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Discussion: Can the Best-Alternative-Justification solve Hume's Problem?

On the Limits of a Promising New Approach

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Abstract In a recent *Philosophy of Science* article Gerhard Schurz proposes meta-inductivistic prediction strategies as a new approach to Hume's Problem [6]. This comment examines the limitations of Schurz's approach. It can be proven that the meta-inductivist approach does not work any more if the meta-inductivists have to face an infinite number of alternative predictors. With this limitation it remains doubtful whether the meta-inductivist can provide a full solution to the problem of induction.

Keywords Induction, Hume's Problem, Best Alternative Justificaion, Prediction Games

1 Introduction

In a recent article on "The Meta-inductivist's Winning Strategy in the Prediction Game: A New Approach to Hume's Problem" [6] Gerhard Schurz proposes the *Best-Alternative-Justification* as a new approach to the problem of induction. As acknowledged by Schurz, the original idea goes back to Hans Reichenbach [3, 4]. But Schurz furnishes this idea with a new technical approach relying on a certain class of prediction strategies which he calls "meta-inductivists". Given that all attempts to solve the problem of induction have hitherto failed [2], what should induce us to reconsider the sceptical conclusion that there is no solution to the problem of induction? Prima facie, Gerhard Schurz has a convincing answer to this question: Most of the proposed solutions to the problem of induction tried to prove the *reliability* of the inductive procedure. But Schurz, following Reichenbach, merely tries to show the *optimality* of a specific inductive strategy, namely his "meta-inductivist" strategy. Demonstrating the optimality of an inductive prediction strategy is a less ambitious task than demonstrating its reliability, because for an inductive strategy to be reliable one would have to prove that it works in any possible world. But an inductive strategy that is merely optimal is allowed to fail in some possible worlds, as long as in the worlds where it does not work all other possible prediction strategies are bound to fail, too. Schurz does not raise the claim that he has solved the problem of induction literally, but the closing paragraph of his article suggests that he believes that he has at least provided a very good candidate for a solution to the problem of induction [6, p. 304].[1]

Schurz discusses the problem of induction within the technical framework of prediction games, where a number of players have to predict the next event in a series of binary valued (0 or 1) or real valued (any real number from 0.0 to 1.0) world events. By proving two theorems regarding the optimality of the prediction strategies of the "weighted meta inductivist" and the "collective weighted meta inductivist", Schurz is at least able to give a partial solution for the problem of induction that accounts for the case of finitely many prediction strategies. But, as shall be demonstrated in the following, Schurz' technical approach meets insurmountable limits once one tries to pass from a finite number to an infinite number of prediction strategies. This raises the philosophical question whether an optimality result demonstrated for a finite number of prediction strategies might suffice to answer the problem of induction. If not, then providing a full solution to Hume's problem remains an open challenge.

In the following, I am going to briefly restate Schurz' central results and then demonstrate that the results for prediction games with finitely many strategies cannot be extended to prediction games with

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[1] Regarding this claim, see also an earlier presentation of Schurz' ideas on the 5th GAP conference [5, p. 256].

infinitely many strategies. Finally, there will be a brief discussion of open questions regarding Schurz' answer to Hume's problem.

2 Schurz' basic approach

The technical framework within which Schurz derives his results consists of a series of events which can either be 1 or 0 (*binary prediction game*) or take any real value in the closed interval from 0.0 to 1.0 (*real valued prediction game*) and of a set of prediction strategies that have to get as many predictions of these events right as possible (in the binary prediction game) or that have to predict the coming events as closely as possible (in the real valued prediction game).

Two kinds of prediction strategies occur in Schurz' framework: *Ordinary predictors* that predict the next event by some arbitrary algorithm without looking at any of the other predictors and *meta-inductivists* that may – although they do not have to – base their own predictions on any of the ordinary predictor's predictions. The ordinary predictor's predictions are considered to be at least *output-accessible*, i.e. the meta-inductivists get informed about the ordinary predictor's predictions before they place their own predictions. In order to chose between the ordinary predictors, the meta-inductivists may take into account the predictor's success rates as well as their previous predictions.

The ordinary predictors are symbolized by Schurz with capital "P"s with an added index, e.g. P_1, P_2, etc. The meta-inductivists are symbolized as xMI_i, where x is a place holder for a string of characters that indicates the type of meta-inductivist and i is, again, an index. There exists a kind of canonical ordinary predictor that Schurz calls the *object-inductivist* and which he denotes as OI. The object-inductivist's algorithm takes either – in the real valued prediction game – the mean value of all past events as its next prediction or – in the binary valued prediction game – the kind of event that had the higher frequency in the past.

If the sequence of events is a *random* sequence and if we exclude demonic predictors that know the coming world event ahead of time, there exists no strategy that can do better than the object inductivist. The meta-inductivist will consequently chose the object inductivist among the ordinary predictors, or, if no object inductivist is present, it will predict according to the object inductivist's algorithm by itself. At any rate the meta-inductivist will either be as good or better than any of the ordinary predictors.

However, in order for the meta-inductivist to be a "winning strategy" of the sort that is required to provide a solution to the problem of induction, it must also be optimal or, at least, approximately optimal in a "deceiving" world, where the event sequence or the other predictors or both "demonically"

conspire against the meta-inductivist. The situation can be described as a game with the following rules:

1. In each round, first, the ordinary predictors predict what the next event in the event sequence will be. For making their predictions the ordinary predictors have access to the following information:

   (a) Complete knowledge about the past of the game, i.e. the past event sequence and the past predictions by all other ordinary and meta-inductivist predictors.

   (b) *Deceiving predictors* know if and by which meta-inductivists their output will be accessed (see point 2). However, they have to deliver their predictions before the meta-inductivists do. As a consequence, deceivers that always have a reliable forenowledge of the predictions of the meta-inductivists cannot exist. For, the case may arise where a deceiver would have to base its evaluation of which prediction to deliver on a meta-inductivist's prediction which is in turn based on the very results of this evaluation. Now, assume a deceiver A predicts 0 whenever a meta-inductivist MI is going to predict 1 and 1 if MI is going to predict 0 and at the same time MI predicts 0 when A has predicted 0 and 1 when A has predicted 1. Then, the prediction of the deceiver would be undefined.

   (c) Deceiving predictors may have the capability of clairvoyance that is, in a *non deceiving world* they know beforehand what the next event will be. For reasons similar as in 1b, permanent clairvoyants cannot exist in a deceiving world.

2. Then, the meta-inductivists make their predictions. In doing so, they may access the "output", i.e. the predictions, of any non meta-inductivist. Also, the meta-inductivists may – just like the ordinary predictors – take into account the complete information about all past events and predictions.

3. Finally, the world event occurs. In a *deceiving* world, the world event may depend in an arbitrary way on what predictions the predictors have made.

4. A meta-inductivist "wins", i.e. is long run optimal, if in the long run the success loss of MI as compared to the best player at the given time converges to zero or a negative number.[2] (Or, simply put, if in the long run it performs at least almost as good as the best player.) Otherwise, the meta-inductivist "looses" the game.

3 Schurz' central results

Within the just sketched framework, is it possible to find a provably optimal meta-inductivist strategy? Schurz believes it is and he presents two important theorems regarding the optimality of certain types

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[2] I am indepted to an anonymous referee for the precise formulation.

of meta-inductivists. But he also relates an argument [6, p. 298] by Cesa-Bianchi and Lugosi [1, p. 67] that points out certain limits. The argument is the following:

Impossibility Theorem 1: In the binary prediction game, a single meta-inductivist cannot be optimal in all possible worlds.

The proof of this theorem can informally be stated as follows: Assume a world with one meta-inductivist and two alternative predictors, one of which always predicts 1 and the other always 0. Whatever the sequence of world events is, the succes rates of the two assumed alternative predictors will always add up to 1. It follows that in any round the success rate of either the one or the other of the two assumed alternative predictors is at least 50%. Now, if the sequence of world events is a demonic sequence that always delivers the event that was not predicted by the meta-inductivist, the meta-inductivist's success rate will always remain 0%. Thus, in any round the meta-inductivist's success rate is significantly lower than that of the best player, which means that the meta-inductivist's strategy is not optimal.

Because of this, Schurz understands the different types of meta-inductivists which he develops in section four through to section six of his article [6, p. 285-296] as only partial solutions to the problem of induction. They are optimal in (large) classes of prediction games but not in all prediction games.

The "impossibility theorem" stated before does not cover the real valued prediction game. And indeed it can be demonstrated for the real valued prediction game that a meta-inductivist that averages over the alternative predictor's predictions weighted by the alternative predictor's relative success-advantage over the meta-inductivist will quickly approximate the maximal success rate of the alternative predictors. This is Schurz' theorem 4 [6, p. 297], which shall be termed "optimality theorem 1" here:

Optimality Theorem 1 for a weighted average metainductivist wMI in the real valued prediction game:

1. *Long run*: The success rate suc_n of wMI approximates the alternative predictors maximal success rate $maxsuc_n(P)$, i.e. $lim_{n \to \infty}(maxsuc_n(P) - suc_n(wMI)) = 0$.
2. *Short run*: In any round n for the success rate of wMI holds: $suc_n(wMI) \geq maxsuc_n(P) - \sqrt{m/n}$ with m being the number of alternative predictors.

Here P denotes the set of all alternative predictors and $maxsuc_n(P)$ denotes the maximal success rate of the alternative predictors in round n. For the precise definition of the strategy of the weighted average meta-inductivist and the proof of the theorem, see Schurz' paper [6, p. 296ff.].

As can be seen, the long run success of the meta-inductivists does not depend on how many alternative predictors are present. Only in the short run, the number or alternative predictors m is

important in so far as the more alternative predictors are present, the more "distracted" the weighted average metainductivist can become in the short run.

Schurz is able to prove a similar theorem for the binary valued prediction game. The impossibility theorem stated earlier precludes that this will work for a single meta-inductivist in the binary prediction game. But, as Schurz is able to demonstrate, the mean success rate of a *collective of meta-inductivists* $cwMI$ can approximate the maximal success of the alternative predictors – be they as demonic as they may – almost equally well as the weighted average meta-inductivist can in the real valued prediction game. The more $cwMI$ predictors are present, the better the approximation. Or, more precisely:

Optimality Theorem 2 for the *mean success rate* of a collective of k collective weighted-average meta-inductivists $cwMI$ in the binary prediction game:

1. *Long run*: The mean success rate $meansuc_n$ of the collective meta-inductivists $\frac{1}{2k}$-approximates the alternative predictor's maximal success rate $maxsuc_n$, i.e. $lim_{n\to\infty}(maxsuc_n(P) - meansuc_n(cwMI)) \leq \frac{1}{2k}$.

2. *Short run*: In any round n for the mean success rate of $cwMI$ holds: $meansuc_n(wMI) \geq maxsuc_n(P) - \sqrt{m/n} - \frac{1}{2k}$ with m being the number of alternative predictors.

For the proof of this theorem and for the precise algorithm of the collective weighted-average meta-inductivists ($cwMI$), see Schurz' article [6, p. 297-299]. Again, the long run success of the collective meta-inductivists (with regard to their mean success rates!) does not depend on the number alternative predictors. This renders the finding non-trivial. For no matter how large the number or alternative predictors is, their maximal success can be approximated by a comparatively smaller collective of meta-inductivists, the precise number of which is only determined by what level of approximation is regarded as satisfactory. It is only in the short run that a large number of possibly demonic alternative predictors can effectively deceive the collective meta-inductivists.

4 Limitations of Schurz' approach: Confinement to the finite

4.1 Why Schurz' approach cannot be extended to the infinite case

Schurz explicitly restricts his investigation to prediction games with finitely many prediction strategies [6, p. 284]. In this case the number of meta-inductivists may even be much smaller than the number of alternative predictors. It will now be shown why a similar argument cannot be made in case the number of alternative predictors is infinite and why, therefore, Schurz' optimality argument is confined to the finite.

Impossibility Theorem 2: If there is an infinite number of alternative predictors, then even a collective of meta-inductivists cannot perform approximately optimal in all possible worlds in the binary prediction game.

Proof: Consider the following scenario: Let there be an arbitrary number of meta-inductivists. As there are only two possible events, namely 1 and 0, at least half of the meta-inductivists predicts the same event. Obviously, in each round there exists an event that is *not* picked by a majority of meta-inductivists. Now, assume a demonic world where the world event is always an event that is not predicted by a majority of meta-inductivists. Then the average success rate of the meta-inductivists never exceeds 50%.

Now we only need to show that there can exist at least one (demonic) predictor that achieves a higher success rate. For this purpose, split the (infinite) set of alternative predictors into two infinite sets in the first round. The predictors in the first set predict 1, the predictors in the other set predict 0. In the following rounds take the (infinite) set of predictors that have always predicted true so far, split it into two infinite sets and again let all predictors from the first set predict 1 and all predictors from the second set predict 0. In any round of the game there is thus an infinite number of predictors left that has a success rate of 100%. Since the meta-inductivist's average success is significantly lower (smaller or equal 50%), their strategy is not optimal.

And, as may be expected, there is a similar impossibility theorem for the real valued prediction game:

Impossibility Theorem 3: If there is an infinite number of alternative predictors then no meta-inductivist xMI can be approximately optimal in all possible worlds in the real valued prediction game.

Proof: Assume a demonic world, where the world event is always 0 or 1, whichever of these two numbers is further away from the predicion that xMI makes. As to the infinite number of alternative predictors: In the first round, let half of them predict 1 and the other half 0. In the following rounds let half of the alternative predictors that have always predicted correctly so far predict 1 and the other half 0. Then at any point in time n, there exist some predictors with complete success, while the average success of xMI does not exceed 50%.

Hence, the conclusion: Neither in the binary nor in the real valued prediction game exists an optimal meta-inductive strategy if the number of alternative predictors is infinite.

4.2 A sidenote: Limitations of "one favorite" meta-inductivists

Just how difficult it is to design a meta-inductivist that covers all possible or at least all desirable scenarios becomes apparent when considering a limitation of Schurz' avoidance meta-inductivist, which is the most universal type in a series of "one-favorite meta-inductivists" that Schurz develops in sections four to six of his article.

As Schurz proves mathematically (his theorem 3), the avoidance meta-inductivist (aMI) ϵ-approximates the maximal success of the *non-deceiving* alternative predictors. However, this proof does not cover all strategies that we might intuitively consider as non-deceivers. For example, a strategy that starts as a deceiver and switches to a non-deceiving clairvoyant prediction algorithm only later in the game (after it has been classified as a deceiver by aMI) will remain classified as a deceiver by aMI. Intuitively, though, we would probably not consider it a deceiver any more after it has switched to a non-deceiving clairvoyant algorithm. Further below it will be demonstrated that this can even happen accidentally for a predictor that never deceives (in an intuitive sense).

This limitation is a consequence of the fact that Schurz' definition of "deception" is purely extensional. It is based on the predictor's overt behaviour and not on the deceptive or non-deceptive algorithm the predictor uses: "A non-MI-player P (and the strategy played by P) is said to *deceive* (or to be a *deceiver*) *at time n* iff $suc_n(P) - suc_n(P|\epsilon MI) > \epsilon_d$" [6, p. 293], where ϵ_d is the "deception-threshold" and $suc_n(P|\epsilon MI)$ is P's conditional success-rate when ϵMI has P as a favorite. So, contrary to what we might intuitively think, it is not a necessary condition for being a deceiver to base the predictions on what favorites the meta-inductivists have.

As Schurz himself notices "even an object-strategy (such as OI) may become a deceiver, namely, when a demonic stream of events deceives the object-strategy"[6, p. 293]. This is of course to be understood in terms of Schurz' previous definition of deception, because the algorithm that, say, OI uses is the same as in a non-demonic world and would intuitively not be considered as deceptive.

But then there is a finite probability that an OI will be classified as a deceiver by aMI even though the stream of world events is not demonic in the sense that the events are computed from the predictions made by the predictors. For there is a finite probability that a random stream of world events accidentally mimics a demonic stream of world events up to round k so that OI appears as a deceiver up to round k. If k is sufficiently large then aMI classifies OI as deceiver. And it will only reevaluate OI's status if OI lowers its unconditional success rate. "For a player P who is recorded as a deceiver will be 'stigmatized' by aMI as a deceiver as long as P does not decrease his unconditional success rate (since P's aMI-conditional success rate is frozen as long as aMI does not favor P)"[6, p.

295]. As OI's success rate reflects the frequency of random world events in the binary prediction game, it is unlikely that it significantly lowers its success rate at a later stage in the game.

In such a situation aMI would fail to ϵ-approximate the maximal success of OI even though no deception was ever intended and the conditions under which this situation can occur are completely natural (i.e. non demonic world, no supernatural abilities like clairvyoance, etc.). Thus, if aMI can fail to be optimal even with respect to OI under completely natural circumstances, the optimality result concerning the performance of aMI with regards to all non-deceivers may not quite deliver what we expect. For example, we cannot not say that aMI is optimal save for demonic conditions or deception, if decpetion is understood in an intuitive sense as described above.

5 Open Questions

Schurz' "New approach to Hume's problem" is both a research program on prediction strategies and a proposed answer to the problem of induction. As a research program it can be pursued more or less independently of its claim to offer a new approach to Hume's problem. Regarding this claim, the central question is of course: Can meta-inductivists solve the problem of induction? This question can be broken down into three separate questions.

1. How is Schurz' model related to the problem of induction, if the latter is understood as the problem of justifying scientific inference?
2. Does it matter that no single meta-inductivist but only a collective of meta-inductivists constitutes an optimal strategy in the (binary) prediction game?
3. Is it acceptable that the best alternative justification works only when the number of predictors is finite?

In order to better understand these questions, each of them shall briefly be discussed:

1) *Justification of scientific inference*: Schurz answers the problem of induction within a technical framework that consists of a highly stylized world that produces a sequence of (binary or real-valued) events at discrete time intervals. Further interpretative work might be necessary to show that the problem of induction that has been solved within Schurz' technical framework matches the problem(s) of induction that occur in the real world. In particular, the kind of induction in Schurz' model is the induction from previous events to the next event – in contradistinction to the inductive or abductive inference from a number of single instances to a general rule. For the justification of scientific inference, the latter type of induction seems to be even more important. After all, we would like to know whether we can rely on a law of nature if it has been confirmed in a finite number of instances and never

disconfirmed. Thus, the question would be whether and how Schurz' answer to Hume's problem can also be transferred to this case.

2) *Admissibility of collectives of meta-inductivists*: In the binary-valued game a *collective* of meta-inductivists is required to assure optimality in the prediction game. Now, if we are looking for a reliable method of induction to the next event, then we are faced with the somewhat puzzling fact that we do not get a single proposal but a multitude of proposals instead. As follows from the impossibility theorem for single meta-inductivists (impossibility theorem 1 above), it is impossible to melt down the different predictions of the collective of meta-inductivists to a single prediction without becoming vulnerable to deception by a demonic world. There is no grave problem involved if the prediction game is considered in a decision theoretical framework [6, p. 301ff.]. For then the primary goal would not be to get the next prediction right, but rather to derive as much utility as possible from a number of predictions. However, this implies a shift of emphasis from the original problem of induction to a closely related decision theoretical problem.

3) *Confinement to the finite*: If only a finite number of prediction strategies is taken into account, then we exclude the overwhelming majority of *possible* prediction strategies from the game right from the beginning. For, given that the sequence of events is infinite, there exists an uncountably large number of posssible event sequences. And even if we take into consideration only those prediction strategies which can be described by an algorithm, there still remains a countable infinity of possible alternative prediction strategies. Unfortunately, neither a single meta-inductivist nor a collective of meta-inductivists can perform optimal in all possible worlds if the number of alternative predictors is infinite (see section 4.1).

Schurz deliberately restricts his investigation to prediction games with finetely many players, because he makes "the realistic assumption that xMI has finite computational means." [6, p. 284]. But in order to justify this restriction one would need to show that an infinite number of alternative players is impossible, rather than arguing that xMI cannot deal with an infinite number of alternative players. Otherwise the notice "that xMI has finite computational means" merely amounts to admitting that under this "realistic assumption" xMI simply cannot always perform optimal.

As there is no logical contradiction involved in the assumption of an infinite number of alternative players, the only grounds upon which Schurz' restriction could be defended would be empirical grounds. But then it is hard to see how the general impossibility of an infinite number of alternative players can be demonstrated without silently or explicitly making use of inductive inference.

Very simply put, there seems to be no good reason why a theoretical framwork for answering Hume's problem that allows for "clairvoyants", "deceivers", "demonic" streams of world events, should not also

admit infinite sets of alternative predictors. Surely, that an xMI which has only finite computational means does not work under this condition is not a sufficient reason.

6 Conclusion

Summing it up, what can be achieved with prediction games with respect to inductive inferences is a non-trivial result which shows that if only a finite number of prediction strategies are involved there exists with the "(collective) weighted average meta-inductivist" a strategy that is an approximately best strategy in all possible worlds. However, it can also be demonstrated that no approximately best strategy exists if an infinite number of alternative prediction strategies is involved. And there is some reason to believe that for a full solution to the problem of induction the restriction to a finite number of alternative prediction methods is insufficient. If this is true, then the best alternative justification cannot offer a full solution to the problem of induction.

7 References

References

1. Cesa-Bianchi, N., and G. Lugosi (2006): *Prediction, Learning and Games.* Cambridge University Press, Cambrdige.

2. Howson, C. (2000): *Hume's Problem. Induction and the Justification of Belief.* Oxford University Press, Oxford.

3. Reichenbach, H. (1983): "Erfahrung und Prognose. Eine Analyse der Grundlagen und der Struktur der Erkenntnis," in *Gesammelte Werke. Band 4*, ed. by A. Kamlah, and M. Reichenbach, Wiesbaden. vieweg Verlag, first published in Chicago 1938.

4. ——— (1994): "Wahrscheinlichkeitslehre. Eine Untersuchung über die logischen und mathematischen Grundlagen der Wahrscheinlichkeitsrechnung," in *Gesammelte Werke. Band 7*, ed. by A. Kamlah, and M. Reichenbach, Wiesbaden. vieweg Verlag, first edition published in Leiden 1935, second edition published in Berkeley 1949.

5. Schurz, G. (2003): "Der Metainduktivist. Ein spieltheoretischer Zugang zum Induktionsproblem," in *Proceedings der GAP.5, Bielefeld 22.-26.9.2003*, ed. by R. Bluhm, and C. Nimtz, pp. 243–257, Paderborn. Gesellschaft für analytische Philosophie, mentis Verlag.

6. ——— (2008): "The Meta-inductivist's Winning Strategy in the Prediction Game: A New Approach to Hume's Problem," *Philosophy of Science*, 75, 278–305.