ON THE SOVEREIGN INDEPENDENCE OF SPACETIME[1]

Graham Nerlich

graham.nerlich@adelaide,edu.au

ABSTRACT: Special Relativity is not a branch of electromagnetism: it does not depend on light's having a constant, limiting speed. If the theory is true, it depends on no matter theory. Rather, very general and familiar symmetries of space and time impose the form of the Lorentz transformation on every matter theory, independently of any that obeys it. I explore this and its metaphysical consequences.

Introduction

Special Relativity (SR) is not a branch of electromagnetism: it does not depend on light's having a constant, limiting speed. If the theory is true, it depends on no matter theory. Rather, very general and familiar symmetries of space and time impose the form of the Lorentz transformation on every matter theory, independently of any that obeys it. Spacetime rules.

I aim to explore this and its metaphysical consequences in a style accessible to non-specialist philosophical readers.

Minkowski first developed this approach in §1 of his path-breaking paper of 1908.[2] That part of his message has retreated from the centre of attention. For some, it entered the basic folklore of SR – e.g. for more mathematically minded physicists. Many more know nothing of it – especially the majority of philosophers, including philosophers of science. For these, SR rests on the principle that nothing outstrips light, made vivid by the sheer brilliance of Einstein's operationalist analysis of light-synchrony and simultaneity. That is apt to be how we learn the theory. It has overshadowed the geometrical approach. [3]

But Minkowski arrived at the Lorentz metric, at spacetime and, therefore, SR 'along a purely mathematical line of thought' – that is, along a purely geometrical path from the symmetries of real - if you like, physical - space and time to the Lorentz transformation and the metric of spacetime (75). What earned first place in Minkowski's lecture is the mathematical purity of the journey rather than where it began. Einstein's revolution in physics could have been gained by bold mathematical conjecture based on known, empirical symmetries of space and time. Minkowski did not improve on Einstein's theory: his theory differs in its foundations. It needs neither light, electromagnetism nor any other matter theory to sustain it. [4]

The opening four-sentence preamble to the lecture acknowledges that the new views of space and time 'have sprung from the soil of experimental physics, and therein lies their strength' (75). But Minkowski then goes straight to the mathematical approach. He concludes '… mathematics, though it can now display only stair-case wit, has the satisfaction of being wise after the event' (79).

Thus, in SR, spacetime itself lies at the foundation of matter theories, imposing its constraints on each one. Its structure yields the broad picture of the world that has shaped physics since 1905. Spacetime is sovereign and independent.

I shall prune, amplify and simplify Minkowski's rather informal discussion so as to track philosophical interests. I think it more intuitive than the rigorous algebraic deductions that followed in 1910 (see §3). Its metaphysical, ontological importance is my main theme.[5]

2 Minkowski's 'purely mathematical line of thought'

Minkowski, 'setting out from the accepted mechanics of the day' (76)[6] begins by noting that two sorts of transformations leave Newton's laws unchanged: changes of position and changes in uniform velocity. The first, in more detail, is the standard group of continuous spatial transformations − translations and rotations. (Reflections aren't relevant here.) This means that the laws of physics are the same (invariant) everywhere and in any direction. The second, in more detail, is the Galilean group of transformations that take us from one inertial frame of reference to another. 'Thus the two groups, side by side, lead their lives entirely apart' (76). The aim is to unify them. How?

Minkowki reminds us that no one observes times except at places or places except at times (76). Thus, space and time although crucially distinct are deeply connected, facts that neither group exploits. The two groups are incongruous in that the first contains an orthogonal rotation of the spatial coordinates. The second group is not rotational, but allows replacement of x, y and z with constant speeds in these spatial directions (79). Time enters into the Galilean group via this relative speed of frames of reference, and so appears in the transformation of x, y and z coordinates. Yet the time coordinate itself is not transformed: t remains t. This leads the Galilean group to be 'treated with disdain' (loc. cit.) since it thereby raises the embarrassing problem of absolute motion. Thus the orientation of the *t* axis to spatial ones is left entirely undetermined.

Connecting the two into a satisfactorily unified group is easy. Choose a parameter, *c*, to tie space and time together in a single metrical structure rather like Euclid's.

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$$

To forge the link, *c* must be a constant with dimensions of both space and time i.e. a speed. It then figures in the metric and transformations as a limiting constant. The new transformations rotate space and time together, not as a pointless spatial rotation in one more dimension, but, by changing the sign of the x differentials, as a rotation in spacetime. That yields the form of the Lorentz transformations, but only its form. The argument assigns no value to the constant, *c*. Light or electromagnetism appear nowhere: that is the crucial feature.

Nothing yet determines the value of *c* – more on this later §5 (ii). It is to be found by observation, not necessarily observations on light. The value must lie in the interval $0 \leq c \leq \infty$. Values within the limits are 'mathematically more intelligible' (79) than either limit is, even though the upper limit defines the familiar Galilean group. A limit of zero speed is unbearably queer.[7] Infinite speed is queer, too: along its line of motion the thing is everywhere at once! A finite, invariant limiting speed forbids velocities, as ordinarily conceived, to add arithmetically. This offends one intuition but rescues another from obscurity: spatiotemporal continuity, and thereby both identity and causality make better sense.[8] Minkowski doesn't mention these metaphysical grounds in seeing a finite speed as mathematically the more intelligible. Perhaps he recognised them. Thus, requirements of sense and elegance lead to a finite constant, *c,* for the Lorentz group, and so to the structure of Minkowski's spacetime. While transformations leave physical laws the same in each frame, they no longer leave the properties of things invariant.

Group theory has general objective criteria for simplicity. The Lorentz group is semi-simple; the classical Galilean group is not.[9]

The upshot is a full rotational transformation group: change of inertial frame (boost) rotates all coordinate axes (through a pseudo-angle for time). A boost is an angular, geometrical, transformation. A thing (particle) is then a material curve in spacetime, a worldline if it is straight.[10]

Here are Minkowski's own words on how an audacious mathematician might have done it.

> Group G_c in the limit when $c = \infty$, that is the group G_∞, becomes no other than that complete group which is appropriate to Newtonian mechanics. This being so, and since G_c is mathematically more intelligible than G_∞, it looks as though the thought might have struck some mathematician, fancy-free, that after all, as a matter of fact, natural phenomena do not possess an invariance with the group G_∞, but rather with the group G_c, c being finite and determinate, but in ordinary units of measure, *extremely great*. Such a

premonition would have been an extraordinary triumph for pure mathematics.

Minkowski, H. op. cit (79) [Original italics.]

Clearly, the line of thought is speculative − fancy-free, not deductive. Neither its premises nor its conclusions are a priori or necessarily true. It has to match the world.

3 An algebraic approach

Newton's title 'Corollary V' for his discussion of the motion of relative spaces implies that his Laws of Motion entail Galilean relativity. It must have seemed obvious that relative motions must inevitably retain, as invariants, all properties of things that could consistently be taken as invariant. That delivers the Galilean transformations. Yet Newton made no serious attempt to derive them. We now know that the conclusion does not follow. Einstein gave the decisive counterexample in 1905. He was motivated by the belief that a satisfactory understanding of Maxwell's electrodynamics required the speed of light to be a finite constant, invariant under change of inertial frame of reference. Yet the Lorentz transformation was never confined to electromagnetism. It led at once to a Lorentz invariant revision of the laws of mechanics. From the start, the transformations were understood as a formal demand on any matter theory in physics. They are at the foundation of all.

A question naturally arises: what constraints on transformations does the Relativity of Motion, just by itself, demand? The principle assumes deep and familiar symmetries of space and time long taken as among a priori necessities in Euclidean geometry. Once we make these assumptions explicit we can find a determinate answer to the question. Explicitly, assume the homogeneity of space and time (laws don't change over time), and isotropy of space[11], already implied in Euclidean geometry and classical views of time. Despite Minkowski's saying he begins 'with the accepted mechanics of the day' his conclusion actually obliges us to revise that mechanics. It is really from these geometric premisses, underpinning the relativity of motion, rather than mechanics in full, that both arguments begin. The algebra machine cranks out a result. It yields the form of the Lorentz transformations with an undetermined constant, *c*, just as in Minkowski's more intuitive, approach.

Work in this style began with Ignatowski as early as 1910. There is now a considerable physics literature[12] proposing more or less minor variants on routes to the same conclusion.

The calculations are not our interest. Again, no appeal is made to electromagnetism or light nor to assumptions about simultaneity.

4 Truthmakers for spatial relations

On the face of it, these arguments imply that space, time and their union in spacetime inhabit the ground floor of ontology as first order members. Yet, since an order of deduction or speculation does not, by itself, entail an order of causality or being, that needs to be argued.

It is tempting to think that spatiotemporal symmetries must rest on matter-physics or, at least, that electromagnetism sneaks in to determine the value of *c*. There are interesting reasons not to yield to this temptation. It is wise at least to contemplate spacetime's independent sovereignty.

In this section, metaphysical arguments precede brief, elementary remarks in later sections on three kinds of matter theory. I sketch a spatial realism that begins as theory of spatial relations among objects; it is intended as a model for temporal and spatiotemporal realism, too. Although realism is almost always stated as a doctrine about points, the metaphysics of this theory ignores them. Spatial relationism has ambitions for the adequacy of just those spatial relations that hold among observables: do they equip us to say all we want about space and time? But there is a primary, deeper question – what is the nature of spatial relations? Unless that can be answered in relationist fashion, the realist may ignore the ambitions of adequacy. They are irrelevant to the argument of this section.

The main ontological question is about the truthmakers for spatial relation propositions. The splendid Lawrence Sklar says: '… attempts to define the "spatiality" of spatial relations are usually taken as ineffective or unintelligible, so I will simply give the relationist his designated family of relations'.[13] Sklar explicitly bypasses the main problem.

One might think that actually Sklar gives a good reason for refusing to hand over the family of relations. The realist is bluntly rebuffed, turned away empty handed. So it may seem. But Sklar's gift is a proposal to ignore the truthmaker problem, so as to get on with a secondary one – are relations among objects adequate for spatial (spatiotemporal) discourse? Sklar's great and rare merit is that he is candid and aware of this. He doesn't presuppose a relationist answer – he confesses that he has no answer. His gift reflects no worse on realist truthmakers for spatial relations than on any other. If the secondary question is merely about the discursive adequacy of a limited class of spatial relation statements, it does not deny realism. The adequacy of Sklar's gift is irrelevant; what matters is its nature. Mere secondary relationism risks being realism in denial.

Let's go back a bit. It is an entrenched belief that nothing intrinsic to an object tells us where it is, whether there are other objects anywhere, how distant it is from any other, whether it is between two other objects and so on. Spatial relations hold independently of anything intrinsic to things they relate. Spatial relations are not grounded in the natures of their terms. That this poses a problem is stark in the particular case of distance. It is a symmetric relation, but, unlike easy symmetric relations, it has nothing at all to do with the intrinsic properties of the things it relates. So what grounds this relation, and what grounds other, e.g. non-metrical relations – ternery ones like 'between', say?

Further, what grounds a spatial separation and connection of a thing to itself, as the North Pole is separated from and connected to itself via infinitely many distinct longitudinal geodesics? In Max Black's well-known counter example to the Identity of Indiscernibles[14], it is assumed that, because indiscernible iron spheres are at a spatial distance, the relation must be irreflexive, and then they are indeed two spheres. But that might fail. What makes them two is not just that they are in the same space, separated and connected, but that there is, for example, no array of distinct spatial relations that gives the space the global topology of S_3, not E_3. Neither the nature of any iron sphere nor the existence of numerous but unspecified spatial relations settles that.

If the entrenched beliefs are true, truthmakers are needed for spatial - and temporal - relations that include more than their terms. Formally, the sentence (where F is an intrinsic property):

$$\exists x\ Fx\ \&\ \exists x\ {\sim}Fx$$

entails that there are non-identical objects, but not that any relation independent of F holds between them. Nor does any necessary ontological principle ensure it. So, something more is demanded to ground spatial relations than the existence and intrinsic properties of the things. What?

The entrenched beliefs allow just two options. First, spatial relations might be definable in terms of other non-spatiotemporal relations grounded in their terms (the main object of Sklar's justified despair). Causal relations have been front runners in this race to explicate spatiality, a race that has never ended with a breasting of the tape, but only with a bogging down in one or another quagmire of the sort Sklar gloomily indicates. Second, they might be mediated relations, i.e. their complete truthmakers include something real but not a property, and thus independent of the terms and their natures. Obviously, if properties don't do the work, it is paths that spatially separate and connect things. Maybe this, too, is an object of Sklar's dismay. But, I will argue, that they are effective and intelligible.

The argument just given yields a minimal spatial realism. It casts entities in the role of mediators for spatial relations among things, entities that are not of the ontic type of things of physics or objects of perception. Thing-terms in spatial relations shed no light on spatiality: the mediator must do that.

The obvious entities may be described, pretty well synonymously, as paths, separators-and-connectors or spatial relators. Space is their fusion, but this omits much. "Path", like space and time themselves, is a primitive ideas - too familiar and basic to admit definition in other terms, yet clear and simple enough to be described. By contrast, it is no good making a primitive of something merely because it is a mystery – as Sklar explicitly does. Just as there may be scholia to definitions, telling us about the thing defined without defining it, primitives need description. Newton limned the essences of space and time, not in the section on definitions, but in its famous sequel, the Scholium. His essentialist style may repel, but his explication of primitives is exemplary.

"Path" is a familiar word and paths are familiar in concrete experience, separating and connecting things. We might see just two objects, exactly similar to perception: they are separated yet, since seen together, connected. True, we do not see separating, connecting paths; they aren't visual objects. Yet we do immediately see that there is a path. We need not see an occupant of it. We move along paths, look along and across them, trace them and so on. They are among the commonest entities encountered in concrete, immediate perception and action. For example, there is no pole star for the southern hemisphere, but there is a night-sky recipe for southing. Project the long axis of the Southern Cross. Where it intersects the perpendicular from the mid-point of the line joining the "pointer" stars, is the rotational south pole of the sky. It's an easy (straight) path-tracing recipe. No intervening stars are easily seen. Thus it seems best to say that they are concrete entities familiar in perception.

Nevertheless, paths are certainly a discursive problem. We are unsure how to talk about them; especially just how to describe their ontic type, obvious in perception though they may be. But if the argument so far is sound, it may be useful to hazard a preliminary description: paths in space (spacetime) are concrete particulars, immediate in perception, sovereign over and independent of matter, formally objects of reference and bearers of properties, but substantial in no richer sense. [15]

Perceptual immediacy does not entail infallibility. For example, we are poor judges of the distance from ourselves of two stars seen as separated and connected, so we misjudge the linking path's orientation. We see the stars as linked by a spatial rather than a spacetime path, since we don't see that the light from one is from an event much earlier than that from the other. What we trace are spacetime paths, linking point events perhaps widely separated and connected both in space and in time. But it is immediate, not inferred, that there are paths: they are not small, not hidden, not elusive, not invisible substances, not theoretical things. There is no need for a mere structural realism of paths. They are quite unlike the hidden entities of micro theories and their properties. They are visibly, tangibly there whenever we see things or move to grasp them.

That tells us so little about paths that it might baffle rather than enlighten: for instance, it does not entail that paths are one-dimensional or straight. I sketch a few plausible features to help domesticate *Modest Realism*.

To confine myself to what is plausibly metaphysics, I stick to mereological remarks.

A multitude of objects requires a multitude of paths separating and connecting pairs of things: some will pass through other objects, some not. Some have paths as proper parts. Paths, I suggest' intersect in paths (not, yet, points). Less modestly – *Bold Realism* if you like – some spatial relations hold between paths and things; others between paths and paths; perhaps all paths have paths as proper parts. Only here do we strike an issue about the adequacy for discourse of spatial relations just among things. These further steps to bold realism look plausible once we take the first. They add no ontic types or conceptual load to ontology, but only more paths. *Rash Realism* is the view that continuity and differentiability are among basic primitive properties of any spacetime. These postulates are pragmatic - there to legitimate the use of differential equations. There is no metaphysical reason to accept them, so none to accept points. None of these realisms presupposes a global topology, nor any metric. Only bold and rash realists quarrel with secondary relationists. Mediation is the core ontological difference between realists and primary relationists. It is modest realism that admits space to ontology as a first order entity, conceptually unique and primitive.

The present section's main conclusion is this: a metaphysical theory of spatiality entails that the properties of space and time are not founded on observables or their properties, but are path-mediated. Paths are not explicated though the nature of their occupants or terminants. Space is their fusion.

5 SR and matter theories

(i) The general structure of field theories

A field is a plenum, not a complex of spatial relations among things. Variable quantities, the values of some matter theory such as electromagnetism, are assigned continuously and smoothly, for classical fields, to points of spacetime.

Spacetime underlies the field and not vice versa. Fields presuppose rash realism. Continuity and smoothness are imposed on path separation and connection to yield points at which field values hold. We have no understanding how matter theories or field values could provide any of this structure. Every field presupposes it, each in the same way. A field structure of paths and points is independent of how the field is sourced physically, or of how complex its values (vectors, tensors) may be at points. Electromagnetic field values do not spread spacetime; spacetime spreads the field values. Our current concept of (non-abstract) field makes spacetime prior.

Quantum physics may change that. One gauge-theoretic approach to GR (General Relativity) would reduce spacetime curvature to a spin-2 particle (graviton) gauge-field. So far, this is more a hope of quantum gravity than an achievement.[16] Various attempts to place GR on a non-geometric (e.g. a bimetric)[17] footing are current but none has yet succeeded. Richer geometric structure with strings would supersede the present approach. But, in standard GR, as we find it in the mainstream chapters of *Gravitation*[18], spacetime, in the sense that Minkowski invented, is the principal entity. I am arguing for a realism that understands "spacetime" in GR as directly referential without a need to reparse it.

(ii) Electromagnetism

The impression may well remain that electromagnetism plays a dominant role in fixing the finitude and the specific value of the fundamental constant *c*. However, for many theories, the observed value of a main constant is not rooted in its structure. Only observation yields the values; they are brute. This is a strong claim, but it is directly acknowledged in the many 'fine tuning' arguments afloat. These note that many fundamental constants are improbably finely adjusted to each other in ways not demanded by the fundamentals of the theory. Without that tuning, life as we know it - indeed any life at all - would be impossible. Thus Paul Davies:

> In the present state of our knowledge the 20 odd parameters that appear in the standard model of particle physics seem to just be completely free, they're not determined by any underlying theory. But what is clear is that if some of them have values even a little bit different from those that they do, then there could be no life in the universe.[19]

So constants could be revalued without insult to the structure of the theories that give rise to them. A derivation of dimensional constants, such as *c*, looks very unlikely. (We need not suppose that constants are somehow arranged to give us an easy ride.)

This view of *c* in particular, is tacitly endorsed in many good popular versions of SR. For instance, Gamow's well-known early story[20] sets *c* at 10 miles per hour. This rewrites the covariant consequences of the theory on domestic scales without changing the structure of electromagnetism or mechanics in SR. For instance, *c* may still be defined as the ratio of the electromagnetic to the electrostatic unit of electricity. All the features of Gamow's story fit the general physics perfectly – it is electromagnetism, but slowed down.

Light moves at the constant invariant maximum speed because it is a zero mass particle; light doesn't determine what that speed is.

(iii) Mechanics.

Newton's first law underpins the relativity motion in its standard form: the laws of physics are invariant under transformation to inertial frames of reference (privileged frames). The law is:

> Every body persists in its state of rest or of moving uniformly straight ahead, except insofar as it is compelled to change its state by forces impressed.[21]

Already this broadly hints that not all of mechanics is matter theory. Remarkably, the law says nothing at all about the properties or causal powers of any body to which it might apply. Remarkably, too, we have good classical reason to think that no body ever actually escapes the (gravitational) causal net or persists in its state of rest or uniform motion (although there might be some bodies on which the resultant of forces is zero, briefly or not). The law is not about bodies; it is about trajectories, spatio-temporal entities, not about what might occupy them. It tells us nothing of how any such trajectory ever comes to be occupied or how an occupying body persists in its force-free state so as to be extended along it or, indeed, to persist at all. That is no part of the law's message. The second law requires that all bodies have mass; the first has no requirement. It demands only that causes are needed to drive a thing off its inertial trajectory - if the thing goes on existing.

The same is true for SR. If any body exemplifies the law, it remains at rest in an inertial frame. It remains, that is, in a state, for which explanation or cause is neither needed nor, in simplest cases, intelligible: nothing is done or suffered by things that simply stay put. That is the importance for dynamics of the non-causal case: the force-free trajectory with zero acceleration. Matter theories are disengaged − their forces and fields are inactive.

The first law really is first. It is conceptually simpler and theoretically deeper than the second. Once we can decide simply when forces are on or off, it identifies the frames of reference (candidate rest states) that are the foundation of the whole theory. To a large extent, Newton decided the issue of forces by seeing uniform motion as free from impressed forces (impacts, pushes and pulls) and gravity. This laid a crucial groundwork: candidate forces should have (i) observable sources, and (ii) regularities governing (a) when and (b) how they are at work. This pretty completely rules out arbitrary, conventional forces. Only when we have the right frames of reference and, by implication, the right transformation group, may we explore accelerations relative to them in a comprehensive way; only then can forces be quantified and oriented and the theory tested. You can apply the second law and verify that second derivatives are at the core of dynamics. That the second law entails the first, does not rob the first of first place.

There is no mystery about this, contrary to Brown[22]. It demands no conspiracy among objects to behave in certain ways. It is not even about objects or their properties. Any object that exemplifies the first law is at rest; that is, it has no behaviour beyond continuing to exist. It need 'know' nothing of the dynamical state of any other object[23].

Now for a quick glance into GR. Its analogue of the first law is that spacetime geodesics are force-free trajectories. Here it really is clear that the law is not about the occupants of these trajectories, nor about any of their properties. The fiction of a point mass allows us to say that the point may be taken as at rest without need to postulate any gravitational force in its local IF to keep it in place. It is in a free float/free fall state. But a strict point mass in GR would be a black hole! More significantly, GR does not characterize the inertial motion of bodies save in quite special geometrical cases. Bodies are extended and their spacelike cross sections are intersected by infinitely many distinct geodesics. In a curved region of spacetime, these geodesics mutually deviate. So worldlines of points within any body tend to move closer or apart in their inertial motions, despite the 4-acceleration on each geodesic being 0 everywhere. This deviation changes the distances of points from each other and thus the equilibrium forces between them. In general, a body's trajectory won't be geodesical, nor will that of any geometric point within it. Thus despite the formally interesting fact that the field equation entails GR's law of motion, there are no free fall trajectories of extended bodies save in special cases where the curvature is 0.

I conclude that mechanics does not imply or suggest that the structures of space, time and spacetime derive from the physics of matter.[24]

6 Conclusion

First, this is an essay in metaphysics, not physics. It's about a concept that plays a major role in GR. There it is a fundamental, highly articulated entity rich in explanatory consequences within the most powerful, elegant and intelligible theory ever to grace physics. GR may fail and probably will. If it fails so as to undermine the role of geometry in relativity, then curved spacetime, like Newton's absolute space, will be history. But, in the long run, we are all dead and everything is history. Till then, the ground-floor place of spacetime in present ontology and its clarity and its central role in one superb theory, has to be of fundamental metaphysical interest, well worth positive exploration rather than the reflex impulse to expel it.

Second, GR was the first theory of physics to be formulated presupposing only minimal structure for its background space. This is the merely differential manifold i.e. pure rash realism. Its weak structure is hospitable to a wide range of geometrically different models. Only separation/connection and smoothness are given in the manifold. Other geometric and matter (dynamic) properties are encoded in tensors (complexes of vectors). Thus the most basic properties of space and time, separation and connection of tensor locations, are clearly primitive postulates of GR. Smoothness and continuity get in, pragmatically, to justify points and the use of differential equations. There is no understanding how to encode these deepest features of spacetime from any tensor field envisaged in GR, else that would surely have been done. They are accessed in familiar experience.

Last, the symmetries to which Minkowski appealed are global in his spacetime. The spacetimes envisaged in the full range of GR models have those symmetries only locally. But, significantly, they hold in the tangent spaces common to each point in any model.

The metaphysics of spacetime deserves closer and more sympathetic attention than is usually given it.

---

[1] I am indebted in this paper to Angas Hurst, Peter Lavskis, Steve Leishman, John Mercier, Colin Mitchell, Chris Mortensen, John Norton, Peter Quigley and Peter Szekeres.

[2] Minkowski, H. "Space and Time" (1908) In Lorentz, H. A. et al. *The principle of relativity* Dover, New York (1923 pp.75-81). Numbers in brackets refer to pages in this paper.

[3] In the vast philosophical literature on the conventionality of simultaneity, the shade is deep indeed. That no signal outstrips light is a dominant theme, I believe a misleading one, pervasive through that work, from Reichenbach onwards. It did much to lend the discussion a spurious epistemological flavour. Conventionalism was the main focus for philosophy of relativity in the 60s and 70s. At the very least, the issue looks different within Minkowski's picture. The frequency of use of 'light cone' lightlike line' etc eclipses, 'null cone' and 'null line'.

[4] For a discussion of the transition from Einstein to Minkowski see John. D. Norton "General Covariance and the Foundations of General Relativity" § 2 *Reports on progress in Physics* **56**, 1993 pp 791-858.

[5] For an informal discussion with a reasonably accessible preface preceding a complete mathematical treatment see S. Cacciatori , V. Gorini and A. Kamenschick "Special relativity in the 21st. century" *arXiuv:08073009v1 [gr-qc] 18July 2008* (2008) pp. 1-40; Dyson Freeman. "Missed opportunities" *Bulletin of the American Mathematical Society* **78** (1972) 635-72 is helpful and informal. H. Bacry and J.–M. Levy-Leblond "Possible Kinematics" *Journal of Mathematical Physics* **9** (1968) 1605−1614 is a succinct and somewhat different analysis, accessible to those familiar with Lie groups and their algebras.

[6] The next section makes clear that Minkowski needed just the symmetries of Euclidean space and the reversibility of velocities.

[7] This defines the Carroll group. "A slow sort of country," said the Queen, "Now, here, you see, it takes all the running you can do, to keep in the same place." Lewis Carroll *Through the looking-glass, and what Alice found there* Macmillan, London (1871) p. 109

[8] J. R, Lucas and Peter Hodgson, P. *Spacetime and electromagnetism* Clarendon, Oxford (1990) pp. 310. §1.1.

[9] Dyson, F. op.cit. p. 642

[10] Dyson (op. cit.) suggests, and Cacciatori et al. (op. cit.) agree, that this line of thought might have been continued past the point where Minkowki left it while remaining purely mathematical. The Lorentz transformations operate at a spacetime point and are homogeneous at the point. The complete group that captures all the symmetries of Minkowski spacetime must include spatial rotations and the group of translations T_4. This defines the Poincaré group. But the inclusion of T_4 makes this group inhomogeneous (Cacciatori et al. (op. cit.) §1.) Mathematically a simple and obvious terminus is at the form of de Sitter spacetime. It contains a further constant, the cosmological constant, λ, its value again to be found observationally. λ did not figure in Einstein's original field equations because he wanted Minkowki spacetime as the mass-free solution of the field equations (Wolfgang Rindler, *Essential relativity: special, general, and cosmological* Van Nostrand Reinhold

Co., New York (1977) §14.5). So, mathematical simplicity yields a de Sitter spacetime which, even though it is matter free, not merely expands but accelerates its expansion (or decelerates it or alternates expansion and contraction). Matter perturbs its structure merely locally. Recent observational evidence suggests that this is close to the structure the universe has. (Clifton, T. and Fereira, P. "Does dark energy really exist?" *Scientific American* **300**, 48-55 (April 2009). Many physicists who fully accept Minkowski's conclusion, pause at the further move to de Sitter spacetime: it lacks the same motive and inevitability. I will not pursue the matter but refer readers to §1 of Cacciatori et al. op. cit.).

[11] This is usually said. But you also need a kinematic isotropy: if F_2 has velocity **v** relative to F_1, then F_1 has **−v** relative to F_2. See R. Torretti *Relativity and geometry Pergamon*, Oxford (1983) pp. 79-80. Feigenbaum, M. "The theory of relativity – Galileo's child" arXiv:class-ph/0806.1234v1 (2008) accessed 6 Jun 2008, notes that standard deductions assume for simplicity that the frames of reference are in standard configuration, thus sidestepping the need for a Wigner rotation (aka Thomas precession) if the relative motions are not all in the same plane. The full Lorentz group is not commutative.

[12] See, e.g.; Mermin, N. "Relativity without light" *American Journal of Physics* **52** (1984) 119-24; Sen, A. "How Galileo could have derived the special theory of relativity" *American Journal of Physics* **62** (1994) 157–16 (with diagrams); Feigenbaum, M. op. cit. For treatments of the deduction by philosophers see Richard T. W. Arthur "Time and Inertia" delivered at 2nd conference of Ontology of Spacetime Society, Montreal (2008); Brown, H. R. *Physical relativity* Clarendon, Oxford (2005) 105-109, M. Lange (forthcoming) "How to explain the Lorentz transformation" in Mind, Occasional Series, Mumford S. ed., Metaphysics of Science , Torretti op.cit. §3.7. Curiously, Pauli, W. *The theory of relativity* Pergamon, Oxford (1965) p. 11, mentions the Ignatowski approach, says that it should have been attainable simply by group considerations, but does not mention Minkowski.

[13] Sklar, Lawrence *Space, Time, and Spacetime.* University of California Press, Berkeley and Los Angeles (1974) pp.168-9

[14] Black, Max. "Identity of Indiscernibles" *Mind* **61**(242) (1952) pp. 153-164

[15] See G. Nerlich *The Shape of Space* Cambridge University Press 2nd edn. 1994 Chapter 1.

[16] See R. Mills "Gauge Fields" *American Journal of Physics* 57 (6). June 1989 pp. 493-507

[17] Brown op. cit. §9.5.2. claims that the bimetric solutions are not geometric, although he notes that Beckenstein, J. "Relativistic gravitation theory for the MOND paradigm" arXiv:astro-ph/0403694v2 (2004) refers to the geometric part of the action.

[18] Misner, C. et al. *Gravitation* W. H. Freeman, San Francisco (1973)

[19] Davies, Paul et al. in "The Anthropic Principle" The Science Show, Australian Broadcasting Commission, Radio National (2006)

[20] Gamow, G. *Mr Thompkins in wonderland: or, stories of c, G, and h* Cambridge University Press (1957)

[21] Newton, Isaac *The Principia: Mathematical Principles of Natural Philosophy* (Trans. I. Bernard Cohen and A. Whitman) Berkeley, University of California Press (1999) p. 416

[22] Brown, H. op. cit. §8.3.2 'Mystery of mysteries'

[23] See g. Nerlich "Why spacetime is not a hidden cause' Phil Sci Archive 2010 for a fuller account.

[24] For an interesting discussion of the prominence of spacetime metric in the foundations of GR see D. Lehmkuhl "Maas-energy-momentum: only there because of spacetime?" Phil-Sci Archive 2010