Concrete Transitions

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Abstract

Following von Wright, "transitions" are needed for understanding agency. I indicate how von Wright's account of transitions should be adapted to take account of objective indeterminism, using the idea of branching space-time. The essential point is the need to locate transitions not merely in space-time, but concretely amid the indeterministic, causally structured possibilities of our (only) world.[†]

1 Background: Concrete transitions

The account of action in von Wright 1963 features a well-developed underlying concept of *transition*; it teaches us that in order to become clear on actions, it is essential to become clear on the transitions that conceptually underlie them. The aim of these remarks is to contribute a little to our understanding of the transitions that underlie actions by focusing on the family of *possible concrete transitions*. A (really) possible concrete transition is distinguished by involving unique, definite, "pointable" regions in the causal structure of our world—our one and only world with its *own* possibilities open toward the future.

2 Generic transitions

I make my target clearer by contrast with the family "generic transitions." A generic transition bears the marks of abstractness: A generic transition, like a color, occurs (or doesn't) at some position in our world, but not typically at a *unique* position; a generic transition can be repeated over and over, and more generally two

[†]This is a "postprint" of Belnap 1999. The page numbers do not, of course, match those of the original. The idea of a transition is invoked in several publications that appeared after this one: Belnap, Perloff and Xu 2001 and three essays by the author on branching space-times that are archived in http://philsci-archive.pitt.edu.

places where a generic transition happens can have temporal separation, or spatial separation, or spatio-temporal separation, or indeed respectively be part of individually possible but jointly impossible courses of events. This last point, the most difficult, is the one, as we later see, that drives this study.

Von Wright studies generic transitions in von Wright 1963, especially ch. II. His notation *pTq* for the transition from generic state of affairs *p* to generic state of affairs *q* highlights an essential feature of generic transitions, that each involves two states of affairs, the *initial* state and the *end* state; and that in any occurrence of a transition, the initial occurs before the end. A transition is not itself a state of affairs, but "an *ordered pair* of two states of affairs," the "occasion" of the initial of which must precede the occasion of its end (von Wright 1963, p. 27). Von Wright labels the state transition itself an "event"—perforce a *generic* event.

Regrettably but essentially, if we are to locate a generic transition, we must always mention two occasions, one for the initial state and one for the end state. In a simple case, however, one can sometimes speak instead of the *region* between the initial occasion and the end occasion, and when it makes sense, I will say that a generic transition occurs *across* that region.

Von Wright uses the "*T*" notation to help make it clear to us that "state transition" does not mean "change of state": The transition *pTp* represents an important type of generic transition, namely, the *remaining* or *unchanging* of *p*, e.g. *(Mary is healthy) T (Mary is healthy)*, across some occasion.

3 Spatio-temporal locations and individual transitions

There is another contrast to be made. Von Wright identifies "occasions" with "spatio-temporal *locations*"; and he says that "occasions are the 'individualizers' of generic propositions" (p. 23). Even so, spatio-temporal locations, including regions, are "generic" rather than "concrete" in their own sort of way. Given a spatio-temporal region, unless determinism be true, there is no (objective) telling which generic transitions occur across that region. This is clearest with respect to the future: Suppose we are doing the Schrödinger experiment in a laboratory. Suppose we stand at the spatio-temporal location, *lab/12:00 noon*, with a live, caged cat, and a quantum spin measurement to be made; and suppose further that at *lab/1:00 p.m.*. the life of the cat depends on whether the measurement outcome is spin-up or spin-down. What shall we say of "across the *lab/noon hour* there is a transition from cat alive to cat dead"? The point is this: Given the spatio-temporal location, *lab/12:00 noon*, there is no determinateness to "cat dead at *lab/1:00 p.m.*" It depends, in this case, on the outcome of a piece of indeterminism during the *lab/noon hour*, namely, the quantum measurement. Therefore, we *cannot* indi-

vidualize the generic transition, (*cat alive*) *T* (*cat dead*), merely by attaching a spatio-temporal region: "The dying of the cat across the *lab/noon hour*" has a reference in the (objectively) possible case in which the measurement comes spin up, but no reference in the equally possible spin down case. Whether or not it has a reference logically depends on an indeterministic measurement, in exact analogy to the Prior/Thomason history-dependent account of truth.

That was a case of spatio-temporally-determined individuality failing by lack of the existence clause. For a companion example, take the same circumstances at *lab/12:00 p.m.*, and consider "across the *lab/noon hour* there is a transition from cat's cage unexamined to cat's cage examined." Since this generic transition occurs across the *lab/noon hour* regardless of spin-up vs. spin-down, there is no "existence" problem. A uniqueness problem nevertheless remains. Consider "the examining of the cat's cage across the *lab/noon hour*." The trouble now is that there are (at least) *two* of them. There is the examination after the cat dies, and there is the examination after the cat remains alive. The two examinations obviously and critically are across the same space-time region, but they have different causal histories. Merely mentioning a spatio-temporal location does not suffice for individuation of generic transitions, if indeterminism be true.[1]

4 Individuality of transitions

It stands the same with von Wright's example, that Brutus killed Caesar, which is said to be an "individual proposition" (p. 24). It is quite true, as von Wright says, that (logically) Caesar can be killed only once. But that only says that in any single possible course of history, brave Caesar dies but once. That truth does not comprehend that the generic transition, (*Caesar unkilled*) *T* (*Caesar killed*), can occur across exactly the same spatio-temporal location in alternate histories consequent on objectively indeterministic transitions. Look at it this way. In one case a coward might fear dying several times in the Senate House; that would exhibit an unrealistic imagination. In another case, a coward might fear dying more than once across exactly the same space-time region—say *Senate House/ Ides of March*. The imagination of such a coward would be not only unrealistic but demented. Consider, however, a third case, a coward who worries about a variety of violent deaths across *Senate House/ Ides of March*, each of which separately is a really, objective possibility (though of course no pair is jointly possible); such a person is neither demented nor even unrealistic, and although doubtless a coward,

[1] It is, incidentally, a grave and widespread mistake to try to sort this out with the help of something like "the actual future," a Thin Red Line uniquely marked among all the manifold possibilities that indeterminism requires as objective realities. See Belnap and Green 1994.

might even be taken to be prudent.

One way of putting this conclusion about "many really possible deaths of Caesar across a single region" is that you cannot get rock-bottom concrete individuality by combining two abstractions. What you will leave out is a unique causal history and a unique causal future of possibilities. Everyone sees that generic transitions can be located across two or more spatio-temporally distant regions, and most folks give lip service to the observation that a given spatio-temporal region can harbor as possibilities two or more incompatible generic transitions. The further *essential* point is that if indeterminism be true, you do *not* reach specificity by combining a generic transition with a spatio-temporal location. "*(Caesar unkilled)T (Caesar killed)* across the region, *Senate House/ Ides of March*," does not describe an individual (unique) transition, just because the generic transition across that region (really) could have occurred with different causal relations to the rest of our world.[2]

None of this makes any difference, of course, if determinism be true. In that case, there is only one objectively possible course of history; so that the idea of "alternate courses of history" is only a philosopher's or logician's fancy; and the fashion has been to find theories in various parts of philosophy (ethics, action theory, philosophy of language, philosophy of mind, etc.) that are "compatible" with determinism. This, I think, has been an unrewarding occupation. Determinism is false, so that there is no credit in finding a theory compatible with it. Our world is in fact indeterministic (I say here without argument, but thinking of agency, free will, radioactivity, quantum measurements, the concept of dread, etc.), so that the real and difficult philosophical challenge is to *find theories that are compatible with indeterminism*. That is the best sort of "compatibilism." For example, how can we rework von Wright's notion of "occasions"—the putative individualizers of generic transitions—to be compatible with indeterminism? That is the problem to which I turn.

5 Branching space-time

The key to a solution is the thought that space-time is not big enough for a serious theory of alternate possible events such as is required by objective indeterminism. Space-time is not the right field in which to find "locations" for generic transitions because space-time location does not locate you amid alternate possibilities. Nor

[2]Space-time positions must be distinguished from events. Do not confuse the lack of uniqueness of "the killing of Caesar occurring across space-time region, *Senate House/ Ides of March*," with the unqualified uniqueness of "the killing of Caesar *in our past* that occurred across the region *Senate House/ Ides of March*." And do not make the Thin Red Line mistake of thinking that one obtains uniqueness when "future" is substituted for "past." To do so is a sign of being unwilling to take objective indeterminism seriously.

is the well-worked theory of "other worlds," even if supplemented with a space-time grid common to all "worlds"; for you will lose the objective causal order between an "occasion" at one time in one world and an "occasion" at another time in another world. Most seriously, you will lose any causal connection with *us*. (Von Wright, happily, does not tend in the direction of other worlds.) Branching space-time, Belnap 1992, is an effort to describe such a field of "locations." In that theory the "locations" are called *point events*, and every last one of them is in some form of causal relation to *us*. These point events are not mere "positions" in space time, since they are ordered by a causal ordering, $<$, that simultaneously generalizes *both* the causal ordering of Minkowski's theory, *and* the causal ordering of the Prior/Thomason theory of so-called branching time (Prior 1967, Thomason 1970, Thomason 1984; see also Belnap and Green 1994). For example, the point events called "space-like related," while (i) not being directly causally related by $<$, always have both (ii) a common lower bound in the causal order, and also (iii) a common upper bound. Inconsistent point events are two thirds similar: (i) they are not directly causally related by $<$, and (ii) they always have a common lower bound, but (iii) they *never* have a common upper bound.

If you locate a generic transition between two point events of branching space-time, you have found true uniqueness, a true individual transition in (not just space-time but) the causal order. The best indicator of this is that you have fixed its entire causal past and also its entire causal future of possibilities. Consider an individual, concrete transition from Caesar unkilled to Caesar killed, that is, an occurrence of the generic transition, (*Caesar unkilled*) *T* (*Caesar killed*) between two point events, e_1 and e_2, of branching space-time, whose space-time locations bound the space-time region, *Senate House/Ides of March*. Then this individualized transition has a unique causal past—the past of e_1, not shared by any other such transition. Given the individual transition, its entire past—what did happen—is a "fact."

The story is more complicated for the future, and much less well understood in our shallow "compatibilist" times. It requires the essential idea of a "history," which, although rigorously definable in terms of just "$<$," should be understood intuitively (and roughly) as a total consistent collection of point events that shares with us some definite point event *in our own past*. The theory of branching space-time says that given the individualized, concrete occurrence of Caesar's being killed that lies in our own past, it makes sense to go further back, still in our own (actual) past, perhaps before the region, *Senate House/Ides of March*, and ask if there are real indeterministic choice points that could also have issued in Caesar's being killed, and being killed across that very same spatio-temporal region, *Senate House/Ides of March*—but in a different causal matrix.[3] If determinism be

[3]I fear to give examples, because the question does not have to do with *interestingly different*

true, there are no such choice points in our past. If there are such choice points, however, then, as a conceptual obligation, we should—rather than easing our conscience with compatibilist waffle and fudge—accept the reality of these alternate concrete possibilities. "Space-time is not enough."

6 Possible concrete transitions

Branching space-time provides a helpful and rigorous theory of point events as causally ordered by $<$. In this theory we can give a satisfying, airtight account of *possible concrete transitions*.[4] To do so smoothly, I have to change jargon a little from von Wright. I need a word for an arbitrary nonempty set of point events, and for this I say just *event*. (The change in jargon is not intended to minimize von Wright's point that common-sense "events" should frequently be identified with transitions (p. 28), which is surely both correct and extremely important.) An *initial* event (adopting word and adapting idea from von Wright, p. 28) is any event that is upper bounded in branching space-time. Each initial event can pass away; there is always a possible concrete point event at which we can truly say "that's over." An *outcome* event (changing word while adapting idea from *ibid*.) is any event that is lower bounded. Each outcome event can come to be; there is always a possible concrete point event at which we can truly say "that hasn't yet come to be, but it might." Notice the difference between the structures of initials and outcomes of a transition; this difference is essential for a constructive general theory of outcomes as in Belnap 1996.

A (possible concrete) *transition* is now well-defined as an ordered pair (this much follows von Wright) of events (but "events" in the jargon sense of branching space-time), $\langle I, O\rangle$ where I is an initial event, and O is an outcome event, and such that every member of the initial, I, is (not just in the "temporal" past but) in the causal past of every member of the outcome, O.[5]

Now a key definition, relying on the concept of history. A transition $\langle I, O\rangle$ is *contingent* if the passing away of its initial, I, does not guarantee the coming to be of its outcome, O. That is, it is contingent if there is a history that totally contains I and totally excludes O. This notion of contingency is absolutely objective

alternate killings, nor with choice points that seem to us *relevant*. The question is about objective indeterminism, not about what we think important.

[4]Does the phrase "possible concrete …" make you nervous? Probably. Certainly the determinists and many compatibilists automatically think that possibilities are, as a matter of logic, abstract, or perhaps tied somehow to our language or imagination or even logic. This is, however, bad logic, on a footing with the faulty logic involved in the improperly-named false doctrine of "logical determinism."

[5]See Szabo and Belnap 1996 for more details, and an application to quantum weirdness.

and absolutely fundamental. It has nothing to do with laws or languages or logic or physics or nature or culture or other worlds, such as "compatibilist" programs suggest. It is based entirely upon the causal ur-structure of our world (and of no "other" world), a structure that is the common ground of natural process and of human action. It is rooted in reality. The theory of agency cannot flourish without such a notion.

Here are two brief examples.

Here we are, having seen a living cat. We are at *lab/1:00 p.m.*, but although unique in the "space-time" grid, these words do not pick out a unique place in the causal order. Let us just call where we are *here/now*, and let us suppose, for expository convenience, a frame of reference in which to keep time. What of the transition from the initial point event *here/(one hour ago)* to the outcome point event, *here/now*, keeping the supposition that the quantum measurement was set to go (but had not yet occurred) at *here/(one hour ago)*? We have a contingent transition (if quantum theory be true), since one of the possible histories that continues *here/(one hour ago)* excludes a living cat, and so excludes the concrete point event, *here/now.*

What if we keep *here/now* as outcome, but consider the initial event *here/(one second ago)*. Then to believe that this transition is *not* contingent is to believe that the happening of *here/(one second ago)* guarantees the happening of *here/now*, in the usual sense that every history containing *here/(one second ago)* also contains *here/now*. Is the belief true? Who today knows the answer to this entirely objective question? Certainly no philosopher.

The single most fundamental transitions in branching space time seem to be the "immediate" transitions, in which there is no room at all between initial and outcome. And of these, the ones that have spawned the most interesting theory to date are transitions whose initials are point events. Thus, an *immediate transition* is always from a point event as initial to a set of point events as outcome, where the initial point event is a proper greatest lower bound of the outcome event. Suppose such a transition is contingent. Then the initial will be a "last moment of indetermination," while in general there will be no "first moment of determination"—like a species of Dedekind cut. We call any initial of a contingent immediate transition a *choice point.* It is "where the action happens," whether it be a moment of choice of a person or a metaphorical moment of choice in a quantum measurement. It is a point event at which past history divides—right there—into more than one possible future history. I firmly believe that it is impossible to have a satisfactory theory of any of objective indeterminism, objective causality, or objectively free action, without studying contingent immediate transitions, or something very much like them. If the theory of branching space-time be true, they serve as "originating

causes."[6]

7 Propositional transitions

What about generic concepts involving "propositions"? Branching space-time gives help here. We adapt a standard jargon by defining a *proposition* as a set of histories. Given a possible concrete transition, there is a "propositional" transition of some interest. There is the set of all histories in which the initial comes to a close; these histories all *contain* the initial. There is the set of all histories in which the outcome begins to be; these histories all *overlap* the outcome. Then the first proposition, call it *p*, "says" that the initial passes away, and the second, call it *q*, "says" that the outcome begins to be. So there is excellent sense in saying that there is a transition from *p* to *q*. The contingency of the transition can be marked by saying that *q* is a *proper* subset of *p*—some histories are dropped in passing from the initial to the outcome.

Propositions as sets of histories will not do, however, for generic transitions in the sense of von Wright, since central examples such as *pTp* and *pT*∼*p* obviously require us to evaluate the *same* proposition, e.g. *p*, on *different* occasions (von Wright 1963 p. 23) in the *same* history. Instructed by the work of Prior and Thomason (*op. cit.*), we know what is wanted for indeterminism, namely, the sort of proposition that can be said to be true (not just in a history but) in a history at a point event.[7] Such propositions are represented by sets of (not histories but) ordered pairs $\langle e, h\rangle$, where e is a point event and h is a history, and where e in fact belongs to h. Of course such a proposition is "true in h at e" iff the pair $\langle e, h\rangle$ belongs to *p*; and we also say that "*p* is settled true [false] at e" iff $\langle e, h\rangle$ is in [not in] *p* for every history to which e belongs.

The whole story of generic transitions involving such propositions must be complicated, and so like von Wright (1963, p. 28), we begin with the simplest case: (i) The initial and outcome propositions are both to be state-like, where a proposition, *p*, is *state-like* iff for each point event, e, *p* is either settled true or settled false at e and (ii) the initial I and the outcome O of the concrete transition $\langle I, O\rangle$ are both single point events: $I = \{e_I\}$ and $O = \{e_O\}$. Our question is, then, when does generic *pTq* occur at the concrete point-to-point transition $\langle\{e_I\}, \{e_O\}\rangle$? The only possible answer—and the one that follows von Wright—seems

[6] If this is correct, then, using "event" in the branching-space-times sense, originating causes are certain ordered pairs of events (transitions), rather than events. This is confusing, and so it is good to remark that originating causes are "events" in *von Wright's sense* that identifies an event with a transition.

[7] This scheme sounds like the "true at a time in a world" scheme; but, as you may infer from our earlier discussion, the causal ordering, $<$, in fact makes a profound difference.

this: *pTq* occurs there just in case (i) *p* is settled true at e_I, and (ii) *q* is settled true at e_O. Thus, for example, *pTp* occurs at the concrete transition, $\langle\{e_I\}, \{e_O\}\rangle$, just in case *p* is settled true at e_I and *p* is also settled true at e_O.

The account of the logic of *pTq* beginning on p. 27, including the surprising reduction on p. 31, exactly fits this case. Naturally, as von Wright in effect notes on p. 28, that will no longer be true, when we pass to more complicated sorts of generic transitions *pTq*, or to more complicated concrete transitions from initial I to outcome O. And we must go on to these cases if we wish to justice to agency; even the very simple form of "sees to it that" agentive transition studied in the 'dstit' theory of Horty and Belnap 1995 does not have this "point event to point event" structure.

The concrete transitions underlying dstit are not point-to-point at all, but essentially both continuous and immediate; they are and must be of the form $\langle I, O\rangle$ where I consists of a single point e_I, while O typically descends infinitely toward e_I. When does a generic transition, *pTq*, occur at $\langle\{e_I\}, O\rangle$? For dstit theory, and I should think for many purposes, we shall need (i) for the initial, that *p* is settled true at e_I, and (ii) for the outcome, that for every history overlapping O, *q* is true throughout some (perhaps very small) beginning portion of O. For example, picture a billiard player with cue in hand, and let *p* be "the cue ball is motionless." Then *pT*∼*p* occurs at $\langle I, O\rangle$ when e_I is the point event of impact, and represents a standard continuous transition from rest to motion. Provided motion in our world is continuous, however, ∼*pTp* can never occur at an immediate transition $\langle\{e_I\}, O\rangle$; for any such occurrence would be a *discontinuous* transition from motion to rest.

To see that the von Wright reduction does not always apply to these immediate transitions, however, we need a different example. Let the general situation be the same, but add that the cue ball is motionless at e_I. Now let *p* be "the velocity of the cue ball exceeds more than one inch per second toward the nine ball." The problem is that given $\langle\{e_I\}, O\rangle$ immediate, *none* of the following four generic transitions will occur there: *pTp*, *pT*∼*p*, ∼*pTp*, ∼*pT*∼*p*. The first two fail because the cue ball is at rest, and the second two fail because you cannot—if motion be continuous—have an *immediate* transition from rest to a velocity of more than one inch per second. The elegant von Wright reduction depends on at least one of these *T*-formulas occurring, as is importantly true for the point-to-point transitions that von Wright considers; but the example shows that the reduction does not and should not hold for the immediate transitions—the "originating causes"—that underlie the dstit theory of action; nor has von Wright implied otherwise.

What does remain true, I think, is the eventual capacity of branching space-time to illuminate much more complicated cases of concrete transitions. The "theory of outcomes" of Belnap 1995 investigates many of these in a simplified setting, but

still includes cases in which initials and outcomes are complex in various important ways. Nevertheless, to my mind we are not yet close to possessing a conceptual structure that can give us a nice, warm feeling about our understandings of initials, outcomes, transitions, causality, causal independence, "strange correlations," etc. …, all of which are preliminary to an appreciation of how agency fits into the causal structure of our world. I believe part of the blame must be put on the regrettable conscience-easing aspect of our philosophical culture that delights in finding ways to avoid the conceptual problems concerning concrete, objectively possible events in a seriously indeterministic world.

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