MATHEMATICAL EXISTENCE DE-PLATONIZED: INTRODUCING OBJECTS OF SUPPOSITION IN THE ARTS AND SCIENCES

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Abstract. In this paper, we introduce a suppositional view of linguistic practice that ranges over fiction, science, and mathematics. While having similar consequences to some other views, in particular Linsky and Zalta's plenitudinous platonism, the view advocated here both differs fundamentally in approach and accounts for a wider range of phenomena and scientific discourse.

1. Introduction

Philosophers of mathematics frequently quarrel over whether entities such as the natural numbers of Peano arithmetic (P) or the sets of Zermelo-Frankel set theory (ZF) [really] exist. Such entities are distinguished by the feature that they are invariant under the automorphisms of the intended models of those systems. Indeed, the intended models have no non-trivial automorphisms. Hardly ever is it discussed whether the entities of, say, Euclidean geometry, exist or in general, of systems whose models have non-trivial automorphisms.

The issue is this. Platonists, who maintain that numbers and sets exist, typically impose as a criterion of individuation on such abstract objects a version of Leibniz's principle of the identity of indiscernibles: if x and y share all the same formal[1] properties, then $x = y$. This principle is clearly satisfied by a structure whose only automorphism is the identity map. However, if the symmetry group of a mathematical structure contains permutations of the domain of discourse other than the identity map, the structure attributes to all points from the same orbit exactly the same properties — else they could not be mapped to one another by one or another member of the symmetry group. One of two conclusions appears to be forced. Either the entities that populate the domain of discourse do not exist or else points from the same orbit must be identified with one another. Clearly the latter will not do, since for such mathematical structures as Euclidean space or Minkowski space-time the domain of discourse then collapses to a singleton set. Thus, Platonists of this stripe are saddled with a double standard. The points of some spaces of Riemannian

[1]The reason for this qualification will become apparent in what follows.

geometry make it into their ontology, but not the points of other such spaces (say, those of constant curvature).

This appears to be an artificially arbitrary ontology. We take it to be a sound methodological constraint that no matter what one's philosophical persuasion that all mathematical entities be treated on a par unless there is some good reason internal to mathematics to do otherwise. If it is held that the points of homogenous spaces strictly speaking do not exist, we want at least a good explanation of why we are ordinarily justified in speaking as if they do.

It may be objected that the Leibnizian principle is itself a good reason internal to mathematics.[2] In what follows, we trace how it happens in practice that individual points from homogenous spaces typically enter into mathematical discourse and explain the sense in which the above sort of Platonism is almost correct. In doing so, though, we show how one can have most of what the Platonist wants while remaining true to naturalism and without buying into metaphysical dogmas. The account we give not only reveals connections with the realm of fictional entities, as some Platonists have noticed, but also extends to the very heart of scientific theory and discovery.

2. Some Other Constraints.

We take as our foil the position of "Platonized" naturalism due to Linsky and Zalta (1995). But before setting out the details of their position, we would like to bring to the forefront three consequences of their position that we believe that any adequate philosophy of mathematics should entail. As will be seen, these are consequences of the view we shall set out.

The first, that mathematical objects should have the formal properties they have necessarily, may seem jejune, but takes on pizazz if it is noted that a view that explains why this is so is preferable to a view that merely stipulates it is so. On our view, mathematical objects necessarily have the formal properties they do in virtue of what they are, otherwise we would not be able to refer to them.

The second constraint is the permissibility of truth value gaps. The status of the continuum hypothesis or Gödel's first incompleteness theorem are often cited as principal reasons. We remind the reader of these, but emphasize the ultimate reason is the nature of the game. No set theorist (or none that we are aware of) would insist on the axiom of foundations. Take it if you like. But if you have a hankering for what life is like with sets that are not well-founded, knock yourself out.

[2]Leibniz himself argued against absolute space as something *real* on the grounds of the indiscernibility of the parts of space but saw this as insufficient reason to reject space as something *ideal*. See Leibniz (2000).

The third (and final) constraint is that the identity of mathematical entities is as fine-grained as can be. This takes some explaining. Any difference in conformal properties indicates distinct mathematical entities. Under this requirement, the Benacerraf question of 'which ordinal is the *real* number 3?' is a meaningless question. The different objects that fit the bill of being the number 3 are in fact distinct mathematical objects because of the additional properties they carry. For instance, the von Neumann ordinal satisfies $3 \in 4$ while the Zermelo does not. Neither of these constitutes a "true" conception of the number 3, but simply describes a property of the respective ordinal.

3. "Platonized" Naturalism.

Meinong (1915) argued that, since there are things that don't exist (the golden mountain, Santa Claus), there is a difference between being and existence. For any collection of properties, there is something that has just those properties. The position on the face of it is inconsistent (see Parsons (1980)). Meinong's student, Mally (1912), observed that the inconsistency can be avoided if a distinction is also drawn between two different ways in which something x can have a property F. On the one hand, x may *exemplify* F, written Fx. This is the sense in which things are normally said to have the properties they do. On the other, x might *encode* F, written xF. Meinong's principle is then to be understood in terms of encoding. For any collection of properties, there is something that *encodes* just those properties.[3]

What Linsky and Zalta call "Platonized naturalism" picks up on Mally's hint to salvage Meinong's jungle. They add that objects that do not exist are *abstract* objects and institute three principles governing them (we quote, Linsky and Zalta 1995, 536).

(1) For every condition on properties, there is an abstract individual that *encodes* exactly the properties satisfying the condition.[4]
(2) If x possibly encodes a property F, it does so necessarily.[5]
(3) If x and y are abstract individuals, then they are identical if and only if they encode the same properties.[6]

[3]Mally also observed that one could instead distinguish between two *types* of properties, nuclear and extra-nuclear. Each property (as conceived pre-analytically) comes in both nuclear and extra-nuclear form. Instead of encoding a property, an object exemplifies the nuclear form of the property. Exemplifying a property is just exemplifying the extra-nuclear form. This option is developed in Parsons (1980)

[4]Symbolically, $\exists(A!x \wedge \forall F(xF \leftrightarrow \phi))$, where x is not free in ϕ and '$A!$' is a predicate constant whose intended interpretation is 'is abstract'.

[5]$\Diamond xF \to \Box xF$.

[6]$A!x \wedge A!y \to (x = y \leftrightarrow \forall F(xF \leftrightarrow yF))$.

Thus, for Linsky and Zalta, the realm of abstract objects covers the entire waterfront, from creatures of myth and fiction, to intentional objects in general, to the complete sequence of increasingly accurate decimal approximations of π. The domain of mathematics is marked out by mathematical *theories*, which must satisfy the "rule of closure" to the effect that they are closed under logical consequence.[7,8] Technically, if p is a theorem of theory T, then T encodes the property *being such that* p. If κ is a variable free term in the language of T, then there is an abstract object κ_T that encodes exactly the properties that the theory states that κ_T exemplifies.

Going back to the three desiderata from the preceding section, it's clear that it follows from (2) that a mathematical object necessarily encodes the properties it in fact encodes. But that seems to be the sole purpose of (2), so that the first desideratum appears to be put in by hand.

The second desideratum follows simply from the fact that many mathematical theories are incomplete in the usual sense that there are sentences such that neither they nor their negations are theorems. Apart from the famous examples of P and ZF, whose incompleteness derives from deep facts about diagonalization and the representability of recursive arithmetic functions, there are mundane examples such as group theory or the theory of vector spaces formulated in first-order languages that do not intend to describe a single structure up to isomorphism but instead an entire category of structures.

The third desideratum is satisfied insofar as each mathematical entity is tied to a unique mathematical theory, and, hence, distinct theories describe distinct mathematical entities. For example, let N be the theory of the intended model of P. Since N is complete while P is not, N and P are distinct theories. Hence, $(0)_N$ is subtly distinct from $(0)_P$.

Alas, it appears that the Linksy-Zalta theory's virtues end there. In particular, postulate (3), or more specifically the half that demands the identity of indiscernibly encoded individuals, entails that there are no individuals populating homogenous spaces. Since the only work this half of the principle appears to be doing is to cap the ontology for fear that it becomes *way* over-bloated, it might be thought that the problem of homogenous spaces can be solved just by jettisoning the requirement. This, by itself, only removes the prohibition on the wanted individuals from existing and leaves it completely undecided whether they exist or not. We take the difficulty

[7]Presumably, Linsky and Zalta take this to be classical, not intuitionistic logical consequence, but do not elaborate whether they have in mind specifically first-order entailment or entailment in a higher-order logic.

[8]The reader may wonder whether this is *sufficient* to demarcate mathematical objects from other varieties of abstract objects. Is it never the case that what's true according to a work of fiction is closed under logical consequence? Our view accounts for the difference in terms of the principles at work in what we call the *interpretive engine* responsible for generating the supposition in question.

to be a symptom of a deeper problem, viz., the status of entities introduced by mere supposition in the course of mathematical argumentation.

4. Objects of Supposition

The standard proof that $\sqrt{2}$ is irrational is a reductio ad absurdum on the supposition that that it is rational. Specifically, suppose that p and q are integers (with no common divisors) such that $p/q = \sqrt{2}$. Then $p^2/q^2 = 2$. Hence, p^2, and in turn p, must be even. Thus, $p = 2r$ for some integer r and $4r^2/q^2 = 2$, i.e., $q^2 = 2$. So q as well as p must be even, contradicting the assumption they have no common divisors.

What is the ontological status of p (respectively, q)? Whatever it is, it appears to fit no traditional category. We have *stipulated* the existence of p and q as a matter of supposition and have proceeded, throughout the proof, to speak of them as though they are definite integers. Moreover, we may continue to speak of them coherently outside the proof, as we have been doing. For lack of anything better, let us say that their ontological status is one of *existence-by-stipulation*. As a matter of linguistic practice we find ourselves able in the context of supposition to introduce new terms, to use them as though they have a definite reference, and to ascribe properties to them. Subsequently, outside the context of supposition, we are able to continue to use the terms as though they are names for the objects introduced in the context of the supposition.

Our emphasis, though, is not on ontology, but on the phenomenology of language use, in particular, on the use of language in a clearly identifiable and pervasive practice that is best described as the practice of *supposition*, as long as we are clear that the term 'supposition' is not meant here to carry any epistemic import but only the import of "suppose for argument's sake" or "suppose just for the hell of it." We also caution that *to suppose* [something] is not just the same as to entertain a proposition. For the act of supposing may involve the extension of our language by the introduction of new terminology, as when the novelist pens a new name or the scientist coins a new term for something merely hypothetical. The fabrication of the supposition itself provides the guide for usage of the newly introduced terminology. Some may find it irresistible to further assimilate objects of supposition to some traditionally established metaphysical category. But we do not see anything inherent in the phenomena that forces this upon us and will continue to speak of existence-by-stipulation, not as some newly invented ontological category, but merely as a descriptive place-holder for a phenomenal realm.

5. The Dynamics of Suppositional Practice

Although the use of supposition in mathematical proof provides a paradigm, the practice of supposition pervades the use of language in many domains, as we have intimated above, including discourse about fictions, the use of idealizations in science, and also the introduction of hypothetical entities in scientific theory. As a general picture of the practice of supposition we see three components. Taking a cue from Walton (1990), one begins with a *prop* — a picture, toy, statue, physical model, segment of fictional text — to which is applied an *interpretive engine* from which is generated a (perhaps fuzzy) set of sentences in some language, which may contain previously uninterpreted terms. The resulting set of sentences is a *representation* of the corresponding supposition.[9] Given the same prop, different interpretive engines can (and typically will) yield distinct suppositions. In the case of fiction and mimetic art, the supposition comprises just those sentences that may be said to be true according to the story or according to what's depicted by the work of art. In the course of the generation of the supposition, the interpretive engine in general works non-monotonically, using a blend of deductive, inductive, and interpretive principles such as authorial intentionality. Suppositions need not be complete nor even necessarily closed under logical consequence. In scientific practice, the prop may be a set of phenomena to be explained, the interpretive engine rely on abductive inference, and the resulting supposition include terms for newly introduced, purported entities (hypothetical entities).

In discourse about an object of supposition outside the context of its generation, it becomes necessary to distinguish between two patterns of predication. One is to predicate *in conformity* with the supposition. Suppose a sentence of the form $\varphi(\tau)$ occurs in the supposition S. It is then common to assert $\varphi(\tau)$, or more explicitly, that *according to* S, $\varphi(\tau)$. Informally, we will then call φ a *conformal* property of τ. This tracks what the Mally-inspired Platonist calls *encoding*. The only difference is that we do not posit properties or a new type of relation between properties and individuals. What Linsky and Zalta call *exemplification* mirrors normal predication, and in cases of normal predication (wherein prefixing 'according to S' perverts the sense of what is meant) we will call the property τ is said to have a *normal* property. So, e.g., being-extremely-bulked-up is a conformal property of Superman, while being-a-marketing-success is a normal property of Superman. Similarly, being-even is a conformal property of both the integers p and q above, while being-used-for-the-sake-of-reductio is a normal property.

The desiderata we introduced in § 2 apply not just to mathematical entities, but to objects of supposition in general. There, we introduced the desiderata as though they were independent of one another. The first, that mathematical objects should have the *con*formal properties they do necessarily, is intimately related to the desideratum of fine-grained identity. Concerning fine-grained identity, when we begin to examine

[9]We do not claim that the supposition *is* the set of sentences. Suppositions are historically located actions involving language use. Sets of sentences are not.

the Superman legend, we can wonder whether the Superman of comic book fame is the same Superman as the Superman of the TV series, and again whether that is the same Superman as the Superman of the movies. Even within these genres, we can ask whether we have in mind a single cumulative supposition or a series of suppositions, one per episode. THE Superman in each alternative is a distinct Superman. The reason we get by in ordinary discourse without having to differentiate is that we seldom drill down deep enough in conformal discourse to find any discrepancies. But if discrepancies arise, then, to straighten things out, we are forced to recur to the content of the separate suppositions in order to individuate in terms of conformal properties. Thus, the conformal properties are necessary properties and the proper theory of names for objects of supposition resembles a description theory. There is no way to track reference other than by the conformal properties of the bearer of the name. So 'Superman wears a red cape' is analytically true.

6. Mathematic Existence and Proof

Insofar as theorems and proofs play an indispensable role in mathematics, mathematics is steeped in supposition. The props of the mathematician are axiom sets. The interpretive engine of mathematics closes them under logical consequence, yielding suppositions otherwise known as theories. That is the global picture. More intricate workings of mathematical supposition can be found under the hood of the interpretive engine.

A common literary device is the supposition within the supposition, often in conscious reflection of the outer supposition. This is a matter of what is outputted by the interpretive engine. In contrast, mathematics puts supposition to further work *inside* the interpretive engine in the generation of the main supposition. This is the role of supposition in the proof techniques of natural deduction: conditional derivation, reductio ad absurdum, separation of cases, universal derivation, and existential instantiation. The last two, because of the need to introduce an *arbitrary* individual, involve an expansion of the language by a new term. The new term spawns an object of supposition. This object of supposition does not make its way into the theory generated. Nonetheless, from the outside, in the metalanguage, the object of supposition survives. This is what enabled us to speak of the integers p and q from the outside of the above reductio argument.

Only what we *say* in the metalanguage about the status of objects of supposition introduced in the course of a derivation differs according to the character of the derivation. In contrast to the above reductio argument, consider Proposition 1, Book I of Euclid and its proof. In original form, the proposition is stated as a construction problem: to describe an equilateral triangle upon a given finite straight line. In indicative form, we can take the theorem to be: for any given finite straight line there exists an equilateral triangle having as one of its sides the finite straight line. The standard proof begins with the strategy of universal derivation. Let AB be a given finite straight line. According to Euclid's postulates, there exist circles of radius AB centered at A and B, respectively, which (and here there is a lapse) intersect at a third point C. And so on.

Outside the proof, we can continue to speak of the endpoints A and B of the given finite line, of the sides AC and BC of the constructed equilateral triangle ABC, and so on. But now, here is the contrast. Outside the reductio argument, we typically assert that p and q do *not* exist.[10] Outside the constructive proof above, we typically do not deny the existence of the geometric points A, B, C, the corresponding line segments or the equilateral triangle. The general principle seems to be this. If an object of supposition introduced in the course of a proof leads to inconsistency, then, in the metalanguage, the existence of the object of supposition is to be denied. If the object leads to a consistent extension, then, in the metalanguage, the existence of the object of supposition is, ceteris paribus, *not* denied.[11]

The connection with non-trivial automorphisms is as follows. We take as our example now a Minkowski space-time, which in the framework of manifolds with pseudo-Riemannian metrics is a structure of the form $\mathfrak{M} = (\mathbb{E}^4, \eta)$, where η is the Minkowski metric. The automorphisms of $\mathfrak{M}$ comprise the Poincaré group, and thus all points of the domain of discourse $\mathbb{E}^4$ are indistinguishable. Thus, according to the Linsky-Zalta account, they are either the same or else do not exist at all. However, in deriving theorems about Minkowski space-time, we are invariably led in our proofs to speak of specific points P, Q, R, and so on. As long as these are not used in reductio proofs, then we are dealing with expansions of $\mathfrak{M}$ of the form $(\mathbb{E}^4, \eta, P^{\mathfrak{M}}, Q^{\mathfrak{M}}, R^{\mathfrak{M}}, \ldots)$ whose symmetries will be a proper subgroup of the Poincaré group, perhaps even the trivial group,[12] since the points $P^{\mathfrak{M}}, Q^{\mathfrak{M}}, R^{\mathfrak{M}}$ must be left invariant.

Were we never to introduce constants for individual points, we might well wonder whether the points of a homogenous space exist. This, however, is a counterfactual nearly as outré as one with 'if 6 were 9' as antecedent. The bottom line is this. The suppositional view, although a description of standing practice, is illuminating

[10]This has a parallel in talk about fictions. Outside the Superman supposition, we say (as a matter of normal, not conformal discourse) that Superman does not exist. However, the underlying reasons in the two cases appear to be different. In the Superman case, it is to emphasize that Superman is *merely* an object of supposition.

[11]We would be remiss not to mention that it is a common practice in geometry texts to state the theorems in a form that tacitly presupposes universal generalization.

[12]We are not hereby suggesting that the justification is that at some stage the Linksy-Zalta principle (3) kicks in. Numerical identity in mathematics is always ultimately a matter of stipulation, not of discovery.

enough to explain our inclination to grant existence to the points of a homogenous space.

7. Internal Critique of Platonized Naturalism

We have *not* shown that the Linsky-Zalta doctrine cannot be repaired by the introduction of auxiliary assumptions, perhaps mirroring in ontology what we have described as practice. There is a reason internal to their scheme, though, that leads us to think their approach is irrescuable.

The difficulty lies in their comprehension scheme (1). Comprehension schemes in mathematics (and axiom schemes in general) invariably appeal to conditions *formulated in some precisely specified language*. But the Linsky-Zalta theory speaks only of "conditions" without specifying a language. One thus has to speculate on their intent. Clearly, they do not have in mind the language of ZF or some other contender for a foundation of mathematics. For they explicitly want their comprehension scheme to generate as abstract objects such creatures of fiction as Santa Claus and Superman. So the language must be cast rather widely. That leaves, as far as we can discern, two possibilities. Either the language is intended to be natural language as it currently is or else some super-language of which current natural language is but a proper sublanguage. The first alternative is not in keeping with the spirit of their platonism, according to which abstract entities are outside space and time. Otherwise abstract individuals will pop into existence as natural language grows over time. This leaves the super-language alternative. But it is hard to see what sort of super-language might be intended, whether it be something like natural language in the limit of all time, or like the union of all possible languages, or some yet further option. We find such notions just as unworkable as appealing to the ultimate completion of Cantor's transfinite hierarchy (not in some formal model but in reality). Without any input for "conditions on properties" the doctrine is inert. Without some circumscription of the "conditions on properties" the view is unfathomable.

8. Methodological Matters

Quite apart from this we find the Linsky-Zalta project, as well as a range of others in analytic metaphysics, suspect on methodological grounds that can be fully appreciated only with further articulation of the suppositional view. These grounds reveal the extent to which Platonized naturalism falls short of naturalism if "naturalism" is to mean conformity with the method of the natural sciences.

As we have intimated above, science too trades in objects of supposition.[13] On the one hand, some are for matters of convenience and are not construed by practitioners

[13]In addition to those of mathematics.

as adding ontological freight to the theories that use them. These are convenient fictions that assist in modeling the phenomena, unifying treatments of various classes of phenomena, giving explanations, or solving problems. Among these are idealizations and auxiliary quantities of a "mixed" mathematical character, e.g., frictionless planes, incompressible fluids, image charges in electrostatics, epicycles and equants in Ptolemaic astronomy, valencies and quantum orbitals, Fermi spheres, Hamiltonians and Lagrangians.

On the other hand are hypothetical entities such as celestial spheres, vital pneuma, Cartesian vortices, phlogiston, and caloric. One can readily verify that these have the typical signatures of objects of supposition: conformal vs. normal properties, truth-value gaps with respect to conformal properties, fine-graining of identity. In addition to its failed hypotheticals, science also has its vindicated hypotheticals (the electron, neutron, positron, neutrino, top quark, black holes) and its current hypotheticals (the Higgs boson, squark, gravitino). One would then like a uniform treatment of hypotheticals in general. This can be done given an account of discovery we call the *appropriation model*.

But before outlining the model, we need to be clear on what we mean by a hypothetical entity. We mean an entity introduced into scientific discourse as if it were an empirical entity, but whose existence is hypothetical in the sense there is no currently established empirical entity that is the referent of discourse. Accordingly, both caloric and the Higgs boson should count as hypothetical entities. As for the electron, neutron, and so on, we take it that presently these have the status of well-established empirical entities, but that prior to their respective discoveries "they" were merely hypothetical entities.

Without the scare quotes a paradox would arise. How can something that is merely an object of supposition suddenly, through an act of discovery, become a concrete empirical entity? Obviously, no such transmogrification transpires. The hypothetical entity itself is not detected during the course of discovery, but rather a concrete empirical entity whose configuration of *normal* properties sufficiently matches certain of the *conformal* properties of the hypothetical entity in such a way as to give someone cause to make an association between the two. There are two principal grounds for maintaining a distinction between the hypothetical entity and its post-discovery counterpart. First, per definition, the term or terms for the hypothetical has no empirical referent. That what we ordinarily call hypothetical entities fit the definition is plausible insofar as it's hard to generate principled and credible causal accounts of reference that don't also entail that 'Cartesian vortex' refers to space-time curvature or 'caloric' to kinetic energy. The second is the modal contrast that arises given that conformal properties are had necessarily whereas normal properties (for the most part) only contingently. This can be seen in particular in the case of the neutron.

The term was coined by William Sutherland 1899, 273: "If electrons are distributed through the aether, we must suppose that in aether showing no electric charge each negative electron is united with a positive electron to form the analogue of a material molecule, which might conveniently be called a neutron." This hypothetical neutron never took off, but in the early 1920s a revamped neutron appeared in the theoretical work of both Rutherford (1920) and Harkins (1921) on the constitution of atomic nuclei. As initially, the core feature of the revived neutron was that it is a composite particle, consisting of an electron and a proton in a tightly bound state. Even in Chadwick (1932), the paper which is popularly regarded as the announcement of the discovery of the neutron, he writes "We may suppose it to consist of a proton and an electron in close combination, the 'neutron' discussed by Rutherford . . . ." (697) Otherwise, he could not have identified it *as* the so-called neutron. An otherwise constituted (or elementary) particle would not have been *the* neutron. With Chadwick, however, the term 'neutron' came to denote the constituent particles of a particular type of beam of radiation emitted by beryllium. Within a few months it was possible for D. Iwanenko to muse whether "the neutrons can be considered as elementary particles," (Iwanenko 1932, 798) a view championed by Blackett shortly thereafter in order to make sense out of quantum spin and statistics.

To the extent that the appropriation model of discovery is on target, theories trading in hypothetical entities are not even candidates to be true or false. Nonetheless, they can be evaluated according to whether or not they are *vindicated*, i.e., whether the hypotheticals they speak of are eventually "discovered" or instead discarded. If a hypothetical, according to the theory itself, is in principle barred from empirical detection as one of its conformal properties, the hypothetical becomes *hopeless*. An example is the version of Lorentz's stationary ether theory that conforms to Poincaré's (limited) Principle of Relativity, viz., that there is no way to detect the motion of a physical system through the ether. One might ask why we should prefer special relativity rather than believing that there is a preferred stationary ether frame even though we will never know which initial frame it is. You might invoke Ockam's razor, but that won't help unless you have a brief that nature must be simple. The suppositional view (with the appropriation model) yields an immediate reason for preferring special relativity.

We now find the Linsky-Zalta view on the horns of a dilemma. Either their Platonism is a daring one according to which the postulation of mathematical entities is a bold move on a par with advocating hypothetical entities in the natural sciences; or else it is a timid one according to which the postulation of mathematical entities is no more ontologically daring than the use of auxiliary constructs in the natural sciences. If it is daring, then mathematical entities are hopeless — there is no analogue for the role of discovery. On the other hand, if it is timid, then its ontological commitments are no deeper than the deflationary ontology of the suppositional view and, frankly, there is no need for a postulational approach. In short, even if some of the consequences of their view appear to be on target, it in no way follows that we should take their metaphysical assumptions seriously. In contrast, we would like to say that these consequences should be taken seriously because they are inductively warranted from an examination of the empirical features of language use in mathematical practice.

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