Bohr’s quantum postulate and time in quantum mechanics

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Abstract

In this paper I shall consider the concept of time used in the quantum theory from the perspective of Bohr’s ideas as presented in what he called the quantum postulate and some of its consequences. In particular, Bohr’s views on the role of the experimental arrangement that defines the context in which we can consider the state of the quantum system to be defined – and the related interpretation of the quantum wave function –, enables a view on the time concept being used in the theory that makes compatible the discontinuous changes in individual physical systems with a continuous description in time of the behavior of an ensemble of systems. A crucial aspect will be to show that Bohr’s ideas are coherent with the use of an external classical time.

1 Introduction

Niels Bohr considered that the “discovery of the *elementary quantum of action*, which revealed a feature of *wholeness* inherent in atomic processes” (Bohr, 1958, p. 2) means a departure from the classical physics description, where we assume that the interaction of a physical system with a measuring instrument can ultimately be disregarded. In the case of quantum phenomena we have to be aware that due to the “indivisibility of the quantum of action” (Bohr, 1934, p. 5) we must associate to all “individual atomic processes an element of discontinuity quite foreign to the fundamental principles of classical physics, according to which all actions may vary in a continuous manner” (Bohr, 1934, p. 4). This means that in the case of all atomic processes every energy change results from an indivisible – and because of that discontinuous – transition between different states that cannot be continuously connected. Bohr’s ideas were summed up and presented in what he called the quantum postulate (and its consequences).

In part two I will give a historical account of the coming to be of the postulate of the quanta and some of its consequences according to Bohr’s views on the quantum theory. In part three, I will consider the temporal description of quantum systems given by the time-dependent Schrödinger equation, taking into account Bohr’s views on the interpretation of the formalism of the theory. In particular, considering Bohr’s interpretation of the wave function – related to the role he gives to the experimental arrangement in the interpretation of the mathematical formalism of the theory –, it is possible to accommodate the discontinuous changes in individual systems with a temporal description of an ensemble of identical systems in a continuous ‘classical’ time (that is external to the quantum systems themselves). But as will be shown we must address the question: are Bohr’s ideas coherent taking into account his quantum postulate and the use in the description of a quantum system of an external classical time? Following the ideas of Don Howard the answer seems to be no. But from a different perspective (compatible with views put forward, for example, by Paul Teller, Simon Saunders or Henrik Zinkernagel) the answer can be yes.

In a nutshell, according to Bohr it is not possible to define a quantum system independently of the experimental context – (at least in part) classically described – that defines the conditions in which the quantum phenomenon is manifested. In this way it is not possible to take the wave function as describing an isolated quantum system – a concept that has no meaning for Bohr – but a quantum system defined in a particular experimental context, which, at least in part, is classically described. Also, Bohr did not consider the need for the wave function collapse during the observation of a quantum system (Howard, 2004, p. 669). This is possible if the wave function is not associated with an individual quantum system but to a probabilistic description of identically prepared quantum systems submitted to the same experimental procedure (Teller, 1980, pp. 211-214).

One point of quantum theory that usually is not considered in connection with Bohr's ideas is the concept of time being used in the theory. In reality the use of a classical concept of time is coherent with Bohr's ideas. Even if Bohr did not develop in detail his views regarding space and time in quantum theory, it is possible to give a coherent reading of his ideas by taking into account the need for a classical description of part of the experiment. This makes possible a classical treatment of time, and this even if the quantum postulate implies the impossibility of conceiving the quantum phenomena independently of the measuring apparatus. In particular it is possible to see the temporal parameter of the Schrödinger equation as an external classical time related to the measuring apparatus. This makes it possible to have simultaneously:

A) A quantum system defined in a way that is dependent on the experimental context (due to the quantum postulate).

B) A space-time macroscopic reference frame associated with the measuring apparatus, but defined without taking into account the indissociability – in every possible experimental context – with the quantum system.

C) The description of the quantum system using a classical clock with which it has no direct interaction.[1]

Some further remarks are presented in the conclusions.

2 The quantum postulate

As is well known Bohr developed around 1912-1913 a non-classical model of the atom without having to address directly the problem of the nature of radiation and Einstein's light-quanta hypothesis (Bohr, 1913; Jammer, 1966, pp. 78-81; Darrigol, 1992, pp. 85-89).[2] In fact, up until 1922 Bohr did not address in any detail the light-quanta hypothesis (Klein, 1970, p. 21). When Bohr finally addressed the concept of light-quanta, his view was that it "excluded in principle the possibility of a rational definition of the

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[1] In this way, I will be presenting a reconstruction (in Howard's sense; see Howard, 1994, p. 203) of Bohr's interpretation of the quantum formalism, which in my view makes compatible Bohr's quantum postulate and Bohr's views regarding space and time in quantum theory.

[2] Bohr's model of the atom is based on the idea of stationary states – not submitted to the consequences of the classical theory of radiation –, where the electron can make a transition (jump) from one stationary state to another by emission or absorption of radiation with a frequency ν given by the relation $h\nu = E' - E''$, where h is Planck's constant, and E´ and E´´ are the energies of each stationary state.

conception of a frequency ν" (Bohr, 1924, p. 35), and that "in spite of its heuristic value, however, the hypothesis of light-quanta, which is quite irreconcilable with so-called interference phenomena, is not able to throw light on the nature of radiation" (Bohr, 1923, p. 32). In this article Bohr made an explicit association of the expression $h\nu = E' - E''$ from his atomic model with the emission or absorption of electromagnetic waves by taking into account the correspondence (analogy) with classical electrodynamics:[3]

> A process of transition between two stationary states can be accompanied by the emission of electromagnetic radiation, which will have the same properties as that which would be sent out according to the classical theory from an electrified particle executing a harmonic vibration with constant frequency. This frequency ν has, however, no simple relation to the motion of the particles of the atom, but is given by the relation $h\nu = E' - E''$, where h is Planck's constant, and $E'$and E'' are the values of the energy of the atom in the two stationary states that form the initial and final state of the radiation process. Conversely, irradiation of the atom with electromagnetic waves of this frequency can lead to an absorption process, whereby the atom is transformed back from the latter stationary state to the former. (Bohr, 1923, p. 33)

Around July of 1925 it was already clear to Bohr that a more classical wave-like picture of the electromagnetic radiation was untenable. This was due to experimental results, related to the so-called Compton scattering, obtained by W. Bothe and H. Geiger. In their experiment, Bothe and Geiger confirmed energy-momentum conservation in individual atomic processes by observing a simultaneous detection (coincidences) of scattered x-rays and recoil electrons in the scattering of x-rays by free electrons (Fick & Kant, 2009, pp. 399-401). Also A. H. Compton and A. W. Simon made an experiment in a cloud chamber that permitted the observation of the track of the recoil electrons and the direction of scattering of the x-rays (due to the occasional production of secondary tracks). They obtained the expected relation, according to the light-quanta hypothesis, between the scattering angles of the x-rays and the electrons (Jammer, 1966, p. 186).

According to Bohr we do not observe directly the scattered x-rays but "photoelectrons released by the scattered radiation" (Bohr, 1925a, p. 204). It is important to take this into account to understand Bohr's reasoning. The experimental result of Bothe and Geiger implies strict energy-momentum conservation in the interaction of the electron and the radiation. This implies that depending on the change of energy and momentum in the recoil electron we will have a change in the energy and momentum of the photoelectron, which is causally dependent on the changes in the first electron due to the strict conservation of energy and momentum in the interaction of the electrons with the radiation. Bohr refers to this situation as a "coupling between the emission of the recoil electrons … and the photoelectrons" (Bohr 1925a, p. 204). Considering this electromagnetic coupling between two individual transition processes (one occurring with the recoil electron and the other with the photoelectron), Bohr concludes that "no space-time mechanism seemed conceivable that permitted such a coupling and at the same time achieved a sufficient connection with classical electrodynamics" (p. 204). That is, it is not possible to maintain a wave-like picture because of the conservation of energy and momentum in individual atomic processes.

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[3] To the purpose of this work it is not necessary to make too precise what we should understand by 'correspondence principle'. As is well-known Bohr's use of this term gained different meanings in time (see e.g. Honner, 1987, pp. 60-64). I consider that the interpretations given for example by Darrigol (1997) or Bokulich (2009) enable a consistent use of the term in the cases being considered here, which, in particular, are related to the 'asymptotic' agreement between the classical and quantum description of parts of a measurement apparatus (see below pages 9 and 10).

This does not mean a renunciation of the wave-like perspective and adoption of the light-quantum concept. According to Bohr the problem is not simply "distinguishing between two well-defined conceptions of the propagation of light in empty space corresponding to either a corpuscular theory or a wave theory of light" (p. 204). The real problem being faced is "to what extent the space-time pictures [wave-like or particle-like], by means of which the description of natural phenomena has hitherto been attempted, are applicable to atomic processes" (p. 204). The paradoxical situation facing Bohr is that there is no simple choice between two incompatible pictures used in the description of natural phenomena. We are in a situation where "the radiative activity of individual atoms is influenced by the presence of other atoms in the sense to be expected in the picture of the wave propagation of light" (p. 204), but on the other hand we have to recall "the coupling between individual atomic processes [due to energy-momentum conservation], which forces upon us the picture of a corpuscular propagation of light" (p. 204). In another article published in December of 1925, Bohr states that one is facing "an essential failure of the [wave-like and particle-like] pictures in space and time on which the description of natural phenomena has hitherto been based" (Bohr 1925b, 848). This situation did not prevent Bohr from grasping the profound conceptual importance of accepting the "individuality of single [atomic] processes" (quoted in Mehra & Rechenberg, 2000, p. 191) due to the strict conservation of energy and momentum in the interaction of matter and radiation, as had been verified in the experiments of Bothe and Geiger and Compton and Simon and proposed by Einstein in 1905 within his light-quanta hypothesis.

In a letter to H. A. Lorentz from 24 June 1926, accepting an invitation to attend a meeting, Bohr mentioned the eventual name of the report he would present: "Le Postulat des Quanta et le Nouveau development de l'Atomistique" (Mehra & Rechenberg, 2000, p. 175). What Bohr meant by 'le postulat des quanta' or as it appears in English 'the quantum postulate' would appear in print only in 1928. According to the Bothe-Geiger and Compton-Simon experiments, it is necessary to use the idea of an elementary quantum in the description of the interaction between matter and radiation. Because of the indivisibility of the quantum we have to consider that in each single atomic process of interaction between the electron and the electromagnetic radiation we have a "discontinuous change of energy and momentum" (Bohr, 1927, p. 93). From this we may speak of the 'individuality of single processes'. Related to this idea of individuality is the possibility of a coupling between spatially separated atomic processes, meaning with the term 'coupling' the causal relation between individual processes due to energy-momentum conservation.[4] But the essential point in the quantum theory is the indivisibility of the quantum, as Bohr stresses with his quantum postulate:

> Its essence may be expressed in the so-called quantum postulate, which attributes to any atomic process an essential discontinuity, or rather individuality, completely foreign to the classical theories and symbolised by Planck's quantum of action. (Bohr, 1928, p. 580)

According to Bohr this has an immediate consequence:

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[4] Scott Tanona gives an account of Bohr's reaction to the Bothe-Geiger experiment that stresses the 'coupling' between different atoms, or more generally the coupling between an atomic system and a measurement instrument (i.e., according to Tanona, a Bothe-Geiger-type coupling with a measurement instrument). In this way Tanona gives to 'individuality' a slightly different meaning than the one adopted here (which relates the term already to the interaction between radiation and matter). Also Tanona stresses Bohr's emphasis on the breakdown of the classical space-time pictures and not on the light-quantum concept (Tanona, 2004, pp. 498-500; see also Murdock, 1987, pp. 29-30).

> The quantum postulate implies that any observation of atomic phenomena will involve an interaction with the agency of observation not to be neglected … the definition of the state of a physical system, as ordinarily understood, claims the elimination of all external disturbances. But in that case, according to the quantum postulate, any observation will be impossible … if in order to make observation possible we permit certain interactions with suitable agencies of measurement, not belonging to the system, an unambiguous definition of the state of the system is naturally no longer possible. (p. 580)

That is, as a consequence of the 'quantized' interaction, there is no way, as in classical physics, to consider a weaker and weaker interaction that in the limit of an 'infinitesimal' exchange of energy-momentum would enable us to define a quantum system independently of the experimental context where it is being observed.

In his later writings the importance Bohr gave to this consequence of the quantum postulate became even clearer. In successive drafts for an article published in 1956, Bohr uses the word 'wholeness', 'indivisibility', and 'unity' (Honner, 1987, p. 69), finally writing "The essential indivisibility of proper quantum phenomena" (Bohr, 1956, p. 87). In several others of his later writings Bohr refers to a 'feature of wholeness' in the atomic processes (Bohr, 1954, p. 71; 1958, p. 2; 1962a, p. 78; 1962b, p. 80). This characterization of atomic processes by their 'wholeness' results from Bohr's insight into the consequences of the "atomistic feature in the energy transmission" (Bohr, 1933, p. 421) that he addressed in published articles since 1928, according to which "all effects of light may be traced down to individual processes, in each of which a so-called quantum is exchanged" (p. 421). We see then that it is Bohr's recognition of the quantized interaction between radiation and matter (embodied in the light-quantum concept) that leads him to the generalization that *all* atomic processes result from indivisible (atomistic) momentum and energy exchanges (i.e. *all* atomic processes are discontinuous).

One of the most relevant aspects of the "impossibility of separating a behavior of atomic objects from the interaction of these objects with the measuring instruments" (Bohr, 1948, p. 313) is the need of redefining the meaning of the term phenomena. According to Bohr we have to limit the "use of the word *phenomenon* to refer exclusively to observations obtained under specified circumstances, including an account of the whole experiment" (Bohr, 1948, p. 317). In this way we are no "longer in a position to speak of the autonomous behavior of a physical object, due to the unavoidable interaction between the object and the measuring instruments" (Bohr, 1937, p. 293). This implies that for Bohr the wave-function of a 'quantum' system does not have an independent meaning on its own, it can only be defined in the context of an experimental arrangement. For example, we cannot speak of the wave-function of an electron by itself, only of its wave function as defined in a particular experimental setup (Teller, 1980, p. 206; Camilleri, 2007, p. 522; Peres, 1995, pp. 24-25). This contextualization of the quantum phenomena brings with it, according to Bohr, the need to use classical concepts. Thus, according to Bohr, we must consider that:

> [An] unambiguous communication of physical evidence demands that the experimental arrangement as well as the recording of the observations be expressed in common language, suitably refined by vocabulary of classical physics. In all actual experimentation this demand is fulfilled by using as measuring instruments bodies like diaphragms, lenses and photographic plates so large and heavy that, notwithstanding the decisive role of the quantum of action for the stability and properties of such bodies, all quantum effects can be disregarded in the account of their position and motion. (Bohr, 1962b, p. 91)

3. The concept of time in quantum theory and Bohr's quantum postulate

To see the role of the concept of time in quantum theory we have simply to consider the quantum-theoretical description of time-dependent phenomena by the time-dependent Schrödinger equation:

$$i\hbar \frac{\partial \Psi(t)}{\partial t} = H\,\Psi(t)\,,$$

where H is the Hamiltonian and ψ the wave function of a system. For the present purpose, I will consider the quantum-theoretical description of the spontaneous emission of radiation by an atom. In this case the Hamiltonian is $H = H_{atm} + H_{rad} + H_{int}$, where H_{atm} is the Hamiltonian for an atom, H_{rad} is the Hamiltonian for the electromagnetic field, and H_{int} is the Hamiltonian describing the interaction between the electromagnetic field and the atom. Considering the case where the electromagnetic field wavelength is much larger than the dimension of the atom, the interaction between the field and the atom can be described using the electric dipole approximation (Loudon, 1973, pp. 42-43). Also, in the case of a weak coupling between the field and the atom, the time-dependent Schrödinger equation can be solved in first-order perturbation theory (Ballentine, 1998, pp. 349-350). To determine the spontaneous emission rate (and from this the lifetime of the excited state), corresponding to the transition of an atom initially in an excited state to the ground state (with the electromagnetic field initially in the vacuum state), a further approximation will be considered; we will consider a time t larger than the inverse of the frequency of the spontaneously emitted photon, but since it is a first-order calculation we must also take t to be smaller than the excited state lifetime (Craig & Thirunamachandran, 1984, pp. 84-86). Solving the time-dependent Schrödinger equation for this case we see that the probability of finding the atom in the excited state follows an exponential decay law, that is, the time dependence is given by $e^{-t/\tau}$, where τ is the excited state lifetime (Allen & Eberly, 1975, p. 167). This means that if we consider an ensemble of equally prepared systems each one consisting of an atom in the same excited state (in a suitable experimental setup), and we measure the time it takes for each atom to decay, for a large ensemble the measured times will fall on an exponential line defined by the lifetime of the atoms. However, as we have seen, according to the quantum theory “the process of transition is indivisible” (Bohm, 1951, p. 426). In this way, when considering just one atom there is only a sole perception of a change in the state of the atom due to the instantaneous emission of radiation at a certain time. Without any external clock there would be no way of timekeeping with just this one atom. There would be no timekeeping before the emission of radiation and after the emission of radiation, only a perception of the change itself due to the emission of a photon. But the time-dependence of the wave function is not related to (what can be called) an internal time related with some dynamical variable, but to a time parameter that is external to the quantum systems. Quantum theory is formulated by considering a background space-time that enters the equations of the theory as parameters not dependent on any physical systems that are described by the theory. In this way the time parameter can be considered as external to the physical systems whose equations of motion are dependent on this space-time parameters.[5] We see then that in the case under consideration of an atom in an

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[5] On the other hand, it is possible, in same cases, to define an internal time by considering some dynamical variable of a quantum system whose behaviour mimics the external time (Hilgevoord, 2005, p. 31). One example is the quantized linear oscillator, described by a quantized angle variable whose

excited state, its description, as just mentioned, presupposes an external time flowing independently of the atom. This means that in the quantum theory we cannot really think in terms of one isolated system: the temporal behavior as described in the quantum theory is not perceptible from the behavior of just one system but only by taking into account an ensemble of equally prepared systems.

The exponential decay law considered in the case of the spontaneous emission of electromagnetic radiation is common in the temporal behavior of different physical systems. The first known example is the disintegration of radioactive substances that follows an exponential law of decay (Rutherford & Chadwick & Ellis, 1930, pp. 4-8). Considering this, we could imagine a situation where "all 'regular' clocks were abolished from our laboratories, and we were forced to use radium clocks, in which the defining events are the disintegration of individual atoms" (Campbell, 1927, p. 779). In this situation, if we had only a few radium atoms (or equivalently in case of the spontaneous emission, a few atoms in an excited state), from the sequence of more or less simultaneous groups of decays, it would not be possible to construct an exponential curve and with it have a rough timekeeping procedure. But for a very large ensemble, we could expect, according to the theory, to have, as time goes by, a clearer notion of the temporal behavior of the ensemble as a whole, even without an explicit reference to an external clock. In similar lines, in the case being considered of the temporal description of the spontaneous emission, each individual atom, according to the quantum postulate, will have a discontinuous change of state at a particular (external) time, and it is only possible to determine the probability for this change. But considering a large ensemble of identical systems, by measuring the time of decay of each one we obtain a distribution that fits an exponential curve. In this way the regularity of the temporal behavior of a quantum system is only made 'visible' by considering a large ensemble of equally prepared systems (Peres, 1995, p. 403). But we must recall that the time dependence of each equally prepared quantum system is determined in terms of an external classical time measured by a 'classical' clock.

I will now address the problem of seeing how this use of a classical external time fits into Bohr's account of quantum theory. For Bohr, the space and time description of a quantum phenomenon is dependent on the definition of a space-time reference frame by "fixed scales and clocks" (Bohr, 1949, p. 40). This coordinate system is "fixed in the ordinary way by means of solid bodies and unperturbable clocks" (Bohr, 1928, p. 584).[6] That is, for Bohr the reference frame is defined by the macroscopic experimental arrangement.

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eigenvalue runs through the interval $[0, 2\pi]$, and whose equation of motion describes "the behaviour we expect of the hand of a clock" (Hilgevoord, 2002, p.304): $U(t)|\phi\rangle = |\phi + \omega t\rangle$, where U(t) is the time evolution operator, ϕ is an angle variable, and ω is the constant frequency of the 'quantum clock' (Larmor clock). It is important to notice that the dynamical variable ϕ is described quantum mechanically. For example, if we try to use this 'clock' to measure the time of decay of an atom in an excited state and consider an ensemble of clocks subjected to the same experimental arrangement (each clock being coupled to an atom that is not directly observed), we obtain a time distribution from the ensemble of clocks (for the time of decay) according to the exponential decay law (see Peres, 1995, pp. 406-412).

[6] In this work I will make the simplifying assumption that we can identify directly the space-time reference frame with the background space-time, i.e. that the reference frame is inertial. This means that I am taking the effective laboratory frame to be an inertial frame or at least to be, for all practical purpose, nearly inertial. It is important to notice that by definition an inertial reference frame is one in which Newton's laws of motion are valid. This implies that the operationally defined inertial reference frame (the material frame of reference) must itself be describable by classical dynamics (for details on this subject see Dickson, 2004).

Not only is the existence of a coordinate system linked to an experimental arrangement, but also for its definition we have to disregard the quantum postulate – in what regards the stipulation of a reference frame ‘connected’ with the experimental setup – and consider it entirely defined on classical terms. This is because, according to Bohr, “if we want to use the idea of space-time we must have watches and roads which are outside and independent of the object under consideration” (Bohr, 1985, p. 369). This classical operationally defined space-time coordinate system (i.e. a coordinate system presupposed by an inertial material frame of reference so large and heavy that all quantum effects can be disregarded so that it has a well-defined position and momentum in relation to the background space-time) not in directly interaction with the quantum system, can be seen as implemented in the formalism of the quantum theory as external space and time parameters that appear in the Schrödinger equation and wave function.

That we must consider, according to Bohr’s view, the space and time parameters as related to the experimental arrangement, can be expected by taking into account Bohr’s interpretation of the quantum formalism. Due to the quantum postulate the wave function (and the Schrödinger equation) cannot be seen as something intrinsic to a quantum system and independent of the experimental arrangement that enables the observation of the phenomena we associate with the quantum system. This means also that part of the conceptual content and mathematical description of the wave function is dependent on the existence of an experimental arrangement. In particular the time parameter that appears in the Schrödinger equation and wave function depends on a background space-time, that we can see as justified, according to Bohr’s interpretation of the quantum formalism, by taking into account – due to the quantum postulate – the unavoidable interaction of the quantum system with the experimental arrangement *with its entirely classically described* “fixed measuring rods and synchronized clocks” (Bohr, 1955, p. 90).

To sum up: on one hand we must take the quantum system – due to the quantum postulate – not to be definable without taking into account the experimental setup; and on the other hand we must take the reference frame to be independent of the quantum system so that it can be described entirely in classical terms. This situation is not inconsistent. In reality it is possible to present a coherent view on the consequences of the quantum postulate in what regards the classical description of space and time, by noticing that, according to Bohr, not all of the experimental arrangement has to be considered in direct interaction with the quantum system, only the “significant parts of the experimental arrangement” (Bohr, 1962b, p. 92). This is what makes it possible to consider that the watches and measuring rods “are outside and independent of the object under consideration” (Bohr, 1985, p. 369). That is, at the same time the quantum system and its mathematical description cannot be defined without taking into account, due to the quantum postulate, the interaction with the measuring instruments, but in what regards the rods and clocks belonging to the experimental arrangement and necessary for the definition of the space-time reference frame, they are independent of the quantum system (i.e. not in direct interaction), and so not submitted to the consequences of the quantum postulate being treated as totally classical entities.

This reading of Bohr’s ideas implies considering the part of the experimental arrangement not directly in interaction with the quantum system as describable by classical physics. What about the parts that are in interaction? According to Paul Teller’s account of Bohr’s interpretation of quantum mechanics:

> Bohr acknowledges that one may include the immediate macroscopic measuring device as part of the object described with the formalism of quantum mechanics, as long as there remains some further part of the total experimental context which receives a classical description. (Teller, 1980, p. 215)

I agree with Teller's reading of Bohr. Let us consider the two-slit electron diffraction experiment. According to Bohr, depending on what we want to measure, the position or momentum of the electron, we must use a first diaphragm rigidly fixed to the apparatus (in the case of a position measurement) or not rigidly connected to the apparatus (in the case of a momentum measurement). In the second case the position and momentum of the diaphragm are treated quantum mechanically:

> In the arrangement suited for the control of the momentum of the first diaphragm, this body can no longer be used as a measuring instrument for the same purpose as in the previous case [(the position measurement)], but must, as regards its position relative to the rest of the apparatus, be treated, like the particle traversing the slit, as an object of investigation, in the sense that the quantum-mechanical uncertainty relations regarding its position and momentum must be taken explicitly into account. (Bohr, 1935, p. 698)

How is it possible to treat classically the diaphragm in one case and in the other to treat it quantum mechanically? According to Bohr, this is made acceptable by taking into account his correspondence principle. Bohr mentions the "necessity of discriminating in each experimental arrangement between those parts of the physical system considered which are to be treated as measuring instruments and those which constitute the objects under investigation" (Bohr, 1935, p. 701). Bohr considers that "the place within each measuring procedure where this discrimination is made is … largely a matter of convenience" (p. 701). Accordingly, there is no "inconsistency in the quantum-mechanical description, connected with a change of the place where the discrimination is made between object and measuring agencies … we only have a free choice of this place within a region where the quantum-mechanical description of the process concerned is effectively equivalent with the classical description" (p. 701).

To provide a more elaborated characterization of the reconstruction being proposed in this paper I will make a contrast with Don Howard's reconstruction of Bohr's interpretation of quantum mechanics, which in my view makes it very difficult to give a coherent reading of Bohr's ideas when taking into account his views on space and time in quantum mechanics. According to Howard:

> Bohr demanded a classical description only of those properties of the measuring instrument that are correlated, in the measurement interaction, with the properties of the observed object that we seek to measure … this implies, as well, a classical description of the associated measured properties of the observed object itself. A quantum description would be possible for the remaining properties of instrument and object, the properties not crucially involved in the measurement. (Howard, 1994, p. 203)

In the articulation of his argument, Howard first makes it plausible, by quoting Bohr, the idea that not all the entire measuring instrument has to be described in classical terms. Howard focuses on the description of the diaphragm in the previously mentioned experiment (which he refers to as diaphragm A), which as mentioned in a particular experimental setup had to be rigidly connected with the rest of the measuring apparatus while in another experimental setup had to be detached from the support that defines the frame of reference. Howard considers that in both cases – of a movable or fixed diaphragm – we can consider the diaphragm as part of the measuring instrument. I think the quotation of Bohr cited by Howard (the passage in page 698 of Bohr's 1935 article quoted above) is ambiguous enough to accommodate also Howard's reading. Also, in a different article, regarding the particular experimental arrangement for measuring the momentum, Bohr mentions that "certain parts of the whole device must naturally be given the freedom to move independently of others" (Bohr, 1949, p. 48), and this goes along the lines of Howard's proposal. But I think that, according to the reconstruction of Bohr's ideas proposed here, this possibility of giving a classical or quantum-mechanical description of the diaphragm is taken care of by taking into account Bohr's line of argumentation related to his correspondence principle: there is a freedom of choosing to treat part of the experimental setup quantum mechanically (usually as part of the object under investigation) or classically (usually as part of the measuring apparatus). That is, we can choose where to make the 'Heisenberg cut' according to our convenience without entailing any inconsistency.[7]

More complicated in my view is another point of Howard's argumentation. Howard considers also that

> In neither arrangement will the whole of diaphragm *A* be given a classical description. In the second arrangement, the position is described quantum mechanically; in the first, we may infer, the momentum will be so described. What will be described classically are, by implication, only those properties of diaphragm *A* that are correlated with the observed system in the measurement. (Howard, 1994, p. 214)

According to Howard, further evidence for this interpretation can be found in two passages from Bohr (1939a). The first one is:

> We must recognize that a measurement can mean nothing else than the unambiguous comparison of some property of the object under investigation with a corresponding property of another system, serving as a measuring instrument, and for which this property is directly determinable according to its definition in everyday language or in the terminology of classical physics. (p. 311)

While this passage cannot be said to contradict or give direct evidence to Howard's claim, the second seems to me to be a typical Bohrian reference to a consequence of the correspondence principle:

> In the system to which the quantum mechanical formalism is applied, it is of course possible to include any intermediate auxiliary agency employed in the measuring process. Since, however, all those properties of such agencies which, according to the aim of the measurements have to be compared with the corresponding properties of the object, must be described on classical lines, their quantum mechanical treatment will for this purpose be essentially equivalent with a classical description. (pp. 315-316)

But again (as in the case mentioned above), no clear conclusion can be given in relation to Howard's interpretation of these quotations without taking fully into account Bohr's views regarding quantum mechanics, or more exactly a 'self-consistent' reconstruction of Bohr's writings making compatible Bohr's quantum postulate and Bohr's doctrine of a classical space and time.[8]

To set my case I will need to consider Howard's full-blown development of his views. In another passage Howard spells out his main point again:

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[7] In the view presented here, the 'Heisenberg cut' is an unavoidable aspect of considering a (classically described) reference frame taken to be, according to Bohr's views, outside and independent of the quantum system under consideration. Accordingly the meaning I give to 'wholeness' or 'unity' is not like in Howard's case of an entanglement between the quantum system and the experimental arrangement in its totality, but results from to the indivisibility of the quantum interaction, which entails – according to the quantum postulate – the impossibility of defining the quantum system independently of the experimental arrangement (see also footnote 4).

[8] However, in this particular case we do not have to look far to notice that this quotation taken out of context can have a broad array of (re)interpretations. Just a few lines below Bohr writes: "the only significant point is that in each case some ultimate measuring instruments, like the scales and clocks which determine the frame of space-time coordination – on which, in the last resort, even the definitions of momentum and energy quantities rest – must always be described entirely on classical lines, and consequently kept outside the system subject to quantum mechanical treatment". (Bohr, 1938a, p. 316)

> This means that the only *essential* use of classical methods of description will be in connection with that property of the instrument that is correlated with the property of the object that the instrument is designed to measure. (Howard, 1994, p. 216)

From this several things will follow:

A) We must interpret Bohr's reference to classical concepts not relating it to the correspondence principle, since, according to Howard, only certain properties of both the measurement apparatus and the object must be considered from a classical perspective. In this way, we must look for some other type of 'essential equivalence' between the quantum and classical description.[9] In Howard's view the kind of convergence between quantum and classical descriptions demanded by the correspondence principle is a wholesale convergence, not an 'essential equivalence' between selected sets of properties (Howard, 1994, p. 217; see also Howard, 2005, pp. 28-29).

B) According to Howard we will find this 'essential equivalence' by looking at the case of an ensemble of identically prepared composite systems (consisting on the measurement setup and the 'quantum object') described by a density matrix built with non-factorizable state functions (due to the entanglement between the measurement setup and the 'quantum object') – the pure case. With a proper selection of sub-ensembles describing factorizable state functions (according to Howard corresponding to the classical idea of separability) the ensemble can be seen as a mixture of elements of the sub-ensembles. According to Howard in each experimental context it is possible to find the proper mixture that gives exactly the same predictions as the original density matrix for the entangled pair (Howard, 1994, pp. 220-222; see also Shlosshauer & Camilleri, 2008, pp. 17-18). Howard considers this approach as the key to a reconstruction of Bohr's views on the need of classical concepts:

> It is upon this disarmingly simple mathematical fact—the equivalence, context by context, of pure cases and mixtures—that I build my interpretation of Bohr's doctrine of classical concepts. I claim that we make the clearest sense out of Bohr's stress on the importance of a classical account of experimental arrangements and of the results of observation, if we understand a classical description to be one in terms of appropriate mixtures. More specifically, I would reconstruct the doctrine of classical concepts as follows. Given any measurement interaction, a description in terms of a pure case is correct, in the sense that it yields all of the right predictions. This is the proper quantum mechanical account of the interaction, and such an account can always be given for all aspects of the interaction, including all parts of both instrument and object. Such a description reflects the essential nonseparability of the quantum mechanical interaction formalism, the nonseparability that Bohr stresses as a fundamental lesson of the quantum mechanical account of the instrument/object interaction; it reflects, too, the non-classical character of quantum statistics. On the other hand, precisely because of its nonseparability, a description in terms of a pure case does not permit us to distinguish instrument and object in the way that Planck and Einstein thought necessary to ensure objectivity. But here is where the concept of an appropriate mixture finds its place. Once we specify the kind of measurement being performed, an appropriate mixture can be

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[9] In Howard's reading of Bohr's Como lecture (Bohr, 1928), Bohr takes the object and instrument to form an entangled pair that is described quantum mechanically. In order to assign a measured value to the quantum system we must consider the object and instrument not to be entangled (in what regards the property of the measuring instrument that is correlated, in the measurement, with the property of the observed object that we seek to measure). According to Howard, "doing that is what Bohr means by a description in terms of 'classical concepts'" (Howard, 2005, p. 28). In this way according to Howard reading of Bohr, "the descriptions are 'classical' simply in the sense that entanglement is denied and separability is affirmed" (p. 28). For more details on Howard's use of the term 'classical' see point B and the quotation within.

> constructed that gives all of the right predictions for the parameters involved in such a measurement; and, at least with respect to those parameters, we can separate the states of the instrument and the object and give a purely classical, ignorance interpretation of their statistics. The proper "classical" description, then, is a description in terms of an appropriate mixture. (Howard, 1994, pp. 222-223)

C) One of the implications of all this is clearly spelled out in the following sentence:

> The fourth question asked why, in the description of a measuring instrument, the only *essential* use of classical methods of description is in the account of that parameter of the instrument that is correlated with the measured property of the object. The answer is that only in connection with these properties need we assume the separability of instrument and object. And the appropriate mixture reconstruction reflects this fact by its context dependence: A different mixture is appropriate to every different context, in the sense that an appropriate mixture yields the correct predictions only for those parameters measurable in that context. All other parameters—of both object and instrument—are correctly described only quantum mechanically, in terms of the pure case density matrix. (Howard, 1994, p. 224)

In this way, we would not need a description of the apparatus in classical terms. The classical description (in Howard's sense) is only necessary for the part that we can consider to be in direct interaction with the object and even in this case only for the parameter involved in the measurement. In the case being considered, for a fixed diaphragm we would have to give a classical description of the position of the diaphragm and the object, and in the case of a moving diaphragm it would be necessary to give a classical description of the momentum. All the rest would be described quantum mechanically.

This interpretation faces problems when we consider Bohr's views on the concepts of space and time in quantum mechanics. As Jan Hilgevoord mentions, there seems to be a problem related to the use of the time concept in quantum mechanics that has posed a challenge to several physicists but to which Bohr makes no reference (Hilgevoord, 2005): the inexistence of a time operator in quantum mechanics. Bohr does not mention this because in his interpretation of quantum mechanics there is simply no motive not to consider space and time from a classical perspective (Hilgevoord, 2005, pp. 47-48). This should be clear from the presentation made above of Bohr's reliance on classical concepts of space and time in his interpretation of quantum mechanics. However, and I must stress this point, it is important to notice that for Bohr the classically described space-time reference frame (which is part of the experimental arrangement) is taken to be outside and independent of the observed quantum system.

This is in direct contradiction with Howard's reconstruction. As we have just seen Howard considers that:

1) The only essential use of classical methods of description (in Howard's sense) will be in connection with that property of the instrument that is correlated with the property of the object that the instrument is designed to measure.

2) A quantum description would be possible for the remaining properties of instrument and object, the properties not crucially involved in the measurement.

As it stands I consider this view incompatible with Bohr's account of reference frames in the experimental arrangements. Bohr wrote *explicitly* that "if we want to use the idea of space-time we must have watches and roads which are outside and independent of the object under consideration" (Bohr, 1985, p. 369), and in similar lines that "some ultimate measuring instruments, like the scales and clocks which determine the frame of space-time coordination … must always be described entirely on classical lines, and consequently kept outside the system subject to quantum mechanical treatment" (Bohr, 1939a, p. 316). As I have shown this view is compatible with Bohr's quantum postulate. As it stands Howard's reconstruction seems to be at odds with Bohr's views on space and time in quantum mechanics. In particular, according to Howard's views, we would have to consider that the entire quantum mechanically described apparatus is entangled with the quantum object (since Howard considers that only for the relevant parameter of the measurement we need a 'classical' separability of apparatus and object). It is not clear how, in these circumstances, an external classical time might fit into Howard's reconstruction.

Parts of the views being developed here have been noticed before. For example by Simon Saunders who made the following remark:

> The conditions of an experiment must ultimately involve rigid connections to bodies of arbitrarily large mass. In that case the uncertainty relations, for the latter bodies, become irrelevant (so long as there is non-zero latitude in *both* position and momentum). Bohr admitted as much when he remarked that the freedom of choice in the divide between quantum and classical was restricted to "a region where the quantum mechanical description of the process concerned is effectively equivalent with the classical description" [5, p.701], and later, when he said that the requirements of unambiguous description of the apparatus "is secured by the use, as measuring instruments, of rigid bodies sufficiently heavy to allow a completely classical account of their relative positions and velocities" [9, p.3]. (Saunders, 2005, p. 24)

In similar lines Henrik Zinkernagel remarked that:

> Bohr actually agreed that the measurement apparatus can also be described by quantum theory. However, he writes (1939, p. 104):
>
> ...in each case some ultimate measuring instruments, like the scales and clocks which determine the frame of space-time coordination – on which, in the last resort, even the definitions of momentum and energy quantities rest – must always be described entirely on classical lines, and consequently kept outside the system subject to quantum mechanical treatment.
>
> The point is that we can treat a measuring apparatus (or part of this) as a quantum system, but only when some other system is then treated classically. (Zinkernagel, 2006, p. 5)

Zinkernagel's and Saunders's reading of Bohr seems to go along similar lines as the one being presented here, as is the case with the previous quotation from Teller (see page 8). Teller mentions that, according to Bohr, even if we have the possibility of describing quantum mechanically the immediate macroscopic measuring device (i.e. the part of the instrument in interaction with the object under investigation), we have to describe classically some further part of the experimental arrangement. Saunders stresses the need of a material frame of reference (bodies of arbitrarily large mass) to describe any experiment, and, quoting Bohr, that this material frame of reference must be described by classical dynamics. Zinkernagel quotes Bohr saying that the material frame of reference must be described entirely by classical dynamics and taken to be outside the system subjected to a quantum mechanical description (in an hypothetical revision of Howard's approach this might imply taking from the start the material frame of reference to be disentangled from the quantum system).

I think it is fair to say that we are all reading Bohr as implying the need for a classical physics account of (at least) part of the experimental arrangement (the one not directly in interaction with the quantum system), stressing in particular the need for a classical account of the reference frame. This view is clearly at odds with Howard's. Howard's reconstruction implies an all-quantum description of the entangled pair instrument & object, giving just a classical description (in Howard's sense), for both the instrument and object, of the property being measured. On the reconstruction being presented here (which I must stress is compatible with the interpretation being given to the quantum postulate), according to Bohr, we must describe classically (in the sense of using classical theories, not in Howard's sense) the material frame of reference, taken to be independent (in my sense this means not in direct interaction; in Howard's sense it might mean disentangled) from the object of observation. In this way I consider Howard's, nevertheless very interesting, reconstruction of Bohr's interpretation of quantum mechanics to be incompatible (at least as it stands at the moment) with Bohr's conception of space and time in quantum mechanics (related to an operationally defined and classically described reference system). In exact opposition to Howard's views, Bohr's doctrine of a classical space and time implies treating classically (in the usual sense) the parts of the measuring apparatus that are not in interaction with the quantum system.

4. Conclusions

In the quantum theory we can only recover a temporal description of the behavior of physical systems when considering an ensemble of identically prepared systems that are subjected to similar experimental arrangements. In fact, we cannot from one sole system have a well-defined sense of temporal flow. This might lead to consider that there is a tension between the idea of a continuous time and the discontinuous jumps between the possible states of a quantum system. In part the tension is avoided by constructing the theory considering a background space-time on which it is not the temporal behavior of an individual system – due to the discontinuity – that is described but the temporal behavior of an ensemble. Another related point is that the discontinuity is implemented in the theory already with the previous 'input' of a background time, that is, we talk about discontinuous changes as implied in the quantum postulate from the perspective of a macroscopic continuous space-time related to the experimental arrangement from which it looks as if the quantum system has discontinuous changes. As Bohr stressed, the point is that, as a consequence of the quantum postulate we cannot regard the physical properties of the quantum system as intrinsic and independent of the experimental arrangement being used (with its related classical reference frame). According to Bohr:

> No result of an experiment concerning a phenomenon which, in principle, lies outside the range of classical physics, can be interpreted as giving information about independent properties of the object; but is inherently connected with a definite situation in the description of which the measuring instruments interacting with the objects also enter essentially. (Bohr, 1939b, p. 269)

> While within the scope of classical physics we are dealing with an idealization, according to which all phenomena can be arbitrarily subdivided, and the interaction between the measuring instruments and the object under consideration neglected, or at any rate compensated for, it was stressed that such interaction represents in quantum physics an integral part of the phenomena, for which no separate account can be given if the instruments shall serve the purpose of defining the conditions under which the observations are obtained … The characteristic new feature in quantum physics is merely the restricted divisibility of the phenomena, which for unambiguous description demands a specification of all significant parts of the experimental arrangement … the whole purpose of the formalism of quantum theory is to derive expectations for observations obtained under given experimental conditions. (Bohr, 1962b, pag. 91-92)

In this way the characterization of the discontinuous behavior of the quantum system, that is, of the phenomena being observed, rests on the previous notion of a time coordinate (associated with clocks that are part of the measurement apparatus). We have no intrinsic notion of discontinuity by itself; this notion arises in the context of a theory that, following Bohr's view on the interpretation of the formalism, does not treat the phenomena we observe and associate with a 'quantum object' (like for example an electron) as something that has a meaning independent of the experimental arrangement that permits its observation. We have a discontinuity in the context of an operational notion of a continuous time that is simultaneously inscribed in the formalism of the theory as an external parameter in the Schrödinger equation. It is from the perspective of this external time, which we use in the description of the functioning of any measurement apparatus, that the discontinuity we associate with the phenomena appears, simultaneously with a statistical description of the (continuous) temporal behavior of an ensemble of equally prepared systems.

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