The Feynman Diagrams and Virtual Quanta

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The received view in philosophical studies of quantum field theory is that the Feynman diagrams are simply calculational tools. Alongside with this view we have the one that takes the virtual quanta to be also simply formal tools. This received view was developed and consolidated in philosophy of physics works by Mario Bunge, Paul Teller, Michael Redhead, Robert Weingard, Brigitte Falkenburg, and others. In this paper I will present an alternative to the received view.

1. Introduction

In a recent overview on philosophy of physics views on the status of virtual particles in quantum field theory (in practice in quantum electrodynamics), Tobias Fox presented what he considered as the main arguments pro and against giving to the virtual quanta a status that in some way approaches – whatever it might be – the status of the so-called real quanta. In particular Fox considers that "no pro-argument is ultimately satisfactory, and that only one contra-argument – that of superposition – is sufficient to deny the realistic interpretation of virtual particles" (Fox 2008, 35). This view is not uncommon; in reality is the most common view in philosophy of physics at the moment. Warming up for the discussion in section 3 let us follow for a moment Michael Redhead's considerations on the theme (Redhead 1988). Redhead considers a typical scattering process. According to current practice, in the initial $\left|\mathrm{IN}\right\rangle$ state we have free particles (described as quanta of the quantum fields), and the same for the final $\left|\mathrm{OUT}\right\rangle$ state (i.e. the $\left|\mathrm{IN}\right\rangle$ and $\left|\mathrm{OUT}\right\rangle$ states are eigenstates of H_0, the free-particle Hamiltonian). The scattering process is described in terms of the S-matrix, defined as $\left\langle \mathrm{IN}\middle|\mathrm{S}\middle|\mathrm{OUT}\right\rangle$. Now, according to Redhead, in the interacting region "the states develop in time under the full

Hamiltonian H" (Redhead 1988, 19). Redhead considers that we can expand the interacting state at time t as

$$\left|\psi(t)\right\rangle = \sum_n C_n(t)\left|\phi_n\right\rangle,$$

where $\left|\phi_n\right\rangle$ are the eigenstates of H_0. According to Redhead, " if $\left|\psi(-\infty)\right\rangle = \left|IN\right\rangle$ then the transition amplitude to the Out state $\left|\phi_n\right\rangle$ is simply given by $C_n(\infty)$"(Redhead 1988, 19). $C_n(\infty)$ can be calculated using a perturbative approach where we consider the "sum of contributions from the relevant Feynman diagrams of all orders" (Redhead 1988, 19). The Feynman diagrams are related to virtual particles, which are "identified with internal lines of the Feynman diagrams" (Redhead 1988, 19). Accordingly, the previous expression for the interacting state "is a mathematical expansion, rather like Fourier analysing the motion of a violin string" (Redhead 1988, 20). In this way, according to Redhead, this mathematical expansion has "no direct physical significance for the component states. To invest them with physical significance is like asking whether the harmonics really exist on the violin string?"(Redhead 1988, 20).

Let as look a little into the violin string. Consider an irregularly vibrating string. We can represent its vibration as a weighted sum of normal modes: a·sin(t) + b·sin(t/2) +…. However it would not be at all right to say that the string is actually vibrating with frequency 1 AND vibrating with frequency 2.... Or, again, suppose we have an object subject to a force pointed due East. We can decompose the force as a sum of forces pointed Northeast and Southeast. But it would not be at all right to say that there is something pulling the object Northeast AND a force pulling the object Southeast. So we must remember the S-matrix expansion is a sum of components of an overall process.

The components are no more actually occurring parts of the process than in the prior two examples. According to this analogy it would seem silly to try to give to the virtual quanta any relevance beyond being simply terms of a mathematical expansion.

In this paper I will leave the *classical analogies* behind and focus on how are the interactions really described within *quantum* electrodynamics and what role is given to the virtual quanta in this description. In this way it will also be possible to address the possible relevance or not of the Feynman diagrams as representations of the interaction processes and not simply as mnemonic pictures helping writing down the terms of the S-matrix expansion. In section 2 I will provide a simplified account of the quantum electrodynamical description of the interaction between matter and radiation. In section 3 I will focus on the current philosophy of physics account of Feynman's diagrams and virtual quanta and show where it goes astray.

2. Introductory quantum electrodynamics

Quantum electrodynamics describes the interaction between two quantized fields, the electromagnetic (Maxwell) field and the electron-positron (Dirac) field. Also we can address from within quantum electrodynamics 'semi-classical' cases where we consider the quantized Dirac field to interact with what appears as a classical electromagnetic potential. This situation result from using the so-called external field approximation (Jauch and Rohrlich 1976, 303), where it seems to be a classical potential within the quantum formalism, but that really is due to a quantum field theoretical description of the interaction with a very heavy charged particle (described by a quantum field) when its recoil is neglected (Schweber 1961, 535). It is within the external field approximation that a Dirac field operator equation with an 'external' field appears, and from which the relativistic one-electron equation with a 'classical' potential can be seen to emerge from the full quantum electrodynamics formalism (Jauch and Rohrlich 1976, 307 and 313).

In quantum electrodynamics we basically consider the description of scattering processes and bound-state problems. In both cases we can use the S-matrix formalism (Veltman 1994, 62–67). In 1948, Freeman Dyson proved the equivalence of Richard Feynman's and Julian Schwinger's (and Sin-Itiro Tomonaga) theories (Dyson 1948). The main contribution of Dyson was to show that the two so seemingly different approaches could be put together by resort to the S-matrix approach. Considering the perturbative solution of the Tomonaga-Schwinger equation in terms of a unitary operator, Dyson realized that when taking the limits for the initial state to infinite past and for the final state to infinite future, Schwinger's unitary operator was identical to the Heisenberg S-matrix. Following Feynman's symmetrical approach between past and future, Dyson used a chronological operator P( ) that enabled him to present the S-matrix in the form

$$S(\infty)=\sum_{n=0}^{\infty}(-i/\hbar c)^{n}[1/n!]\int_{-\infty}^{+\infty}dx_{1}\ldots\int_{-\infty}^{+\infty}dx_{n}P(H^{I}(x_{1}),\ldots,H^{I}(x_{n})),$$

where $H^{I}(x)$ is the term in the Hamiltonian corresponding to the interaction between the Maxwell and Dirac fields (Dyson 1948, 492).[1]

[1] The S-matrix program was originally developed by Werner Heisenberg as an alternative to quantum field theory. His idea was to sidestep the problem of divergences in quantum field theory – in his view due to the point-like interaction between fields – by considering only what he saw as measurable quantities (Miller 1994, 97). Heisenberg's idea was to retain only the basic elements of quantum field theory, like the conservation laws, relativistic invariance, unitarity, and others, and to make the S-matrix

How do the Feynman diagrams (and in the process, virtual quanta) come into play in this approach? Let us follow Dyson's presentation on this. In Feynman's original approach he made use of a set of rules (associated to his diagrams) to calculate matrix elements for scattering processes. Dyson gives a more formal derivation of the Feynman rules, for problems with an initial (and final) charged particle (electron or positron) and with no photons in the initial and final states. Accordingly, Dyson aims to

obtain a set of rules by which the matrix element … between two given states may be written down in a form suitable for numerical evaluation, immediately and automatically. The fact that such a set of rules exists is the basis of the Feynman radiation theory; the derivation in this section of the same rules from what is fundamentally the Tomonaga-Schwinger theory constitutes the proof of equivalence of the two theories. (Dyson 1948, 492–493)

Dyson considers the contribution of the nth order term of the transition matrix (S-matrix) element, which is a sum of terms of the form

$$M = \varepsilon' \prod_{i \neq k} \frac{1}{2} S_F(x_i - x_{r_i}) \prod_j \frac{1}{2} \hbar c D_F(x_{s_j} - x_{t_j}) \psi^*(x_k) \psi(x_{r_k}),$$

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the central element of a new theory (Pais 1986, 498). This was not done because in practise it was not possible to define an S-matrix without a specific use of the theory it was intended to avoid (Cushing 1986, 118). The S-matrix later reappeared in mainstream physics with Dyson's use of it as a calculational tool. In Dyson's view the "Feynman theory will provide a complete fulfilment of Heisenberg's S-matrix program. The Feynman theory is essentially nothing more than a method of calculating the S-matrix for any physical system from the usual equations of electrodynamics" (quoted in Cushing 1986, 122).

where D_F and S_F are the Feynman propagators for an electron and a photon, and $\psi(x)$ is the electron-positron field operator and $\psi^*(x)$ its adjoint operator. Following Feynman, Dyson calls the attention to the fact that each term in the matrix element can be associated to a graph (a Feynman diagram), and that there is "a one-to-one correspondence between types of matrix elements and graphs" (Dyson 1948, 495). Also Dyson mentions that "in Feynman's theory the graph corresponding to a particular matrix element is regarded, not merely as an aid to calculation, but as a picture of the physical process which gives rise to that matrix element" (Dyson 1948, 496).

In reality Feynman's own view, following John Archibald Wheeler view (based on his scattering theory), was that all physical phenomena could be seen as scattering processes (Schweber 1994, 379). In his paper (published after Dyson's account on his theory), Feynman considered the mutual interaction of two electrons as a fundamental interaction described by his fundamental equation for quantum electrodynamics (Feynman 1949b, 772), which is directly related to the second order term of the S-matrix series expansion when taking into account Pauli's exclusion principle. Feynman presented his diagram as a representation of the electron-electron interaction in which there was an exchange of a virtual photon (Carson 1996, 128–129).

Dyson then gives a prescription for writing down the terms of the S-matrix series expansion. The Feynman diagrams had a one-to-one correspondence to a S-matrix element. More than this, each component of the diagram could be related with a particular factor in the mathematical expression. In particular the ones that consider us must here are the internal lines of the diagrams, which corresponds to photon and/or electron (positron) propagators. This means that we can easily translate a diagram into a complex mathematical expression. The use of Feynman diagrams provides an easy and systematic procedure to write down mathematical expressions that increase in complexity in higher order calculations. To Dyson the diagrams were nothing but “a conventional representational scheme with no pretensions to picturing actual particles’ real scatterings” (Kaiser 2000, 61). However on what regards the S-matrix as a whole, Dyson’s view was that quantum electrodynamics was the perturbative series expansion of the S-matrix (Schweber 1994, 565). It is ironic that having Dyson this view it was he who soon after (in the summer of 1951) found a physical heuristic argument that suggested that after all the series expansion of the S-matrix was divergent (Dyson 1952). According to Dyson “all the power-series expansions currently in use in quantum electrodynamics are divergent after the renormalization of mass and charge” (Dyson 1952, 631). At best it can be an asymptotic expansion (Schweber 1961, 644). In the last decades Dyson’s result has been corroborated (e.g. Aramaki 1989, 91–92; West 2000, 180–181; Jentschura 2004, 86–112; Caliceti et al 2007, 5–6). We will see in the next chapter that this result brings down the ‘superposition argument’ that Fox (and others) regards as sufficient to take the virtual quanta as mere formal tools.

Before analysing the philosophical arguments related to the virtual quanta let us see in more detail the description of a scattering process within quantum electrodynamics. As an example let us consider the electron-electron scattering. In the second-order expansion of the S-matrix the electron-electron interaction results from a photon exchange. In the overall space-time approach of Feynman’s (implicit in Dyson’s S-matrix approach) we are considering virtual photon propagation between all the Minkowski space-time points. The Feynman photon propagator is given by

$$\langle 0 | T\{A^{\mu}(x)A^{\nu}(x`)\} | 0 \rangle = i\hbar c D_{F}^{\mu\nu}(x - x') \quad \text{(Mandl and Shaw 1984, 86).}$$

This expression means that we are considering a virtual photon 'created' at one space-time point and 'annihilated' at another. The use of the time-ordered product T{ } means that in this covariant expression we are already considering, depending on the time order, a propagation from one electron to the other or vice versa, since $T\{A^{\mu}(x)A^{\nu}(x')\}= A^{\mu}(x)A^{\nu}(x')$ if t > t', and $T\{A^{\mu}(x)A^{\nu}(x')\}= A^{\nu}(x')A^{\mu}(x)$ if t' > t. Loosely speaking, we have contributions in which the 'emitter' and 'receiver' change roles. The transition amplitude for the electron-electron scattering in the second-order expansion of the S-matrix (the simplest for this process) results from a contribution of all possible localized interactions of Dirac and Maxwell fields 'connected' by a photon propagator (Mandl and Shaw 1984, 113):

$$S^{(2)}(2e^{-} \rightarrow 2e^{-}) = \frac{-e^{2}}{2!}\int d^{4}x_{1}d^{4}x_{2}N\left[(\overline{\psi}^{-}\gamma^{\alpha}\psi^{+})_{x_1}(\overline{\psi}^{-}\gamma^{\beta}\psi^{+})_{x_2}\right]iD_{F\alpha\beta}(x_{1}-x_{2})\,.$$

This means that the overall process we call 'interaction' (i.e. virtual photon exchange) results from the contribution of photon propagation from one electron to the other and vice versa: it is a two-way process in all space-time (in Fig. 1 is depicted the Feynman diagram for this term of the S-matrix).

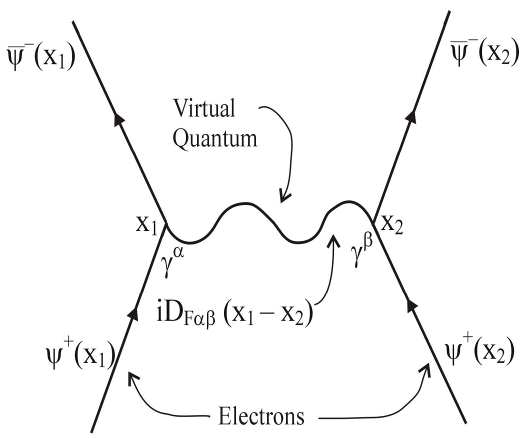

Figure 1: Electron-electron scattering in second order, resulting from a virtual photon exchange (direct diagram).

The label 'virtual' attached to the photon is related to two things. In the space-time points where the photon is created or annihilated we have conservation of energy and momentum between the photon and the electrons. But the energy-momentum relation for the virtual photon is not $k^2 = (k^0)^2 - \mathbf{k}^2 = 0$ corresponding to a zero mass photon, it is different from zero due to the fact that in the expression for the propagator $\mathbf{k}$ and k^0 are independent of each other (Mandl and Shaw 1984, 86). In a certain sense it is as if the 'dynamics' of the virtual photon are all messed up (the same occurs with the electron when it is in the role of a virtual quantum), because it is as if it has a mass during the virtual process. At the same time the 'kinematics' come out wrong also, because the propagator is non-vanishing at space-like separations (Björken and Drell 1965, 388–389). The second point is that this virtual quantum is supposed by definition not to be observable – it is part of the internal machinery of the description of the interaction. In the case of the photon in the electron-electron scattering it seems impossible to avoid this situation, as the idea that this is the must elemental process possible is implicit in the formalism of the theory: in the case of scattering processes we *only* have experimental access to the cross-section that "as an empirical quantity, it is the measured relative frequency of scattering events of a given type" (Falkenburg 2007, 107). In quantum electrodynamics the scattering cross-section (as a theoretical quantity) is calculated from the transition probability per unit space-time volume, which is related to the S-matrix in a simple way (Jauch and Rohrlich 1976, 163–167). In this way it is not possible experimentally the parsing of amplitude terms (i.e. Feynman diagrams). Quantum theories only describe the probability distribution for the outcome of measurements of physical observables in specific experimental setups; and, to compare quantum predictions with experimental results we need to obtain relative frequencies from repeated measurements (Falkenburg 2007, 106). If we imagined that we could see how the interacting was going on, we would be in a different – in fact impossible – experimental setup from the actual existing experimental setup that permits the analysis of scattering processes.

3. The philosophical debate

From looking into the S-matrix treatment of interaction, the question arises: are the virtual quanta simply the result of the perturbative treatment of interaction, i.e. simply mathematical terms, or they convey as Dyson remarks about Feynman's views, "a picture of the physical process which gives rise to that matrix element" (Dyson 1948, 496). The contemporary view in philosophy of physics is that they do not. According to Fox, the virtual particles serves "to symbolise the interaction" (Fox 2008, 35), and they "are merely pictorial descriptions of a mathematical approximation method" (Fox 2008, 35). Reviewing the arguments of several authors[2], Fox centres in what he considers the argument that proves the status of virtual particles, "as only pictorial symbols for mathematical terms" (Fox 2008, 36). This so-called argument of superposition rests on

[2] For example, Mario Bunge takes the virtual quanta (and interaction processes described as exchange of virtual quanta) to be "fictions and as such have no rightful place in a physical theory" (Bunge 1970, 508); Paul Teller's view is that "a Feynman diagram is only a component in a much larger superposition" (Teller 1995, 139); and Fritz Rohrlich considers that "virtual particles are an artifact of the perturbation expansion into free particle states" (Cao 1999, 363).

the (wrong) idea that in the S-matrix description of interactions we have to consider an infinite expansion of the S-matrix that results in "the infinite superposition of Feynman diagrams of higher and higher order" (Fox 2008, 38), even if according to Fox, "due to practical reasons – the perturbation progression is stopped sooner or later" (Fox 2008, 37).

One between several variants of this argument was set forward by Robert Weingard. If, when calculating say the amplitude for electron-electron scattering, the complete S-matrix was (somehow) considered, then there would be an infinite number of terms corresponding to an infinite number of combinations of different quanta. One could say that in this case the quanta "type and number are not sharp" (Weingard 1988, 46). The quanta description of interactions, as quanta exchange, would then appear to be a mathematical fiction due to the use of perturbation theory in the calculation of the scattering amplitude. However, when considering the scattering as described in the applications of the theory, we can only use a few terms of the S-matrix expansion. There simply is no possibility of considering the (unexisting) exact S-matrix, only the (probably) asymptotic S-matrix.[3]

In similar lines, but with important differences in relevant details, Brigitte Falkenburg considered the virtual particles as "formal calculational tools" (Falkenburg 2007, 223). According to Falkenburg the virtual particles come into play within time-dependent perturbation theory: "the propagators of the virtual field quanta are mathematically components of a quantum theoretical superposition. Operationally, it is

[3] There might appear to be ways of sidestepping this type of approach considering the Feynman path integral approach (Weingard 1988, 54). But again, when considering the specific applications of the theory there is no infinite expansion of the transition amplitudes. In the mathematical expression for the transition amplitudes there are propagators, and the interpretation of the propagators relating them to quanta cannot be overturned in a (finite expansion) application based on path integrals.

by no means possible to resolve them into single particle contributions. They are nothing but the mathematical contributions to an approximation procedure: like the harmonics of the oscillators of a mechanical string, the Fourier components of a classical electromagnetic field, or the cycles and epicycles in Ptolemy's planetary system" (Falkenburg 2007, 234). However Falkenburg after concluding that "virtual field quanta are nothing but formal tools in the calculation of the interactions of quantum fields" (Falkenburg 2007, 237), calls attention to the fact that "this does not mean, however, that the perturbation expansion of the S-matrix in terms of virtual particles is *completely* fictitious" (Falkenburg 2007, 237). According to Falkenburg,

The virtual processes described in terms of the emission and absorption of virtual particles contribute to a scattering amplitude or transition probability. Hence, *infinitely many* virtual particles, *together* may be considered to cause a *real collective effect*. In *this* sense, they obviously *have* operational meaning. What is measured is an S-matrix element or the probability of a transition between certain real incoming and outgoing particles. The transition probability stems from *all* virtual field quanta involved in the superpositions of the relevant lowest and higher order Feynman diagrams.

In the low energy domain, it is sometimes even possible to single out the contribution of one single Feynman diagram to the perturbation expansion. There are even several well-known high precision measurements to which mainly one Feynman diagram or the propagator of a virtual particle corresponds. This is demonstrated in particular by the best high precision tests of quantum electrodynamics, the measurement of the hydrogen Lamb shift and the (g – 2)/2 measurement of the gyromagnetic factor g of the electron or muon. Dirac theory alone incorrectly predicts the fine structure of the hydrogen spectrum (no splitting of the levels $S_{1/2}$ and $P_{1/2}$ for n = 2) and a gyromagnetic factor g = 2 for the electron or muon. Measurements reveal the Lamb shift of the hydrogen fine structure and the anomalous magnetic moment of the electron.

The anomalous difference (g – 2)/2 between the prediction of the Dirac theory and the actual magnetic moment was measured with high precision from the spin precession of a charged particle in a homogeneous magnetic field. The next order quantum electrodynamic correction stems from a single

Feynman diagram which describes electron self-interaction. Here, theory and experiment agree at the level of 1 in 10^8, with a tiny discrepancy between theory and experiment in the eighth digit. In such a case, the experiments are *for all practical purpose* capable of singling out the real effect of a single Feynman diagram (or virtual field quantum). The case of the Lamb shift is similar. Here the next order perturbation theory gives a correction based on two Feynman diagrams, namely for vacuum polarization and electron self-interaction. The correction shows that only 97% of the observed Lamb shift can be explained without the vacuum polarization term. A textbook on experimental particle physics tells us therefore that the missing 3% are "a clear demonstration of the actual existence of the vacuum polarization term". Any philosopher should counter that this is *not really* the case. The virtual field quanta involved in this term cannot be *exactly* singled out.

Hence, the above conclusions remain. Virtual particles are formal tools of the perturbation expansion of quantum field theory. They do not exist on their own. Nevertheless they are not fictitious but rather produce collective effects which can be calculated and measured with high precision. (Falkenburg 2007, 237–238)

I find this presentation by Falkenburg very enlightening, even if I do not agree with the view of the virtual particles as formal mathematical tools. The motive is that this conclusion by Falkenburg, like the others mentioned, rests on what Fox called the argument of superposition. This argument does not hold in quantum electrodynamics. We do not have an infinite expansion of the S-matrix, what we have are applications of the theory resting in an approximate scheme of description of the interaction between two distinct fields that cannot be taken beyond a few order calculations. I would characterize the state of affairs in quantum electrodynamics by considering that it has a limited domain of applicability, i.e. it does not provide a full description of the interaction of radiation and matter, only an approximate one resting on an asymptotic S-matrix[4]. We do not ‘stop the perturbation progression due to practical reasons’; we only really have a few lower order terms to count on. In this way it makes sense to single out a few or even just one Feynman diagram. That is, it is not that “the experiments are *for all practical purpose* capable of singling out the real effect of a single Feynman diagram” (Falkenburg 2007, 237–238), on the contrary, the experiments are, *due to the limited domain of applicability of the theory,* capable of singling out the ‘real’ effect of a single Feynman diagram. Concluding, I agree with the view implicit in Feynman’s presentation (e.g. Carson 1996, 128–129), and stressed by Dyson, that “in Feynman’s theory the graph corresponding to a particular matrix element is regarded, not merely as an aid to calculation, but as a picture of the physical process which gives rise to that matrix element” (Dyson 1948, 496). In my view, this is the correct view for quantum electrodynamics; it describes interactions in terms of virtual quanta exchange, and we can single out one Feynman diagram (I have more to say about this below).

That Feynman’s diagrams convey a physical description of the interactions in quantum electrodynamics, and with it ‘physical meaning’ to the virtual quanta beyond being simply calculational tools, does not mean that there are not limitations in what regards the kind of ‘physical description’ being provided by quantum electrodynamics through its Feynman diagrams. Franz Mandl and Graham Shaw called attention to this: “the reader must be warned not to take this pictorial description of the mathematics as a literal description of a process in space and time” (Mandl and Shaw 1984, 56). The problem is that the propagator for the photon is non-vanishing for a space-like separation. This would apparently imply the possibility of an electron-electron interaction with a speed greater than the velocity of light. As we will see in a moment

[4] The philosophical implications of this situation are certainly important. However any tentative treatment of this subject would go far beyond the subject of this paper.

this is not the case, when considering in detail how the scattering is really described in quantum electrodynamics. However this situation forces the question of what to make of Feynman's overall space-time description of physical processes and the role of Feynman's diagram within this approach? To give an answer I will now return to the analysis of the description of the electron-electron scattering in quantum electrodynamics.

The crucial aspect of the description of scattering in quantum electrodynamics is that there really is no description *in time* of the interaction. This is due to the fact that in the application of the S-matrix method we are always considering the free particle initial state (at t = –∞) and free particle final state (at t = +∞), while disregarding the detailed description of the intervening times. In this sense we have an overall temporal description of the scattering processes. Feynman did not consider this as a limitation; on the contrary, his view was that "the temporal order of events during the scattering … is irrelevant" (Feynman 1949a, 749).

To see how Feynman's overall space-time approach works out, 'solving' the possible problems due to the fact that the propagator is non-vanishing at space-like separations, let us consider a counterfactual realistic picture of the virtual 'processes' involved in the calculation of the S-matrix. When considering the interaction between two electrons, the S-matrix element is constructed with an underlying idea of an elapsing time. A (virtual) photon is emitted by one electron, which means that due to the localized interaction of the Dirac and Maxwell fields it is created at a specific space-time point. This photon propagates and is luckily absorbed by an electron expecting him. We have a sort of effect 'next': the quantum 'knows' what is going to happen and behaves accordingly so that we have a smooth adjustment between the electrons and the photon. In reality the sequence of creation and absorption of the photon is adjusted '*ab* *initio*' in a mathematical expression – the S-matrix – that provides an overall temporal (and spatial) description of what we consider to be an *in time* temporal phenomenon. In a certain sense the problem lies not in the adjustment of the creation and annihilation of the photon but in the use of temporal language in an overall description of the interaction in quantum electrodynamics, like when Feynman considered a situation where it was supposed that "one electron was created in a pair with a positron destined to annihilate the other electron" (Feynman 1949b, 773).

Exploring a little more the counterfactual realistic picture of the virtual 'processes', if we try to maintain an *in time* temporal perspective considering, in contradiction to the usual interpretation of quantum theories, a submicroscopic 'observer'– say Alice –, then the cat – our propagator – will reveal peculiar behaviours. The fact is that, as mentioned, the propagator does not vanish for a space-like separation. This means that we would have an interaction between space-time points not connectable with a classical electromagnetic wave. However in this quantum word the photons and electrons (or positrons) being propagated between two points are not restricted by the usual energy-momentum relations, so we are beyond any classical dynamical description of the 'propagation', and, as mentioned previously, we refer to these quanta as 'virtual' while use the ontologically charged word 'real' for the quanta whose energy-momentum relations are $k^2 = 0$ in the case of the photon and $p^2 = m^2$ in the case of fermions. For a submicroscopic 'observer' located in the space-time point where a quantum is emitted we can imagine that an objective notion of present (emission) and future (absorption) exists. The problem is that for a space-like separation, a moving 'observer' – Alice – might see the absorption before the emission. In the case of electron propagation this would imply seeing a positron. The cat would be changing its form. Considering Einstein's kinematical interpretation of relativity (Einstein 1905, 48; see also Smith

1995), from the perspective of a moving 'observer' – Alice –, (which we imagine to make her 'observations' using a 'submicroscopic' classical electromagnetic wave, i.e. 'respecting' Einstein's relativity), in the situation described above it would seem as if there is an interchange of the creation and annihilation points. In the case of photon propagation, this only makes her think that the direction of propagation is the opposite. In the case of electron propagation it will seem as if the (unobserved) quantum is now a positron. But even Alice, taking into account relativity theory, can only see the points of interaction between the fields, not the propagation 'process' itself. In this way the 'true' virtual electron only appears to be a positron due to 'kinematical' relativity, but it is 'really' an electron.

When considering the *overall* amplitude the problem fades away. The case is that the S-matrix is covariant. So, different 'observers' will obtain the same result for the scattering amplitude, with their identical submicroscopic experimental devices, when considering the propagation between all space-time points (a 'real' observer cannot make these space-time experiments to determine the scattering amplitude, he can only obtain experimental cross-sections). We can express the covariant S-matrix in two alternative forms (Sakurai 1967, 204):

$$S_a^{(2)} = (-i)^2 \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{t_1} dt_2 H_I(t_1) H_I(t_2)\text{, and } S_b^{(2)} = (-i)^2 \int_{-\infty}^{\infty} dt_1 \int_{t_1}^{\infty} dt_2 H_I(t_2) H_I(t_1)\,.$$

To see the content of these formulas let us consider localized 'observers' of 'processes' that can be described by S_a. In this case we are considering 'processes' where $t_2 < t_1$. Now, a passer-by might think, in relation to a spacelike propagation, that she is seeing a 'process' where $t_1 < t_2$ as described in S_b. But she will also think that another 'process', which for the localized 'observer' is from S_b, is described in S_a. The overall result will be the same for both ‘observers’. The possible time inversion problem does not occurs as is sweeped under the covariance of the S-matrix:

$$S^{(2)} = (-i/2)^2 \int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \{H_I(t_1)H_I(t_2)\theta(t_1 - t_2) + H_I(t_2)H_I(t_1)\theta(t_2 - t_1)\},$$

where $\theta(t) = 1$ if $t > 0$, and $\theta(t) = 0$ if $t < 0$ (Sakurai 1967, 204).

Going back to real observers, the fact is that we do not have a submicroscopic experimental access to the theoretical point-like interaction between the fields. In the case of scattering processes we *only* have experimental access to cross-sections, calculated from the S-matrix (Jauch and Rohrlich 1976, 163–167). The point is that with the (experimentally accessible) cross-section calculated from the S-matrix – the only possible theoretical approach to scattering processes within quantum electrodynamics – we are not considering time as its goes by, but an overall temporal (and spatial) calculation of the interaction processes: all of the past and future is put into it.

We see then that in quantum electrodynamics we can ‘save’ the concept of virtual quanta exchange as a valid description of the interactions. However, the Feynman diagrams must be seen as a representation of an *overall* space-time description of scattering processes (as an exchange of quanta) in which “the scattering process itself is a *black box*” (Falkenburg 2007, 234). We do not have an in space and *in* (*through)* time description of the interactions.

For some it might yet seem that mine is a straw-man position. The fact that I am using Dyson’s result against the superposition argument might seem not enough according to some views on this argument. It is correct that we cannot consider any more that the quanta “type and number are not sharp” (Weingard 1988, 46). However it might be the case that there are versions of the superposition argument where the fact that we still may consider several Feynman diagrams in the description of the interactions is enough to relegate the diagrams and virtual quanta to the role of accessory tools not given in any sense a physical description of the interactions. I think that Teller's argumentation can be seen as an example of this 'finite'-superposition argument. The part of Teller's argumentation not depending explicitly on *classical analogies* is sustained on the following points:

1) As we have already seen, in the description of the electron-electron interaction, when considering one Feynman diagram (in second order) we "have x_1 and x_2 as free variables, which must be integrated before we get the diagram's final contribution to the scattering amplitude. The processes allegedly described by the diagram [(which I referred to as 'processes')] must be superimposed for all values of x_1 and x_2 before we get a description of what is still only a contribution of a quantum-mechanical amplitude for a real scattering process" (Teller 1995, 142). Now, the point is that, as I mentioned, we have only one global process described by this one Feynman diagram, corresponding to the black box calculation of localized 'processes', which are mathematical artifacts resulting from the point-like description of the interaction between the quantum fields (which has no operational meaning). This does not entail that the exchange of quanta cannot be seen as a physical process, simply that it is not a physical process in the classical sense of space-time processes that we have in classical theories. Teller recognizes this much, since he mentions that, in his view, "in quantum theories the components represent potentially but no actually existing states" (Teller 1995, 141). I will not discuss Teller's interpretation of quantum field theories, but simply remember that we are not dealing with 'classical pictures' of physical processes occurring in space and *in* time. Reversing the order of arguments, we can take the intricate description of one Feynman diagram (representing, in the case being

considered, the exchange of one virtual quantum) as the physical description of interactions in quantum electrodynamics, giving at the same time the only available physical-mathematical meaning to what we understand by the term 'exchange of quanta'. We cannot have implicit in our argumentation classical analogies when interpreting the physical-mathematical description of interactions in quantum electrodynamics (in this part, Teller's point is dependent on a supposed superposition of different 'processes' occurring in different points of space-time).

2) Even if we cannot count on an infinite series expansion of the S-matrix we still have a superposition of several terms (related to different Feynman diagrams). According to Teller, "the full scattering amplitude, is, in principle, given only when the results from second order are further superimposed with contributions from all even higher orders" (Teller 1995, 142). Now, we do not have a *full* scattering amplitude that is, in principle, given by further superimposing *all* higher order terms. However it is in general necessary to consider higher order contributions to the S-matrix to get a good agreement with experimental results (recall Falkenburg's presentation). Again, my view is that we must not fall into the trap of classical analogies when addressing a quantum electrodynamical description of the interaction of radiation and matter (described within the theory as quantum fields). The fact that we can (and sometimes have to) describe the scattering by considering simultaneously several Feynman diagrams (corresponding each to a particular type of exchange, where the quanta type and number *are* sharp) does not imply that we must see them as simply abstract mathematical tools. My view is that we can see them as describing the interaction of radiation and matter as a quantized exchange of energy and momentum even if an *intricate one* (i.e. resulting from the contribution of different Feynman diagrams to the S-matrix), in which there is no place for an account relying on classical-like analogies, i.e. it is not like we have two tennis

players playing with several sets of balls at the same time, as Fox refers to (Fox 2008, 42). As mentioned, in the description of the interaction of radiation and matter (described as quantum fields), applied for example in the case of the description of electron-electron scattering, we have conservation of energy and momentum between the virtual photon and the electrons (i.e. in the 'emission' and 'absorption' of virtual quanta). In fact, in *all cases* (involving real or virtual quanta) we must consider the interaction of radiation and matter as resulting from a *quantized exchange of energy and momentum*, i.e. even when considering the description of scattering processes (in which it is impossible in principle to observe the virtual quanta), which we can see, following Falkenburg, as resulting from the *collective effect* of virtual processes (which nevertheless involve the exchange of quanta whose type and number *are* sharp).

4. Conclusion

I have defended that the main argument against virtual quanta – the superposition argument, cannot be made to stand in quantum electrodynamics. In this way by a reconsideration/reformulation of Falkenburg's argument 'against' virtual quanta, I have defended that, following Feynman, we can individuate one Feynman diagram and take it as representing the description of interactions that we have in quantum electrodynamics. In the case of electron-electron scattering, this means taking the interaction as resulting from virtual quanta exchange, *primarily* from a photon exchange. It is true that we can consider further terms of the S-matrix series expansion as *corrections* to this (in Feynman's words) 'fundamental interaction', but since the series expansion of the S-matrix is at best asymptotic we know that this is a finite correction of the main term describing the interaction, i.e. simultaneously with the photon exchange we can consider a finite number of higher order processes involving other virtual quanta.

Accepting the Feynman diagram as a representation of a physical interaction as given by quantum electrodynamics, does not mean that we must see the Feynman diagram as giving a space-time description of the interaction. As shown, we must be careful in not giving an immediate space-time reading of the Feynman diagram. According to David Kaiser, referring to the S-matrix theory developed in the 1950s and 1960s, in particular by Geoffrey Chew, there was an "association of 'realism' with Feynman diagrams … based on their simple similarity to 'real' photographs of 'real' particles" (Kaiser 2000, 75). This resulted from a misinterpretation of lowest-order Feynman diagrams as depictions, as a sort of Minkowski diagrams, representing a schematic reconstruction of bubble chamber photographs; according to Kaiser, "the Feynman and Feynman-like diagrams that were taken over into S-matrix theory *were* not the high-order loop corrections … but rather *lowest-order* and, most frequently, *single-particle exchange* diagrams. And what were the 'visual ingredients' of these particular classes of Feynman diagrams? Nothing but vertices and propagation lines" (Kaiser 2000, 74). I must stress that this is not the view being proposed here. Letitia Meynell called attention to the fact that in Feynman's work it is not enforced a 'bubble chamber view' of the Feynman diagrams. Meynell asked the question if "Feynman diagrams prescribe imaginings of definite trajectories through and positions in space-time?" (Meynell 2008, 53). Now, Feynman presented his approach to quantum electrodynamics in two papers from 1949 that are strongly interrelated. According to Meynell, "the quintessential Feynman diagram pictured in the second paper drew on the physical interpretations and visual schemata of the first" (Meynell 2008, 53). In this first paper, Feynman illustrates the scattering of an electron with two equivalent pictures (that Meynell calls pre-Feynman diagrams). In one case Feynman gives a wave description of the electron scattering and in the other a particle description (Feynman 1949a). Thus, According to Meynell, Feynman was not trying to enforce a reading of the diagram as representing a trajectory in space-time. I agree (as can be seen from the previous sections). This is clear in Feynman's second paper. Feynman clearly indicates that he was developing an overall space-time approach; and in this approach the main physical aspect of the Feynman diagram was the representation of the interaction as an exchange of virtual quanta (Feynman 1949b).

The virtual quanta are in principle non-observable. As mentioned, in the case of scattering experiments we have no 'insider' making observations; we have the initial state (corresponding experimentally to a determined preparation of the system) and the final calculated state, which will enable us to make comparisons with the experimental results (Peres 1984, 647). To have a path in space-time we cannot consider an 'elemental' interaction, because we cannot observe the *internal* 'configurational' space (the Minkowski space-time) where the interaction 'occurs' through the mathematical device of the propagator. To make this point clear, let us recall Heisenberg's description of the appearance of tracks of α-particles in a Wilson cloud chamber. Heisenberg considers that each successive ionisation of molecules of the medium, due to a α-particle, is "accompanied by an observation of the position" (Heisenberg 1930, 69). This sequence of observations reveals a 'path' in space. However, in between each ionisation, the particle is described by a wave function. There is no quantum mechanically described microscopic trajectory. Each observation corresponds to a state preparation for the next one. It is the sequence of observations controlled by us that gives the impression of a trajectory in space-time. In the case of an electron-electron scattering we do not have that. It is a unique and global process associated with a sole experiment. It is not possible to visualize this process as something that is going on as we speak and following a particular trajectory in space. Minkowski space-time has to be seen, when used in the context of S-matrix calculations, as a mathematical abstract space, where mathematical objects like the propagators are used as part of calculation machines. If we consider the scattering process as a black box (Falkenburg 2007, 234), it is the space-time itself that is this black box.

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