

# The Limits of Modeling

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**Abstract.** First, I propose a new argument in favor of the Dappled World perspective introduced by Nancy Cartwright. There are systems, for which detailed models can't exist in the natural world. And this has nothing to do with the limitations of human minds or technical resources. *The limitation is built into the very principle of modeling: we are trying to replace some system by another one.* In full detail, this may be impossible.

Secondly, I'm trying to refine the Dappled World perspective by applying the correct distinction between models and theories. *At the level of models*, because of the above-mentioned limitations, we will always have only a patchwork of models each very restricted in its application scope. *And at the level of theories*, we will never have a single complete Theory of Everything (or, a complete pyramid of theories) allowing, *without additional postulates*, to generate all the models we may need for surviving in this world.

**Keywords:** models and theories, model templates, models in statistical physics, simulation.

This article is written from an anti-Platonist point of view.

The Dappled World perspective was introduced by [Nancy Cartwright](#) (1999: [The Dappled World: A Study of the Boundaries of Science](#), Cambridge University Press, p. 1): "... we live in a world rich in different things, with different natures, behaving in different ways. The laws that describe this world are a patchwork, not a pyramid."

I will propose a new argument in favor of the Dappled World perspective, and will try to refine it by applying the correct distinction between models and theories.

## What is a Model?

Surprisingly, in philosophical texts, this question is asked (sometimes), but almost never – answered. Instead of a general answer,

usually, some classification of models is considered. A general answer can't be found even in the prominent account given by [Roman Frigg](#), and [Stephan Hartmann](#) (2006: [Models in Science](#). *Stanford Encyclopedia of Philosophy*). Should this mean that a sufficiently general definition of modeling is impossible?

Computer scientists are proposing very broad definitions of modeling since many years.

The following general definition of the term "model" was proposed by [Marvin Minsky](#) (1965: [Matter, Mind and Models](#). *Proceedings of IFIP Congress 65*, 1: 45-49) on p. 45: "We use the term "model" in the following sense: To an observer B, an object A\* is a model of an object A to the extent that B can use A\* to answer questions that interest him about A. The model relation is inherently ternary. Any attempt to suppress the role of the intentions of the investigator B leads to circular definitions or to ambiguities about "essential features" and the like."

A similar definition was proposed by [Jeff Rothenberg](#) (1989: [The Nature of Modeling](#). *Artificial Intelligence, Simulation, and Modeling*, John Wiley and Sons, 75-92) on p. 75: "Modeling in its broadest sense is the cost-effective use of something in place of something else for some purpose."

Among philosophers, such a broad definition of modeling is advocated, to my best knowledge, only by [Paul Teller](#) (2001: *Twilight of the Perfect Model Model*. *Erkenntnis*, 55: 393–415) on p. 397: "... in principle, anything can be a model, and that what makes the thing a model is the fact that it is regarded or used as a representation of something by the model users. ... it would be a mistake for the general account of the use of models in science to specify more narrowly what can function as a model."

Thus, the broadest possible definition of modeling could sound as follows: *a model is anything that is (or could be) used, for some purpose, in place of something else.*

Could such a broad definition be useful? Can one derive any useful consequences from it? See below...

According to this definition, a model is a *single concrete system* replacing another *single concrete system*. For example, a model of the Solar system may include: a specified number of planets (8, or 9), a specified diameter and mass of Sun ( $1.5 \times 10^8$  km,  $2.0 \times 10^{20}$  kg), and for each planet: specified diameter, mass, initial location and velocity. No satellites, no asteroids, no comets. To allow computer simulation, the Newtonian mechanics with the gravitation law also should be included in the model as means of reasoning. The “model” of an “arbitrary” system of  $n$  planets, is *not* a model in the above sense (it is a model template, see below).

### Theories as Means of Model-Building

None of theories is modeling a single concrete system. Thus, theories are not models, are they “nobler” than models? No, *theories are useful only as a means of model-building*, no more than that! Newtonian mechanics is not a model, but it allows generating of models of various concrete mechanical systems. For references on this point see my (2009: Towards Model-Based Model of Cognition, *The Reasoner* 3(6): 5-6).

### Model Templates

Between models and theories, there is an important intermediate concept that is called *model template* by computer scientists. The term “parametric model” also would be appropriate.

A *model template* is a structure containing parameters. By setting these parameters to concrete values, we obtain models (*instances* of the template). The above-mentioned “arbitrary” system of  $n$  planets is a model template having as parameters:  $n$  – the number of planets,  $D$ ,  $M$  – diameter and mass of the central body,  $d_i$ ,  $m_i$  – diameter and mass of  $i$ -th planet etc. By setting these parameters appropriately ( $n=8$  etc.) we can obtain, for example, the computer simulation of Solar system running on my laptop.

Some theories can be represented almost entirely as model templates. For example, the Hamiltonian formulation of Newtonian mechanics can be regarded as a model template of an arbitrary mechanical system having  $6n+2$  parameters:  $n$  – the number of components,  $H$  –

Hamiltonian function of the system,  $6n$  initial coordinates and velocities. Thus, some theories are “models” indeed – but parametric models.

The idea that some theories are, in fact, model templates can be found already in [Ronald Giere](#) (1979: *Understanding Scientific Reasoning*, Holt, Rinehart and Winston). See also R. Giere (1985: *Constructive Realism*. In: [Images of Science: Essays on Realism and Empiricism](#), University Of Chicago Press, 75-98), where on p. 78 model templates are called *general models*.

In a sense, model templates represent the “homogeneous” extreme of model-building. The “heterogeneous” extreme is represented by less organized means of model-building discussed, for example, by [Mauricio Suarez](#) and [Nancy Cartwright](#) (2008: [Theories: Tools versus Models](#). *Studies in History and Philosophy of Modern Physics*, 39, 62-81) on p. 79.

### The Limits of Modeling

Now, the main point: there are systems that *can't be modeled in full detail even in principle*.

As an example, let's consider an isolated container containing one litre of air. How detailed can be a *simulatable computer model* of such a system? If we believe that air consists of molecules, then our container includes about  $10^{22}$  molecules. Thus, to represent the state of this system at a particular moment of time, we need to store at least  $6 \times 10^{22}$  numbers (coordinates and velocities). And to simulate the evolution of state in real time, we need to compute – at a very high speed – the solutions of  $6 \times 10^{22}$  Hamiltonian equations. We can't build such a computer. Moreover – such a computer can't exist in the natural world! And even moreover: in the natural world, *two systems each consisting of  $10^{22}$  separable components simply can't be “isomorphic enough” to represent each other in every detail* (whatever it means)!

If the number  $10^{22}$  doesn't seem convincing enough, replace the air container by the entire Universe, *all* the photons and neutrino included. How detailed can be a *simulatable computer model* of the Universe? Could it simulate *every*

photon and *every* neutrino? Could this model include the model of the simulating computer itself?

Thus, it makes sense to ask the following question: for which values of  $N$ , two natural world systems each consisting of  $10^N$  separable components can't be "isomorphic enough" to represent each other in every detail? My hypothesis: already  $N=22$  is large enough.

Pierre Simon Laplace was aware of this problem in 1812: he concluded that despite having a "Theory of Everything" (Newtonian mechanics), we will never be able to predict the future in every detail: "... our efforts will always fall infinitely short of this mark" – an English translation from p. 101 of [Kevin D. Hoover](#) (2001: [Causality in Macroeconomics](#), 311 pp.)

Thus, there are systems, for which detailed models can't exist in the natural world. And – contrary to Laplace – this has nothing to do with the limitations of human minds or technical resources. The *limitation is built into the very principle of modeling*: we are trying to *replace some system by another one*. In full detail, this may be impossible. Many systems are "too unique" to be modeled in full detail.

This represents another argument in favor of the Dappled World perspective: *at the model level, we will always have only a patchwork of models each very restricted in its application scope*.

But at the level of theories? Will we have, some time, a *single* complete Theory of Everything (or, at least, a complete pyramid of theories) not only "explaining everything in the world", but also allowing, *without additional postulates*, to generate *all* the models we may need for surviving in this world? According to the Dappled World perspective, the answer is negative.

This represents the refinement of the Dappled World perspective announced at the beginning.

### **Templates do exist, but instances don't?**

Now, a possible Platonist objection: the above-mentioned detailed models of systems

containing  $10^{22}$  separable components do not exist in the natural world, but we can *imagine* them! There are "imagined systems" that "would be physical things if they were real" – as put by [Roman Frigg](#) (2010: [Models and fiction, Synthese](#), 172(2), 251-268) on p. 253.

But how could one imagine a computer simulation of  $10^{22}$  molecules, if such a computer can't exist in the natural world even in principle? For an anti-Platonist, it means that, in fact, one is using some *mathematical axioms* to derive the "existence" of such a huge and powerful computer (axioms of first order arithmetic would suffice here). But many things the "existence" of which follows from your axioms, can't exist in the natural world.

What really exists in the natural world, are *definitions of model templates* – because they can be written down on a paper. For example, one can write down a definition of the following model template: an isolated container containing the so-called *hard-ball-gas* having as parameters:  $n$  – the number of spherical molecules; dimensions of the container; and for each molecule: diameter, mass, initial location and velocity. The Hamiltonian function is defined here as a simple sum of kinetic and potential energies of molecules, assuming elastic collisions. This *model template* written down on a paper exists in the natural world in the most solid sense possible.

But, if we set  $n=10^{22}$ , then there is no real way of assigning particular values even to the initial coordinates and velocities of each molecule. Thus, while the model template itself exists in the natural world, for  $n=10^{22}$ , its *instances* "exist only in axioms". One can prove *general* theorems about these instances, but most of *particular* instances are not accessible to analysis.

### **What could physicists do in such a situation?**

At the model level, they can try computer simulation only of very small models of gas containers: for  $n=10^3, 10^4$  etc. For impressive educational programs of this kind see [Paul Falstad](#) (2009: [Gas Molecules Simulation](#)) and John I. Gelder et al. (2000: [Chemistry Web](#)

[Server](#)). For a research application involving simulation – see [Yakov G. Sinai](#) et al. (2008: [Statistical Properties of the Cluster Dynamics of the Systems of Statistical Mechanics](#), *ESI Lecture Notes in Mathematics and Physics: Boltzmann's Legacy*, *European Mathematical Society*, 203-216).

But, perhaps, the most striking simulation experiment of this kind – [The Millennium Simulation Project](#) was performed by [The Virgo Consortium](#). The simulated Universe of this experiment consists of about  $10^{10}$  big gravitating “particles” (each about  $10^9$  solar masses in the first simulation, and about  $10^7$  – in the second one). Trillion times less details – when compared to one litre of air!

And, at the level of theory? For realistic numbers of molecules, most particular instances of the corresponding model templates are not accessible to analysis. Thus, physicists may try deriving only the average properties of all (or most) instances of these templates. This is exactly what is going on in statistical mechanics (micro-canonical ensemble etc.).

One can obtain some information about the location and velocity of some particle only by shooting another particle at it. At the micro-level, this implies the famous *uncertainty principle*: a micro-particle possessing a definite location and velocity as its “state” is a wrong intuition. Couldn't a similar argument be applied to the “states” of gas containers? Which information and how can be obtained about these “states”? For example, the locations of most molecules can be determined only up to the dimensions of the container. Hence, *each molecule of gas possessing a more definite location is a wrong intuition*? And, gas consisting of separate molecules is, perhaps, a correct intuition, but to what extent?

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