Emergence and Singular Limits

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Abstract

Recent work by Robert Batterman and Alexander Rueger has brought attention to cases in physics in which governing laws at the base level "break down" and singular limit relations obtain between base- and upper-level theories. As a result, they claim, these are cases with emergent upper-level properties. This paper contends that this inference—from singular limits to explanatory failure, novelty or irreducibility, and then to emergence—is mistaken. The van der Pol nonlinear oscillator is used to show that there can be a full explanation of upper-level properties entirely in base-level terms even when singular limits are present. Whether upper-level properties are emergent depends not on the presence of a singular limit but rather on details of the ampliative approximation methods used. The paper suggests that focusing on explanatory deficiency at the base level is key to understanding emergence in physics.

1. Introduction

Relations between phenomena at different levels in physics have been much discussed recently by philosophers of science. Particular attention has been paid to the question of under what conditions properties or behaviours described by upper-level (e.g., macroscopic) theory may be said to be *emergent* relative to a base-level (e.g., microscopic) account. Robert Batterman and Alexander Rueger have taken what seems the most sensible approach to this problem. They engage in detail with the physical theories about which physicists and philosophers of science make emergence claims. They then bring results of this engagement directly to bear on the issue of intertheoretic relations in philosophy of science. One of their most important results is the recognition that emergence claims are often made where governing laws at the base level, which work well in other physical systems, “break down,” indicated by the presence of a singular limit relation. Another result is that emergence is best understood as an epistemic failure, and more specifically a failure of reductive or explanatory practices that, again, work so well elsewhere. This contrasts with approaches that take emergence to be a matter of conceptual or ontological novelty, a contrast we shall return to below.

Batterman and Rueger aim to give precise meaning to the concept of emergence in physics and to show that emergent properties are far more widespread than is usually assumed. Crucial to their argument is the association of emergence with the presence of a singular limit relation between basal and upper-level theories (Rueger 2000a; 2000b; 2001; 2004; 2006; Batterman 1997; 2002a; 2002b; 2005a; 2005b; 2009). As Batterman puts it, "what is essential is the singular nature of the limiting relations between the 'base' theory and the theory describing the emergents" (Batterman 2002b, 6). His central claim about emergence is that “the epistemic aspects of emergence—tenets 3 and 4 [*i.e.*, “the unpredictability of emergent properties” and “the unexplainability/irreducibility of emergent properties”]—result from a failure of the reductive schema (6.1) [*i.e.*, the presence of a singular limit relation between theories]” (Batterman 2002b, 126). Similarly, Rueger claims that "*every singular perturbation problem implies a…* transition to ‘novel’ [emergent] behaviour," (Rueger 2000a, 308). The idea is that emergence—a methodological concept related to the novelty, irreducibility and/or unexplainability of upper-level properties in basal terms—is due to the occurrence of a singular limit. Batterman and Rueger emphasize that it is not that emergent properties are unexplained; it is just that they cannot be explained exclusively in terms of the base-level theory. An adequate explanation of an emergent phenomenon necessarily invokes the terms and structures of upper-level theory.

This paper contends that the connection between singular limits and emergent phenomena is not as close as it has been taken to be. As we shall see, the Batterman/Rueger type inference—from singular limits to explanatory failure, novelty or irreducibility, and then to emergence—is mistaken. The breakdown of base-level approximations, and the consequent presence of a singular limit, is not sufficient for emergence, as follows. The paper describes a simple case, that of the van der Pol nonlinear oscillator, in which regular approximation schemes break down and singular limits are used (Section 3). Rueger claims that this system exhibits emergent properties. Attention to the details of the case shows that there is, in fact, a full explanation of the putatively emergent upper-level properties entirely in base-level terms (Section 4). What of the other two commonly invoked criteria for emergence, novelty and irreducibility? Section 5 shows that these criteria, as elaborated by Batterman and Rueger, do not provide an independent basis for claims about emergence. Rather, whether upper-level properties in a singular limit system are emergent depends sensitively on details of the ampliative approximation methods used to construct upper-level theory. The paper suggests that explanatory deficiency at the base level is key to emergence (Section 6). First, however, it will be helpful briefly to place the argument of this paper in the context of criticisms of the singular limits approach that have received some attention in the literature.

2. Asymptotic explanation

A key feature of the models of emergent phenomena of interest to Batterman and Rueger is their structural stability, stability under perturbations at the base level, a feature physicists call *universality*. These structural features are revealed through analyses in which base-level details are systematically eliminated using asymptotic mathematical techniques. What explains the common characteristic shape of droplets at breakup, as when water drops fall from a dripping faucet?

> We can explain and understand (for large scales) why a given drop shape at breakup occurs and why it is to be expected. The answer depends essentially upon an appeal to the existence of a genuine singularity developing in the equations of motion in a finite time. It is because of this singularity that there is a decoupling of the breakup behaviour (characterized by the scaling solution) from the larger length scales such as those of the faucet diameter. Without a singularity, there is no scaling or similarity solution. Thus, the virtue of the hydrodynamic singularity is that it allows for the explanation of such universal behaviour. The very break-down of the continuum equations enables us to provide an explanation of universality (Batterman 2009, 442-443).

This is analogous to the way in which renormalization group techniques are used to analyze thermodynamic systems at critical points and to derive structural features such as the critical exponent β (Batterman 2002b, pp. 37-42; Batterman 2005a). In this way, asymptotic analyses enable idealized models to explain underlying structural or universal features in cases where these features are not explained at a more fundamental level. Batterman has dubbed these "asymptotic explanations" (Batterman 2002b, Ch. 4).

Some have raised concerns about this putatively new type of scientific explanation. Gordon Belot has objected that upper-level theories have only heuristic value, and they can in principle be eliminated in scientific explanation (Belot 2005). The mathematics of the upper-level theories are definable in terms of base-level theories, so in these singular limit cases all results can be explained exclusively in base-level terms. Belot and Michael Redhead (Redhead 2004) object to the further claim that upper-level theories themselves may be explanatorily adequate. Invoking upper-level theory as part of the explanans requires us to reify its mathematical structure, they argue, but upper-level theories are themselves approximations or idealizations, and so false. Upper-level theories are best thought of as superseded and incorrect descriptions of the world with continuing heuristic value. They can play no part in the true premises needed in acceptable deductive-nomological explanation.

Batterman has responded that in order to derive upper-level theory from base-level theory, elements are required that are describable only using upper-level terms and concepts (Batterman 2005b). These elements connect concepts between theories, they establish a correspondence of base-level and upper-level properties, and they enable us to describe the appropriate initial and boundary conditions for the base-level problem. Elements of the derivation from base-level theory are themselves theory-laden, and the theory they are laden with is upper-level theory. The explanatory adequacy of base-level theory in these cases is, according to Batterman, simply illusory; its explanations break down without contributions from upper-level theory. This is particularly true for explanations of patterns or regularities at the upper level. Batterman agrees that upper-level theories are idealizations and that idealizations can play no part in D-N explanation, but argues that this is indicative of a problem with the D-N account of explanation and not with the explanations themselves. Upper-level theory functions in novel asymptotic explanations that forgo traditional D-N requirements about the truth of the premises in the explanans or deductive derivation of the explanandum from the explanans. Thus we can have explanation in terms of an idealized upper-level theory without reifying the theory by demanding that it be a true description of the world.

The present paper is not concerned with whether Batterman is right with respect to the detailed case studies that are his main focus: drop formation in hydrodynamics, critical phenomena in thermodynamics, catastrophe optics, and the semi-classical limit of quantum mechanics. In each instance Batterman argues that concepts and structures of base-level theory are inadequate to produce derivations and explanations of upper-level phenomena. In his analysis of the rainbow in catastrophe optics, for instance, wave optics becomes singular (wavelength $\lambda \rightarrow 0$) and breaks down at the boundary between base-level wave theory and upper-level ray theory (Batterman 2002b, Ch. 6). Yet this singularity at the boundary is also responsible for the rainbow phenomenon. In this singular regime, Batterman claims, characterizing boundary conditions and associating them with relevant physical details (the shape of the raindrop, its reflective and refractive properties) necessarily goes beyond base-level explanatory resources. In this case, as in his others, Batterman's claims are plausible. It seems, however, that further work may need to be done on the details of Batterman's examples to determine whether his key premise is correct in each case: does derivation of the upper-level phenomenon *ineliminably* require appeal to upper-level theory?

Nor is the present paper concerned with whether, if Batterman is right, we are left with any explanation at all of the rainbow or other putatively emergent phenomena. The idea that highly idealized models may underwrite bona fide scientific explanations goes against orthodox views of scientific explanation. Physicists seem to appeal to idealizations in their explanations, but we lack an account that makes sense of this practice—or even a clear view about whether these idealized accounts are intended to be part of *bona fide* explanations. What is needed is a well-developed theory of scientific explanation that requires neither truth of the theories being appealed to nor deductive derivation in the explanation. If Batterman is right, some highly idealized models in physics—even extreme "asymptotic" idealizations—may have genuine explanatory power. If he is wrong, "asymptotic explanation" is not bona fide explanation at all and emergent phenomena, specifically structural regularities, simply remain unexplained. Rather than supplementing base-level theory with upper-level structures, as Batterman proposes, perhaps it would be more fruitful to look in the other direction for scientific explanation. It may be that any satisfactory explanation can only come from a novel sub-base-level theory that explains the base-level theory, its "breakdown" in the asymptotic regime *and* the upper-level phenomena previously thought to be emergent. Clearly, further work on the role of idealization in scientific explanation is needed.

Rather, the present paper is concerned with sharpening our account of emergence and intertheoretic relations by focusing on the role of explanation therein. To start, a clarification may be helpful. As we have seen, Rueger and Batterman make claims about the breakdown of theories, as in Batterman's recent assertions that "the governing laws 'breakdown'" (Batterman 2009, 432) and there is a "break-down of the continuum equations" of hydrodynamics in the drop formation (quoted above). Such talk may be misleading. It is not the case that the laws of nature stop working in these systems. Nor are our theories about these systems shown to be false; neither the continuum equations of hydrodynamics nor wave optics are falsified in the singular limit cases. Where such a "breakdown" occurs, it would be more accurate to say that certain procedures to derive observational consequences (such as regular approximation techniques), which worked effectively in other contexts, cease to work. For example, the procedures may yield infinities for what are observed to be finite values, such as the intensity of light in the rainbow. These cases are outside the regular domain of applicability of the theories, and this necessitates the use of other procedures, typically involving singular limits, asymptotic expansions and ampliative inferences.

3. The van der Pol oscillator

The one-dimensional van der Pol oscillator was originally investigated as a model of the human heart (van der Pol and van der Mark 1928), and it has been used subsequently to describe some oscillatory vacuum tube and electronic circuits. The oscillator is a classic example of singular limit problem. Rueger uses this example to show the connection between singular limits and emergent properties. Rueger shows that the van der Pol oscillator exhibits properties at distinct levels, that there is a singular limit relation between the levels, and that some upper-level properties are qualitatively different from base-level properties. He concludes that these upper-level properties are emergent (Rueger 2000a, Rueger 2001). We shall see that this last claim is mistaken, and more generally that the breakdown of base-level approximations and presence of singular limits are not sufficient for emergence. What the van der Pol example shows is that there may be full explanation of upper-level properties in basal terms even with the presence of a singular limit. And as I shall argue, a full basal explanation of upper-level properties precludes counting such properties as emergent. This section describes the relevant phenomena before moving on to a brief theoretical treatment.

The upper diagram of Figure 1 shows a typical phase space diagram of an undamped or simple harmonic oscillator. The horizontal axis is the oscillation variable and the vertical axis is its time derivative. The lower diagram of Figure 1 illustrates a phase space trajectory of a weakly-damped van der Pol oscillator, a typical trajectory for a self-excited nonlinear oscillator. The trajectory begins where the line begins closest to the centre of the diagram, and it converges by spiralling outward to the heavy line. Unlike in the simple harmonic oscillator, the oscillation amplitude is not constant but varies slowly relative to the period of oscillation. Experimenting with this system one finds that whatever initial non-zero value of the amplitude, over time it converges to the stationary amplitude indicated by the heavy line. Notice that the phase-space portrait of the short-timescale behaviour of the van der Pol oscillator looks like the ellipse in the upper diagram. Indeed, for any short-timescale observation of the van der Pol oscillator, of the order of a small number of oscillations, the dominant behaviour will be near-harmonic oscillation. By contrast, over the long timescale, of the order of convergence to the limit cycle, the behaviour is dominated by the rate of change of amplitude. These are two very different sorts of behaviour, suggesting two different models will be appropriate to represent the behaviours of interest in this nonlinear system.

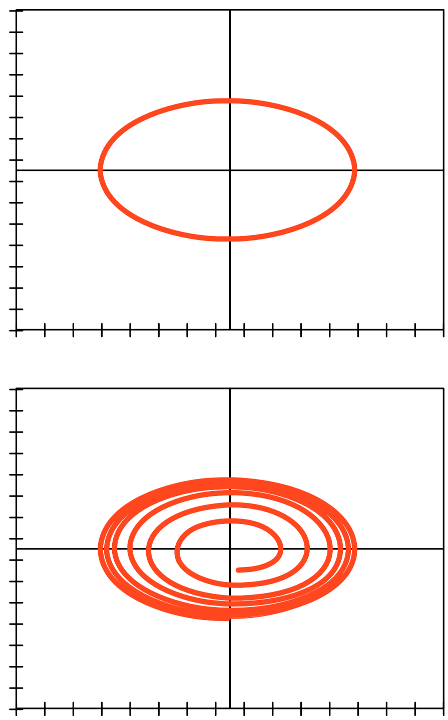

Figure 1. Typical phase space diagrams. Upper: a simple harmonic oscillator traces an ellipse through its initial point. Lower: from any initial point in phase space except the origin, a van der Pol oscillator converges to the unique ellipse indicated by the outer edge of the heavy line (for diagrammatic clarity the rate of convergence has been exaggerated).

Figure 2 presents a schematic view of these two sorts of models. At the base or microscopic level, short-timescale behaviour is simplified by treating it linearly as a simple harmonic oscillator. Long-timescale models are obtained by reducing the temporal resolution or coarse graining, resulting in a linear model of the change in amplitude over time and ignoring the harmonic motion entirely. The two families of models describe distinct aspects of the same physical system. Indeed, this process of modelling simple, linear behaviours of interest in complex nonlinear systems is typical, and physics is full of families of models at different levels such as these. Models of statistical mechanics treat fluids as composed of interacting point particles, while, at a larger spatial scale, hydrodynamics treats the same system as composed of a continuous fluid. It must be emphasized that not all divisions between models occur along spatial scales. The modelling technique in the effective field theory program in quantum field theory uses distinct models for distinct energy scales. And in the kind of models of interest here, base-level models describe short timescale behaviours while upper-level models describe long timescale behaviours. The short-timescale and long-timescale models are related, of course. But how? One might be inclined to think that a theory of the long-timescale behaviour reduces to a theory of the short-timescale behaviour. This is a question about intertheoretic relations, and answering it requires a theoretical account, to which we now turn.

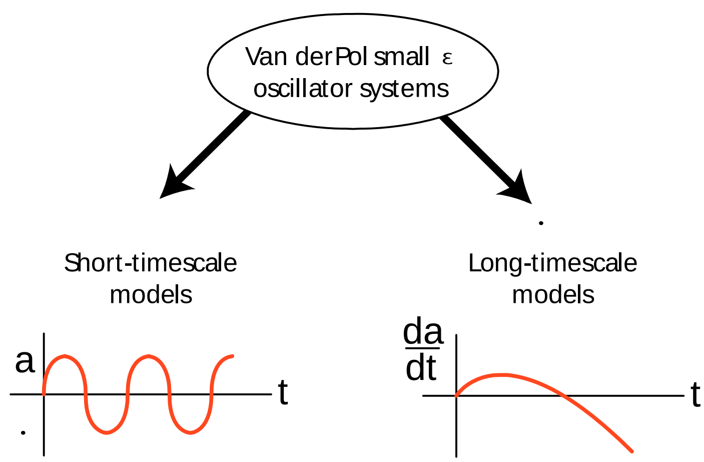

Figure 2. Phenomenological models of the van der Pol oscillator

The van der Pol oscillator is exactly described by van der Pol's equation

(1) $$y'' - \varepsilon(1 - y^2)y' + y = 0\,,$$

where $y(t)$ is the oscillation variable, $y' = dy/dt$, $y'' = d^2y/dt^2$ and ε is a parameter related to the strength of the damping. As is characteristic of a self-excited oscillator, this is a system with non-linear friction. For $y < 1$ the "$-\varepsilon y$" term dominates so there is negative damping (so-called "negative friction"), and this increases the amplitude of oscillation. For $y > 1$, the "$+\varepsilon y$" term dominates and oscillations are damped. Eq. (1) is of the form

$$(2) \qquad y'' - \varepsilon F(y, y') + y = 0 \qquad 0 < \varepsilon << 1,$$

where F is a nonlinear polynomial. This nonlinearity means that the oscillator equation cannot be solved exactly, nor, in general, can it be solved using approximation techniques involving regular limits, such as regular perturbation methods. Over a long timescale, equations of the form (2) "break down," in the sense that they (plus regular perturbation methods) fail to yield results anywhere close to the observations. Nonetheless, the behaviour of the van der Pol oscillator over the long timescale can be derived to an arbitrary degree of accuracy using any one of several singular perturbation techniques.

The Krylov-Bogoliubov-Mitropolsky (KBM) method, for instance, can be used to determine a solution of any differential equation of form (2) to an arbitrary degree of approximation. The method involves one substantive assumption about the solution: the amplitude and phase of the solution vary slowly, if at all, with respect to the period of oscillation. Thus, the KBM method assumes a solution of the form

$$(3) \qquad y = a\cos\psi + \varepsilon u_1(a,\psi) + \varepsilon^2 u_2(a,\psi) + \ldots$$

where the u_i terms are periodic functions of ψ with period 2π, a is the amplitude of the first fundamental harmonic (the fast oscillation) as a function of time, and ψ is the frequency of the first fundamental harmonic as a function of time. The time derivative of a and ψ can be written

(4) $$a' = \varepsilon A_1(a) + \varepsilon^2 A_2(a) + \ldots$$

and

(5) $$\psi' = 1 + \varepsilon B_1(a) + \varepsilon^2 B_2(a) + \ldots$$

The functions u_i, A_i and B_i are chosen so that when the expressions for a and ψ obtained from integrating eqs. (4) and (5) are substituted into eq. (3), it becomes a solution of eq. (2). So the problem of finding a solution to eq. (2) reduces to the more tractable problem of solving eqs. (4) and (5). In practice, eqs. (3), (4) and (5) are limited to a finite number of terms, and in many applications, including the van der Pol oscillator, consideration of terms up to order ε^2 is sufficient. Curiously, the success of the KBM method does not depend on whether the infinite series eqs. (3), (4) and (5) converge, but rather on their asymptotic properties for a fixed finite order in the limit $\varepsilon \rightarrow 0$. The KBM method provides a general mathematical procedure to determine the functions u_i, A_i and B_i to any finite order for any $F(y, y')$ in eq. (2) because the finite analogues of the series (3), (4) and (5) are increasingly accurate approximations as $\varepsilon \rightarrow 0$ and as the number of terms in the series increase.

The KBM method to order ε^2 can be used to derive limit cycle properties. The full second-order solution is complicated, but properties of interest can be investigated by looking at the variation of amplitude over time

(6) $$a' = \left(\frac{\varepsilon a}{2}\right)\left(1 - \frac{a^2}{4}\right).$$

Figure 3 shows a graph of eq. (6), rate of change of amplitude da/dt versus amplitude a, and it illustrates several of the novel upper-level features of the van der Pol oscillator. The curve approximates the observed curve in the right-hand diagram of Figure 2. It is worth repeating that, as is typical of singular problems, in the van der Pol case the solution includes ampliative steps, that is, steps in which additional substantive information is required beyond the basic equation of motion and initial and boundary conditions. We might be inclined to say that the limit cycle solution is *contained in* the van der Pol equation although not *deducible from* it.

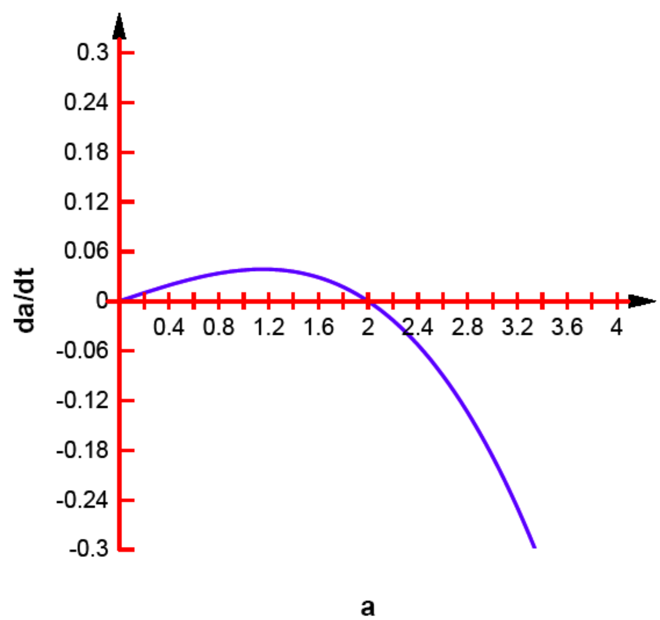

Figure 3. Variation in amplitude to order ε^2 for the van der Pol oscillator

4. Singular limits and basal explainability

Batterman and Rueger claim that in singular limit cases, upper-level phenomena cannot be explained exclusively in base-level terms. Their examples, from dynamical systems theory, condensed matter physics, statistical mechanics and quantum mechanics, give the impression that somehow the singular limit is responsible for the failure of basal explanation. Batterman does not put forward a definite account of basal explanation, but he focuses on cases in which it is plausible that basal explanation fails. As we have seen his main concern is slightly different, namely to characterize asymptotic explanation at the upper level. Rueger has a clear account of the relevant sort of what he calls “reductive explanation”: for an upper-level theory $T_{\varepsilon=0}$ and a base-level theory T_ε, “T_ε explains $T_{\varepsilon=0}$ *because* lim $T_\varepsilon = T_{\varepsilon=0}$” (Rueger 2001, 506, my notation). Basal explanation is identified with a regular limit relation, so by definition basal explanation is impossible in any singular limit case. Clearly, this is far too quick.

A more careful look at the van der Pol oscillator shows that the presence of a singular limit does not imply explanatory failure at the base level. On the contrary: while the singular limit indicates a breakdown of regular approximation techniques, and while it does preclude deductive derivation of upper-level phenomena from base-level theory, *this does not mean that upper-level concepts and structures must be invoked to fill the gap*. Rather, as we have seen, the gap may very well be filled by additional conditions that are themselves entirely characterizable in base-level terms. The additional substantive assumption that goes into solving the van der Pol case is that amplitude and phase do not vary quickly relative to the period of oscillation. The key point is that this assumption is characterized exclusively in basal terms. To say that amplitude and phase vary slowly does not require concepts or structures foreign to base-level theory, that is, concepts or structures imported from another (e.g., upper-level) theory. This is not to say that base-level theory somehow contains this assumption. The base-level theory is of a simple harmonic oscillator with fixed amplitude and phase, and this is inconsistent with the assumption of slowly varying amplitude and phase. Rather, the claim here is that the assumption can be articulated using concepts exclusively at the base level: harmonic oscillation, amplitude and phase. The novel upper-level properties of the van der Pol oscillator—limit cycles, their precise structure and their stability—are derived and explained in terms of the base-level simple harmonic oscillator, an initial condition (a starting point in phase space), the assumption of slowly-varying amplitude and phase, and the techniques of asymptotic analysis sketched in the previous section. None of these explanatory resources makes reference to upper-level structures or any other physical concepts beyond those characterizable at base level. Thus the novel upper-level properties of the van der Pol oscillator are entirely basally explainable—indeed, they are basally explained.

One might object, as Patrick McGivern does, that we do not truly have an explanation entirely in base-level terms (McGivern 2007). The asymptotic KBM method appears to derive long-timescale behaviour from base-level considerations, McGivern notes, "[b]ut arriving at that solution in the first place is not possible without [first] distinguishing between the system's two characteristic timescales" (McGivern 2007, 6). The explanation of upper-level limit cycles begins with the assumption that there is some phenomenon involving change in amplitude that can only be characterized over the long timescale, he claims.

> In terms of the distinction between 'levels' suggested by the perturbation technique itself – the fast and slow timescales – basal explainability seems to fail since predicting the limit cycle behaviour involves explicitly recognizing the different timescales characterizing the system (McGivern 2007, 7).

But it is *not* the perturbation technique that suggests the division into base and upper levels. Rather, as we have seen, that division is a consequence of a phenomenological investigation of the system, our interest in the novel limit cycles and our observations of various behaviours of the system under a range of conditions. The characterization of base-level models is predicated on recognizing different behaviours of interest at distinct timescales. Prior recognition of distinct levels in the van der Pol case is a typical—I would say ineliminable—element of inter-level scientific explanation. For example, a paradigm case of a non-emergent property is the total (macro) force on a non-relativistic classical object. A satisfactory basal explanation of the total force on the object appeals to facts about the base-level (micro) forces and a vector addition law for forces that itself presupposes the micro-macro distinction. The explanatory resources, nonetheless, can be characterized in base-level terms, and clearly macro forces reduce to, are predicted from, and are explained by micro forces. If prior recognition of distinct levels vitiates basal explanation, as McGivern suggests, then no upper-level property is basally explainable. What is needed, rather, is a distinction between cases in which upper-level resources are required for the explanation and those in which they are not, a distinction which depends sensitively on the details of the ampliative approximation methods used to construct upper-level model.

To see this last point another way, it will be instructive to contrast the van der Pol case with one of Batterman's paradigm cases of the failure of basal explainability mentioned above, rainbows (Batterman 2002b, 77-97). Rainbows are described by the asymptotic limit of wave optics (base theory) in a singular asymptotic regime as wavelength goes to zero. The theory of rainbows uses geometrical optics (upper-level theory). This boundary regime, where base theory breaks down, is only characterizable in upper-level theory terms (Batterman 2005a, 161). In the rainbow case, setting up specific initial and boundary conditions that correspond to this limit case, and that correspond to the specific phenomenon of the rainbow, ineliminably involves reference to concepts of geometrical optics (upper-level theory), Batterman claims. Hence rainbows are not explainable in base terms.

Granted that both limit cycles and rainbows are upper-level phenomena whose derivations involve singular limit relations; that in these kinds of cases, upper-level properties cannot be derived and explained exclusively on the basis of basal theory plus purely deductive perturbation methods (*i.e.*, regular perturbation analysis); and that additional substantive assumptions about the system must be made. These are important considerations. It is just that they are less relevant to the issue of explanation, and ultimately emergence, than proponents of the singular limits approach take them to be. Whether an upper-level phenomenon can be explained in basal terms depends, rather, on the particular details of the explanatory resources brought to bear after breakdown at the singular limit, including initial conditions, boundary conditions and empirical premises in the asymptotic methods used at singularity. In the van der Pol case, the additional assumptions are completely characterizable in basal terms, and we have seen how limit cycles are, in fact, basally explained. By contrast, in the rainbow case some of the additional assumptions needed to explain the upper-level phenomena themselves are (if Batterman is right) only characterizable in upper-level terms.

5. Novelty and reduction

We have seen that the presence of a singular limit does not imply explanatory failure at the base level. But explanatory failure is not the only criterion regularly invoked in discussions of emergence. Batterman and Rueger follow common practice in also claiming that the *novelty* and *irreducibility* of upper-level properties are sufficient for emergence. However, Batterman and Rueger's accounts of novelty and irreducibility are closely linked with the presence of a singular limit, and they provide no independent support for the claim that the presence of a singular limit is an adequate criterion for emergence.

It is no easy task to make precise the intuitive idea that certain properties described by upper-level theory are truly novel with respect to base-level theory, whereas other properties are merely different. Rueger begins with a distinction between "quantitative" and "qualitative" differences between upper- and base-level properties (Rueger 2000a). In the case of the van der Pol oscillator, although its short-timescale behaviour is quantitatively close to that of an undamped oscillator, the limit-cycle behaviour over a long timescale is qualitatively different from the undamped case. Rueger makes the notion of a qualitative difference precise in terms of the topological inequivalence of the phase-space trajectories of the undamped and van der Pol oscillators (Rueger 2000a, 303). But why should topological inequivalence be sufficient for qualitative difference, and thus for novelty and emergence? Because topological inequivalence captures the singular limit relation, according to Rueger.

> The problem we encountered in the cases of singular limits now can be phrased as the lack of limits of such [topologically equivalent] sequences of models of [base-level theory] which are themselves models of [base-level theory].... Such a change in the topology is reflected, in our examples, in giving up on the requirement of uniform convergence of perturbation expansions in order to accommodate the singular limit cases (Rueger 2001, 517-8).

Batterman reaches a similar conclusion. "Novelty," he notes, is commonly used, especially in the context of physics, to describe phenomena that are unexplainable or irreducible and so does not provide an independent criterion for emergence. The one notable exception is Jaegwon Kim's account of emergence in terms of the novel causal powers of the upper-level properties (Kim 1999). As Batterman argues, however, the expansion of the notion of levels to include scales and other non-spatial relations, along with a general lack of causal models of systems of interest, makes this criterion largely inapplicable in physics (Batterman 2002b, 127). Batterman suggests that the concept of novelty is best understood within physics in terms of properties playing novel explanatory roles: "novel" properties are those that cannot be explained exclusively in base-level terms. Batterman concludes: "*the novelty of the emergent phenomena I am discussing is a direct result of the singular nature of the correspondence relations between the two theories*" (Batterman 2002b, 120).

With respect to reduction, Rueger asserts that the upper-level limit-cycle behaviour in the van der Pol oscillator is irreducible to the base undamped harmonic oscillator because of the presence of a singular limit (Rueger 2000a), and Batterman makes similar claims about the cases he studies. On their approach, reductive failure is simply identified with the presence of a singular limit. A base-level theory T_ε reduces to a coarser upper-level theory $T_{\varepsilon=0}$ in the limit as parameter ε tends to zero (Rueger 2000a, 305; Batterman 2002b, 78; Nickles 1973),

(7) $$\lim_{\varepsilon \to 0} T_\varepsilon = T_{\varepsilon=0} .$$

In this way, for example, the special theory of relativity (T_ε) reduces to Newtonian mechanics ($T_{\varepsilon=0}$) as $(v/c)^2 \rightarrow 0$. Sometimes dubbed the "physicists' notion of reduction," on this approach base-level theories reduce to upper-level theories through a process of coarse-graining using a regular limit procedure (this is opposite in direction to most philosophical notions of reduction, in which the upper-level theory reduces to base-level theory). In the van der Pol case, base-level theory fails to reduce to upper-level theory; as we have seen, the theory of time-dependent amplitude and convergence to limit cycles cannot be derived from base theory using a regular limit relation. Clearly, however, this sort of reductive failure provides no independent support for an emergence claim beyond the presence of the singular limit relation itself.

It is tempting to respond that the physicists' notion of reduction fails to capture our intuitions about reduction and is simply too weak to provide an independent criterion for emergence; a more robust account of reductive failure would surely fit the bill. This line of thinking should be resisted, I submit: reductive failure does not give any better criterion for emergence on the main philosophical approaches to reduction. Orthodox thinking about reduction in physics is based on an approach going back to Nagel (Nagel 1961; cf. Schaffner 1976, Schaffner 2008). Here, reduction is a derivational relation between upper-level and base-level theories. Where upper-level theory has terms not already in the base-level one, the terms must be connected using bridge laws, which are universally quantified conditional or bi-conditional sentences. One of the problems with derivational approaches to reduction is that failure can always be remedied in an *ad hoc* way, as the British emergentist C.D. Broad well recognized (Broad 1925). Broad's paradigm case of an emergent phenomenon was chemical composition. He recognized that it is possible to gerrymander a "law of composition" that oxygen combines with hydrogen in fixed proportions to form water with a given set of properties, such as liquidity. But this "law" relies on upper-level information specific to this case—facts about the very upper-level phenomena the "law" is supposed to cover. The "law" is not general and it is gerrymandered *post hoc*, so properties of water are emergent, not resultant. Chemical compounds more generally, Broad believed, require new "laws of composition" for each particular compound, and it is for this reason that he took their properties to be emergent (this was prior to the development of quantum mechanics).

The obvious response is to introduce constraints on bridge laws that preclude gerrymandered or case-specific laws. This can be done in such a way as to make it plausible that all and only Nagel-irreducible upper-level properties are emergent. The important point for our purposes is that all the work relevant to the concept of emergence is being done by these additional constraints. For example, one could introduce the condition that bridge laws used in reductions must themselves be fully explainable in base-level terms. The result would be a notion of reduction that matches a plausible notion of emergence. This fails to restore any substantive link between reduction and emergence, however, because a condition has been introduced into the definition of reduction simply to achieve this match. Without such additional emergence-related conditions, I suggest, the success or failure of Nagel-type reduction has no implications for emergence.

On the main alternative to Nagelian reduction developed by Jaegwon Kim, an upper-level property is emergent only if it is not functionally reducible to basal properties (Kim 1999; 2005; 2006). Kim's functional reduction requires that one find a base-level (micro) property that fulfills the causal role of the upper- level (macro) property to be reduced. The account presumes that the causal powers of the macro and micro properties are identical—and so the properties are identical—as the first step towards functionalizing and thus reducing the macro property. In physics, however, micro-based macro properties fulfilling these macro causal roles are generally not to be found, and so Kim's functional reductions fail to get off the ground (Rueger 2006). As well, Kim has focused on spatial relations between levels, and as we have seen, the expansion of the notion of levels in the context of physics to include energy scales, timescales and other non-spatial relations makes the causal criterion largely inapplicable. Here again, failure of reduction, in this case of functional reduction, gives us no good reason to infer that macro properties are emergent. On the contrary, in some fields of physics virtually no macro property can be functionally reduced, and we surely do not want to say of these fields that virtually all of their macro properties are emergent.

6. To BE or not to BE, that is the question

Batterman and Rueger acknowledge that emergent properties are associated with cases in which base-level theory is, in Batterman's term, "explanatorily deficient" (Batterman 2002b, 109). They claim that the explanatory deficiency of the upper level is a consequence of the presence of a singular limit relation. This paper shows this not to be the case; a singular limit does not imply the failure of basal explanation. The moral of the story is that whether a particular property is emergent depends not just on the breakdown of a regular approximation scheme, but more devilishly on the finer details of the explanation. The presence of a singular limit is less relevant to the issue of intertheoretic relations than Batterman and Rueger have taken it to be.

To basally explain or not to basally explain, that is the question. I suggest that failure of basal explanation is the key to a useful account of emergence in physics (this suggestion is developed in Wayne and Arciszewski 2009). For one thing, it makes sense of intuitions about the differences between the van der Pol oscillator and Batterman cases. It seems reasonable to think that upper-level phenomena in the van der Pol oscillator and similar nonlinear dynamical systems are not emergent, and this is the unanimous opinion of physicists with whom I have discussed the case. This contrasts markedly with views on Batterman's rainbow and critical phenomena cases, where physicists' opinions were diverse but many were willing to consider these as interesting and open questions about emergence. What motivates this contrast between the van der Pol case and seemingly more robust candidates for emergence, I am inclined to think, is that in the former we have a complete basal explanation and in the latter, well, we are not so sure.

What I began by calling the "sensible approach" to emergence taken by Batterman and Rueger is surely along the right lines. Emergence is best understood as an epistemic failure, and I have proposed that this should be cashed out more specifically as a failure of basal explanation. The proposal is not without its challenges, of course. Basal explanation in physics does not fit orthodox accounts of scientific explanation. As the example of the van der Pol oscillator shows, such explanations may fail to trace causal structure, and they may make use of false premises and a non-deductive derivational structure. Clearly, what is needed is an account of scientific explanation that makes sense of physicists' widespread appeal to highly idealized models in their explanatory practices (Wayne 2010).

Another set of challenges comes from alternative accounts of emergence, of which Paul Humphreys usefully distinguishes three types (Humphreys 2008). On inferential approaches to emergence, an entity is emergent with respect to a base-level domain if and only if it is impossible to predict or compute that entity on the basis of base-level theory. Mark Bedau's account of "weak emergence" is an example of this approach (Bedau 1997, Bedau 2008). On conceptual approaches to emergence, an entity is emergent with respect to a base-level domain if and only if a conceptual apparatus is required to represent the entity that is not available in base-level theory. Finally, ontological approaches to emergence try to capture the idea that an entity is emergent when it is genuinely novel and ontologically irreducible to the base-level domain. These include Humphreys' own fusion model of emergence (Humphreys 1997), the ontological account most relevant to emergence in physics. *Prima facie*, the account of emergence as failure of basal explanation suggested here fails to fit comfortably into any of these categories. The relation between explanans and explanandum may be inferential, but only in an attenuated sense, and the inferences involved differ fundamentally from the predictive, deductive or computational inferences that have heretofore characterized this approach. In any event, what is needed is an account that provides a basis for constructive engagement with a wide range of cases in physics, both contemporary and historical. Interesting and substantive questions undoubtedly remain yet to be explored about emergence in physics.

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