SPONTANEOUS SYMMETRY BREAKING (II): Variations in complex models

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Abstract

This paper, part II of a two-part project, continues to explore the meaning of spontaneous symmetry breaking (SSB) by applying and expanding the general notion we obtained in part I to some more complex and, from the physics point of view, more important models (in condensed matter physics and in quantum field theories).

1. Introduction

In another paper[1] -- part I of the same project -- I have examined the concept of spontaneous symmetry breaking (SSB) from a simple mechanical model -- the ring-bead model, the result of which can be summarized as follows.

1. Two different meanings exist for what people call 'spontaneous symmetry breaking' or 'spontaneously broken symmetry'. One meaning, SSB_1, refers to the fact that a system has *stable* and *degenerate* ground (i.e. lowest energy) states, each of which breaks the symmetry(-ies) of the Lagrangian (or the dynamic law); and the other, SSB_2, that the breakings of symmetries are caused by nothing more than random perturbations.

2. The two meanings distinguish the *formal/mathematical* aspect of SSB from its *causal/physical* aspect. The idealized models usually display to us the former, which via rigorous mathematical arguments show us the *possibility* of an SSB, while the latter aspect tells us via physical arguments what must happen to or in a system for an SSB to actually take place there. From the formal arguments we typically see a *one-parameter* controlled dynamical process in which the crossing of a critical value of the parameter produces a *bifurcation* or a *spread* of the original ground state into a set of degenerate ground states, which together preserve the symmetry of the dynamics but singly breaks it, so that the system when nudged by a particular perturbation, however minute, may fall into one of such states.
3. No actual breakings in SSB are therefore uncaused -- if spontaneity is mistook for such -- for the perturbations are the antecedent asymmetries that cause them, and yet the *possibility* of SSB does emerge without any causes in the form of antecedent asymmetries.

I have also argued there, on general grounds, that this result must hold in general for all cases of SSB. But modifications and limits will have to be introduced, as we shall see in this paper. The main aim of this part II is to see what, if any, we need to add to our understanding of the concept by going through some of the most intensely discussed models of SSB in the areas of condensed matter physics and high energy physics.

I will then be able to approach the remaining questions I asked in the introduction section of part I: why is the size of a system relevant to whether an SSB takes place in it? Is there a place of arbitrary choice in some SSBs? Does the existence of SSB provide a straightforward argument for 'emergentism'? And does our understand of SSB help us to understand quantum measurement?

2. Phase transitions as SSB

One of the main concerns in condensed matter physics is to understand how matter makes the transition from one phase to another and what happens exactly at the transitional regions. The study of how liquid turns into gas when enough heat is introduced and what happens at boiling is an example of it, so is the study of magnets making the transition from paramagnetic to the ferro- (or antiferro-) magnetic phase, and so are the studies of transitions from a normal fluid to a superfluid and from a normal conductor to a superconductor. the former two types, as shall be made clear later, can be seen as SSBs of external symmetries, and the latter two, internal symmetries (e.g. gauge symmetries).

First, ferromagnetism. Experimental results show that some metals have the property that when at relatively high temperatures, the magnetization of them by an external magnetic field, B, disappears when the field is withdrawn, but when the temperature drops below a certain 'critical value', T_c, a nonzero magnetization remains even when B is reduced to zero. We call the former the paramagnetic, and the latter the ferromagnetic, phase; and it is further noted that the phase transition region at T_c has a singularity in the sense that the magnetization as a function of B develops a discontinuity at $B = 0$: the value of the function switches from $+m(\neq 0)$ to $-m$ when it goes from $B \to 0^+$ to $B \to 0^-$ (cf. Stanley 1971; Goldenfeld 1992; Liu 1999).

Two types of theories are used to account for phase transitions: the *mean-field* models, which assume micro-structure but only deal with averaging effects, and the *lattice* models, which deal with idealized but truly microscopic processes.

Different mean-field models are devised, some for particular systems; but there is a generic model, which I will call the *Landau model* ('the Landau-Ginsburg theory' in some literature), that aims at covering all phase transitions (cf. Goldenfeld 1992). Assuming that the crucial independent variables for the study of phase transitions are (besides the temperature) the *coupling constants* -- representing the nature and intensity of interactions between a system's constituents -- and the *order parameter* -- representing the transition between an ordered and a disordered state (the generic feature of all phase transitions), the Landau model seeks to construct a function -- the Landau free energy density, L -- of these two variables. Assuming again that the order parameter is small, one may consider the expansion of L in terms of it only up to its 4th power; and then through some general considerations, such as of symmetry, one sees that all the coefficients of odd powered terms vanish; and hence, we have,

$$\mathrm{L} = a_0(J,T) + a_2(J,T)\eta^2 + a_4(J,T)\eta^4 \,,$$

where J is the coupling constant, T temperature, and η the order parameter. After some general analysis of the coefficients, we have a simplified form:

$$\mathrm{L} = a(J)t\eta^2 + \frac{1}{2}b(J)\eta^4 \,, \tag{1}$$

where, $t = (T - T_c)/T_c$, $a > 0$ and $b > 0$ are two constants (given a fixed coupling constant, J), whose values are left for experiments to determine.

If η is a scalar, then L, when $B = 0$, is invariant under reflection: $\eta \to -\eta$, while its ground state (the lowest free energy state) is not necessarily so. To see this, we look for the minimum of L, namely

$$\partial \mathrm{L}/\partial \eta = 2\eta(at + b\eta^2) = 0 \,.$$

From this some familiar results follow:
(i) for $t > 0$ (i.e. $T > T_c$), $\eta = 0$ is the only ground-state solution, but
(ii) for $t < 0$ (i.e. $T < T_c$), we also have the symmetry breaking solution, $\eta^2 = -(a/b)t$ (or $\eta = \pm\sqrt{-(a/b)t}$).
These are exactly analogous to those of the mechanical model we saw in part I.

If the order parameter is a vector, e.g. $\eta = (\eta_1, \eta_2, \ldots \eta_n)$, it can represent systems of continuous symmetries, such as of n-dimensional rotations. Equation (1) is invariant under a group of n-dimensional rotations, and solution (ii) breaks this symmetry. This is analogous to the case of Poincaré's inverted cone we discussed in part I (see also figure 1).

To apply this model to the phenomenon of ferromagnetism or antiferromagnetism, one only needs to identify the order parameter as the total magnetization **S** (i.e. $\eta = \mathbf{S} = (S_1, S_2, S_3)$) and the coupling constant as the short-range interaction between neighboring spins.

Unfortunately, the Landau model is an defective theory both quantitatively and qualitatively. Quantitatively defective because it gives the wrong numerical values for such quantities as the critical exponents, which tell us the behavior of a system arbitrarily near the critical temperature. And qualitatively defective because it does not give us a microscopic account of the order parameter.

Were it the right model for phase transitions, it would enable us to say the following, given its striking similarity with the mechanical model (cf. table 1)[2]. It is the random perturbations of the order parameters -- whatever they denote -- that cause a system to make the transition, with equal probability, from one (symmetrical) to another (symmetry-breaking) phase.

Table 1. The formal parallelism between the two models

Mechanical model (ring-bead system)	**Landau model** (known to be defective)
Extremum equation: $\partial V / \partial\theta = \sin\theta(1-\beta\cos\theta) = 0$.	Extremum equation: $\partial L/\partial\eta = 2\eta(at + b\eta^2) = 0$.
When $\beta < 1$, $\theta_0 = 0$.	When $t > 0$, $\lvert\eta_0\rvert = 0$.
When $\beta > 1$, $\theta_0 = \pm\theta_1 \neq 0$.	When $t < 0$, $\lvert\eta_0\rvert = \eta_1(t) \neq 0$.
SSB of reflectional (or rotational) symmetries.	SSB of continuous rotational symmetries.

A simple but still realistic microscopic model that can accurately account for ferromagnetism is the Heisenberg model; and it can be reduced to an even simpler model -- the Ising model, which figures in most of the rigorous arguments in the literature (cf. Thompson 1972; Goldenfeld 1992). Below are the basics of the model:

(i) a lattice of N fixed sites of equal distance, each of which is occupied by a particle of certain spin, $\mathbf{s}_i$.

(ii) an interaction between any two sites are given by the term, $J_{ij}\mathbf{s}_i \bullet \mathbf{s}_j$, where, i *or* $j = 1,2,...,N$, and $J_{ij} \neq 0$ for specified sites (e.g. nearest neighboring sites) and $J_{ij} = 0$ otherwise. ($J_{ij} > 0$ for ferromagnets and $J_{ij} < 0$ for antiferromagnets.)

The Hamiltonian of the system, which is its total energy, is an expression of the following kind:

$$H_N \propto \sum_{<i,j>} J_{ij} \mathbf{s}_i \bullet \mathbf{s}_j \qquad plus \qquad B \sum_i s_i^B \,, \tag{2}$$

where $<i,j>$ means pairs of sites with specified relations only, B is the external field, and s_i^B the spin component in B's direction.[3]

Next, then, is the partition function:

$$Z_N = \sum_{[\mathbf{s}_1]} \ldots \sum_{[\mathbf{s}_N]} exp(-H_N / kT) \,, \tag{3}$$

(where k is the Boltzmann constant and T the temperature and each summation is over all the values of each spin). It is the sum of $exp(-H_N / kT)$ for every possible value of the spin on every site in the lattice.

If this function, *per impossibile*, is computed, we can then recover all thermodynamically interesting quantities with well-established rules; for instance, the most important in our case are the fee energy,

$$F_N = U_N - TS_N = -kT \ln Z_N \,, \tag{4}$$

where U_N is the internal energy and S_N the entropy; and the magnetization per site,

$$m_N = \frac{1}{N} \sum_{i=1}^{N} \langle \mathbf{s}_i \rangle = -\frac{1}{N} \frac{\partial F_N}{\partial B} \,. \tag{5}$$

Then one can plot a set of curves in the $m_N - B$ space parameterized by the temperature T, and the curves ought to agree, within the limit of approximation, with the empirical generalizations from experiments. Especially, one should find the singularities in those curves with $T < T_c$, as I mentioned above.

If this were true, the phase transition models would be no more puzzling, or interesting, to theorists than the simple mechanical models we saw in part I. We have no such luck. First, as represented by the Heisenberg model (or any other lattice models), a finite system can be proven to harbor no phase transitions in that no singularities can possibly be found in those with $T < T_c$ even if an exact calculation of (5) is obtained. Second, it is then proven that if one takes the model system in question to the *thermodynamic limit*, the singularities reappear. I now explain these points in some depth in turn.

First, no phase transitions can appear in finite systems. There are two (types of) arguments for this claim, one (cf. Griffiths 1972: 50-55) uses a simplified version of our model and show that its partition function when expanded as a polynomial with a finite number of terms is *everywhere analytic* (i.e. differentiable to at least the first few degrees), and the other (cf. Goldenfeld 1992: 49-52; Griffiths 1972: 59-63) uses a general symmetry argument, which I give a sketch below.

The symmetry in question is the symmetry of time-reversal, $\mathbf{T}$, which not only flips every spin but also $\mathbf{B}$; in other words,

$\mathbf{T}: (\mathbf{s}_i, \mathbf{B}) \rightarrow (-\mathbf{s}_i, -\mathbf{B})$. If we apply time reversion to a finite Heisenberg system, we have from (2) to (5) the argument as:

$$H_N(\mathbf{B}, J, \{\mathbf{s}_i\}) = H_N(-\mathbf{B}, J, \{-\mathbf{s}_i\})^4 \;\Rightarrow\; Z_\Lambda(\mathbf{B}, J, T) = Z_\Lambda(-\mathbf{B}, J, T) \;\Rightarrow\; F_N(\mathbf{B}, J, T) = F_N(-\mathbf{B}, J, T)\,.$$

And then from (5) we have,

$$m_N(-B) = -\frac{1}{N}\frac{\partial F(-B, J, T)}{\partial(-B)} = -(-\frac{1}{N}\frac{\partial F(B, J, T)}{\partial(B)}) = -m_N(B)\,. \qquad (6)$$

Obviously, $m_N(0) = -m_N(0) = 0$, namely, the magnetization with the absence of external field must vanish, which means that there cannot be any spontaneous magnetization at any non-zero temperature. This is a general result since the only condition, besides the system being described by a finite Heisenberg model, is the application of a time-reversal operation. It also shows that any phase transitions of this type must break the time-reversal symmetry of the Hamiltonian.

Second, phase transitions appear in thermodynamic limit (cf. Ruelle 1969; Goldenfeld 1992; Emch & Liu 2002). Taking the thermodynamic limit of a system, such as a Heisenberg system, is an act of idealization that takes, in a well-controlled manner, the volume V and the number of sites N of the system to infinity with the assurance that the density N/V remains finite. Its justification aside (cf. Liu 1999), its benefits are numerous: not only singularities (re-)appear in the limit, but one also gets, among other details of the transitions, the correct numerical values of the critical exponents.

The reason that a finite system fails to exhibit a phase transition is, as I mentioned earlier, that its free energy F_N or $f_N = F_N/N$ (the free energy per site) is analytic. When the thermodynamic limit is taken, $f_\infty = \lim_{N\to\infty}(f_N)$ is still a continuous function of its independent variables (i.e. B, J, T), but it may not be analytic, e.g. it may be discontinuous at certain values of, say, B, such that at those values the left and right limits of the derivative, $\partial f_\infty/\partial B$, are not equal. One of such points is $B=0$ at any $T<T_c$, and we have

$$m_0^+ = \lim_{B\to 0+} \partial f_\infty/\partial B \qquad \neq \qquad \lim_{B\to 0-} \partial f_\infty/\partial B = m_0^- \qquad (7)$$

But from (5) we have[5]

$$m_0^+ = -m_0^- \neq 0\,, \qquad (8)$$

which means the appearance of spontaneous magnetization or the SSB of time-reversal symmetry.

This is no more than a possibility argument; however, to calculate any quantity, such as the value of T_c or of one of m_o^+ and m_o^- in Heisenberg model, is next to the impossible. Simpler models have to be used and among them the Ising model is the one used most often. An Ising model is a Heisenberg model with the restriction: $\mathbf{s}_i \to (1/2)\sigma_i$, where σ_i is the Ising spin and has only two possible values: $\{+1,-1\}$ or {up, down};

and the Onsager transform-matrix method in giving a rigorous study of the model is one of the most celebrated results in contemporary condensed matter physics. It shows, *inter alia*, that a 1-dimensional Ising model harbors no phase transition if the inter-site interactions are between nearest neighbors only or decrease exponentially (the two most plausible kinds) -- phase transitions are dimension-sensitive -- and that the 2-dimensional Ising model with a nearest-neighbor interaction is perhaps the simplest model that exhibits a phase transition. However, as we are frequently reminded in the literature, the Onsager solution does not quite solve the problem (cf. Emch & Liu 2002: §12.2), for to do that the calculation has to handle not one but two limits, one is the thermodynamic limit, $N \to \infty$, and the other the limit of $B \to 0^+$ or $B \to 0^-$ (because magnetization is conceptually understood, as mentioned earlier, as the *remaining* magnetic moment of a system when the external field B is reduced to zero). The Onsager solution handles with complete rigor the former but not the latter because it is not yet possible to calculate the magnetization of a 2-d Ising model at $B \neq 0$. However, one can calculate another closely related important quantity of the model, the correlation function of distant spins at $B = 0$ and below the critical temperature, and then use its direct relation to magnetization to obtain latter's values. So finally we have the result:

$$m_o^{\pm} = \{ \begin{array}{ll} \pm M(J,T), & T < T_c \\ 0 & T > T_c \end{array} \qquad (9)$$

where $M(J,T)$ is a complex non-zero function.

We are yet to see a way of calculating with the same degree of rigor a 3-d Ising model, which is closer to ordinary ferromagnets, although a 2-d model is by no means less 'real' than the 3-d model because both are equally idealized.

This concludes my sketch of the (rigorous) formal arguments for the possibility of phase transitions (as manifested in ferromagnetism). When one realizes that the $M(J,T)$ in (9) is very similar to $\eta_1(t)$ in table 1, one may think that the above is an unnecessary long detour; but it is a detour in which the truth about phase transitions (up to a certain degree of idealization) is revealed. In other words, whether the Landau model may be on the right track is something without the detour no one can judge. Moreover, the formal analogy between the Heisenberg model and the mechanical model is still strong: both involve a one-parameter controlled dynamical process, which has a critical value for its parameter, beyond which a bifurcation (or a spread if the symmetry is a continuous one) occurs and which leads to some new, *degenerate* ground states that together preserve the symmetry of the Hamiltonian (or Lagrangian) while separately break it.

Should we expect that the analogy holds at the causal level as well? Now that we have the exact, albeit idealized, micro-structures -- unlike in the case of the Landau model -- we should be able to answer this question. If the formal analogy with the mechanical model is not superfluous, the SSB here ought to be caused by some kind of perturbations (as understood in condensed matter physics), and if the Landau model is not totally on the wrong track, the perturbations must have something to do with whatever the order parameter refers to.

A ground state of a condensed-matter system is one that has the lowest *free energy* (not the Landau free energy), which, as seen in (4), is defined as $F = U - TS$, (unlike the case in mechanics where the ground state is the lowest *energy* state). Without thermal agitation (e.g. if, *per impossibile*, $T = 0$), stationary spins on a lattice have a tendency to align themselves in the same direction. (Consider the interactional energy term $-J\sigma_i\sigma_j$ in a 1-d model. If both spins are in the same direction: $\sigma_i = \sigma_j = -1/+1$, the energy is then $-J$, but if they are in opposite directions, $\sigma_i = -\sigma_j$, then the energy is J, which is greater.) The ground state would be one in which all the spins point to a single direction, which in an infinite system is referred to as having a 'long-range order.' For $T > 0$ and as it increases, the free energy ($F = U - TS$) becomes smaller (S usually increases with T). In a 2-d (or higher dimensional) Ising system, for instance, the contention between the interactional tendency (as in U) to align and the thermo-motion[6] (as in TS) to dis-align results in a two-phase pattern for $T > 0$ with a phase transition at T_c. Above T_c, the thermo-motion of the spins wins over the tendency for alignment such that the ground state becomes a paramagnetic one, while below T_c, the opposite holds such that the ground state becomes a ferro- (or antiferro-) magnetic one. This explains qualitatively why the magnetization in $T_c < T < 0$ (or the long-range order) increases its magnitude as the temperature drops.

A puzzle seems to arise: the mechanical case seems to show that the thermo-motion inside a system is the source of perturbations (or fluctuations) that cause the actual SSB, while here the presence of thermo-motion seem to 'cause' the breaking-up, rather than the formation, of long-range orders, which are responsible for the SSBs and new phases.

To resolve this puzzle involves the identification of the right kind of perturbations that does the breaking of the relevant symmetry. In the case of ferromagnetism it is the inter-spin interaction that is responsible for the *formation* and *maintenance* of long-range orders; but it is not a randomly distributed element in the system, nor is it necessarily small; hence it cannot be identified as the source of perturbation (or fluctuation) by any stretch of the latter's meaning. However, the interaction is only the *potential*, but not the *actual*, cause of the presence of long-range orders. The thermo-motion $T > 0$ is the constant presence, against which we may conceive the SSBs in ferromagnetism either *negatively* as the lack of disruption of residual magnetization when $B \to 0^+$ or $B \to 0^-$ or *positively* as the chancy formation of long-range orders at $B = 0$. The latter is fully support by the rigorous solution of Onsager for the 2-d Ising model.

When $T > T_c$, the thermo-motion is so strong that any incidentally formed cluster of aligned spins will be destroyed long before it can reach the critical size from which an actual long-range order can grow. As $T \to T_c$ and with $B = 0$, it becomes physically possible for the clusters to get over the critical size. However, the thermo-motion is still strong enough to make it a matter of chance, namely, the probability of clusters getting over the critical size directly depends on the random thermo-motion that tends to prevent such. Nor is it possible to quantitatively estimate what the critical size of such clusters is or in which direction (in 3-space) its net spin points. Therefore, as in the mechanical case, when $T < T_c$, the condition of the system is such that the formation of long-range orders is possible, while which order actually obtains depends on which cluster of aligned spins grows by chance over the critical size.

Therefore, there are at least two different ways that a transition to a ferromagnetic phase can take place: one is to take the system below the critical temperature with the external field present, and then diminish the field until it vanishes. The system, for lack of the thermo-energy to destroy the long-range order established by the field before it vanishes, will remain in that ferromagnetic phase; and the other is to decrease the temperature as above without the presence of the external field. When the temperature goes below T_c, perturbations in the form of the emergence of spin-aligning clusters in the system appear, which eventually cause the occurrence of long-range orders. In the former case, the direction of the magnetization is determined by the external field, while in the latter, it should be a matter of chance. Strictly speaking, only the latter can be regard as a case of SSB.

3. The spontaneous breaking of gauge symmetries

What remains to be discussed is the largest class of complex and recondite models of SSB which has attracted a great deal of attention in recent decades. Some models of this class belong to the condensed matter physics and some to the high energy physics; what identifies them is that they are the result of what is referred to by this section's title.

I begin with a generic case of the spontaneous breaking of a gauge symmetry in a quantum field ($\phi(x)$) of the form:

$$\phi = \frac{1}{\sqrt{2}}(\phi_1 - i\phi_2) \text{ and } \phi^* = \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2),$$

where ϕ_1 and ϕ_2 are two real scalar fields, and which may represent a charged particle-field of zero spin. It is well-understood that quantum fields can be studied as classical fields for certain of their properties and then as quantum fields for others. Very roughly (cf. Itzykson & Zuber 1980; Huang 1998) before quantization $\phi(x)$ behaves very much like a classical field potential that obeys the relativistic quantum equations of motion, where $|\phi(x)|^2$ tells us the field strength at x, and then upon quantization it becomes an operator in the Fock space as functions of creation and annihilation operators that acts on vacuum state, $|0\rangle$, to yield quantum excitation states at x. The field is supposed to be of infinite extension, so that some regard quantum fields as infinite many-body systems (cf. Strocchi 1985; Martin & Rothen 2002). For the classical part, we have its Lagrangian consisting of two parts: the dynamical and the interactional: $L = K - V$, where V, the interactional potential, is a function of $|\phi|^2 = \phi^* \phi$, which when expanded has the following form (cf. Ludwig & Falter 1996: 374):

$$V(|\phi|) = \frac{\varepsilon}{2}\mu^2 |\phi|^2 + \lambda |\phi|^4, \qquad (10)$$

where $\varepsilon = \pm 1$.[7] (I will return to discuss the meanings of $\mu^2 > 0$; and for the potential to remain bounded from below, it must be that $\lambda > 0$).

Very similar to the mechanical case in part I, we know that the lowest energy state (= the ground state) of the field is when

$$\left.\frac{\partial V}{\partial \phi}\right|_{\phi=\phi_0} = 4\phi_0 * (\frac{\varepsilon}{4}\mu^2 + \lambda |\phi_0|^2) = 0\,; \qquad (11)$$

(i) when $\varepsilon = +1$, the only solution is,

$$|\phi_0| = \frac{1}{\sqrt{2}}\sqrt{\phi_{10}^2 + \phi_{20}^2} = 0\text{; and hence, } |\phi_{10}| = |\phi_{20}| = 0$$

(ii) when $\varepsilon = -1$, there is another solution:

$$|\phi_0| = \frac{1}{\sqrt{2}}\sqrt{\phi_{10}^2 + \phi_{20}^2} = v = \frac{1}{2}\sqrt{\mu^2/\lambda} \neq 0\,,$$

which means that the potential has its minimum, (ϕ_{10}, ϕ_{20}), at any one of the points on the $\phi_1 \sim \phi_2$ plane with a radial distance of $|\phi_0| = v$ from the origin (see figure 1), and which implies that the ground state is

$$\phi_0 = v e^{-i\alpha}\,, \qquad \alpha = \text{const.} \qquad (12)$$

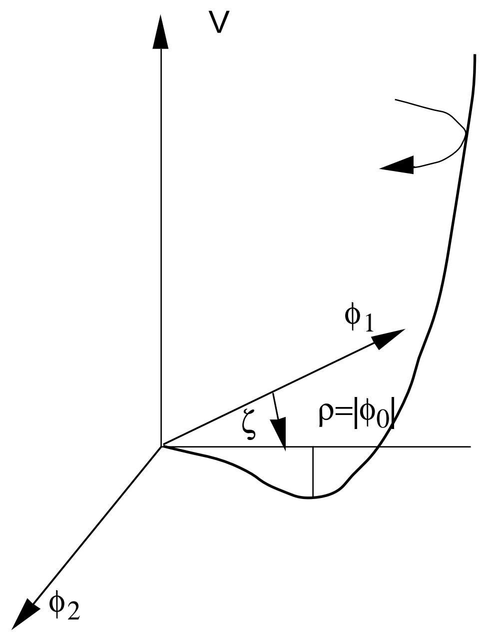

Figure 1: the potential for $\varepsilon = -1$. The minimum is at any point of (ρ, ζ), where $\rho = |\phi_0|$ and $0 \le \zeta < 2\pi$ (i.e. imagine the result of the shown curve sweeping 2π).

We can now see that while the potential (10) is invariant (also is the Lagrangian) under the group (U(1)) of gauge transformation: $\phi \rightarrow \phi' = \phi e^{i\theta}$, while the ground state in solution (ii) (i.e.(12)) is not, i.e. $\phi_0' = \phi_0 e^{i(\theta-\alpha)} \neq \phi_0$.

These results can be directly translated into quantum field results when $\phi(x)$ is canonically quantized (i.e. when $\phi(x)$ becomes an operator)[8] and is brought to act on the vacuum state, $|0\rangle$, of zero number of particles as the ground state of the field. Therefore, in solution (i) we have $\langle 0|\phi_0|0\rangle = 0$, and in solution (ii), $\langle 0|\phi_0|0\rangle = v = (1/2)\sqrt{\mu^2/\lambda}$, which implies that $\phi_0|0\rangle = ve^{-i\alpha}$, containing an arbitrary phase factor which breaks the gauge symmetry seemingly without any cause. I will return to discuss the meaning of this SSB later; for now it is a formal result (cf. Bernstein 1974).

As in the mechanical and ferromagnetic models, the symmetry-breaking ground states also appear to be infinitely degenerate in this case, with different values of α in (11) such that the gauge transformations take one such state into another. However, there is a possible complication for quantum systems (cf. Weinberg 1996: 163-167) because of the possibility there of superposed states and quantum tunneling; for instance, if $\phi_0 \neq 0$ and $-\phi_0$ are the two symmetry-breaking vacuum states, why should one believe that the 'real' ground state is one of them but not a superposition of the two, which does not breaking the symmetry? The same worry applies to the continuous symmetries as well. It turns out that for infinite quantum systems such superposed states do not exist (cf. Coleman 1975). This result further supports the idea of 'thermodynamic' limit and strengthens the similarity between the condensed matter cases and the quantum field ones (see also, Strocchi 1985; Anderson 1997).

To see then how a massless field emerge, we note (cf. figure 1) that the new degenerate ground states form a circle, which makes it simpler to use a polar expression of ϕ such that $\phi = \rho e^{i\zeta}$ (= $\frac{1}{\sqrt{2}}(\phi_1 - i\phi_2)$), where ρ and ζ, just like ϕ_1 and ϕ_2, are two real spinless fields where ρ corresponds to the length and ζ and angle in the $\phi_1 \sim \phi_2$ plane (see figure 1). Now, expanding in the neighborhood of the new ground state, $\rho_0 = \phi_0 = v$, in terms of $\rho' = \rho - v$, we get a Lagrangian in terms of the two new fields, $\rho'(x)$ and $\zeta(x)$, such that we can directly read off the mass distribution situation from it. The result is that the ρ'-field is massive and the ζ-field massless (i.e. having zero rest mass).

The last result can be generalized to a theorem (the Goldstone theorem), which has a relativistic version for quantum fields and a non-relativistic one for condensed matter systems. The former says essentially that if the Lagrangian density of some fields is invariant under a *continuous* (discrete groups may not have this feature) *global gauge* group, $G(\alpha_s)$ (where $s = 1,\ldots,m$ indexes the number of independent constants that parametrize the transformations), and a 4-current density, $j_s^\mu(x)$ (where $\mu = 0,1,2,3$), exists and is conserved, then for each field, $\phi^i(y)$ (where $i = 1,\ldots,n$), such that its vacuum state does not vanish (i.e. $\langle 0|\phi^i(y)|0\rangle = v^i \neq 0$) there exist a massless (and spinless) particle which has the same quantum number as $\phi^i(y)$'s. If one needs to picture such massless boson-fields, one is usually advised to think of them as fields that rotates the asymmetrical vacuum state (cf. (12) and figure 1) from one phase (e.g. $e^{-i\alpha}$) to another (e.g. $e^{-i\alpha'}$); and since these states are degenerate, no energy is need to do the rotation, and hence the bosons are massless. (Conversely, one may say that the masslessness of the Goldstone bosons entails that the corresponding SSB results in a set of degenerate symmetry-breaking vacuum states.[9])

(Historically, the massless and spinless Goldstone bosons were a problem because there were good reasons to believe that they do not represent any real quantum fields nor can any known quantum fields be represented by them. Some years earlier, the original Yang-Mills proposal for characterizing the strong interaction by the gauge field theory of isotropic spin ran into essentially the same problem. It was then realized (Englert & Brout 1964; Higgs 1964a, b) that the Goldstone theorem no longer holds if local, rather than global, gauge groups are applied to quantum fields. Logically, one of the premises for the Goldstone theorem is only valid for global gauge groups, a 'loophole' from which massive bosons become possible (cf. Schwinger 1962). If one uses either the U(1) or the SU(2) local gauge group to 'introduce' either the electromagnetic field or the Yang-Mills field, respectively, one obtains a 'coupling' of it with the Goldstone boson field. The result of such an 'interaction'[10] turns out to be the cancellation of the two massless fields and the emergence of a new field with the appropriate number of massive and massless components. This model -- the Higgs model (or mechanism) -- is indeed a rare triumph of scientific ingenuity (even by the standard of theoretical physics), but its details do not shed new light on the nature of SSB in quantum fields, since the SSB that produces the Goldstone bosons is assumed in the Higgs model.[11] Hence, I will not discuss it here.

Nor is it necessary for our purposes to give an account of the more complex models that supposedly represent real force-fields. There is nothing in the SSB of, for instance, the gauge theory of the unified field of the weak and the electromagnetic interaction that may offer insight into its nature which the simple model cannot.)

4. The meaning of the SSB of gauge symmetries

Let us first see some striking similarities among the three models (some of which I have alluded to earlier) -- the mechanical, the Landau, and the quantum-field model -- as listed in table 1.

Table 1. The formal parallelism among the three models.

Mechanical model (ring-bead system)	**Landau model**	**Quantum field model** (complex scalar)
Extremum equation: $\frac{\partial V}{\partial \theta} = \sin\theta(1-\beta\cos\theta) = 0.$	Extremum equation: $\partial L/\partial\eta = 2\eta(at + b\eta^2) = 0.$	Extremum equation: $\frac{\partial V}{\partial \phi} = \phi^*(\frac{\varepsilon}{2}\mu^2 + \lambda\|\phi\|^2) = 0.$
When $\beta < 1$, $\theta_0 = 0$.	When $t > 0$, $\|\eta_0\| = 0$.	When $\varepsilon = +1$, $\|\phi_0\| = 0$.
When $\beta > 1$, $\theta_0 = \pm\theta_1 \neq 0$.	When $t < 0$, $\|\eta_0\| = \eta_1(t) \neq 0$.	When $\varepsilon = -1$, $\|\phi_0\| = v \neq 0$.
SSB of reflectional (or rotational) symmetries.	SSB of continuous rotational symmetries.	SSB of global continuous gauge symmetries.
Degenerate ground states but no Goldstone modes.	Goldstone modes, e.g. spin waves.	Goldstone bosons.

One can see that the striking similarity between the mechanical and the Landau model, on the one side, and the quantum field model, on the other, is only formal -- by 'formal' here I mean merely syntactic or uninterpreted -- which cannot tell us whether the SSBs are of similar nature. Here is where the problem seems to lie. In the mechanical model, the physics of its SSB is in the relation between the centrifugal force acted on the bead (proportional to ω^2 or to β) and the deviation of the bead from its 'symmetric' position (measured by the deviation angle θ). When the strength of the force passes a certain magnitude, the condition of the ring-bead system becomes such that it would cost less energy to keep the bead stationary at an angle, θ_1, than keeping it at $\theta_0 = 0$. And hence a perturbation from the system's internal thermo-motion may actually cause the bead to move to θ_1. A similar story holds for the Landau model (when corrected by the lattice model). When the temperature, which indicates the amount of thermo-motion in a system, passes a certain magnitude, the balance of the thermo-motion and the tendency for a long-range order due to inter-molecular interactions becomes such that it costs less Landau free energy to form than not to form the long-range order. And hence a perturbation in the form of a lack of sufficient thermo-motion may actually allow some randomly formed cluster to grow to a long-range order.

No such stories can be told in the quantum field case despite the formal similarities. We do not know what the bi-valued ε may mean in physics. One of course may eliminate it by directly taking μ^2 to be capable of assuming positive and negative values. But to what does μ^2 refer? Does it make sense for it to have negative values? As mentioned earlier (see (10)), the term $\mu^2|\phi|^2$ in the Lagrangian is best conceived as a mass term for the quantized field (since $|\phi(x)|^2$ is the probability of finding the field quanta at x, μ^2 should be the net mass term). If so, what does it mean to have negative mass? Even if negative mass can be made meaningful, what physical picture of SSB can it offer? What relation can its change between being positive and being negative have with ϕ such that a causal picture for the actual transition between $|\phi_0| = 0$ and $|\phi_0| = v \neq 0$ can be had? I do not see a plausible answer to this question, nor can I find one in the literature.[12]

Nevertheless, when it comes to the nature of the SSBs in quantum fields, models of phase transitions are frequently invoked in the literature to supply heuristic or substantive ideas (cf. Anderson 1963; Aitchison 1982; Li 2000; Strocchi 1985). Indeed, one is not far off in observing that the only offering for the physical cause of the SSB in quantum fields comes from various arguments by analogy between the SSBs in condensed matter and in quantum fields. The conceptual justification at the most general level is apparently the idea that quantum fields are in essence 'many-body' systems of infinite degrees of freedom, and hence there ought to be some substantive -- not just formal -- similarities between models treated in the two areas.

Different models of phase transitions are used in this connection, the closest being the models of superconductivity and of superfluidity since both are instances of SSBs of gauge symmetries (cf. Anderson 1997; Aitchison 1982; Moriyasu 1983). But the problem is we do not really understand these two phenomena any better than we do of quantum fields. The BCS model was thought to be the correct one for superconductors until the discovery of high-temperature superconductivity, of which the model gives no adequate account. And the relation between superfluidity and the Bose-Einstein condensation is so intricate that current researches are constantly revising our understanding of it (cf. Emch & Liu 2001, ch. 14).[13]

Nor can the Landau model -- thought it covers all phase transitions -- be regarded as adequate, for it is known to be false. Without the lattice models and the detailed arguments and calculations, one would not know whether the Landau model is even approximate. However the formal similarities may be taken to suggest that our model is also a 'mean-field' model for quantum fields, meaning that it uses variables that only represent, and equations that are only true for, the average effects of quantum fields. If so, a negative value of μ^2 and the transition to it from a positive value -- a puzzle at this level -- may receive either a proper interpretation or a correction at the next level, as in the case of η in the Landau model. But at our present understanding of quantum fields, there is no indication that this will be the case: the Lagrangian (see (10)) may be an approximate one, but there is no reason to believe that its independent variables are not fundamental to the quantum field it represents.

All considerations so far seem to indicate that the knowledge of the true causes of SSBs in quantum fields is not forthcoming. The following addition observations further support this point. First, all SSBs we have studied before those in quantum fields, either in detail or in passing, can be regarded as phase transitions (one can regard the mechanical SSB as a transition between two stable equilibrium positions as ω changes). The SSB in our generic quantum-field model cannot be viewed as such because the model gives us no idea of what kind transition process might take place there. Some take the result, $\langle 0|\phi_0|0\rangle = v$, to mean that '[t]he vacuum is filled by a Bose condensate ($\sim v$), or in other words, the field has a fixed orientation in the space of the internal degrees of freedom of ϕ.' (Ludwig & Falter 1996: 375) But this is borrowing from condensed matter physics without rigorous arguments (cf. Moriyasu 1983 for a similar view explained in greater length). Even if this is the right interpretation, it still does not explain how or by what a normal, symmetrical vacuum state may get into such a symmetry-breaking state -- a vacuum filled with a Bose condensate or with a field with a fixed orientation. The model's answer, by passing from the positive μ^2 to the negative μ^2, seems entirely inappropriate.

Second, it is not clear, because of what has just been said, how a particular symmetry-breaking solution, $\phi_0 = ve^{-i\alpha}$, is 'chosen.' Many regards this a matter of convention (let us recall the Salam metaphor mentioned in the introduction of part I, and see also Martin & Rothen 2002: 317) and such an attitude seems justified by the observation that no such solutions are directly observable. In fact, neither the phase-independent solution, $|\phi_0| = v$, nor the massless Goldstone bosons (as a result of the former) are observable. However, it is difficult to make sense of such a view. There certainly are appropriate cases in which a choice of a phase is truly conventional; but those must be cases in which the gauge symmetry is not broken in any sense. Then for theoretical (or computational) reasons, one may choose a convenient phase to work on -- mostly commonly a phase condition rather than a particular phase; but that is the same as choosing a (set of) coordinate system(s) in studying certain processes which are invariant under transformations among such coordinate systems. If the symmetry (or invariance) is truly broken, in whatever forms, it makes no sense to talk about its result being arbitrarily chosen or chosen by some conventions.

Third, because of the Higgs mechanism that rehabilitates the SSB model in connection with the Goldstone theorem, little attention is paid in the literature on what the 'physics' is for whatever goes before the Higgs mechanism; all the talks about the 'interactions' between the Goldstone bosons and the gauge field photons that result in the appearance of massive gauge fields (not to mention the grosser expressions, such as that 'the gauge fields eat the Goldstone bosons and thereby become massive') seem to be dressing the purely formal results with metaphorical clothes.

Despite all this, *there is no suggestion that the SSBs in quantum fields, if they are genuine physical processes of symmetry breaking, are of a different nature from the ones in mechanics and condensed matter physics*. There is no reason to believe, for instance, that the symmetry-breaking vacuum states are not some kind of 'long-range order' of the individual phases of the field quanta in the vacuum. The question is what it means to have a long-range order of quantum-field phases and what the cause of it is. Even if, by analogy with the Landau model, we may assume that the symmetrical vacuum state, $\langle 0|\phi_0|0\rangle = 0$, represents the lack of an order of phases, and the symmetry-breaking states, $\langle 0|\phi_0|0\rangle = v$, the presence of it, it can hardly help us in figuring out what the cause of such an order of phrases in a vacuum state might be. The lack of a clear picture of what a long-range order in terms of phase-coordination in a vacuum state is blocking the analogy from yielding any real insight into what the nature and cause of SSBs in quantum fields are.

What have we learned about SSB?

1. From the epistemological point of view.

Different levels of theorizing fare differently in terms of being able to satisfy the requests for an explanation of how certain SSBs arise. Classical mechanics, in which the ring-bead model is studied, is in fact incapable of providing a satisfactory explanation of its SSB. It tells us via a simple and rigorous derivation how an SSB is possible, but to explain how it arises and whether it is in fact caused, one has to go beyond mechanics and study it as a system in statistical mechanics. (The same is true with Poincaré's cone.) Quantum statistical mechanics -- the dominating theory used in condensed matter physics -- in which the Landau and the Heisenberg model are studied seems at the moment the most adequate level of specificity for both the possibility and the actuality of the spontaneous breaking of a symmetry. It also provides analogical examples for the study of quantum-field SSBs, without which no explanation -- in the full-blooded sense of explanation -- of such SSBs can even be conjectured. The murkiest water for SSB *per se* is the realm of quantum field theory. From the analogy between the Landau model and the generic model of the quantum-field SSB (see table 2) it seems that the latter may also be a half-way house for the phenomenon. With Heisenberg model, we are able to see which are, and which are not, the right conjectures in the Landau model, but where could the 'Heisenberg model' in the quantum field theory be?

2. From the ontological point of view.

We now have a good idea of what the causes of actual SSBs must be: given the possibility of SSB, it is likely that all causes of the actual breakings have something to do with the random perturbations (or fluctuations) of the systems in question. This is not yet clearly demonstrated in the quantum-field model, but it is difficult to imagine that the nonvanishing of $\langle 0|\phi_0|0\rangle$ is not due to some kind of long-range orders as the result of vacuum fluctuations, although its precise mechanism and laws are not yet known. whence comes the *possibility of SSB*? (In other words, how is it possible that a symmetry-breaking solution has a lower energy, i.e. stabler, than the symmetrical solution.) Without an answer to it, we really do not know what it really means for the Lagrangian or the equation of motion of a system to obey certain symmetries and for the system's ground states to break them. For the mechanical model, the possibility comes from a battle between gravity and the centrifugal force acting on the bead: when the rotating bead reaches certain speed, the latter, which is a function of the speed while the former is not, wins over the former and opens the possibility for the bead to be stable higher up the ring. For the condensed matter model, the battle that makes the SSB in it possible is between the tendency to align due to the inter-spin interaction and the tendency to dis-align due to the thermo-motion. When the former wins over the latter, the possibility of long-range orders materializes. Again, there is no reason to believe that some such stories will not hold for quantum-field models.

5. Conclusion

Let me conclude this paper by addressing the remaining questions from part I (see section **1**). Should the size of a system matter as to whether SSB occurs in it or not? In particular, how should we understand Coleman's remark (1975) that while SSBs in finite systems are common and not interesting the ones in infinite systems are the opposite? By now we should know that it is not the size of a system *per se* that matters, for most systems in which the thermodynamic limit is taken in order to account for the phase transitions taking place in them are macroscopically finite; but rather whether the boundaries of the systems should be taken into consideration. If the boundaries matter and they are the reason for breaking the symmetries of the Lagrangians, then naturally we do not have SSBs there, while the genuine SSBs can only happen in those systems whose boundaries are such that they, though finite in size, should be treated as infinite systems.

Is there a place of arbitrary choice in some SSBs? The answer should be 'no,' although it is not yet clear in the quantum-field cases. Arbitrary choices (e.g. by convention) are only justifiable when the symmetry is intact.

Does the existence of SSB provide a straightforward argument for 'emergentism'? Anderson has argued forcefully for an affirmative answer on several occasions (cf. Anderson 1997). There is an ambiguity about the notion of emergent properties that may have let to a confusion in this case. 'Emergent property' is sometime inappropriately used to mean the emergence of a new property at the end of a process which the system undergoing the process does not have previously. This is clearly not the meaning that can be used in association with the notion of emergentism, a philosophical view that says that there exists at a certain time properties of a system which cannot be accounted for by the intrinsic properties of the parts and their relations within the system *at the same time*. That SSB is an agency for new properties in the former sense may well be a reasonable claim, but it is simply not true if the latter meaning is implied. There is simply no way of construing, for instance, the spontaneous magnetization of a ferromagnet as a property not accountable by the spins and their interactional relations at the time when the magnetization occurs. In other words, if such results of SSBs are 'emergent properties,' then it seems that any thermodynamical property is emergent as well.

And lastly, can the notion of SSB help us to resolve the quantum measurement problem? What I have discussed in these two parts is obviously not nearly sufficient to answer this question. The connection to the measurement problem is made by Anderson (1997: 50-51) in the form of long remark. The idea is that the transition in quantum measurement from a superpositional state to a determinate one may be regarded as an SSB in the following sense. The superposed states of a quantum system before a measurement may be regarded as states that are transformable by a group of transformations under which the laws that the system obeys are invariance. After the measurement, this symmetry is spontaneously broken when the system settles into one of the superposed states (NB: the system's Lagrangian still preserves the symmetry). And this happens because during the measurement the quantum system becomes part of a macroscopic solid system -- the measurement apparatus or the observer -- which by default is one of the 'broken-symmetry objects' (ibid. 50). It is obvious that to realize this idea one must be able to say what the symmetry is for the superposed states and how becoming part of an apparatus produces a spontaneous breaking of that very symmetry. And even if this, which is not a simple task, can be formulated, it is still not clear that it solves the problem of quantum measurement.

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[1] The paper is currently available at the website: philsci-archive.pitt.edu.

[2] To remind our readers, the mechanical model comprises a vertically suspended metal ring rotating frictionlessly with a bead threaded frictionlessly on it. θ is the angle of the bead from the ring's downward vertical radius and $\beta = \omega^2 R/g$, ω is the angular velocity, and R the radius, of the ring, and g is the gravitational constant.

[3] This 'non-equation' is to highlight the two energy terms, the first on the RHS being the energy from spin interations and the second the energy from the interation of the spins with the external field. Omitted are several constants that do not concern us here.

[4] Here I use $-\mathbf{P}$ to represent $< -P^x, -P^y, -P^z >$.

[5] Taking the thermodynamic limit on the system, i.e. making $f_N \rightarrow f_\infty$, should not affect the validity of (6) for all non-singular values of f_∞. And given no other singular points exist in the neighborhood of $B=0$, (which must be true because the singular points are not dense for f_∞), (6) holds in it on both sides of $B=0$, which implies that when the value of B approaches 0 from the opposite sides, the magnetization value of the system will always be of the same magnitude but in opposite signs; hence (8).

[6] I will use this term to refer to any random motion of the consituents of a macroscopic system, whose increase or decrease is accounted for solely by the addition or subtraction of heat or by the increase or decrease of temperature.

[7] Most texts do not use $\varepsilon = \pm 1$ but instead let μ^2 admit negative values (cf. Aitchison 1982; Quigg 1983; Rajasekaran 1989). I will return to this point later.

[8] It is worth noting that one may question the rigor of this translation, for the most rigorous way of obtaining the quantum-field results is to derive the vaccuum solutions in the two cases directly from a quantized field Lagrangian (or potential as in (10)). This 'seems not to be available. We shall accept it as an *assumption*.' (Aitchison 1982, 85)

[9] For different proofs and discussions of the Goldstone theorem, see Guralnik et al 1968; Weinberg 1996; and O'Raifeartaigh 1986 for a proof in classical fields.

[10] I use 'coupling' and 'interaction' (with quotes) to indicate the purely theoretical nature of what they refer to. It is not at all clear whether there are massless Goldstone bosons or Yang-Mills gauge fields, not to mention whether they actually couple with each other.

[11] It is indeed customary in the literature of Higgs mechanism to begin with some assumptions, one of which is to assume, rather than to prove or derive, the existence of some symmetry-breaking vacuum states, $|\phi_0| = v$.

[12] Referring to another model (the σ-model), Ling-Fong Li wrote, '[i]n the frame work [sic] of relativistic field theory,..., spontaneous symmetry breaking seems to be put in by hand, i.e. setting the quadratic terms to have negative sign in the scalar potential in order to develop vacuum expectation value. This is rather ad hoc and no physical reason is given for why this is the case.' (Li 2000: 22) This is, to my best knowledge, the only explicit allusion to this problem. Li then proceeds to give two models, the Ising model and the superfluid model, to show the physics of SSB

[13] I am by no means challenging the attempt to obtain a unified understanding (and theories) of the phenomena in these two areas or to reduce one area to the other or to use analogical features across the areas for heuristic purposes. I only argue against the move of using whatever we now understand of the nature of SSB in superconductivity or

superfluidity to directly say what the nature of SSB in quantum fields is or is not.