}} &= [\tilde{t}^{K'}(A)]_{LT} \end{aligned}$$

According to relativity theory, the $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$ tags in K' and in K are related through the Lorentz transformations. From (3)–(4) we have

$$[\tilde{t}^{K'}(A)]_{\text{relativity}} = \frac{\hat{t}^K(A) - \frac{v\hat{x}^K(A)}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (10)$$

$$[\tilde{x}^{K'}(A)]_{\text{relativity}} = \frac{\hat{x}^K(A) - v\hat{t}^K(A)}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (11)$$

14. On the other hand, taking the immediate consequences of (L2a) and (L2b) that the measuring-rod suffers a contraction by factor $\sqrt{1 - \frac{v^2}{c^2}}$ and the standard clock slows down by factor $\sqrt{1 - \frac{v^2}{c^2}}$ when they are gently accelerated from K to K' , one can directly calculate the $\widetilde{\text{space}}$ tag $\tilde{x}^{K'}(A)$ and the $\widetilde{\text{time}}$ tag $\tilde{t}^{K'}(A)$, following the descriptions of the operations in (D5) and (D6).

First, let us calculate the reading of the clock slowly transported in K' from the origin to the locus of an event A . For the sake of simplicity we continue to restrict our calculation to one space dimension. (For the general calculation of the phase shift suffered by moving clocks, see Jánosy 1971, pp. 142–147.) The clock is moving with a varying velocity

$$\hat{v}_C^K(\hat{t}^K) = v + \hat{w}^K(\hat{t}^K)$$

where $\hat{w}^K(\hat{t}^K)$ is the $\widehat{\text{velocity}}$ of the clock relative to K' , that is, $\hat{w}^K(0) = 0$ when it starts at $\hat{x}_C^K(0) = 0$ (as we assumed, $\hat{t}^K = 0$ and the transported clock shows 0 when the origins of K and K' coincide) and $\hat{w}^K(\hat{t}_1^K) = 0$ when the clock arrives at the place of A . The reading of the clock at the time \hat{t}_1^K will be

$$T = \int_0^{\hat{t}_1^K} \sqrt{1 - \frac{(v + \hat{w}^K(\hat{t}))^2}{c^2}} d\hat{t} \quad (12)$$

Since \hat{w}^K is small we may develop in powers of \hat{w}^K , and we find from (12) when neglecting terms of second and higher order

$$T = \frac{\hat{t}_1^K - \frac{\left(\hat{t}_1^K v + \int_0^{\hat{t}_1^K} \hat{w}^K(\hat{t}) d\hat{t}\right)v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{\hat{t}^K(A) - \frac{\hat{x}^K(A)v}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (13)$$

(where, without loss of generality, we take $\hat{t}_1^K = \hat{t}^K(A)$). Thus, according to the definition of \tilde{t} , we have

$$\left[\tilde{t}^{K'}(A)\right]_{LT} = \frac{\hat{t}^K(A) - \frac{v\hat{x}^K(A)}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad (14)$$

which is equal to $\left[\tilde{t}^{K'}(A)\right]_{relativity}$ in (10).

Now, taking into account that the length of the co-moving meter stick is only $\sqrt{1 - \frac{v^2}{c^2}}$, the distance of event A from the origin of K is the following:

$$\hat{x}^K(A) = \hat{t}^K(A)v + \tilde{x}^{K'}(A)\sqrt{1 - \frac{v^2}{c^2}} \quad (15)$$

and thus

$$\left[\tilde{x}^{K'}(A)\right]_{LT} = \frac{\hat{x}^K(A) - v\hat{t}^K(A)}{\sqrt{1 - \frac{v^2}{c^2}}} = \left[\tilde{x}^{K'}(A)\right]_{relativity}$$

This completes the proof. The two theories make completely identical assertions not only about space and time tags \hat{x}, \hat{t} but also about space and time tags \tilde{x}, \tilde{t} .

15. Consequently, there is *full agreement* between the Lorentz theory and special relativity theory in the following statements:

- (a) $\widehat{velocity}$ —which is called “velocity” by relativity theory—is not an additive quantity,

$$\widehat{v}^{K'}(K''') = \frac{\widehat{v}^{K'}(K'') + \widehat{v}^{K''}(K''')}{1 + \frac{\widehat{v}^{K'}(K'')\widehat{v}^{K''}(K''')}{c^2}}$$

while $\widehat{velocity}$ —that is, what we traditionally call “velocity”—is an additive quantity,

$$\hat{v}^{K'}(K''') = \hat{v}^{K'}(K'') + \hat{v}^{K''}(K''')$$

where K', K'', K''' are arbitrary three frames. For example,

$$\hat{v}^{K'}(light\ signal) = \hat{v}^{K'}(K'') + \hat{v}^{K''}(light\ signal)$$

- (b) The $(\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{t})$ -map of the world can be conveniently described through a Minkowski geometry, such that the \tilde{t} -simultaneity can be described through the orthogonality with respect to the 4-metric of the Minkowski space, etc.
- (c) The $(\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{t})$ -map of the world, can be conveniently described through a traditional “space-time geometry” like $\mathbb{E}^3 \times \mathbb{E}^1$.
- (d) The $\widehat{velocity}$ of light is not the same in all inertial frames of reference.

- (e) The $\widetilde{\text{velocity}}$ of light is the same in all inertial frames of reference.
 - (f) $\widetilde{\text{Time}}$ and $\widetilde{\text{distance}}$ are invariant, the reference frame independent concepts, $\widetilde{\text{time}}$ and $\widetilde{\text{distance}}$ are not.
 - (g) \hat{t} -simultaneity is an invariant, frame-independent concept, while \tilde{t} -simultaneity is not.
 - (h) For arbitrary K' and K'' , $\hat{x}^{K'}(A), \hat{t}^{K'}(A)$ can be expressed by $\hat{x}^{K''}(A), \hat{t}^{K''}(A)$ through a suitable Galilean transformation
 - (i) For arbitrary K' and K'' , $\tilde{x}^{K'}(A), \tilde{t}^{K'}(A)$ can be expressed by $\tilde{x}^{K''}(A), \tilde{t}^{K''}(A)$ through a suitable Lorentz transformation.
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To sum up symbolically, the Lorentz theory and special relativity theory have identical assertions about both \hat{M} and \tilde{M} : they unanimously claim that $G_1(\hat{M}) \& G_2(\tilde{M})$.

16. Finally, note that in an arbitrary inertial frame K' for every event A the tags $\hat{x}_1^{K'}(A), \hat{x}_2^{K'}(A), \hat{x}_3^{K'}(A), \hat{t}^{K'}(A)$ can be expressed in terms of $\tilde{x}_1^{K'}(A), \tilde{x}_2^{K'}(A), \tilde{x}_3^{K'}(A), \tilde{t}^{K'}(A)$ and vice versa. Consequently, we can express the laws of physics—as is done in special relativity—equally well in terms of the variables $\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{t}$ instead of the $\widehat{\text{space}}$ and $\widehat{\text{time}}$ tags $\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{t}$. On the other hand, we should emphasize that the one-to-one correspondence between $\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{t}$ and $\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{t}$ also entails that the laws of physics (so called “relativistic” laws included) can be equally well expressed in terms of the traditional $\widehat{\text{space}}$ and $\widehat{\text{time}}$ tags $\hat{x}_1, \hat{x}_2, \hat{x}_3, \hat{t}$ instead of the variables $\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, \tilde{t}$. In brief, physics could manage equally well with the classical Galileo-invariant conceptions of $\widehat{\text{space}}$ and $\widehat{\text{time}}$.

The Lorentz theory and special relativity are completely identical theories

17. Although special relativity does not tell us anything new about space and time, both special relativity and the Lorentz theory enrich our knowledge of the physical world with *the physics of objects moving at constant velocities*—in accordance with the title of Einstein’s original 1905 paper. The essential physical content of their discoveries is that physical objects suffer distortions when they are accelerated from one inertial frame to the other, and that these distortions satisfy some uniform laws.

In the Lorentz theory, the laws of deformation (L2a) and (L2b) apply to both the measuring-equipment and the object to be measured. Therefore, it is no surprise that the “length” of a moving, consequently distorted, rod measured by co-moving, consequently distorted, measuring-rod and clock, that is the $\widetilde{\text{length}}$ of the rod, is the same as the $\widetilde{\text{length}}$ of the corresponding stationary rod measured with stationary measuring-rod and clock. The $\widetilde{\text{duration}}$ of a slowed down process in a moving object measured with a co-moving, consequently slowed down, clock will be the same as the $\widetilde{\text{duration}}$ of the same process in a similar object at rest, measured with the original distortion free clock at rest. These regularities are nothing but *particular instances* of the relativity

principle. Similar observations lead Lorentz and Poincaré to conclude with the general validity of the relativity principle:

- (RP) For any two inertial frames of reference K' and K'' , the laws of physics in K' are such⁵ that the laws of physics empirically ascertained by an observer in K'' , describing the behavior of physical objects co-moving with K'' , expressed in variables $\tilde{x}_1^{K''}, \tilde{x}_2^{K''}, \tilde{x}_3^{K''}, \tilde{t}^{K''}$, have the same forms as the similar empirically ascertained laws of physics in K' , describing the similar physical objects co-moving with K' , expressed in variables $\tilde{x}_1^{K'}, \tilde{x}_2^{K'}, \tilde{x}_3^{K'}, \tilde{t}^{K'}$, if the observer in K'' performs the same measurement operations as the observer in K' with the same measuring equipments transferred from K' to K'' , ignoring the fact that the equipments undergo deformations during the transmission.

Therefore, in its most general form the Lorentz theory consists of (L1) + (L2) + (RP).

On the other hand, (RP) is the basic premise of special relativity theory.⁶ And, as is well known from Einstein's 1905 paper or any textbook on special relativity, (RP) implies the Lorentz transformation equations for $\tilde{x}_1^{K'}, \tilde{x}_2^{K'}, \tilde{x}_3^{K'}, \tilde{t}^{K'}$ and $\tilde{x}_1^{K''}, \tilde{x}_2^{K''}, \tilde{x}_3^{K''}, \tilde{t}^{K''}$ between arbitrary two inertial frames. Applying these transformations between the reference frame of the *etalons* K and an arbitrary inertial frame K' we can derive (see Point 11) the rules of classical kinematics, that is, (L1). It also follows from (RP) that the characteristic dimensions and the characteristic periods in a physical system co-moving with the moving reference frame K' , expressed in terms of $\tilde{x}_1^{K'}, \tilde{x}_2^{K'}, \tilde{x}_3^{K'}, \tilde{t}^{K'}$ are the same as the similar dimensions and periods of the same system when it is at rest relative to K , expressed in terms of $\tilde{x}_1^K, \tilde{x}_2^K, \tilde{x}_3^K, \tilde{t}^K$. Combining this fact with the Lorentz transformations, we have (L2a) and (L2b). (See the quotation from Einstein 1905 in Point 5.) That is, (RP) \Rightarrow (L1) + (L2). In other words, Lorentz theory, too, reduces to (RP); the two theories are identical in all sense.

18. (RP) not only implies the classical rules of kinematics and the relativistic rules of kinematics, but also implies some uniform laws governing the behavior of physical objects when accelerated from one inertial frame to the other. Let \mathcal{E}' be a set of differential equations describing the behavior of the system in question in an arbitrary reference frame K' . Let ψ'_0 denote a set of (initial) conditions, such that the solution $[\psi'_0]$ determined by ψ'_0 describes the behavior of the system when it is, as a whole, at rest relative to K' . Let ψ'_v be a set of conditions such that the corresponding solution $[\psi'_v]$ describes the same system in uniform motion at velocity \tilde{v} relative to K' . To be more exact, $[\psi'_v]$ is a solution of \mathcal{E}' that describes the same behavior of the system as $[\psi'_0]$ but in superposition with a collective translation at velocity \tilde{v} . Denote \mathcal{E}'' and ψ''_0 the equations and conditions obtained from \mathcal{E}' and ψ'_0 by substituting every $\tilde{x}^{K'}$ with $\tilde{x}^{K''}$ and $\tilde{t}^{K'}$ with $\tilde{t}^{K''}$. Denote $\Lambda_{\tilde{v}}(\mathcal{E}')$, $\Lambda_{\tilde{v}}(\psi'_v)$ the set of equations \mathcal{E}' and conditions ψ'_v expressed in terms of—not substituted by—the double-primed variables, applying the Lorentz transformations. Now, what the relativity principle (RP) states is that the laws of physics describing the behavior of moving objects are such that they satisfy the following relationships:

$$\Lambda_{\tilde{v}}(\mathcal{E}') = \mathcal{E}'' \quad (16)$$

$$\Lambda_{\tilde{v}}(\psi'_v) = \psi''_0 \quad (17)$$

To make more explicit how this principle provides a useful method in the description of the deformations of physical systems when they are accelerated from one inertial frame K' into some other K'' , consider the following situation: Assume we know

the relevant physical equations \mathcal{E}' and conditions ψ'_0 such that the solution $[\psi'_0]$ of the equations describes the physical properties of the object in question when it is at rest in K' . We now inquire as to the same description of the object when it is moving at a given constant velocity relative to K' . If (16)–(17) are true, then we can solve the problem in the following way. Simply take \mathcal{E}'' , ψ''_0 and the known solution $[\psi''_0]$ —by putting one more prime on each variable. Then, according to (17), express the solution $[\psi'_0]$ from $[\psi''_0]$ by means of the inverse Lorentz transformation.

Thus, within the scope of validity of the relativity principle,⁷ the Lorentz theory and special relativity provide the *same* powerful problem solving *method* in the physics of moving objects—no matter if variables \tilde{x} and \tilde{t} are called “space” and “time” tags or not.

This completes the proof: special relativity and the Lorentz theory are completely identical theories. They are identical theories about space, time, space, and time; and they provide identical descriptions of the behavior of moving physical objects.

This is, of course, an astounding conclusion, in contrast to how people think about the revolution brought about by Einstein’s special relativity. Therefore, the rest part of the paper is devoted to the discussion of the possible objections.

Are relativistic deformations real physical changes?

19. Many believe that it is an essential difference between the two theories that relativistic deformations like the Lorentz–FitzGerald contraction and the time dilatation are *real* physical changes in the Lorentz theory, but there are no similar *physical* effects in special relativity. Let us examine two typical argumentations.

According to the first argument the Lorentz contraction/dilatation of a rod cannot be an objective physical deformation in relativity theory, because “it is a frame-dependent fact whether the rod is shrinking or expanding”. Consider a rod accelerated from the state of rest in reference frame K' to the state of rest in reference frame K'' . According to relativity theory, “the rod shrinks in frame K' and, at the same time, expands in frame K'' ”. But this is a contradiction, the argument says, if the deformation was a real physical change. In contrast, the argument says, the Lorentz theory claims that the length of a rod is a frame-independent concept. Consequently, in the Lorentz theory, “the contraction/dilatation of a rod” can indeed be an objective physical change.

However, we have already clarified, that the terms “distance” and “time” have different meanings in relativity theory and the Lorentz theory. We must differentiate dilatation from dilatation, contraction from contraction, and so on. For example, consider the reference frame of the *etalons*, K , and another frame K' moving relative to K , and a rod accelerated from the state of rest in reference frame K —*state*₁—to the state of rest in reference frame K' —*state*₂. Denote $\hat{l}^K(\textit{state}_1)$ the length of the rod in *state*₁ relative to K , $\tilde{l}^K(\textit{state}_1)$ the length of the rod in *state*₁ relative to K , etc. Now, the following statements are true about the rod :

$$\hat{l}^K(\textit{state}_1) > \hat{l}^K(\textit{state}_2) \quad \widehat{\text{contraction in } K} \quad (18)$$

$$\hat{l}^{K'}(\textit{state}_1) > \hat{l}^{K'}(\textit{state}_2) \quad \widehat{\text{contraction in } K'} \quad (19)$$

$$\tilde{l}^K(\textit{state}_1) > \tilde{l}^K(\textit{state}_2) \quad \widetilde{\text{contraction in } K} \quad (20)$$

$$\tilde{l}^{K'}(\textit{state}_1) < \tilde{l}^{K'}(\textit{state}_2) \quad \widetilde{\text{dilatation in } K'} \quad (21)$$

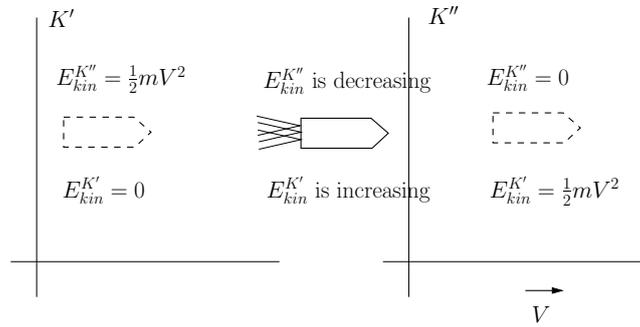
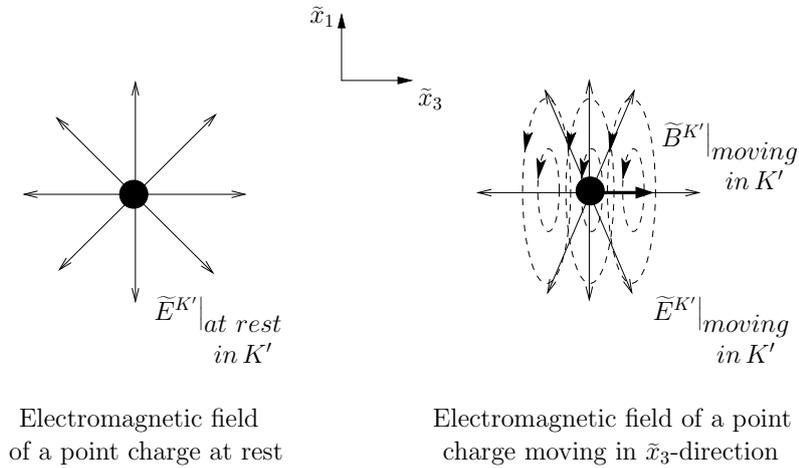


Figure 1: *One and the same objective physical process is traced in the increase of kinetic energy of the spaceship relative to frame K' , while it is traced in the decrease of kinetic energy relative to frame K''*



Electromagnetic field of a point charge at rest

Electromagnetic field of a point charge moving in \tilde{x}_3 -direction

Figure 2: *The change of the electromagnetic field of a point charge*

And there is no difference between relativity theory and the Lorentz theory: *all* of the four statements (18)–(21) are true *in both theories*. If, in the Lorentz theory, facts (18)–(19) provide enough reason to say that there is a real physical change, then the same facts provide enough reason to say the same thing in relativity theory. And vice versa, if (20)–(21) contradicted to the existence of real physical change of the rod in relativity theory, then the same holds for the Lorentz theory.

20. It should be mentioned, however, that there is no contradiction between (20)–(21) and the existence of real physical change of the rod. Relativity theory and the Lorentz theory unanimously claim that length is a relative physical quantity. It is entirely possible that one and the same objective physical change is traced in the increase of the value of a relative quantity relative to one reference frame, while it is traced in the decrease of the same quantity relative to another reference frame (see the example in Fig. 1). (What is more, both, the value relative to one frame and the value relative to the other frame, reflect objective features of the objective physical process in question.)

21. According to the other widespread argument, the relativistic deformations cannot be real physical effects since they can be observed by an observer also if the object is at rest but the observer is in motion at constant velocity. And these relativistic deformations cannot be explained as real physical deformations of the object being continuously at rest.

However, there is a triple misunderstanding behind such an argument:

1. Of course, no real distortion is suffered by an object which is continuously at rest relative to a reference frame K' , and, consequently, which is continuously in motion at a constant velocity relative to another frame K'' . Contrary to the argument, none of the inertial observers can observe such a distortion. For example,

$$\begin{aligned} \tilde{\gamma}^{K'} \left(\begin{array}{c} \text{distortion free} \\ \text{rod at } \tilde{t}_1 \end{array} \right) &= \tilde{\gamma}^{K'} \left(\begin{array}{c} \text{distortion free} \\ \text{rod at } \tilde{t}_2 \end{array} \right) \\ \tilde{\gamma}^{K''} \left(\begin{array}{c} \text{distortion free} \\ \text{rod at } \tilde{t}_1 \end{array} \right) &= \tilde{\gamma}^{K''} \left(\begin{array}{c} \text{distortion free} \\ \text{rod at } \tilde{t}_2 \end{array} \right) \end{aligned}$$

2. It is surely true for any \tilde{t} that

$$\tilde{\gamma}^{K'} \left(\begin{array}{c} \text{distortion} \\ \text{free rod at } \tilde{t} \end{array} \right) \neq \tilde{\gamma}^{K''} \left(\begin{array}{c} \text{distortion} \\ \text{free rod at } \tilde{t} \end{array} \right) \quad (22)$$

This fact, however, does not express a contraction of the rod—neither a real nor an apparent contraction.

3. On the other hand, inequality (22) is a *consequence* of the real physical distortions suffered by the measuring equipments—with which the $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$ tags are empirically defined—when they are transferred from the BIPM to the other reference frame in question. (For further details of what a moving observer can observe by means of his or her distorted measuring equipments, see Bell 1983, pp. 75–76.)

22. Finally, let me give an example for a well known physical phenomenon which is of exactly the same kind as the relativistic deformations, but nobody would question that it is a real physical change. Consider the electromagnetic field of a point charge q . One can easily solve the Maxwell equations when the particle is at rest in a given K' .

The result is the familiar spherically symmetric Coulomb field (Fig. 2):

$$\tilde{E}_1^{K'} \Big|_{\substack{\text{at rest} \\ \text{in } K'}} = \frac{q\tilde{x}_1^{K'}}{\left(\left(\tilde{x}_1^{K'}\right)^2 + \left(\tilde{x}_2^{K'}\right)^2 + \left(\tilde{x}_3^{K'}\right)^2\right)^{\frac{3}{2}}} \quad (23)$$

$$\tilde{E}_2^{K'} \Big|_{\substack{\text{at rest} \\ \text{in } K'}} = \frac{q\tilde{x}_2^{K'}}{\left(\left(\tilde{x}_1^{K'}\right)^2 + \left(\tilde{x}_2^{K'}\right)^2 + \left(\tilde{x}_3^{K'}\right)^2\right)^{\frac{3}{2}}} \quad (24)$$

$$\tilde{E}_3^{K'} \Big|_{\substack{\text{at rest} \\ \text{in } K'}} = \frac{q\tilde{x}_3^{K'}}{\left(\left(\tilde{x}_1^{K'}\right)^2 + \left(\tilde{x}_2^{K'}\right)^2 + \left(\tilde{x}_3^{K'}\right)^2\right)^{\frac{3}{2}}} \quad (25)$$

$$\tilde{B}_1^{K'} \Big|_{\substack{\text{at rest} \\ \text{in } K'}} = 0 \quad (26)$$

$$\tilde{B}_2^{K'} \Big|_{\substack{\text{at rest} \\ \text{in } K'}} = 0 \quad (27)$$

$$\tilde{B}_3^{K'} \Big|_{\substack{\text{at rest} \\ \text{in } K'}} = 0 \quad (28)$$

How does this field change if we set the charge in motion with constant velocity \tilde{v} along the \tilde{x}_3 axis? Maxwell's equations can also answer this question. First we solve the Maxwell equations for arbitrary time-depending sources. Then, from the retarded potentials thus obtained, we derive the Lienart-Wiechert potentials, from which we can determine the field. (See, for example, Feynman, Leighton and Sands 1963, Vol. 2.) Here is the result:⁸

$$\tilde{E}_1^{K'} \Big|_{\substack{\text{moving} \\ \text{in } K'}} = \frac{q\tilde{x}_1^{K'} \left(1 - \frac{\tilde{v}^2}{c^2}\right)^{-\frac{1}{2}}}{\left(\left(\tilde{x}_1^{K'}\right)^2 + \left(\tilde{x}_2^{K'}\right)^2 + B^2\right)^{\frac{3}{2}}} \quad (29)$$

$$\tilde{E}_2^{K'} \Big|_{\substack{\text{moving} \\ \text{in } K'}} = \frac{q\tilde{x}_2^{K'} \left(1 - \frac{\tilde{v}^2}{c^2}\right)^{-\frac{1}{2}}}{\left(\left(\tilde{x}_1^{K'}\right)^2 + \left(\tilde{x}_2^{K'}\right)^2 + B^2\right)^{\frac{3}{2}}} \quad (30)$$

$$\tilde{E}_3^{K'} \Big|_{\substack{\text{moving} \\ \text{in } K'}} = \frac{qB}{\left(\left(\tilde{x}_1^{K'}\right)^2 + \left(\tilde{x}_2^{K'}\right)^2 + B^2\right)^{\frac{3}{2}}} \quad (31)$$

$$\tilde{B}_1^{K'} \Big|_{\substack{\text{moving} \\ \text{in } K'}} = -\frac{\tilde{v}}{c} \tilde{E}_2^{K'} \quad (32)$$

$$\tilde{B}_2^{K'} \Big|_{\substack{\text{moving} \\ \text{in } K'}} = \frac{\tilde{v}}{c} \tilde{E}_1^{K'} \quad (33)$$

$$\tilde{B}_3^{K'} \Big|_{\substack{\text{moving} \\ \text{in } K'}} = 0 \quad (34)$$

where

$$B = \frac{\widetilde{x}_3^{K'} - \widetilde{X}_3^{K'}(\widetilde{t})}{\sqrt{1 - \frac{v^2}{c^2}}}$$

and $\widetilde{X}_3^{K'}(\widetilde{t})$ is the position of the charge at time \widetilde{t} . The electromagnetic field of the charge *changed*: earlier it was like (23)–(28), then it *changed* for the one described by (29)–(34). *There appeared* a magnetic field (turning the magnetic needle, for example) and the electric field *flattened* in the direction of motion (Fig. 2). No physicist would say that this is not a real physical change in the electromagnetic field of the charge, only because we can express the *new* electromagnetic field in terms of the variables of *another* reference frame K'' , in which it has the same form as the old electromagnetic field expressed in the original variables—even if this K'' happens to be the new co-moving frame of reference.

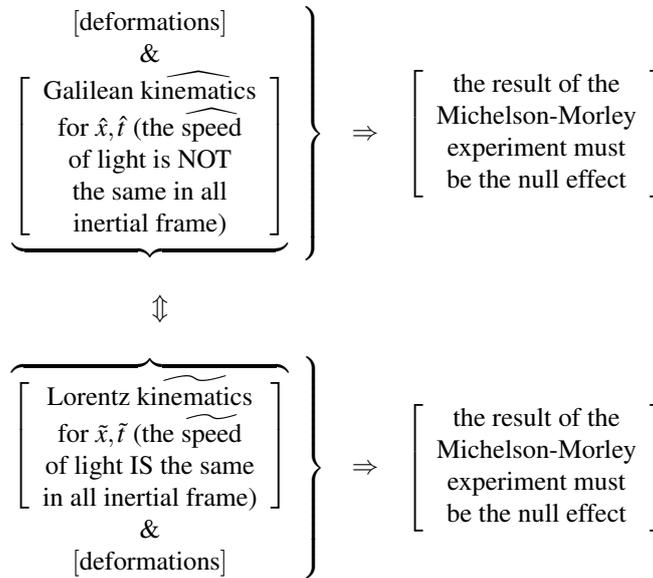
23. Thus, relativistic deformations are real physical deformations also in special relativity theory. One has to emphasize this fact because it is an important part of the physical content of relativity theory. It must be clear, however, that this conclusion is independent of our main concern. What is important is the following: The Lorentz theory and special relativity have identical assertions about length and length, duration and duration, shrinking and shrinking, etc. Consequently, whether or not these facts provide enough reason to say that relativistic deformations are real physical changes, the conclusion is common to both theories.

On the null result of the Michelson–Morley experiment

24. Consider the following passage from Einstein:

A ray of light requires a perfectly definite time T to pass from one mirror to the other and back again, if the whole system be at rest with respect to the aether. It is found by calculation, however, that a slightly different time T^1 is required for this process, if the body, together with the mirrors, be moving relatively to the aether. And yet another point: it is shown by calculation that for a given velocity v with reference to the aether, this time T^1 is different when the body is moving perpendicularly to the planes of the mirrors from that resulting when the motion is parallel to these planes. Although the estimated difference between these two times is exceedingly small, Michelson and Morley performed an experiment involving interference in which this difference should have been clearly detectable. But the experiment gave a negative result — a fact very perplexing to physicists. (Einstein 1920, p. 49)

The “calculation” that Einstein refers to is based on the Galilean “kinematics”, that is, on the invariance of “time” and “simultaneity”, on the invariance of “distance”, on the classical addition rule of “velocities”, etc. That is to say, “distance”, “time”, and “velocity” in the above passage mean the classical distance, time, and velocity defined in (D3), (D4), (D7), and (D8). The negative result was “very perplexing to physicists” because their expectations were based on the traditional concepts of space and time, and they could not imagine other that if the speed of light is c relative to one inertial frame then the speed of the same light signal cannot be the same c relative to another reference frame.



Schema 1: *The null result of the Michelson–Morley experiment simultaneously confirms both, the classical rules of Galilean kinematics for \hat{x} and \hat{t} , and the violation of these rules (Lorentzian kinematics) for the \tilde{x} and \tilde{t} .*

25. On the other hand, Einstein continues this passage in the following way:

Lorentz and FitzGerald rescued the theory from this difficulty by assuming that the motion of the body relative to the aether produces a contraction of the body in the direction of motion, the amount of contraction being just sufficient to compensate for the difference in time mentioned above. Comparison with the discussion in Section 11 shows that also from the standpoint of the theory of relativity this solution of the difficulty was the right one. But on the basis of the theory of relativity the method of interpretation is incomparably more satisfactory. According to this theory there is no such thing as a “specially favoured” (unique) co-ordinate system to occasion the introduction of the aether-idea, and hence there can be no aether-drift, nor any experiment with which to demonstrate it. Here the contraction of moving bodies follows from the two fundamental principles of the theory, without the introduction of particular hypotheses; and as the prime factor involved in this contraction we find, not the motion in itself, to which we cannot attach any meaning, but the motion with respect to the body of reference chosen in the particular case in point. Thus for a co-ordinate system moving with the earth the mirror system of Michelson and Morley is not shortened, but it is shortened for a co-ordinate system which is at rest relatively to the sun. (Einstein 1920, p. 49)

What “rescued” means here is that Lorentz and FitzGerald proved, within the framework of the classical $\widehat{\text{space-time}}$ theory and Galilean kinematics, that if the assumed deformations of moving bodies exist then the expected result of the Michelson–Morley experiment is the null effect. On the other hand, we have already clarified, what Einstein also confirms in the above quoted passage, that these deformations also derive

from the basic postulates of special relativity.

Putting all these facts together (see Schema 1), we must say that the null result of the Michelson–Morley experiment simultaneously confirms *both*, the classical rules of Galilean kinematics for \hat{x} and \hat{t} , and the Lorentzian kinematics for the $\widetilde{\text{space}}$ and $\widetilde{\text{time}}$ tags $\widetilde{x}, \widetilde{t}$. It confirms the classical addition rule of velocities, on the one hand, and, on the other hand, it also confirms that velocity of light is the same in all frames of reference.

This actually holds for all other experimental confirmations of special relativity. That is why the only difference Einstein can mention in the quoted passage is that special relativity does not refer to the aether. As a historical fact, this difference is true. Although, as we will see in Points 30–34, the concept of aether can be entirely removed from the recent logical reconstruction of the Lorentz theory.

The conventionalist approach

26. According to the conventionalist thesis, the Lorentz theory and Einstein’s special relativity are two alternative scientific theories which are equivalent on empirical level (see Friedman 1983, p. 293; Einstein 1983, p. 35). Due to this empirical underdeterminacy, the choice between these alternative theories is based on external aspects. (Cf. Zahar 1973; Grünbaum 1974; Friedman 1983; Brush 1999; Janssen 2002.) Following Poincaré’s similar argument about the relationship between geometry, physics, and the empirical facts, the conventionalist thesis asserts the following relationship between the Lorentz theory and special relativity:

$$\begin{aligned} \left[\begin{array}{c} \text{classical} \\ \text{space-time} \\ \mathbb{E}^3 \times \mathbb{E}^1 \end{array} \right] + \left[\begin{array}{c} \text{physical} \\ \text{content of} \\ \text{Lorentz} \\ \text{theory} \end{array} \right] &= \left[\begin{array}{c} \text{empirical} \\ \text{facts} \end{array} \right] \\ \left[\begin{array}{c} \text{relativistic} \\ \text{space-time} \\ \mathbb{M}^4 \end{array} \right] + \left[\begin{array}{c} \text{special} \\ \text{relativistic} \\ \text{physics} \end{array} \right] &= \left[\begin{array}{c} \text{empirical} \\ \text{facts} \end{array} \right] \end{aligned}$$

Continuing the symbolic notations we used in Points 1–2, denote Z those objective features of physical reality that are described by the alternative physical theories P_1 and P_2 in question. With these notations, the logical schema of the conventionalist thesis can be described in the following way: We cannot distinguish by means of the available experiments whether $G_1(M) \& P_1(Z)$ is true about the objective features of physical reality $M \cup Z$, or $G_2(M) \& P_2(Z)$ is true about the *same* objective features $M \cup Z$. Schematically,

$$\begin{aligned} [G_1(M)] + [P_1(Z)] &= \left[\begin{array}{c} \text{empirical} \\ \text{facts} \end{array} \right] \\ [G_2(M)] + [P_2(Z)] &= \left[\begin{array}{c} \text{empirical} \\ \text{facts} \end{array} \right] \end{aligned}$$

27. However, we have clearly seen that the terms “space” and “time” have different meanings in the two theories. The Lorentz theory claims $G_1(\hat{M})$ about \hat{M} and relativity theory claims $G_2(\widetilde{M})$ about some other features of reality \widetilde{M} . Of course, this

terminological confusion also appears in the physical assertions. Let us symbolize with \hat{Z} the objective features of physical reality, such as the $\widehat{\text{length}}$ of a rod, etc., described by physical theory P_1 . And let \tilde{Z} denote some (partly) different features of reality described by P_2 , such as the $\widetilde{\text{length}}$ of a rod, etc. Now, as we have seen, both theories actually claim that $G_1(\hat{M}) \& G_2(\tilde{M})$. It is also clear that, for example, within the Lorentz theory, we can legitimately query the $\widetilde{\text{length}}$ of a rod. For the Lorentz theory has complete description of the behavior of a moving rigid rod, as well as the behavior of a moving clock and measuring-rod. Therefore, it is no problem to predict, in the Lorentz theory, the result of a measurement of the “length” of the rod, if the measurement is performed with a co-moving measuring equipments, according to empirical definition (D6). This prediction will be exactly the same as the prediction of special relativity. And vice versa, special relativity would have the same prediction for the $\widehat{\text{length}}$ of the rod as the prediction of the Lorentz theory. That is to say, the physical contents of the Lorentz theory and special relativity also are identical: both claim that $P_1(\hat{Z}) \& P_2(\tilde{Z})$. So we have the following:

$$\begin{aligned} [G_1(\hat{M}) \& G_2(\tilde{M})] + [P_1(\hat{Z}) \& P_2(\tilde{Z})] &= \begin{bmatrix} \text{empirical} \\ \text{facts} \end{bmatrix} \\ [G_1(\hat{M}) \& G_2(\tilde{M})] + [P_1(\hat{Z}) \& P_2(\tilde{Z})] &= \begin{bmatrix} \text{empirical} \\ \text{facts} \end{bmatrix} \end{aligned}$$

In other words, there are no different theories; consequently there is *no choice*, based neither on internal nor on external aspects.

Methodological remarks

28. It is to be noted that our analysis is based on the following very weak operationalist/verificationist premise: physical terms, assigned to *measurable physical quantities*, have different meanings if they have different empirical definitions. This premise is one of the fundamental pre-assumptions of Einstein’s 1905 paper and is widely accepted among physicists. Without clear empirical definition of the measurable physical quantities a physical theory cannot be empirically confirmable or disconfirmable. In itself, this premise is not yet equivalent to operationalism or verificationism. It does not generally imply that a statement is necessarily meaningless if it is neither analytic nor empirically verifiable. However, when the physicist assigns time and space tags to an event, relative to a reference frame, (s)he is already after all kinds of metaphysical considerations about “What is space and what is time?” and means definite physical quantities with already settled empirical meanings.

29. In saying that the meanings of the words “space” and “time” are different in relativity theory and in classical physics, it is necessary to be careful of a possible misunderstanding. I am talking about something entirely different from the incommensurability thesis of the relativist philosophy of science (Kuhn 1970, Chapter X; Feyerabend 1970). How is it that relativity makes any assertion about classical $\widehat{\text{space}}$ and $\widehat{\text{time}}$, and vice versa, how can the Lorentz theory make assertions about quantities which are not even defined in the theory? As we have seen, each of the two theories is a sufficiently complete account of physical reality to make predictions about those

features of reality that correspond—according to the empirical definitions—to the variables used by the other theory, and it is no problem to *compare* these predictions. For example, within the Lorentz theory, we can legitimately query the reading of a clock slowly transported in K' from one place to another. That exactly is what we calculated in Point 14. Similarly, in special relativity theory, we can legitimately query the space and time tags of an event in the reference frame of the *etalons* and then apply formulas (1)–(2). This is a fair calculation, in spite of the fact that the result thus obtained is not explicitly mentioned and named in the theory. This is what we actually did. And the conclusion was that not only are the two theories commensurable, but they provide completely identical accounts of the same physical reality; they are identical theories.

Privileged reference frame

30. Due to the popular/textbook literature on relativity theory, there is a widespread aversion to a privileged reference frame. However, like it or not, there is a privileged reference frame in both special relativity and classical physics. It is the frame of reference in which the *etalons* are at rest. This privileged reference frame, however, has nothing to do with the concepts of “absolute rest” or the aether; it is not privileged by nature, but it is privileged by the trivial semantical convention providing meanings for the terms “distance” and “time”; by the fact that of all possible measuring-rod-like and clock-like objects floating in the universe, we have chosen the ones floating together with the International Bureau of Weights and Measures in Paris. In Bridgman’s words:

It cannot be too strongly emphasized that there is no getting away from preferred operations and unique standpoint in physics; the unique physical operations in terms of which interval has its meaning afford one example, and there are many others also. (Bridgman 1936, p. 83)

31. Many believe that one can avoid a reference to the *etalons* sitting in a privileged reference frame by defining, for example, the unit of time for an arbitrary (moving) frame of reference K' through a cesium clock, or the like, co-moving with K' . In this way, it is claimed, one needs not to refer to a standard clock accelerated from the reference frame of the *etalons* into reference frame K' .

In this view, however, there appears a methodological nonsense. For if this operation is regarded as a convenient way of *measuring* time, then we still have time in the theory, together with the privileged reference frame of the *etalons*. If, however, this operation is regarded as the empirical *definition* of a physical quantity, then it must be clear that this quantity is not time but a new physical quantity, say time. In order to establish any relationship between time tags belonging to different reference frames, it is a must to use an “*etalon* cesium clock” as well as to refer to its behavior when accelerated from one inertial frame into the other, or, in some other way, to describe the other clocks’ behaviors in term of the physical quantity defined with the *etalon*.

The aether

32. Many of those, like Einstein himself (see Point 25), who admit the “empirical equivalence” of the Lorentz theory and special relativity argue that the latter is “incomparably more satisfactory” (Einstein) because it has no reference to the aether. As it

is obvious from the previous sections, we did not make any reference to the aether in the logical reconstruction of the Lorentz theory. It is however a historic fact that, for example, Lorentz did. In this section, I want to clarify that Lorentz's aether hypothesis is logically independent from the actual physical content of the Lorentz theory.

Consider, for example, Lorentz's aether-theoretic formulation of the relativity principle—to touch on a sore point.

33. Let us introduce the following notation:

$A(K', K'') :=$ The laws of physics in K' are such that the laws of physics empirically ascertained by an observer in K'' , describing the behavior of physical objects co-moving with K'' , expressed in variables $\tilde{x}_1^{K''}, \tilde{x}_2^{K''}, \tilde{x}_3^{K''}, \tilde{t}^{K''}$, have the same forms as the similar empirically ascertained laws of physics in K' , describing the similar physical objects co-moving with K' , expressed in variables $\tilde{x}_1^{K'}, \tilde{x}_2^{K'}, \tilde{x}_3^{K'}, \tilde{t}^{K'}$, if the observer in K'' performs the same measurement operations as the observer in K' with the same measuring equipments transferred from K' to K'' , ignoring the fact that the equipments undergo deformations during the transmission.

Taking this statement, the usual Einsteinian formulation of the relativity principle is the following:

$$\left[\begin{array}{l} \text{Einstein's} \\ \text{Relativity} \\ \text{Principle} \end{array} \right] = (\forall K') (\forall K'') A(K', K'')$$

Many believe that this version of relativity principle is essentially different from the similar principle of Lorentz, since Lorentz's principle makes explicit reference to the motion relative to the aether. Using the above introduced notations, it says the following:

$$\left[\begin{array}{l} \text{Lorentz's} \\ \text{Principle} \end{array} \right] = (\forall K'') A(\text{aether}, K'')$$

It must be clearly seen, however, that *Lorentz's principle and Einstein's relativity principle are equivalent to each other*. On the one hand, it is trivially true that

$$\begin{aligned} \left[\begin{array}{l} \text{Einstein's} \\ \text{Relativity} \\ \text{Principle} \end{array} \right] &= (\forall K') (\forall K'') A(K', K'') \\ &\Rightarrow (\forall K'') A(\text{aether}, K'') \\ &= \left[\begin{array}{l} \text{Lorentz's} \\ \text{Principle} \end{array} \right] \end{aligned}$$

On the other hand, it follows from the *meaning* of $A(K', K'')$ that

$$(\exists K') (\forall K'') A(K', K'') \Rightarrow (\forall K') (\forall K'') A(K', K'')$$

The reason is that the laws of physics in K' completely determine the results of the measurements performed by a moving—relative to K' —observer on moving physical

objects with moving measuring equipments. Consequently,

$$\begin{aligned}
 \left[\begin{array}{l} \text{Lorentz's} \\ \text{Principle} \end{array} \right] &= (\forall K'') A(\text{aether}, K'') \\
 &\Rightarrow (\exists K') (\forall K'') A(K', K'') \\
 &\Rightarrow (\forall K') (\forall K'') A(K', K'') \\
 &= \left[\begin{array}{l} \text{Einstein's} \\ \text{Relativity} \\ \text{Principle} \end{array} \right]
 \end{aligned}$$

Thus, it is Lorentz's principle itself—which refers to the aether—that renders any claim about the aether a logically separated hypothesis outside of the scope of the factual content of both the Lorentz theory and special relativity. It is Lorentz's principle itself—again, which refers to the aether—that implies that the role of the aether could be played by anything else; the aether does not constitute a privileged reference frame.

As the Lorentz theory and special relativity unanimously claim, physical systems undergo deformations when they are transferred from one inertial frame K' to another frame K'' . One could say, these deformations are caused by the transmission of the system from K' to K'' . You could say they are caused by the “wind of aether”. By the same token you could say, however, that they are caused by “the wind of *anything*”, since if the physical system is transferred from K' to K'' then its state of motion changes relative to an arbitrary third frame of reference.

34. On the other hand, it must be mentioned that special relativity does not exclude the existence of the aether. (Not to mention that already in 1920 Einstein himself argues for the existence of some kind of aether. See Reignier 2000.) Neither does the Michelson–Morley experiment. As we have seen in Point **25**, if special relativity/Lorentz theory is true then there must be no indication of the motion of the interferometer relative to the aether. Consequently, the fact that we do not observe indication of this motion is not a challenge for the aether theorist. Thus, the hypothesis about the existence of aether is logically independent of both the Lorentz theory and special relativity.

Heuristic and explanatory values

35. The Lorentz theory and special relativity, as completely identical theories, offer the same symmetry principles and heuristic power. As we have seen, both theories claim that quantities $\tilde{x}^{K'}, \tilde{t}^{K'}$ in an arbitrary K' and the similar quantities $\tilde{x}^{K''}, \tilde{t}^{K''}$ in another arbitrary K'' are related through a suitable Lorentz transformation. This fact in conjunction with the relativity principle (within the scope of validity of the principle) implies that laws of physics are to be described by Lorentz covariant equations, if they are expressed in terms of variables \tilde{x} and \tilde{t} , that is, in terms of the results of measurements obtainable by means of the corresponding co-moving equipments—which are distorted relative to the *etalons*. There is no difference between the two theories that this space-time symmetry provides a valuable heuristic aid in the search for new laws of nature.

36. It is sometimes claimed that the main difference between the Lorentz theory and Einstein's special relativity is that the Lorentz theory is *constructive*, in the sense that it tries to explain the relativistic effects from the laws of the detailed underlying physical

processes, while special relativity deduces the same result from some basic *principles* (cf. Bell 1992, p. 34; Brown and Pooley 2001). As we have seen in Point 17, the basic principles of the two theories are logically equivalent; both reduce to (RP); the two theories are identical; the statements of “both theories” can be derived from (RP). So, if the fact that the statements of special relativity can be derived from (RP) provides enough reason to say that special relativity is a principle theory, then the same fact provides enough reason to say the same thing about the Lorentz theory. And vice versa, if the statements of the Lorentz theory—all derivable from (RP)—provide enough reason to say that it is a constructive theory, then the same fact provides enough reason to say the same thing about special relativity.

Though, it is a historic fact that Lorentz, FitzGerald, and Larmor, in contrast to Einstein, made an attempt to understand how these laws actually come about from the molecular forces. These are perfectly legitimate *additional* questions.⁹

37. With these comments I have completed the argumentation for my basic claim that special relativity and the Lorentz theory are completely identical. Again, the historical questions are not important from the point of view of our analysis. What is important is the logical possibility of a Lorentz-type theory: classical spatiotemporal conceptions + the relativity principle (governing the physics of moving objects). And, what we proved is that such a theory is completely identical to special relativity: relativistic spatiotemporal conceptions + the relativity principle (governing the physics of moving objects). They are not only “empirically equivalent”, as sometimes claimed, but they are identical in all sense; they are identical physical theories.

Consequently, in comparison with the classical Galileo-invariant conceptions, special relativity theory tells us nothing new about the spatiotemporal features of the physical world. As we have seen, the longstanding belief that it does is the result of a simple but subversive terminological confusion.

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Notes

¹By “slowly” is usually meant that we move the clock from one place to the other over a long period of time, according to the reading of the clock itself. The reason is to avoid the loss of phase accumulated by the clock during its journey.

²Straight line is usually defined by a light beam.

³I mean the tradition of classical physics according to which it must be taken into account that, for example, the pendulum clock slows down as the temperature rises.

⁴This was first recognized by Bridgeman (1927, p. 12), although he did not investigate the further consequences of this fact.

⁵It must be emphasized that relativity principle characterizes the laws of physics in one single frame of reference; in other words, the laws of physics in one single inertial frame of reference pre-determine whether relativity principle holds or not. This is what Bell calls “Lorentzian pedagogy” (Bell 1987, p. 77).

⁶Applying (RP) for electrodynamics, one finds that the velocity of a light signal is the same in all inertial reference frames. This is certainly true if, in the application of the principle, the meaning of the phrase “the Maxwell equations have the same form” includes that constant of nature c has the same value. This is sometimes regarded as the “second basic principle” of special relativity.

⁷According to the standard views, the solution thus obtained really describes the same object when it is in motion at a given constant velocity relative to K' . In fact, the situation is much more complex. Whether or not the solution obtained in this way is correct, that is whether or not relativity principle holds, depends on further details of the physical system in question (Szabó 2004).

⁸This is the result long after the acceleration (Szabó 2004).

⁹Both the Lorentz theory and special relativity are principle theories, but the main “principle” on which they are based is not a general principle. It does not hold for the whole range of validity of the Lorentz covariant laws of relativistic physics, but only for the equilibrium quantities characterizing the equilibrium states of dissipative systems (Szabó 2004). Yet, it is suitable for some derivations; for example, for the derivation of the transformation rules of space and time tags. It is because space and time tags are obtained by means of measuring-rods and clocks co-moving with different inertial reference frames, and the principle of relativity holds for such equilibrium quantities as the length of a measuring-rod and the characteristic period of a clock.

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