

All alone in the Universe: Individuals in Descartes and Newton

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Abstract

In this paper we argue that the primary issue in Descartes' *Principles of Philosophy*, Part II, articles 1-40, is the problem of individuating bodies. We demonstrate that Descartes departs from the traditional quest for a principle of individuation, moving to a different strategy with the more modest aim of constructing bodies adequate to the needs of his cosmology. In doing this he meets with a series of difficulties, and this is precisely the challenge that Newton took up. We show that Descartes' questions and his strategy influenced not only Newton's account of physical bodies, but also the structure of his mechanics.

1. The 'dissolution of bodies'

One important part of the conceptual change that took place in the seventeenth century has been successfully labelled the 'dissolution of the Aristotelian cosmos'.¹ This is not all that was dissolved. A deeper challenge is what we will call the *dissolution of physical bodies*. Having rejected the traditional Aristotelian resources of prime matter, form, and so forth, seventeenth century natural philosophers had then to find new answers old questions, such as 'What is the nature of physical bodies?' and 'What distinguishes this body from the surrounding uniform, homogeneous

¹ Koyre (1957).

matter?'. In addition, the new sceptical trend gave rise to additional metaphysical questions, such as 'Do physical bodies (or indeed physical reality) exist at all?', along with various epistemological counterparts, such as 'How do I know the nature of physical bodies?', or 'How do I know that physical bodies (or physical reality itself) exist?', and 'How can I distinguish one individual body from another or from all the rest (especially if we consider the unreliability of the senses)?'.

A new account of physical bodies was needed, and there were two main suggestions: physical bodies are either specific aggregates of material particles, or shapes cut in a continuous matter. However, both suggestions face problems: in the former case, given the principle of the uniformity of matter, it is unclear what makes the particles stick together; in the latter case it is equally unclear what distinguishes one body from all the other bodies around it. Moreover, given the passivity of matter in the 'new' philosophy, no source of activity is left in the material world to account for the interaction between its physical parts.² Thus, in each case it is far from obvious what individuates a body, giving it unity and distinguishing it from the rest of the material universe. In this way, the problem of the dissolution of physical bodies leads directly to one of the traditional problems of metaphysics: the problem of individuation.³ Matter as an actual, infinite and homogeneous substratum is no longer able to play its traditional role of a principle of individuation (just as forms are no longer the principle of intelligibility), and a new principle of individuation is needed.⁴ In

² See, for example, Hutchinson (1983).

³ By individuation in this context we mean both the 'principle of individuation', by which an individual is uniquely picked out (at an instant), and the problem of the identity of that individual over time.

'Individuation' in seventeenth century philosophy often meant quite different things (see especially Thiel, 1998). By the *problem of individuation* we understand a complex of questions bearing the two issues of individuation at an instant and identity over time. Although there is fairly general agreement in the recent literature concerning what is meant by this problem, some aspects of the debate are perhaps less familiar. See for example Des Chene (1996) pp. 367-371, who distinguishes between 'static individuation' and 'dynamic individuation' in Descartes' *Principles of Philosophy*.

⁴ A similar problem undermines the individuation of human beings. If man is an "aggregate" of soul and matter, what makes him different with respect with any other such aggregate, or even with respect with other parts of matter, or other spiritual beings? Moreover, if what is "individual" is the human

response, new entities were added – the void and active principles in the case of atomism (and bodies as aggregates), geometrical properties in the case of extended matter.

The outcome was not simply a replacement of any of the received principles of individuation with a new principle of individuation, even though there were several such attempts in seventeenth century metaphysics. As has been described by Ariew, in the new philosophy the principle of individuation was transformed into a principle for dividing homogeneous actual matter into the individual parts that are the physical bodies of the new philosophy. Moreover, as we hope to show in what follows, in Descartes and Newton the problem of individuation undergoes a further crucial transformation, from a metaphysical issue tied to questions concerning the intrinsic nature of bodies, to a problem of how to generate the bodies that are to be the subject-matter of the new physics.

2. Descartes and the principle of individuation

2.1 The current status of the debate

Much has been said about Descartes' attempts to provide a principle of individuation for the metaphysical foundations of his physics, or indeed for his physics. Various different authors, including Garber (1983, 1987, 1992), Woolhouse (1996), and Gaukroger (1999), have emphasised Descartes' difficulties in finding a principle of individuation, and the consequences of this for Cartesian physics.⁵ The importance of the issue is vividly formulated by Garber in the following way:

“Everything in Descartes' world must be explained in terms of bodies in motion. But motion itself is defined in terms of individual bodies. ...

soul, unique and immortal, what is the place of the human body in this construction? Again, the principle of individuation needed re-examination.

⁵ See also Thiel (1998); Grosholtz (1991), esp. p. 65-71; Ariew (1999); and Des Chene (1996), pp. 367-382. We will address some of them in the following sections of the paper.

And so, it appears, the explanatory adequacy of Descartes' physics depends upon making sense of the notion of an individual body.”
(Garber, 1992, p. 175)

In many respects, the modern-day debate mirrors that between Descartes' contemporaries, and especially the criticisms made by Leibniz and Newton.⁶ As seen in Garber, one outcome of this trend of interpretation is that if Descartes fails to provide a principle of individuation then the basis of his physics, the theory of bodies in motion, lacks its very object (or, at best, deals with a very badly defined object). The attempt to individuate bodies by appeal to motion has received a great deal of attention in the recent literature, and there is widespread agreement that this attempt is, indeed, a failure (partly because of Descartes' relational definition of motion, partly because of his theory of matter). There is, however, less agreement concerning the precise way in which Descartes attempted to reconstruct the physical world and the existence of individual bodies in it, and therefore concerning the *causes* of his failure. One reason for this is that, although there is a consensus that he fails to provide a principle of individuation, there remains a range of possibilities for understanding what is meant by a principle of individuation. We discuss what he himself said in the next section, but first some further remarks about the current literature on Descartes and the principle of individuation.

A wide variety of issues come under discussion with respect to the principle of individuation. One group of issues is epistemological; for example, there are various epistemological possibilities for distinguishing between substances or between parts of matter (Garber, 1992; *Principles*, I.60; II.55). Others are metaphysical; for example, how can the existence of parts of matter be reconciled with the continuity postulate? Furthermore, if matter is no more than extension, if we have just one

⁶ There are three main points that recur: (i) matter is essentially passive; (ii) there is a circularity in the definitions of matter and motion; and (iii) a body can't be kept track of once it starts moving. See Leibniz, *Discours Metaphysique*, in Woolhouse (1998) p. 12; or Leibniz's letter to Arnauld, 30 April 1687, also in Woolhouse (1998) p. 123; and Leibniz, *De ipse natura*, in Leibniz, *Oeuvres Choisies*, Ed. L. Prenant, Paris, Garnier, s.a, p. 432; Newton (c.1684), and Garber (1992, 1983), Woolhouse (1996).

infinite and homogeneous "world", what is the basis for the division of this matter into parts capable of interacting with one another? These problems lead to questions concerning the objects that Descartes' laws and rules of motion apply to. Recent papers by Dutton (1999), McLaughlin (1993), Hatfield (1998), as well as less recent ones by Garber (1983) and Blackwell (1996), for example, have even questioned the relevance of the laws of nature for the account of physical bodies, suggesting instead that the laws concern metaphysical issues of God's interventions into the world⁷, the foundations of physics, and a holistic framework for cosmology. Contributors to the current debate can be divided (albeit rather crudely, and perhaps even somewhat unfairly) into two broad camps. One camp tends to eliminate from Descartes' physics the very notion of individual bodies, interactions, forces or causes, and to subscribe to what we may label the occasionalist interpretation of Descartes⁸. The other camp is less homogeneous, but tends to insist on the existence of some means of individuation sufficient for the construction of "physical bodies", albeit very different indeed from the physical bodies of our reality.⁹

In what follows we will attempt to move the debate forwards by broadening the context in which it takes place. We will show how Descartes' attempts to solve the problem of individuation relate to the issues we have labelled the "dissolution of bodies", and then against this background we will trace his various attempts to reconstruct physical reality. This process sheds interesting new light on the consequences (both inside and outside his system of natural philosophy) of Descartes' attempts to solve the problem of individuation. In particular, by showing how these two issues are related it becomes clear that the (alleged) failure to provide a *principle of individuation* drives Descartes towards an alternative and highly influential approach: the reconstruction of physical bodies through a *model of individuation*. In

⁷ Blackwell (1966), p. 223; Dutton, (1999), p. 57.

⁸ Garber (1983); Garber *et. al.* (1998); Nadler (1993); etc. See for example the comments by Osler (1994) on the subject.

⁹ There are various ways in which this can be done. See, for example, Gaukroger (2000) for a fluid-model, Slowick (1996) for further suggestions concerning the idealization involved in Descartes' reconstruction of bodies.

the context of the mechanical philosophy a principle of individuation gives necessary and sufficient conditions for the existence of an individual. In contrast, a model of individuation gives sufficient conditions for a part of matter, say, to be treated *as if* it is an individual with respect to some given purpose (doing physics, for example).

2.2. Individuals and the real distinction

The first way that Descartes addressed questions of individuation is common to seventeenth-century scholastic metaphysics¹⁰, as we see for example in the celebrated example of the piece of wax, where the issue of what counts as an individual is present (and pressing) in the background¹¹. Descartes distinguishes between the two fundamental issues of, what we would call nowadays, individuation and identity over time. As in the traditional natural philosophy, he sees these as parts of the same question concerning the “essence” of an individual body (or mind).¹² According to Descartes, identity over time is preserved by God’s action of conservation. However, individuality with respect to other substances, or parts of substances – indeed, the individuality with respect to all the rest of the world – is an issue which is not clarified by the second and third *Meditations*. All we can say at this point in the *Meditations* is that we are facing one of his main metaphysical difficulties: the general question “What makes a body an individual?”

In addressing this question the starting point, and indeed the main point, is the definition of the *real distinction*. Here what is at stake is the essence of material reality, the essence of my own thinking substance, and the distinction between them. However, later in the Sixth Meditation, the *real distinction* is used in the context of Descartes’ explicit attempt to reconstruct physical reality. His purpose is to offer a criterion of physical existence of a strong type: everything I can clearly and distinctly

¹⁰ See especially Roger Ariew (1999, p. 152-3), and also Des Chene (1996).

¹¹ There are numerous authors who have been dealing with the issue. See for example Mijuskovic’s, (1991) discussion of the literature, esp. pp. 322-3.

¹² For discussion of theories of individuation by essence see Gracia (1984, p. 14-15).

perceive is capable of being created by God, therefore my intellect is offering me at least the possibility of the existence of the real world. Moreover, he writes:¹³

“Hence the fact that I can clearly and distinctly understand one thing apart from another is enough to make me certain that the two things are distinct, since they are capable of being separated, at least by God. The question of what kind of power is required to bring about such a separation does not affect the judgement that the two things are distinct.”

As such, the real distinction bears upon the issues concerning the separation of substances, namely of mind and body (*my mind* and *my body*). In the second set of replies to the objections, in the section entitled *Arguments proving the existence of God and the distinction between the soul and the body arranged in geometrical fashion* Descartes clarifies the distinction he wants to set between body and mind through a set of definitions.

“VI. The substance in which thought immediately resides is called *mind*.”

“VII. The substance which is immediately the subject of local extension and of the accidents which presuppose extension, such as shape, position, local motion and so on, is called *body*.”

Thus, body and mind are really distinct, as two *substances*. This time, the real distinction relates to a condition of existence:¹⁴

“Two substances are said to be *really distinct* if each of them can exist apart from other.”

¹³ AT VII, 78. Passages concerning the real distinction can be found in several replies to the first, second and fourth set of objections. See, for example, AT VII, 120-121, 132, 162, and 220-221. See also the *Principles*, I.60, for a definition to be discussed in what follows. For discussions in the recent literature concerning the real distinction and its connection with the principle of individuation see, for example, Garber (1992) and Thiel (1998).

¹⁴ AT VII, 162.

These definitions allow the formulation of a theorem stating the fact that there is a real distinction between body and mind.

As we can see, the whole issue has relevance for the question of my own existence, essence and individuality. This thinking substance which is my mind can be separated from everything else, including other substances, such as body. Its individuality is given through its essence; its identity over time is granted by God recreating it in every instant.¹⁵ In such a way, Descartes is constructing the first *isolated individual*. The human individual is not an individual *as such* with respect to some other similar human mind or body, but with respect to God and the way in which we can conceive it distinct from whatever else can exist in the Universe and in direct relation with its creator. We can say that the model of individuation for the human being is that of an isolated individual soul, all alone in the Universe, facing only God who is preserving this state of isolation through time.

At the same time, this definition of an isolated individual allows us to presume that there is something else in the world, essentially different and clearly distinct from it, namely something called body. This route towards a principle of individuation will not lead us too far, though. As some of Descartes' critics pointed out, the real distinction may get us to the existence of "body", as distinct from mind, but it does not get us to a multiplicity of bodies.¹⁶ In the light of this, the enlarged definition of the real distinction from the first part of the *Principles of Philosophy* is particularly interesting:¹⁷

¹⁵ Third Meditation, AT VII, 49. "For a lifespan can be divided into countless parts, each completely independent of the others, so that it does not follow from the fact that I existed a little while that I must exist now, unless there is some cause which as it were creates me afresh at this moment – that is, which preserves me."

¹⁶ See, for example, the Sixth set of Objections, AT VII, 417-18, which refers to the surface said to characterise and define a body, pointing out that this is not enough for a good definition. However, the comment on the objection postpones a judgement about physical objects "until they see whether you propose to demonstrate it in the treatise on physics which you promise us..."

¹⁷ *Principles of Philosophy*, I., 60. It is important to notice that Descartes is struggling here with

Real distinction properly exists between two or more substances: and we perceive these to be really distinct from one another from the sole fact that we can clearly and distinctly understand one without the other. For, knowing God, we are certain that He can accomplish whatever we distinctly understand. For example, from the sole fact that we now have the idea of an extended or corporeal substance....we are now certain that it can exist; and that if it exists, each part of it {which can be} delimited by our mind is really distinct from other parts of the same substance.

If we can clearly and distinctly perceive two substances, *or two parts of the same substance*, they are really distinct. The argument has several steps: firstly, whatever we can understand clearly and distinctly as separated substances are really distinct; secondly, whatever we can separate in our intellect, God can create as separate objects, and, therefore, they can exist *as separate objects*.¹⁸

It is quite clear that this represents the basis of a discussion of the problem of individuation¹⁹ in, as it were, traditional terms. Garber (1992), for example, offers an extensive discussion of it, together with further evidence from Descartes' letters concerning the connection between the real distinction and the problem of individuation of physical bodies.²⁰ His argument emphasises the difference between the argument for the existence of bodies in the *Meditations* (where bodies are said to affect my 'passive faculty' for sensing) and the *Principles of Philosophy*, where the

different meanings of the concept of substance. Although in the proper sense of the term there are only two substances, the thinking and the extended, he seems to consider that "substance" applies, as in the traditional philosophy, to everything that can exist in and through itself. With this meaning he moved the discussion on the physical domain, talking about parts of matter (extension).

¹⁸ Thiel (1998), pp. 224-5.

¹⁹ Together with articles as II. 54 and 55.

²⁰ See, for example, the letter to Gibieuf, 19 January, 1642, quoted by Garber (1992), p. 175.

Considering the two halves of a portion of matter, Descartes asserts that, providing there is a real distinction between them, we can see "the two halves of a portion of matter, however small it may be, as two complete substances".

existence of external bodies is exclusively grounded in the principle of divine veracity.²¹ However, the main problem with Descartes' proofs of the existence of bodies is that they are irrelevant for his own theory of matter.²² On the one hand, we cannot clearly and distinctly conceive of the parts of matter (due to the indefinite divisibility of matter, the plenum and the continuous motion of the material plenum). On the other hand, divisibility in thought is not enough for developing a theory of matter, where we seek to describe the evolution in time and the interactions of bodies (see p.10, below).

It is worth pausing here to notice that the issue of the real distinction and its metaphysical counterpart - the fact that God guarantees the actual physical distinction between what he can separate and what we can conceive as really distinct - plays an important role in defining what the problem of individuation is, and this is for several reasons. First, Descartes' definition of an individual does not imply criteria of distinguishing it from others of the same kind, but instead a criterion of distinguishing it according to its own 'essence', or essential properties. Secondly, in attempting to do this, Descartes marks the first step towards reconstructing the *isolated individual*, on the model developed in the metaphysical construction of the Ego in the *Meditations*, where the human individual is not defined with respect to his fellow humans (if any such humans exist), nor with respect to other minds or immaterial substances, but as a unique mind facing its creator, absolutely and essentially distinct from anything else, alone in the universe. Thirdly, for Descartes, finding a principle of individuation does not imply only a general criterion of separation, but also the very issue of the existence and "reality" of the physical world. Thus, the whole issue of the metaphysical reconstruction of the world is related to the complex of questions we have called the "dissolution of bodies".

If we now turn our attention to Descartes' natural philosophy in his *Principles of Philosophy*, Part I (where the issues of substance and the real distinction are discussed) might be expected to provide the resources necessary for moving forward

²¹ Garber (1992), pp. 72-4.

to a discussion of physical bodies and their interactions. Instead, what we find at the beginning of Part II appears to be an acknowledgement of failure. The real distinction plus the argument based on divine veracity cannot do more than persuade us of the existence of.²³

a certain substance, extended in length, breadth and depth ... and it is this extended substance that we call body or matter.

The divisibility of this substance is not enough to give us actual parts of matter, or bodies. Descartes needs to say in what way they are distinguished inside the continuum, both at an instant and over time. In other words, from the point of view of doing physics, we don't have the bodies we need, and the whole construction seems useless.

3. Descartes' model of individuation

It has been pointed out by Garber (1992) and others²⁴ that the first articles of Part II of the *Principles* deal with the individuation of physical bodies. Our claim is that this issue extends much further into Part II: all the articles up to and including those on the laws of nature can be systematically understood as a special kind of attempt to solve the problem of the dissolution of bodies. During this attempt, the problem of individuation changes: instead of bearing on the nature of bodies, it focusses on their existence and, moreover, on their existence as bodies suitable for the construction of a mechanical cosmology. What we are seeing here is a shift from *matter theory* as the basis of constructing a cosmology, to *mechanics*; matter theory is concerned with the *nature* of bodies (and thence their behaviour), whereas mechanics proceeds without

²² Thiel (1998), p. 225.

²³ *Principles*, II. 1

²⁴ Thiel (1998), McLaughlin (1993).

commitment to the underlying nature of bodies.²⁵

Descartes begins with an appeal to the external surface of the bodies. This is a way of reducing the problem to a geometrical one, using ‘shapes and measures’ in a geometrical sense. If a body is no more than a definite part of extension in motion, our only interaction with it is through its external surface.²⁶ The surface of one of these shapes would be distinguishable from the others if, at least in principle, a ‘measure’ could be associated with it;²⁷ that is, we need a principle by which a number is associated to a geometrical figure²⁸.

It is interesting that Descartes starts with a thought experiment, proposing a model of physical bodies, or what we can call the “sponge-model” of a body. The whole point is to give an account of rarefaction-condensation in terms of the extension of a body conceived as a “sponge”, namely constructed from various pieces of different sizes between which there are a numbers of holes.²⁹ Starting from this model, we can imagine a measure of the surface of the sponge - apparently including in it the measure of the holes - as an invariant with respect with the experiment of rarefaction and condensation. As a result, the *extension*, properly defined, can be viewed as the only essential property of a body and as its *principle of individuation*. A body is an individual through the measure associated to its surface. The problem is to specify this

²⁵ See also, for example, Gaukroger (2001), p. 166.

²⁶ See the fourth set of replies, AT VII, p. 250-51, the sixth set of replies, AT VII, p. 417-18 etc.

²⁷ *Principles of Philosophy*, II.8.

²⁸ This idea that we should, in principle, be able to associate a number to a geometrical property or to a property as such is tremendously important not only for Descartes’ own construction but for the whole physics to come. Instead of talking about essences and nature, we can talk about coordinates and measures, and this is almost enough to distinguish one body from the rest.

²⁹ The issue at stake here bears upon a traditional controversial problem of natural philosophy: finding an explanation for the phenomena of rarefaction and condensation. Aristotelian philosophy failed to provide one. Atomistic explanations of rarefaction and condensation imply the existence of the void. Descartes is facing here the challenge of offering an alternative explanation, in terms of a new theory of matter, involving extension alone. See II.5, 6, and 7. An interesting alternative explanation of rarefaction and condensation can be found in Charleton (1654) p. 17.

principle precisely. It is clear from Descartes' example that he does not mean by the measure the external surface of the body. The measure is more complicated than that, because it involves holes and also bits that are disconnected. The measure is equated with extension in II.8, but sometimes Descartes speaks of quantity (II.7).

It is perhaps worth noticing that a similar solution is presented in a number of letters concerning the explanation of transubstantiation. There, the question of individuation is explicitly addressed, both with respect to the human body, and to physical bodies. For example, in Descartes' letter to Mesland of 9 February 1645 the principle of individuation for physical bodies is said to be the number associated with the measure of the surface.³⁰ In the reply to the fourth set of objections, which also concerns transubstantiation, Descartes constructs an extended account of what he means by surface (of the bread and wine involved in the consecration):³¹

But we must note that our conception of the surface should not be based merely on the external shape of a body that is felt by our fingers; we should also consider all the tiny gaps that are found in between the particles of alcohol, water, vinegar and lees or tartar that are mixed together to form wine; and the same applies to the particles of other bodies. For, since these particles have various shapes and motions, they can never be joined together, however tightly, without many spaces which are not empty but full of air or other matter....Hence the surface of the bread is not the area most closely marked out by the outline of an entire piece of bread, but is the surface immediately surrounding its individual parts.

This is a very interesting direction of thought, unfortunately abandoned. In the published *Principles*, only the suggestion of the above model is kept for the

³⁰ AT III, 161-72, see especially pp. 163-4, concerning the definition of the surface and the way in which physical bodies can be defined through the "superficie moyenne", which is the boundary of the body and lies between the internal and external place.

³¹ AT VII, 250-251.

explanation of the rarefaction/condensation experiment. Instead of pursuing the definition of surface and the way to associate a number to it, Descartes moves on to other attempts at solving the problem of individuation.³²

The next proposal is that matter might be individuated through motion³³. Here Descartes states explicitly that what he is looking for is a metaphysical principle for dividing matter into parts: mere division in thought is not sufficient. He writes:

If the division into parts occurs simply in our thought, there is no resulting change; any variation in matter or diversity in its many forms depends on motion. This seems to have been widely recognised by the philosophers, since they have stated that nature is the principle of motion and rest. And what they meant by ‘nature’ in this context is what causes all corporeal things to take on the characteristics of which we are aware in experience³⁴.

However, as is well known, the attempt to use motion to provide a solution to the problem of individuation for the parts of matter meets with an obvious and immediate difficulty, due to the fact that while attempting to individuate bodies by appeal to motion, Descartes simultaneously defines motion relationally by appeal to bodies. The result is a blatant circularity:³⁵

... we may say that motion is the transfer of one piece of matter, or one body, from the vicinity of the other bodies which are in immediate contact with it, and which are regarded as being at rest, to the vicinity of other bodies. By ‘one body’ or ‘one piece of matter’ I mean whatever is transferred at a given time, even though this may in fact consist of many

³² The possibility of associating a measure to the external surface of the body is nevertheless important for the final reconstruction, and we should bear it in mind.

³³ *Principles* II.23 All the variety in matter, all the diversity of its forms depends on motion.

³⁴ *Principles* II.23; AT VIII, 53.

³⁵ *Principles* II. 25; AT VIII,54

parts which have different motions relative to each other.

This circularity has been deemed by many commentators to be a fatal flaw in Descartes' project of re-constructing the cosmos³⁶. For our purposes, the consequence of the circle is clear: if motion and location are relational, they cannot by themselves serve as a means of individuating bodies. At best, something more must be added.³⁷

Most of the recent debates in the literature terminate at this point, stressing Descartes' failure to offer a principle of individuation through motion.³⁸ Despite this, the ensuing paragraphs of the *Principles* seem to deal with physical bodies and their properties: the very next step that Descartes makes is to talk about the cause of motion and the laws of nature as applying to bodies or parts of matter,³⁹ as if the problem of the individuation at an instant has been solved. There are two possible interpretations: either Descartes believed that he had successfully provided a principle of individuation at an instant, or he opted to put to one side the approach of using a principle of individuation to generate bodies, and to take a new tack. On the latter interpretation, the remaining articles of Part II of the *Principles* are to be understood as an attempt to leave behind the quest for a principle of individuation as such, turning instead to the development of a *model of individuation*,⁴⁰ whereby bodies are constructed and identified by associating *numbers* to certain properties. Which properties are relevant is determined by the physical theory needed to describe the

³⁶ See for example Garber (1992) and Barbour (1989).

³⁷ There are other difficulties, too. For example, as Grosholz (1991), pp. 68-9, points out: "The definition of "one part of matter" allows for some very peculiar entities, since it does not require that the parts be contiguous, only that they share a common motion. A flock of particles dispersed over the whole solar system might count as one part of matter...".

³⁸ Some commentators suggest that Descartes' final solution is to search for something outside motion, in the ways in which God acts into the natural world. As a matter of fact, we are facing a tricky problem: given the identity over time, we would be able to use body's extension and motion to define its individuality in an instant. But, as several of Descartes contemporaries and mostly Leibniz suggested, the individuation over time presupposes something other than extension.

³⁹ Note that Descartes appears to use the terms 'parts of matter' and 'bodies' interchangeably. See in particular *Principles* II.25: "By *one body*, or *one part of matter*, I here understand..."

behaviour of those bodies. On this interpretation there are, according to Descartes, several such “numbers” encoding properties of bodies. The first may very well be extension, or the measure of the external surface of the body. Descartes’ move from this to the conclusion that there might be other properties of a body which can be expressed mathematically, having an associated measure, and that *together* these might specify a body adequately for the purposes of physics, is an important step in the history of physics. For, on this account, Descartes’ approach is strikingly modern: we see in embryonic form the concept of the state of a system, specified by numbers associated with each of the dynamically relevant properties of that system.

Turning now to the details of what Descartes does when he introduces the laws of nature, we see that the concept of motion gets refined. The motion of bodies is characterised in terms of two very important components: the “quantity of motion”⁴¹ and the *determinatio*⁴². They are two separate aspects of one and the same motion⁴³. Both can be considered properties of any part of matter. Moreover, the first law of nature includes the quantity of motion in a more general concept, the concept of “state” of a body, and so we have significantly enriched the resources with which to divide matter into its parts. In this way, a body is characterised by:

⁴⁰ See section 2.1, final paragraph, above.

⁴¹ The quantity of motion depends on “speed” and volume, but also on the way in which two bodies collide, and possibly on other factors too. See *Principles* II.43 and also, for example, Slowik (1996).

⁴² *Determinatio* is in general considered to be the internal tendency to move in a straight line. It appears in the second law of nature, but also in other passages too. It is a key concept for understanding how motion is inherent in bodies, as Descartes argues. See *Dioptrics*, AT IV, pp. 94-5, Letter to Clerselier, 17 February 1645, AT IV, p. 186. This distinction between the motion of a body and its *determinatio* is essential to Descartes’ physics. Although this distinction does not explicitly appears in the laws, it is nevertheless present as, for example Garber (2001), p. 137 has shown.

⁴³ See for example *The World*, Chap. I, AT XI, pp. 8-9. “.. that the power to move and that which determines the direction in which the motion must take place, are two quite different things and can exist one without the other (as I have explained in the *Dioptrics*)..”. See also the letter to Mersenne, 21 April 1641, AT 3, 355.

- 1.the “state”, a complex “device” which includes shape, size and motion, all of which Descartes has previously explored in connection with individuation.
- 2.the *determinatio* which characterises the instantaneous tendency of a body to move in a straight line

During the second part of his *Principles of Philosophy*, Descartes seems to imply that the addition of properties like measure, motion, quantity of motion and tendency to move in a straight line, provides the theory with sufficient resources to individuate a physical body. If this is the case, then it is very significant, because Descartes is individuating physical bodies *without* using a principle of individuation: we no longer have an instance of a general principle of individuation, applying to non-material entities as well as to parts of matter. Descartes has moved from attempting to provide a single all-encompassing metaphysical principle of individuation that can be applied to everything including matter, to a completely different strategy according to which he is trying to construct a model of material bodies that will provide him with a means of individuating them, but a means which is peculiar to material bodies.

Meanwhile, providing means for distinguishing bodies at an instant is not enough. All the quantities associated with a body are instantaneous. Which leaves open the question of identity over time.

What makes a body the same body at the next instant? Why can we still talk about the *same* body at a later instant of time? In the context of the problem of the dissolution of bodies and the need for a principle of individuation, it is natural to interpret the role of Descartes’ first law as being precisely to provide a means of individuating bodies over time. To see this, consider the first law in more detail.

The laws receive their first formulation in *The World* (c1633)⁴⁴, and the first law reads:

⁴⁴ We extend our consideration to include *The World* because this helps to make clear Descartes’ strategy.

The first is that each individual part of matter continues always to be in the same state so long as collision with others does not force it to change that state.

What we need now is an account of what it is to stay in the same state, of what it is to change state, and also of under what conditions change so defined can take place. Descartes continues as follows:

That is to say, if the part has some size, it will never become smaller unless others divide it; if it is round or square, it will never change that shape unless others force it to; if it brought to rest in some place, it will never leave that place unless others drive it out; and if it has once begun to move, it will always continue with an equal force until others stop or retard it⁴⁵.

The first law and the discussion in the *Principles of Philosophy* is essentially the same as that in *The World*⁴⁶. What we have is a conservation law applying to individual parts of matter, which states that each part of matter remains in the same state unless acted upon externally. The next step is to see how this connects with the problem of individuation.

⁴⁵ Garber (1992) points out that although in *The World* and the *Principles of Philosophy* the law of conservation of quantity of motion is presented as a special case of the more general principle that a system will conserve its state unless acted upon externally, chronologically Descartes had the special case first and the general case appears for the first time in *The World*.

⁴⁶ “The first law of nature: that each thing, as far as is in its power, always remains in the same state...” And then expanding: “The first of these laws is that each thing, provided it is simple and undivided, always remains in the same state as far as in its power, and never changes except by external causes. Thus, if some part of matter is square, we are easily convinced that it will always remain square unless some external intervention changes its shape. Similarly, if it is at rest, we do not believe that it will ever begin to move unless driven to do so by some external force. Nor, if it is moving, is there any significant reason to think that it will ever cease to move of its own accord and without some other thing which impedes it. We must therefore conclude that whatever is moving always continues to move as far as is in its power.” (*Principles of Philosophy* II.37)

We have seen that when he is addressing the problem of individuation at an instant, Descartes seems to end up achieving individuation by appeal to the state of a body, by ascribing numbers to specific properties of parts of matter. This method of individuation is repeated for individuation over time. Suppose we consider a part of matter that is not acted upon externally; specifying the state of that part of matter at a time is then sufficient to specify its state at the next instant, as guaranteed by the first law.⁴⁷ Thus, if specifying the state of a body is sufficient for its individuation, the conservation law ensures that body persists through time and is available as an object of our knowledge.

An important feature of Descartes' model of individuation is that these two solutions can be added together. Even if each on its own fails, there remains the possibility of dividing up homogeneous matter in whatever way gives us parts by ascribing numbers to regions in such a way that the resulting 'objects' satisfy the laws of motion. These are the physical bodies. Although this is a circular solution to the problem of individuation it is not viciously circular.⁴⁸

The next step in Descartes' project is to move from the consideration of an isolated individual body to an analysis of what would happen if a second body was added to the conceptual structure. The eventual target is the indefinitely extended cosmos in which motion is constantly re-distributed in accordance with the general principle that the total quantity of motion in the universe is conserved. This is where the second law of *The World* comes in:

I suppose as a second rule that when one body pushes another it cannot give the other any motion unless it loses as much of its own motion at the same time; nor can it take away any of the other's motion unless its own is

⁴⁷ Notice that the concept of an *isolated* individual, part of matter, or body, is fundamental to Descartes' solution to the problem of individuating bodies. Nevertheless, since Descartes' cosmos is a plenum, there are in actuality no isolated individual bodies.

⁴⁸ See also Grosholz (1994), pp. 50-51.

increased by as much.

The second law of *The World* is a law of conservation of the total quantity of motion of a composite system: it extends the first law *from* the single body case *to* the case of a pair of bodies, and provides us with a law for a composite system of colliding bodies *considered as isolated* from the rest of matter. However, this law is not always characterised in this way and some further discussion is useful⁴⁹.

The law is usually characterised as a law of impact, and as such is judged by its success at determining the outcome of collisions. But this law is not sufficient to determine the outcome of a collision because it does not determine how the total quantity of motion will be distributed after the collision and it says nothing about the subsequent directions of the bodies⁵⁰. The primary purpose of the collisions law is to generalise the first law for single bodies to the case of a pair of interacting bodies, so that we can individuate this composite system over time; and this it does successfully. However, the failure to determine the redistribution of the quantity of motion amongst the bodies following collision has the consequence that Descartes has no means of individuating the bodies that are the components of the composite system. Viewed from the point of view of the “dissolution of bodies” and the principle of individuation, this is the underlying problem that drives Descartes’ search for rules by which motion is redistributed and directions are altered during a collision. In the *Principles*, the law of collisions remains a conservation law for the composite system considered as an isolated system, but Descartes takes a new approach to the

⁴⁹ Different people have treated different selections of the laws as unproblematically conservation laws. For example, compare Garber (1992) and Gaukroger (1995). Garber views the first law and the law of rectilinearity as principles of persistence, with the tendency of a body to move in a straight line at a being conserved along with the speed of the body. But he finds it much more problematic to treat the law of impact as a conservation law. Gaukroger, on the other hand, views both the first and second laws of *The World* as conservation laws, but he does not describe the third law, the law concerning rectilinearity, as a conservation law.

⁵⁰ Garber (1992), p. 231-2 writes, “At best the supposed law of impact is a special case of the conservation principle...” that is, the global conservation principle. And hence, Garber concludes, “Descartes’ purported impact law in *The World* is, thus, no impact law at all.”

redistribution of motion in this law, and supplements the law with his rules of collision. Gabbey (1980) and Garber (1992) have argued that this version of the so-called ‘impact law’ is best viewed not as a conservation law but as a law about collisions based on the idea that a collision is a contest between the two bodies⁵¹. A better interpretation, perhaps, is that the law remains a conservation law for the composite system as a whole, but the problem of redistribution is tackled in a new way in terms of a contest. Gaukroger (2000) has offered a distinct and powerful interpretation of the approach Descartes takes to the problem of redistribution, arguing that Descartes is using the model of statics, and in particular a balance, to work out the rules of collision.⁵²

Thus, Descartes’ first law and his collisions law can be interpreted as attempts to solve the problem of individuation of bodies over time, a problem that arises as a consequence of the “dissolution of physical bodies”. Furthermore, this solution involves appeal to the concepts of an isolated individual or system, and the state of that individual or system. However, at the end of Descartes’ construction these individuals are not what we might expect: there are no actual bodies⁵³, subject to human experience, but instead highly abstract bodies, intelligible through their geometrical properties,⁵⁴ and intended as the subject-matter from which theoretical physics and the construction of a cosmology should proceed.

⁵¹ He writes (p. 234-5), “... the impact law, law B of The World, appears as law 3 of the *Principles of Philosophy*, considerably changed from its initial statement. The contest view, at best implicit in the earlier discussion, becomes the heart of the law, now clearly distinguished from the conservation principle...”. For more on this contest view of forces see Gueroult (1980).

⁵² Thus, a lighter body will never raise a heavier body placed at an equal distance from the pivot point. The ‘balance’ account is made even more convincing by the fact that Wren and Huygens both used balance analogies in their attempts to solve the problem of collisions (in response to the Royal Society challenge); see Radelet (2000).

⁵³ For an extended discussion on the notions of *materia*, *corpus*, *substantia* in the *Principles* see Des (1996), pp. 348-50.

⁵⁴ This is one of the main points of Garber’s demonstration in Garber (1992). See Sixth Meditation, AT VII, 80.

4. Newton and the problem of individuation

The issue of whether Cartesian natural philosophy contains physical bodies is not a twentieth-century problem. Our purpose here is to show that Newton's criticisms of Descartes' natural philosophy originate in a reading of Part II of the *Principles* that is similar to the one argued for above. As we will see, Newton found Descartes' discussion useful for his own concerns over individuation – his own approach is strongly Cartesian, both in its starting point and in much of its subsequent development.

Newton was interested in issues concerning individuation and space from very early on. One of the Trinity notebook entries, bearing the title *Of Space*, reads:⁵⁵

Extension is related to places as time to days, years etc. Place is the *principium individuationis* of straight lines, and of equal like figures; the surfaces of two bodies becoming but one where they are contiguous, because both in one place.

In other words, we can see space as being introduced as a reply to the problem of individuation of physical bodies⁵⁶. This same problem is addressed in the manuscript now referred to as 'De Gravitatione'⁵⁷, this being the *locus classicus* for Newton's criticisms of Descartes⁵⁸. In this context, Newton offers a significantly enriched solution to the

⁵⁵ McGuire and Tamny (1986), p.351.

⁵⁶ This is by no means a peculiar way of answering the question concerning the individuation of bodies. We can find the same kind of approach in Charleton, for example (whose influence on Newton's early writings has been emphasized by McGuire and Tamny, 1986). See Charleton (1654).

⁵⁷ See Hall and Hall (1962), pp. 89-156. For recent discussion of the importance of this manuscript, and its redating, see Cohen in Newton (2000), and also McGuire (2000), and Dobbs (1992).

⁵⁸ Much had been said, in recent years, about the extent of Descartes' influence upon Newton. Little of it was acknowledged by Newton himself, who was always keen on distancing himself from, and criticising, Descartes. See McGuire and Tamny (1983); I.B. Cohen (1964); Bechler (1991). Most recently, Scott Mandelbrote wrote: "His first, great master was Rene Descartes and he only emerged from the Frenchman's shadow once he was past his fortieth birthday.", in Mandelbrote (2001), p. 10.

problem of individuation. Newton starts from Descartes' question of how to reconstruct an appropriate concept of individual bodies, and from the fundamental premise that we don't know – and we are not even interested in⁵⁹ – the 'real' nature of a physical body.⁶⁰

Moreover, since body is here proposed for investigation not in so far as it is a physical substance endowed with sensible qualities but only so far as it is extended, mobile and impenetrable...I have postulated only the properties required for local motion.

However, although we may not know what the nature of bodies is⁶¹, we have a way of defining them as individuals through their properties, which are shape, impenetrability, motion and the capacity of affecting our senses⁶². The extended discussion that follows of physical bodies and their relationship to absolute space shows Newton facing the problem of the dissolution of bodies and proposing an alternative solution to Descartes'. According to Newton's argument in 'De Gravitatione', whatever the nature of body, we can understand bodies as solidified shapes created by God in absolute space, and moved around in absolute space according to certain laws. What individuates a body, on this

The criticisms in 'De Gravitatione' are usually understood as being directed at Descartes' theory of motion. While this is undoubtedly true, it is also the case that Newton is concerned with Descartes' account of body – see, for example, p. 126 “the fundamental definition of motion errs, therefore, that attributes to bodies that which only belongs to surfaces”.

⁵⁹ See also statements like: “...it seems probable to me, that God in the Beginning from'd Matter in solid, massy, hard, impenetrable, moveable Particles, of such Sizes and Figures, and with other Properties and in such Proportion to Space, as most conduced to the End for which he from'd them” *Opticks*, p. 400. In other words, the inner nature, the metaphysical status and the “purpose” of bodies lies beyond the boundaries of “sound” natural philosophy, and bears upon finalists and theological questions.

⁶⁰ Hall and Hall (1962), p. 122.

⁶¹ The nature of explanation concerning bodies is different from that concerning space or geometry because bodies “does not exist necessarily but by divine will”. Hall and Hall (1962), p. 138. The next passage reads: “hence, I am reluctant to say positively what the nature of bodies is, but I rather describe a certain kind of being similar in every way to bodies, and whose creation we cannot deny to be within the power of God, so we can hardly say that it is not body.”

⁶² Hall and Hall (1962), p. 139-140. It is very interesting that one of the main properties for something to be a physical body is an “active faculty” through which it can affect my senses.

account, is its position in absolute space: the individuation of bodies is parasitic upon the individuation of the points of absolute space. This is Newton's "string model" of individuation. By analogy with time or with mathematical strings of numbers, the points of space are individuated by their positions in a regular structure.⁶³

For just as the parts of duration derive their individuality from their order, so that (for example) if yesterday could change places with today and become the later of the two, it would lose its individuality and would no longer be yesterday but today; so, the parts of space derive their characters from their positions; so that if any two could change their positions, they would change their character at the same time, and each would be converted numerically into the other. The parts of duration and space are only understood to be the same as they really are because of their mutual order and position; nor do they have any hint of individuality apart from that order and position which consequently cannot be altered.⁶⁴

The same ordering principle can be found in the *Principia*: the order of parts of space and time is said to be immutable, the parts maintain the same position "from infinity to infinity"⁶⁵.

As the order of the parts of time is immutable, so also is the order of the parts of space. Suppose those parts to be moved out of their places, and they will be moved (if the expression may be allowed) out of themselves. For times and spaces are, as it were, the places as well of themselves as of all other things. All things are placed in time as to order of succession; and in space as to

⁶³ See for example McGuire and Tamny (1986) p.343 (389r): "To help the conception of the nature of these leasts, how they are indivisible, how extended, of what figure etc., I shall all along draw a similitude from numbers, comparing mathematical points to ciphers, indivisible extension to units, divisibility or compound quantity to number, i.e. a multitude of atoms to a multitude of units." The same model is offered in 'De Gravitatione' for parts of space.

⁶⁴ 'De Gravitatione', in Hall and Hall (1962), p. 136.

⁶⁵ Scholium to the Definitions, *Principia*, Newton (1934), p. 9: "Now no other places are immovable but these that, from infinity to infinity, do all retain the same given position on one another; and upon this account must ever remain unmoved; and do thereby constitute the immovable space."

order of situation. It is from their essence or nature that they are places; and that the primary places of things should be movable, is absurd. These are therefore the absolute places; and translations out of those places, are the only absolute motions⁶⁶.

In both ‘De Gravitatione’ and the *Principia*, bodies are individuals because of their different positions in absolute space. Space contains all the mathematical shapes in an actual way, and God solidifies these shapes out of space, making them sensible⁶⁷.

The appeal to the sensible properties of bodies is an important difference between Newton and Descartes.⁶⁸ If shape, mobility and the “place” in space are properties suitable for a mathematical description in a Cartesian way, the capacity of affecting our senses – the active power of bodies to impress upon my perception – is a very different type of quality. How can it be quantified or represented in any way? How can I recognize that a body which is affecting my perception now is the same body which was affecting my senses a minute earlier?

So, although the problem of individuation at an instant is solved, there is still a question of identity over time. What makes a body the same body at the next instant of time? The initial position in ‘De Gravitatione’ is similar to Descartes’: God conserves the “shape”, re-creating or moving the bodies around while preserving their “formal nature”. Despite the later attempt to replace this approach with an appeal to the arrangements of the particles⁶⁹ (a line of thought that gets into considerable difficulty due to the fact that we

⁶⁶ Scholium to the Definitions, *Principia*, Newton (1934), p. 8.

⁶⁷ ‘De Gravitatione’, in Hall and Hall (1962), p.139: “Thus we may imagine that there are empty spaces scattered through the world, one of which, defined by certain limits, happens by divine power to be impervious to bodies, and ex hypothesi it is manifest the this would resist the motions of bodies and perhaps reflect them, and assume all the properties of a corporeal particle, except that it will be motionless”. And also, a body "is nothing more than the product of divine mind realised in a definite quantity of space” .

⁶⁸ Note that there is a difference between Descartes’ *Principles* (read by Newton) and his *Meditations* – in the latter, bodies are capable of affecting my senses.

⁶⁹ *Opticks*, 394.

do not know what the forces that preserve the bodies are), in the end Newton is unable to escape Descartes' solution to this problem, and he is forced to reintroduce conservation laws.

In manuscript IIe, dated by Herivel at around 1665/6, Newton states:

Ax. 100. Every thing doth naturally persevere in that state in which it is unless it bee interrupted by some externall cause, hence axiome 1st and 2^d and [2?] A body once moved will always keepe the same celerity, quantity and determination of its motion.⁷⁰

Here, Newton has conceptually united the first two laws of Descartes' *Principles*⁷¹. As with Descartes, conservation of linear motion is derived from a more general conservation principle; that is, from the more general claim that a body will conserve its state unless acted upon externally. So, what was eventually Newton's first law applies to *isolated* bodies and is derived from a general conservation principle for such bodies. Given an *isolated individual body* at some instant, what makes this body the same body as a later instant is that it has conserved its state. As for Descartes, the concepts of isolated individuals and the state of an individual are crucial to solving the problem of individuating bodies over time. What makes this interpretation of the genesis of Newton's first law in the problem of individuation plausible is the context: Descartes was clearly concerned with this problem in Part II of his *Principles*; Newton read Descartes' *Principles*, and in 'De Gravitatione' he is clearly starting from Cartesian problems, including the problem of individuation.

The similarity does not end here. Newton also follows Descartes' strategy of attempting to generalise his law of conservation of quantity of motion from a single body to a pair of

⁷⁰ The first and second axioms are: "1. If a quantity once move it will never rest unlesse hindered by some externall caus. 2. A quantity will always move on in the same streight line (not changing the determination nor celerity of its motion) unlesse some externall cause divert it." These are the first two laws of Descartes' *Principles of Philosophy*, but with the consideration of rest removed, and Newton has yet to combine them into a single law.

⁷¹ We leave aside here discussion of the development of Newton's concept of mass.

interacting bodies. In Newton, the process of generalization to composite isolated systems becomes more sophisticated, as we will see in a moment. The central point, however, is this: although the problem of individuation does not show itself in its “traditional” form on the face of Newtonian mechanics, three of the most important concepts of Newtonian mechanics - the isolated individual body/system, the state of a body/system, and conservation laws applying to isolated bodies/systems - emerged directly out of the problem of individuation.

5. Newton’s Laws and Composite Systems

How does Newton progress from the motion of a single isolated body to the behaviour of interacting bodies? His general strategy in the *Principia* is exactly that found in Descartes: we proceed by construction from the behaviour of isolated individuals to the behaviour of composite systems via conservation laws, but in Newton the strategy is implemented with dramatically greater success.

From the beginning of his consideration of individual bodies, Newton is interested in saying precisely how the state of a body changes as a result of a collision. Newton’s second law tells us in what way a body’s state will be changed by the action of an external force, and, crucially, this change is *quantifiable*. It is the third law, however, that allows Newton to extend his analysis to the behaviour of bodies interacting with one another. Newton succeeds in providing a rule which determines uniquely and quantifiably the outcome of collisions and interactions between bodies.

The role of the third law is to determine the behaviour of the component bodies of a system, behaviour that must be consistent with the first law continuing to hold for the composite interacting system as a whole. This is perhaps most clearly seen by looking at corollaries III and IV to the laws of motion in Newton’s *Principia*, and at Newton’s treatment of planetary motion.⁷² In this way, we see that the new cosmology is built from

⁷² A case for this view of the role of the third law can also be made by considering the historical process by which Newton came to his third law. As Westfall (1971), pp. 344-7, discusses, it is in Newton’s attempt to

isolated subsystems which preserve their state unless acted upon by a force, by means of conservation principles.

We begin by considering the way in which Newton solved the problems that Descartes had with extending his laws to composite interacting systems. We have seen that Descartes sought to extend his discussion of single bodies to a consideration of interacting bodies by means of a conservation law applying to the composite system as a whole. However, this conservation law is insufficient to determine the outcome of a collision between two bodies, since it does not determine the distribution of the total quantity of motion between the two bodies after the collision, and nor does it say anything about the direction in which the bodies move off after the collision. With respect to the first issue, Descartes leaves us with the following question concerning the generalisation of his laws to composite systems:

By what rule is the total quantity of motion redistributed in an impact between two bodies?

With respect to the second issue, we note that although Descartes extended his conservation of quantity of motion from the single body case to the composite system case, he did not extend his conservation of direction of motion from the single body case to the composite system case. This extension was also needed in order to solve the problem of what happens when two bodies collide.

By means of the Third Law, Newton achieves an answer to the distribution question and an extension of the conservation of the linearity of motion from single bodies to composite systems.⁷³

solve the problem of collisions that he develops his concept of force. In this way we get (a) a measure of the external cause of changes of motion of a body, and (b) the separation of the concept of force from the concept of quantity of motion, and so from Descartes' law of conservation of motion for colliding bodies, giving us Newton's third law as the underpinning of the redistribution of the total quantity of motion (where quantity of motion is now the vector quantity momentum) in a collision, such that momentum is conserved.

⁷³ The genesis of this solution can be traced in the way Newton's laws develop through earlier manuscripts to their final incarnation in the *Principia*. For discussion of the development of the laws see especially Westfall (1971) p. 439ff and Herivel (1965).

Corollary III contains the first use of the Third Law in the *Principia*. Newton uses the Third Law to demonstrate that the total quantity of motion before and after a collision between two bodies is conserved. He is demonstrating Descartes' law of conservation of motion for two colliding bodies, but he is also doing more than this. For Newton, quantity of motion is not Descartes' scalar notion but rather the vectorial concept, momentum. Unlike Descartes' concept, this concept in conjunction with the Third Law allows us to go beyond the claim that the total quantity of motion is conserved to the redistribution of the total quantity of motion, both in terms of the magnitude of the momentum and in terms of the direction of the motion. We have a quantified solution to the distribution problem, and an extension of the conservation of linearity of motion from single bodies to pairs of colliding bodies.

So much for solving this problem. We now consider Newton's own constructional strategy for composite systems. In Corollary IV Newton shows that redistribution of motion in interactions by means of the Third Law is consistent with the First Law holding for a composite system treated as a single body via the centre-of-mass of the system.⁷⁴ The structure of Newton's argument is to build up from the behaviour of a set of mutually isolated bodies, via a pair of interacting bodies, to a many-bodies system of interacting bodies. In detail, Newton begins with a set of bodies each of which is freely moving and straightforwardly argues that:

the common centre of gravity of them all is either at rest or moves uniformly in a right line.⁷⁵

Then, he considers an isolated system of two interacting bodies. By the Second and Third Laws, any change in the momentum of one body will be accompanied by an equal and

⁷⁴ Again, there is a long history to this discussion in the *Principia* which can be found in Newton's manuscripts. For discussion of this history see Herivel (1965).

⁷⁵ Newton, *Principia*, (1962 edition), p. 19.

opposite change in the momentum of the other, and hence the centre-of-mass of the two-body system remains at rest or in uniform motion.⁷⁶

Next, he adds to this pair of interacting bodies the remainder of the set of mutually isolated bodies with which he began. Combining the above results for the set of non-interacting bodies and the pair of interacting bodies, he concludes that the motion of the centre-of-mass of the combination will be unaffected by the interaction of the pair.

Finally, we need to extend this to composite systems in which three or more bodies are interacting. Newton says:

But in such a system all the actions of the bodies among themselves either happen between two bodies, or are composed of actions interchanged between some two bodies.⁷⁷

and from here he concludes that

therefore they never do produce any alteration in the common centre of all as to its state of motion or rest ... And therefore the same law takes place in a system consisting of many bodies as in one single body, with regard to their persevering in their state of motion or of rest.⁷⁸

Conservation of linear momentum is shown to hold for a composite isolated system of interacting bodies via redistribution of motion according to the third law, and the method is to generalise by construction from a single isolated body to a composite isolated system.

⁷⁶ Or, as Newton writes, “Therefore since the changes which happen to motions are equal and directed to contrary parts, the common centre of those bodies, by their mutual action between themselves, is neither accelerated nor retarded, nor suffers any change as to its state of motion or rest.” (Ibid, p. 19)

⁷⁷ Ibid, p. 20.

⁷⁸ Ibid, p. 20.

6. Newton's constructional strategy in practice

This method of building the cosmos is put into practice in Newton's discussion of planetary motion.⁷⁹ For example, in discussing the motion of the satellites of planets⁸⁰ Newton writes that they will move around their planet but that this motion will be disturbed from a perfect ellipse by the influence of the Sun. We can construct the actual motion of a planetary satellite by beginning from a consideration of the satellite plus its planet as a two-body composite system isolated from all other influences.⁸¹

Newton then goes on to describe the way in which the Moon deviates from an elliptical orbit of the Earth, and in Proposition XXV of Book 3 he shows how to "find the forces by which the sun disturbs the motions of the moon" by considering a system consisting of the Moon and Earth only, and then analysing the actual motion of the moon as a deviation from this idealisation.

We end by noting one final feature of this constructional strategy. We have seen that according to Newton the behaviour of the three-body system can be analyzed in terms of how the two-body system would have behaved plus a disturbing factor. In other words, the interaction between the Sun and the Earth is completely blind to whether or not the Moon is present. The overall behaviour of the Earth results from its own behaviour as an isolated system, plus the contribution arising from its interaction with the Sun, plus the contribution from its interaction with the Moon, and so forth, and each of these contributions is completely unaffected by whether or not the other contributions are present. In this way, we can proceed to re-construct the entire universe, adding one body at a time, and nothing that we add will ever require us to go back and recalculate how the Sun and the Earth interact.

⁷⁹ I. B. Cohen (1980) discusses this process in detail, and he attributes to the third law the role of allowing Newton to move from consideration of the motion of a single planet about a fixed centre of force, to a pair of interacting planets, to a many-bodies interacting system, thereby constructing the motions of the planets in the manner we have described. See in particular p. 44.

⁸⁰ Newton, *Principia*, Book 3, Proposition XXII, Theorem XVIII.

⁸¹ "But then their motions will be in several ways disturbed by the action of the sun, and they will suffer such inequalities as are observed in our moon."

In conclusion, then, at the heart of the Newtonian cosmos of the *Principia* lies Newton's solution to Descartes' problem of the individuation of material bodies, many crucial aspects of which (taking isolated individual bodies as the starting point, the concept of the state of the body specified numerically and without appeal to the 'underlying nature of matter', conservation laws, and the constructional strategy) are found also in Descartes' own solution.

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