THE MEANING OF SPONTANEOUS SYMMETRY BREAKING (I): From a simple classical model

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Abstract

This paper, part I of a two-part project, aims at answering the simple question 'what is spontaneous symmetry breaking?' by analyzing from a philosophical perspective a simple classical model. Related questions include: what does

it mean to break a symmetry spontaneously? Is the breaking causal, or is the symmetry not broken but merely hidden? Is the meta-principle, 'no asymmetry in, no asymmetry out,' violated? And what is the role in this of random perturbations (or fluctuations)?

1. Introduction

It is probably not immediately obvious, unlike for instance the concept of quantum measurement, that the concept of spontaneous symmetry breaking (SSB) calls for a philosophical discussion. It may involve complex and difficult theories which challenge the ingenuity of a physicist or a mathematician, but why is it also philosophically interesting? Two answers come forth quite readily. First, what is interesting, and perhaps also puzzling, about the type of symmetry breaking to which SSB refers is the 'spontaneousness.' There are different types of symmetry breaking, most of which I would say are not obviously philosophically interesting. The breaking of a symmetrical object by some external influences, such as cracking a perfectly spherical ball by a jack hammer and thus making the fragments anisotropic, is a symmetry breaking that may take a complicated physical theory to explain but does not generate any philosophical puzzlement. Nor is it puzzling about the breaking of the symmetry of a natural law when a term that does not obey that symmetry is added to the Lagrangian (or Hamiltonian) of the system that the law governs. But in those cases involving SSB no discernible external forces or impacts nor symmetry violating terms are present to explain the phenomena. Are the breakings of this type perhaps 'without any causes?' Krieger (1996) when explaining the word 'spontaneous' in 'spontaneous magnetization' -- a species of SSB -- says, '[it] is here used to mean without a direct and deliberate influence, on its own, arising without a necessarily given direction.(p. 285)' The language is kept ambiguous perhaps on purpose.

Second, although there are rigorous theories about several kinds of SSB phenomena (most of which, such as those for ferromagnetism, superconductivity, and the standard model of the electroweak interactions, are very abstract and complex), when it comes to the general conception of SSB there appear to be some confusing or misleading discussions in the literature. When introducing the subject of SSB and before going into the specific theories, some authors like to offer general remarks or simple examples and metaphors that purportedly show that there is really nothing mysterious or terribly uncommon about SSB. Weinberg (1996) says, '[w]e do not have to look far for examples of spontaneous symmetry breaking.(p. 163)' A chair, he says, is a case of SSB, because '[t]he equations governing the atoms of the chair are rotationally symmetric, but a solution of these equations, the actual chair, has a definite orientation in space.(p. 163)' By the same token, we may say that the macroscopic world is full of finite objects that break spatial isotropy (see also, Anderson 1997, 263-266, where he took Samuel Johnson's retort of Berkeley by kicking a stone and thus demonstrating its existence for a demonstration of the existence of SSB as well!). Coleman (1975), when introducing the notion of SSB, says, 'there is no reason why an invariance of the Hamiltonian of a quantum-mechanical system should also be an invariance of the ground state of the system.(p. 141)' Hence, symmetry breaking of this type is not only common but also theoretically unremarkable; however, this is true only with finite objects. But, Coleman continues, SSB in systems of infinite size is not only less common but also theoretically interesting and deep. Should we not wonder why on both counts: why is it 'a triviality' for finite systems but significant for infinite ones?

Abdus Salam is reported to have offered the following metaphor to explain SSB (Moriyasu 1983, 99, also see Brout & Englert 1998). We are asked to imagine some dinner gusts sitting at a round table on which plates are set at an equal distance, and between any two plates a single spoon is placed. Given each guest is seated directly in front of each plate, the left and right symmetry for each guest of using a spoon holds. However the symmetry is broken, and bound to be broken, when the first guest picks up a spoon, any spoon. What does this metaphor tell us about SSB? Does the spontaneousness refer to something analogous to a choice that the guest makes who first picks up a spoon? We do see such metaphorical language being used in the technical discussion of SSB; for instance, the spins in a ferromagnet are said to 'choose' to align themselves along a certain direction when the temperature drops below the critical value. Connection is also made to the case of Buridan's ass (Moriyasu 1983, 86-87, 101), suggesting that what saves the ass from starving between the two stacks of hay is perhaps an SSB.

One also often hear the complaint that the term 'spontaneous symmetry breaking' is misleading; the right term should be 'hidden symmetries,' which refers to systems in which some symmetries of the law are not visible -- hence, hidden -- from the lowest energy solution(s) of the law-equations (cf. Coleman 1975, 142). This seems to suggest that no symmetry is broken in such systems; rather different symmetries apply to different aspects of them.

Still others (cf. Ross 1985, 59-60) regard the results of SSB not so much as broken symmetries than as approximate ones. Should we then regard SSB as an epistemic notion rather then a notion that refers to a physical property?

Furthermore, is SSB connected to the concept of quantum measurement, supposing that one regards the measurement as the breaking of the unitary evolution of a quantum system?[1] Does SSB give support to the existence of genuinely *emergent* properties, as some have so argued (cf. Anderson 1997, 263-266)? Does van Fraassen's general discussion of symmetry principles (cf. van Fraassen 1989, chapters 10-11) need any amendment in view of SSB? What becomes of the claim that only the gauge invariant are physically real if both global and local gauge symmetries may be spontaneously broken?

I hope the above is sufficient to justify an in-depth philosophical investigation of SSB, which I shall carry out in a two-part project. I begin this first part by bring to the attention of the philosophical community a beautiful and elementary model of SSB in classical mechanics (cf. Greenberger 1978, Sivardière 1983, Drugowich de Felício & Hipólito 1985). This simple model (and its variants) reveals some of the elementary properties that all cases of SSB share, and they are probably the most interesting properties to examine from a philosophical point of view, or so shall I argue.

2. A model of spontaneous symmetry breaking

Imagine a circular wire ring vertically suspended as shown in Figure 1. It rotates freely, and on it is frictionlessly threaded a bead. The only forces acting on the bead are gravity, mg, and the normal force, N, the bead receives from the wire.

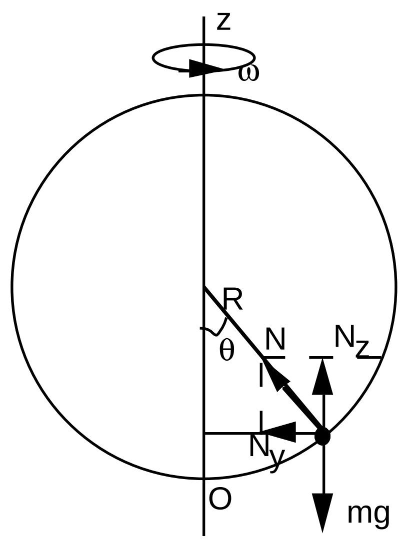

Figure 1. A bead with mass, $m \neq 0$, moves frictionlessly on a circular wire of radius, R, which rotates around the z-axis with a constant ω.

Question: when is the bead stationary on the wire? In other words, what is (are) the equilibrium state(s) of this rotating system? The bead will be stationary whenever the forces are balanced out, which means,

In z-direction: $N_z - mg = N\cos\theta - mg = o$, (1)

In y-direction: $N_y - ma = (N - m\omega^2 R)\sin\theta = o$, (2)

where, a is the linear acceleration of the bead, which is $a = \omega^2 r = \omega^2 R\sin\theta$.

When (1) is true, $N = mg / \cos\theta$. Substitute this into (2) to eliminate N, we have

$$m(g / \cos\theta - \omega^2 R)\sin\theta = 0 . \quad (3)$$

There are obvious two solutions to this equation:

i. $\theta = \theta_0 = 0$ such that $\sin\theta_0 = 0$, which therefore satisfies (3).

ii. $\theta = \pm\theta_1 \neq 0$ such that $g / \cos\theta_1 - \omega^2 R = 0$; or $\cos\theta_1 = g / \omega^2 R$.

Solution (**ii**), θ_1, has a further property: $cos\theta_1$ increases when ω decreases (with R and g being given as constants), and yet $cos\theta_1$ has an upper bound of 1. Hence, there is a *critical value*, $\omega = \omega_1$, where $cos\theta_1 = 1 = g / \omega_1^2 R$, such that below this value solution (**ii**) makes no sense in physics. The standard way of dealing with such a situation in physics is to say that then solution (**i**) takes over, so that for all ω such that $0 \leq \omega \leq \omega_1$, $\theta = \theta_0 = 0$ is the solution at which the system is in equilibrium; and for those ω such that $\omega > \omega_1$, solution (**ii**) -- $cos\theta_1 = g / \omega^2 R$ -- takes over so that all angles θ_1, where $0 < \theta_1 < \pi/2$[2], are possible equilibrium positions for the bead on the wire.

Or equivalently, though perhaps less transparently, the same results can be reached from the Lagrangian (or Hamiltonian) formulation of this problem. Besides obtaining the two solutions, it also gives us the potential energy (or a dimensionless function of it) as a function of the angle, θ. Without going into the actual derivation (see Greenberger (1978) for details), let me summarize the main steps of the argument. The Lagrangian of the bead is of the usual form: L = (kinetic energy) - (potential energy). Since we are only concerned with the stationary situations, we only need to look at the potential energy term, which has two components: one due to the bead's weight and the other its centrifugal tendency. Writing everything out explicitly, we have the potential energy in the following form:

$$= mgRU(\theta), \qquad (4)$$

where $U(\theta)$ is the dimensionless potential. Some simple calculations yields that

$$U = 2sin^2(\theta / 2)[1 - \beta cos^2(\theta / 2)], \qquad \beta = \omega^2 R / g \,.$$

Now, for the bead to be in equilibrium on the wire at a give ω is for its potential to be an extremum, which means that

$$\partial U / \partial\theta = sin\theta(1 - \beta cos\theta) = 0 \,. \qquad (5)$$

This equation, one should realize, is essentially the same as (3); and so we can derive the same two solutions as given above. There are two further benefits from this approach: an exact knowledge of the solutions' stability and a diagram -- in which $U(\theta)$ is plotted against θ -- that vividly shows the SSB in this model.

First, the solutions from (5) are stable if $\partial^2 U / \partial^2\theta > 0$; and now $\partial^2 U / \partial^2\theta = cos\theta - \beta cos2\theta$. So,

iii. for $\theta = \theta_0 = 0$, we have $\partial^2 U / \partial^2\theta = 1 - \beta$, which means that the bead is stable for $\beta < 1$; and

iv. for $\theta = \pm\theta_1$ or $cos\theta_1 = 1/\beta = g / \omega^2 R$, we have $\partial^2 U / \partial^2\theta = \beta - 1/\beta$, which means that the bead is stable for $\beta > 1$.

In other words, $\beta_c = 1$ or $\omega_1 = \sqrt{g/R}$ is a critical value where the equilibrium solution switches from $\theta = \theta_0 = 0$ to $\theta = \theta_1 > 0$.

From either approach we see an SSB at the critical point, $\omega_1 = \sqrt{g/R}$, in that, as shown in Figure 2, for β (or ω) to pass the point from below, the lowest energy solution for the bead passes from $\theta = \theta_0 = 0$ to $\theta = \theta_1 > 0$; and while $U(\theta_0)$ preserves the system's rotational and reflectional symmetries[3] with respect to the z-axis, $U(\theta_1)$ preserves neither, although the Lagrangian and the dynamical equations of the system preserve the symmetries on both sides of the critical point.

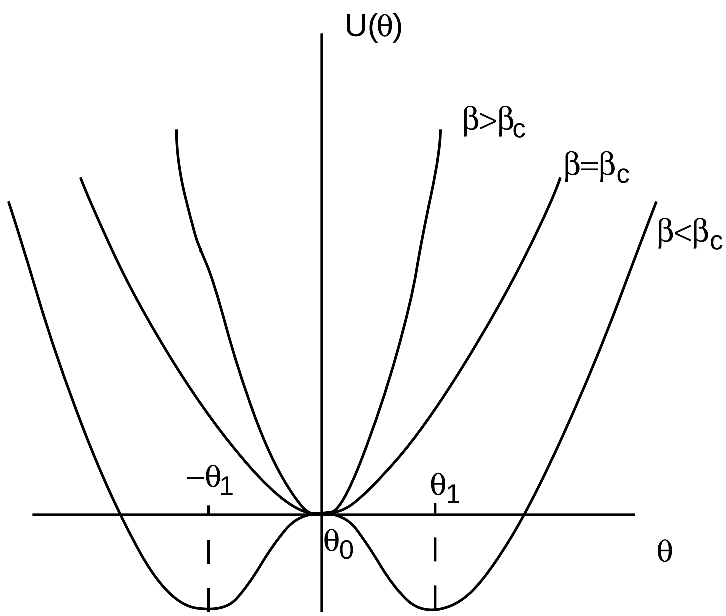

Figure 2. The dimensionless potential, $U(\theta)$, against θ. Note, $\beta = \omega^2 R / g$ and $\beta_c = \omega_1{}^2 R / g = 1$.

3. The meaning of SSB

The model described and the argument given in the previous section are to my best knowledge the simplest and purest case of SSB.[4] Moreover, it has most of the formal (e.g. mathematical) properties that the more important models of SSB in the literature have.[5] In another paper -- part II of the project, I will discuss the relevant similarities and differences between this model and the others, and it will show that the most basic philosophical questions about SSB can all be addressed in this model. Therefore, without further ado, let us begin to consider the questions in their (semi-) logical order.

1. Why is the symmetry breaking in the model an SSB rather than an ordinary symmetry breaking? Or what is an SSB?

A general answer one frequently sees in the literature, as mentioned in section 1, is that an SSB occurs in a system when its Lagrangian or dynamical law has certain symmetries which the ground state solutions do not preserve (cf.e.g. Anderson 1984, 266; Coleman, 1975, 141-142). To see what this claim means, let us examine our simple model to which the claim certainly applies. Given the model, the independent variables which determine the bead's states are, in polar coordinates, $\theta, \dot{\theta}; \varphi, \dot{\varphi} = \omega$,[6] which are, respectively, the vertical angle from the z-axis, the velocity; the horizontal angle of rotation, the angular velocity. Now φ does not explicitly appear in the Lagrangian, which implies that the Lagrangian has the rotational symmetry (i.e. the transformations: $\varphi \to \varphi' = \varphi + \alpha$, where α is an arbitrary angle do not affect the Lagrangian) (see note 3 for a caveat). The Lagrangian is such a function of θ and $\dot{\theta}$ that it is reflectionally symmetrical (i.e. invariant under $\theta \to -\theta, \dot{\theta} \to -\dot{\theta}$).[7] But from Figure 2 we see that $U(\theta_1)$ -- a clear example of a ground state -- has neither.

This is clearly different from the breaking of a symmetry when the Lagrangian in question contains a term that does not preserve the symmetry.

2. *But do all cases of non-spontaneous symmetry breaking have their symmetries broken by the presence of an asymmetrical term in the Lagrangian?*

What the Lagrangian represents may be regarded as the *lawlike properties* of a system, which include both its intrinsic properties and enduring interactions with its environment. This is why from its Lagrangian one can derive the law-equations of the system in question. However, a particular state of a system should be the result (or evolution) of its lawlike properties with a given set of initial and/or boundary (i/b) conditions (cf. Wigner 1979). It is certainly possible that a symmetry is broken not because of any asymmetrical lawlike properties -- which would show up in the Lagrangian -- but because of some special i/b conditions -- which would not (ibid.). A natural question then would be:

3. *is a symmetry breaking due exclusively to i/b conditions an SSB or not?*

It seems that the answer would have to be a 'no', for what we have in our simple model does not *appear* to be a case of symmetry breaking by any special i/b conditions. To see this point, just imagine that we begin with $\omega = 0$ and let it very slowly (e.g. adiabatically) increase. The argument in the previous section is taken to have told us that once ω passes the critical value, the bead will depart from O and ascend the wire and thus break the symmetries. Nowhere in this process is any asymmetrical i/b conditions introduced, and yet the symmetries are broken. Perhaps it is in this sense the symmetries are said to be truly spontaneously broken in this model (and presumably in all those other models that share this feature). Whether this is accurate is a question I shall return later.

Note that it is not a legitimate objection to simply say that because the model is idealized so that it does not tell us what happens in a real wire-and-bead system where any number of asymmetrical causal factors may exist to actually break the symmetries. The bifurcation of the lowest energy state (or ground state) beyond the critical point is a result for the model as it is given, idealized and totally symmetrical. Whatever asymmetrical factors a real system may contain cannot be responsible for the fact of this bifurcation, which is derived without even mentioning any such factors.

We now ask the question:

4. Is an SSB a symmetry breaking that is without any cause?

Since neither a lawlike property nor any i/b conditions of a system may be regarded as responsible for a true SSB, the answer may obviously appear to be affirmative. But the matter is not so simple, so let us approach it more carefully. First, van Fraassen (1989, chapters 10-11) discussed two general forms of what he calls 'the symmetry arguments.'

> There are two forms of argument which reach their conclusion 'on the basis of considerations of symmetry.' One, the symmetry argument proper, relies on the meta-principle: that structurally similar problems must receive corresponding similar solutions. A solution must 'respect the symmetries' of the problem. The second form, rather less important, assumes a symmetry in its subject, or assumes that an asymmetry can only come from a preceding asymmetry. (p. 233)

Since the second form is rather a strategy in theory constructions than a principle (and one may argue that it follows, or is strongly suggested by, the contrapositive of the meta-principle), I shall only consider the first form in connection with our question above. Let us then look at the following question:

4.1. Is the meta-principle -- structurally similar problems must receive corresponding similar solutions -- true among deterministic systems?

Here is another way of putting the meta-principle which makes it a bit more precise. The principle says that problems that can be transformed into one another by a symmetry must receive solutions that are related by the same symmetry. A symmetry is taken here to be a group of transformations that leaves what is transformed 'essentially the same' or 'the same in all essential properties. (ibid. pp. 235-236)' And we only consider deterministic systems because (i) our model is a deterministic system, or it is certainly regarded as one in the literature, and (ii) there is a sense in which the principle is already violated by indeterministic systems.[8]

Prima Facie, the answer is 'no' and the SSB in our simple model is sufficient for such an answer. The 'problems' in our model would be the different equations or Lagrangians connected by the rotational or the reflectional transformations given above (which are the two symmetries for the problem). If the principle holds, the ground-state solutions should also preserve the symmetries; but they do not in the region of $\omega > \omega_1$. Hence, the meta-principle fails.

Second, as mentioned in the introduction, some people argue that SSB is a misnomer; the phenomena it refers to should properly be called 'hidden symmetries.' The laws which govern the behavior of a system has one set of symmetries and the behavior another, usually lesser, set. Since what we measure is behavior, the symmetries of the laws are therefore *hidden* from us. Now the question:

4.2. Is there a breaking of symmetry in the case of hidden symmetries?

As our quote from Coleman in section 1 indicates, there seems to be nothing odd about this situation, or there is no reason why symmetries should not be so hidden. In other words, no symmetry is 'broken' in the so-called spontaneous symmetry breaking, and all the worry about an uncaused 'breaking' of something is a misguided exercise caused by an inappropriate name! If this is right, the answer to our previous question -- whether the meta-principle holds -- is still a 'no' since the symmetries are still not preserved; however, it seem to suggest that 'there is no reason why' such a principle should hold or why anybody should regard it a kind of violation to have symmetries in a problem (e.g. a Lagrangian) which are absent in its ground-state solutions.

In a limited sense this is just right for our model: when you look at the Lagrangian or the two equations (of balance) in the z- and y-directions, you see rotational and reflectional symmetries, and then when you look at the stable ground-state solutions with $\omega > \omega_1$, you do not see them any more. The solutions are mathematical results from the equations -- nothing suspicious are introduced to the derivation -- and yet the symmetries are hidden. However, if you look at the model and consider the movement of the bead as ω increases, something extraordinary does happen and hence a symmetry breaking in some sense is present. To repeat what I said earlier in response to the 3rd question, as the wire ring begins to rotate and very slowly increases its angular velocity, at first the bead sits at the lowest point of the ring, motionless; but as ω passes its critical value ω_1 the bead starts climbing the ring and thus breaks the symmetries. However one regards the symmetries of the Lagrangian or the laws in this model (e.g. as 'hidden' or otherwise), there is presumably a physical process during which the system goes from preserving them ($0 \le \omega \le \omega_1$) to not preserving them ($\omega > \omega_1$). Therefore, calling the symmetries 'hidden' rather than 'broken' does not make the legitimate quest for an explanation of why the bead sudden departs from O goes away. From a physical point of view, the ascending of the bead on the wire ring is caused by the centrifugal tendency of the bead; but how does the bead acquires a such a tendency while remaining motionless at the lowest point of the rotating ring? Hence the main question (question 4) is still with us.

Third, if there is a breaking of a symmetry, it might not be too far-fetched to ask: what then is its dynamics? So here comes the crucial question concerning the nature of SSB.

4.3. Is there a dynamic process for SSB (in general) and, if there is, do we, or are we likely to, have a theory for it?

In our model there does seem to have a dynamic process through which the bifurcation occurs (or the symmetries are broken). When we rotate the vertically suspended wire ring faster and faster, the bead would eventually depart from its symmetrical ground state and settle into an asymmetrical one. And we do have a theory which tells us when the bifurcation occurs and what the asymmetrical ground states are. As we shall see later that this is true for almost all models of SSB: a bifurcation which breaks the symmetry occurs when the values of some (controllable) parameter crosses its critical value.

And yet, I want to argue that there is no dynamics for SSB; or to put it another way: if by a dynamics we mean a set of laws governing the processes in question, then there is not such a thing for SSB.

First, one should not think that if there is ever a real system exactly like our simple model, then the bead, if it initially sits motionlessly at the lowest point of the wire ring, would ever depart from that point however fast the ring rotates. This is because it is one thing for that position (i.e. O in Figure 1) to no longer be the stable ground (or equilibrium) state for the bead when $\omega > \omega_1$ but quite another for the bead to actually move out of that position (and to be transferred into one of the new stable ground states). One can easily see by looking closely at Figure 2 that O is still an equilibrium state when $\omega > \omega_1$; only it is no longer a *stable* one. The difference between these two kinds of states is essentially that a system in the latter will return to it when subject to perturbations, while it in the former will not. In other words, if there are perturbations present in our model, the bead will not stay at O when $\omega > \omega_1$ but will stay at θ_1 or $-\theta_1$. Since, strictly speaker, there are no perturbations of any kind in our idealized model, there should not be any movement of the bead which causes the (spontaneous) breaking of any symmetries of its Lagrangian or its laws.[9]

Therefore, second, what the theory of our model tells us is really only the *possibility* of SSB, not what it is and how it happens. It tells us precisely under what condition there is a change of the stable ground state solutions to the problem given by the model; but it does not give us an account of the dynamic process of a system's actual breaking of the symmetry, namely, the system's making the transition from a symmetrical state to an asymmetrical one. The actual breaking can happen only if some asymmetrical causal factors are introduced to the model. Because of the instability of position O (or $\theta_0 = 0$), neither a sustained force nor a disturbance of measurable magnitude is necessary; small, and usually random, perturbations or fluctuations are sufficient to do the job. Because of the ubiquitous presence of such perturbations, people often neglect to mention them and sound as if an SSB may occur even in an idealized model as ours.[10]

Therefore, the dynamic process in our model which is parameterized by the angular velocity of the wire ring and which tells us a complete story of how the stable ground state solution of the system bifurcates from a single symmetrical state to two asymmetrical states is not the process by which an SSB occurs, if it actually occurs at all. The 'real' (or efficient) cause must lie among the perturbations present in a real wire-and-bead system, for which the theory of the system has no account. Now, to have a dynamics which deterministically accounts for a certain type of processes in physics means, in general, that a dynamic law (or a set of such laws) exists, which given the i/b conditions of the system in question at one instant of time determines its states at any other instants. We do not know any dynamic laws which tell us how random perturbations cause particular breakings of the symmetry, nor is it likely that we will ever discover such laws.[11]

Hence, there are two ways -- one may think of it as SSB_1 and SSB_2 -- of giving the notion of SSB a precise meaning: (i) it means that there are stable ground states that do not preserve some of the symmetries of the Lagrangian (or the laws); or (ii) it means that any actual breaking of a system's symmetry can be achieved by random perturbations. Of course, had the symmetrical ground state not become unstable, perturbations would not have been capable of breaking the symmetry. Or to put it in another way:

> *the difference between a spontaneous symmetry breaking and a non-spontaneous one is that the former can be broken by perturbations while the latter cannot, even though perturbations are present in both cases.*

With this much said, we can now answer our main question (i.e. question 4): whether SSB is uncaused. The answer appears to be negative, if I am right so far.

Someone may object to the answer as follows. Is it not true according to my own account of SSB that even without taking into account of the random perturbations, the symmetry is broken in the solution of a problem that preserves it in the sense that the symmetrical solution is not stable while the stable one is not symmetrical? In other words, would not the demonstration -- without introducing the perturbations -- that an increase of ω across its critical value produces a bifurcation of the ground state be sufficient for a genuine symmetry breaking?

The point would be justified if an interesting feature of the ground states which are suppose to comprise the 'symmetry breaking' solution is not present. From Figure 2 one can clearly see that the ground states when $\omega > \omega_1$ always come in pairs: $(\theta_1, -\theta_1)$, which taken together is reflectionally symmetrical, and if one imagine a 3-d extension of the figure, i.e. with $0 \le \varphi < 2\pi$, one can see that the solution is also rotationally symmetrical. This is what physicists call a (continuously) degenerate solution, meaning that from a formal point of view any of these infinite number of ground states -- $< \theta_1, \varphi >$ -- is a possible stable ground state. Of course, each of these states is asymmetrical, but taken together the solution, which is what our highly abstract model offers us, still has a symmetrical 'shape.' Hence, again, the formal argument from our model only provides us with the *possibility* of symmetry breaking (in the ground-state solutions).

Do all cases of SSB have this feature? To my best knowledge they do. This is a remarkable feature of SSB: it shows that even with the possibility of symmetry breaking, an SSB respects at some level the symmetries in the problem. In fact, one should expect SSB to have this feature, for otherwise it would not be consistent with the fact we realized earlier that an actual breaking of the symmetry is caused by some random perturbations. If the ground states do not form a degenerate set whose members are transformable from one to another by the same symmetry that each is supposed to break, it would be a genuine puzzle as to how random perturbations could produce a set of non-randomly distributed possibilities. For instance, if in our model, state $U(\theta_1)$ is possible but $U(-\theta_1)$ is not, then it would be a puzzle as to why the perturbations in the direction of θ_1 can cause the bead to ascend the wire ring while the perturbations in the direction of $-\theta_1$ can not; or it would be equally puzzling why there are perturbations in the direction of θ_1 but none in the direction of $-\theta_1$, given the symmetries assumed in the model. On the other hand, if it ever turns out that there are such SSB's, then the metaphysical landscape of physics would be radically changed. So far, we do not see such a change.

In connection with van Fraassen's meta-principle or to visit question 4.1 again, we may say the following. 'No asymmetry in, no asymmetry out' holds in spite of the existence of SSB. If we banish the random perturbations (which we do for all idealized models), the 'broken symmetrical' solution comprise a set of degenerate ground states, each of which breaks the symmetry but all of which together preserve it. If we count the random perturbations, then 'no asymmetry in' is no longer true.

And in connection with our question (3) of whether SSB may be a case of symmetry breaking by i/b conditions, we now see that the answer will depend on our conception of random perturbations or fluctuations. According to Poincaré (1952, 64ff), being the results of random perturbations is what it means to be the results of chance.[12] Along this line, we should not count in i/b conditions such things as random perturbations, for it clearly does not fit the schematic division of dynamical laws and i/b conditions that Wigner (1979) has emphasized. And hence, SSB is not a case of symmetry breaking by i/b conditions. However, this short remark does not do justice to this rich subject about chance and perturbations, which I will take up in another paper (see note 11).

Finally, one may think that we also deserve an answer to the question: why should the increase of the angular velocity in our model, nowhere during which is any asymmetry introduced, 'cause' a transition from a ground state, $<\theta_0, 0>$, which makes an SSB impossible to a set of ground states, $<\theta_1, \varphi>, 0<\varphi<2\pi$, which makes it possible? It is not at all clear whether an answer to this question is possible. What would be the answer like which says more than something to the effect that the lawlike properties of the models tell us -- by entailment -- that it is so. Since, as we argued earlier, there is a sense in which no symmetry is broken from this transition, namely, each $<\theta_1, \varphi>, 0<\varphi<2\pi$ is of equal chance, either the question does not make proper sense or the answer is simply this: the transition is caused by the crossing of the angular velocity over the critical value.

In contrast, if it turns out that not all $<\theta_1, \varphi>, 0<\varphi<2\pi$ are of equal chance, then a quest for a causal explanation, *with some asymmetry in the cause*, will be justified, and we are again under the spell of van Fraassen's meta-principle.

4. Conclusion

From a simple classical model we are able to see that what people usually refer to as SSB has in fact two different meanings: SSB_1 means having stable and degenerate ground states each of which taken singly breaks the symmetries of the Lagrangian (or the laws); and SSB_2 means that the breakings of symmetries can be done by random perturbations or fluctuations; and also how in such phenomena, a symmetry in question is and is not broken, depending on which level one is considering them. Thus, some of the seemingly puzzling remarks, some of which I sampled in section 1, can now be seen as the result of confounding these two meanings of SSB. When people argue that SSB is a misnomer and that it should be called 'hidden symmetry,' they are thinking of SSB_1 but not SSB_2, for it is not proper to call a symmetry hidden if SSB_2 is meant. The metaphorical image of objects 'choosing' to break a symmetry without apparently having a sufficient reason can now be understood as alluding to the fact that, according to SSB_2, the symmetry is not broken by anything that can be counted in physics as a proper cause. Of course no object really makes a decision in such cases; but analogous to the dinner guest who first picks up his/her spoon or Buridan's ass which goes for one of the stacks of hay, the move the object makes into one of the symmetry-breaking ground state is arbitrary. Other questions, such as whether the size of a system has anything intrinsically to do with SSB and whether SSB demonstrate the existence of emergent properties, cannot be answered by this analysis. For that one may want to read the other paper -- part II of this project.

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[1] Simon Saunders posed this question in a roundtable discussion (Cao 1999, 382-383). Anderson (1997, 50-51) gives a short and sketchy comment on the connection.

[2] I excludes $\theta_1 = \pi/2$ because only when $\omega \to \infty$ do we have $\theta_1 \to \pi/2$.

[3] Strictly speaking this wire-ring model only has reflectional symmetry, which is sufficient for our discussions in the rest of the paper. But I shall include rotational symmetry, because one can easily extend this model to a spherical system by sweeping the wire-ring once around the z-axis; or think of a sphere inside which the bead can only move frictionlessly along a great circle. This makes the connection with the other models a lot more transparent.

[4] For other related mechanical models in which spatial symmetries are spontaneously broken, see Alben 1972, Aravind 1987, Johnson 1997, Racz and Rujan 1975, Sivardière 1985, Sivardière 1997. Much more widely mentioned, but seldom explicitly demonstrated, cases of SSB include Euler's rod, which buckles under a perfectly vertical load, and Poincaré's rotating 'nebula' in self-gravitating equilibrium, which separates into a two-body system when a critical value of its angular velocity is passed.

[5] The main difference between the mechanical models and those in statistical mechanics is that the latter deal with many-particle systems which, under thermodynamic limit, have an infinite number of degrees of freedom; and the main difference with the models of quantum gauge fields is that there the symmetries in questions are no longer spatial symmetries but rather gauge symmetries of internal spaces.

[6] Because of the wire's constraint, the bead has only 2 degrees of freedom, and therefore 4 independent coordinates.

[7] The Lagrangian for our model is:
$L = T - V = (1/2)mR^2\dot{\theta}^2 + (1/2)m\omega^2 R^2 \sin^2\theta - mgR(1-\cos\theta)$ (Greenberger 1978).
The sine and cosine functions are symmetrical with respect to θ and -θ.

[8] See the discussion on pp. 239-240 of van Fraassen 1989; also see pp. 250-257 for his discussion of the connection between the symmetry principle and determinism. Ismael (1997) argued against van Fraassen and concluded that Curie's principle holds in the indeterministic contexts as well (pp. 176-178). However, such a conclusion derives from Ismael's particular reading of the principle, which is more restrictive than van Fraassen's. I regard Ismael's reading of the principle a bit too narrow, but I will not argue the point in this paper; see also note 11 below.

[9] Poincaré (1952, 67) makes the same point on a model of a stationary cone balanced on its point.

[10] To my best knowledge, there are two exceptions: Radicati 1987 and Ismael 1997, both are discussions of Curie's principle of symmetry and each contains a brief section on SSB. Both in broad outlines give the right analysis of SSB but reach opposite conclusions as to whether Curie's principle is violated by, inter alia, SSB. Radicati: yes; while Ismael: no. On SSB itself, Radicati states that to be an SSB 'two conditions' must be 'satisfied: (i) the system is nonlinear and possesses bifurcation points where a set of stable solutions of lower symmetry branch off the original symmetrical solution; (ii) the system is subject to external chance perturbations. (p. 204)' Similarly, Ismael writes, 'In general, if a system is non-linear and possesses bifurcation points where a set of stable solutions of lower symmetry branch off from the original symmetrical solution and the system is subject to external chance perturbations, a very small chance perturbation may switch the solution to an asymmetrical one. (p. 180)' It is not clear whether SSB can only occur in nonlinear systems or what is meant by 'external' in 'external chance perturbations.' In quantum field theories with regard to gauge symmetries it should be possible to have linear models that contain SSB and random perturbations may well be internal to the system in question.

[11] This point is connected to the relationship between SSB and chance (or indeterminism), which will be the topic of another paper soon to be completed. The upshot is this: whenever the final states are the results of SSB, which state among them the system in question will end up is a matter of chance. See e.g. Poincaré (1952, 64ff), van Fraassen (1989, 233ff) and Ismael (1997, 176-178) for discussions of the relations among symmetry breaking, chance, and indeterminism.

[12] Poincaré's notion of chance is weaker; it takes all effects to be chancy if they are the results of very small, perhaps imperceptible, causes, whether or not such causes are randomly distributed.