

A Two-Sided Ontological Solution to the Sleeping Beauty Problem

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ABSTRACT. I describe in this paper an ontological solution to the Sleeping Beauty problem. I begin with describing the hyper-entanglement urn experiment. I restate first the Sleeping Beauty problem from a wider perspective than the usual opposition between halfers and thirders. I also argue that the Sleeping Beauty experiment is best modelled with the hyper-entanglement urn. I draw then the consequences of considering that some balls in the hyper-entanglement urn have ontologically different properties from normal ones. In this context, considering a Monday-waking (drawing a red ball) leads to two different situations that are assigned each a different probability. This leads to a two-sided account of the Sleeping Beauty problem. On the one hand, the first situation is handled by the argument for $1/3$. On the other hand, the second situation corresponds to a reasoning that echoes the argument for $1/2$.

1. The hyper-entanglement urn

Let us consider the following experiment. In front of you is an urn. The experimenter asks you to study very carefully the properties of the balls that are in the urn. You go up then to the urn and begin to examine its content carefully. You note first that the urn contains only red or green balls. By curiosity, you decide to take a sample of a red ball in the urn. Surprisingly, you notice that while you pick up this red ball, another ball, but a green one, also moves simultaneously. You decide then to replace the red ball in the urn and you notice that immediately, the latter green ball also springs back in the urn. Intrigued, you decide then to catch this green ball. You notice then that the red ball also goes out of the urn at the same time. Furthermore, while you replace the green ball in the urn, the red ball also springs back at the same time at its initial position in the urn. You decide then to withdraw another red ball from the urn. But while it goes out of the urn, nothing else occurs. Taken aback, you decide then to undertake a systematic and rigorous study of all the balls in the urn.

At the end of several hours of a meticulous examination, you are now capable of describing precisely the properties of the balls present in the urn. The latter contains in total 1000 red balls and 500 green balls. Among the red balls, 500 are completely normal balls. But 500 other red balls have completely astonishing properties. Indeed, each of them is linked to a different green ball. When you remove one of these red balls, the green ball which is associated with it also goes out at the same time from the urn, as if it was linked to the red ball by a magnetic force. Indeed, if you remove the red ball from the urn, the linked green ball also disappears instantly. And conversely, if you withdraw from the urn one of the green balls, the red ball which is linked to it is immediately removed from the urn. You even try to destroy one of the balls of a linked pair of balls, and you notice that in such case, the ball of the other colour which is indissociably linked to it is also destroyed instantaneously. Indeed, it seems to you that relative to these pairs of balls, the red ball and the green ball which is linked to it behave as one single object.

The functioning of this urn leaves you somewhat perplexed. In particular, you are intrigued by the properties of the pairs of correlated balls. After reflection, you tell yourself that the properties of the pairs of correlated balls are finally in some respects identical to those of two entangled quantum objects. Entanglement (Aspect & al. 1982) is indeed the phenomenon which links up two quantum objects (for example, two photons), so that the quantum state of one of the entangled objects is correlated or anti-correlated with the quantum state of the other, whatever the distance where the latter is situated. As a consequence, each quantum object can not be fully described as an object per se, and a

pair of entangled quantum objects is better conceived of as associated with a single, entangled state. It occurs to you that perhaps a pair of correlated balls could be considered, alternatively, as an ubiquitous object, i.e. as an object characterised by its faculty of occupying two different locations at the same time, with the colours of its two occurrences being anti-correlated. Setting this issue aside for the moment, you prefer to retain the similarity with the more familiar quantum objects. You decide to call “hyper-entanglement urn” this urn with its astonishing properties. After reflection, what appears specific with this urn, is that it includes at the same time some normal and some hyper-entangled balls. The normal red balls have nothing different with our familiar balls. But hyper-entangled balls do behave in a completely different way. What is amazing, you think, is that nothing seemingly differentiates the normal red balls from the red hyper-entangled ones. You tell yourself finally that it could be confusing.

Your reflection on the pairs of hyper-entangled balls and their properties also leads you to question the way the balls which compose the pairs of hyper-entangled balls are to be counted. Are they to be counted as normal balls? Or do specific rules govern the way these pairs of hyper-entangled balls are to be counted? You add a normal red ball in an hyper-entanglement urn. It is then necessary to increment the number of red balls present in the urn. On the other hand, the total number of green balls is unaffected. But what when you add in the hyper-entanglement urn the red ball of a pair of hyper-entangled balls? In that case, the linked green ball of the same pair of hyper-entangled balls is also added instantly in the urn. Hence, when you add a red ball of a pair of hyper-entangled balls in the urn, it also occurs that you add at the same time its associated green ball. So, in that case, you must not only increment the total number of red balls, but also the total number of green balls present in the urn. In the same way, if you withdraw a normal red ball from the urn, you simply decrement the total number of red balls of the urn, and the number of green balls in the urn is unaffected. But if you remove the red ball (resp. green) of a pair of hyper-entangled balls, you must decrement the total number of red balls (resp. green) present in the urn as well as the total number of green balls (resp. red).

At this very moment, the experimenter happens again and withdraws all balls from the urn. He announces that you are going to participate in the following experiment:

*The hyper-entanglement urn** A fair coin will be randomly tossed. If the coin lands Heads, the experimenter will put in the urn a normal red ball. On the other hand, if the coin lands Tails, he will put in the urn a pair of hyper-entangled balls, composed of a red ball and a green ball, both indissociably linked. The experimenter also adds that the room will be put in absolute darkness, and that you will therefore be completely unable to detect the colour of the balls, no more that you will be able to know, when you will have withdrawn a ball from the urn, whether it is a normal ball, or a ball which is part of a pair of hyper-entangled balls. The experimenter tosses the coin and while you catch a ball from the urn, he asks you to assess the likelihood that the coin felt Heads.

2. The Sleeping Beauty problem

Consider now the well-known *Sleeping Beauty problem* (Elga 2000, Lewis 2001). Sleeping Beauty learns that she will be put into sleep on Sunday by some researchers. A fair coin will be tossed and if the coin lands Heads, Beauty will be awakened once on Monday. On the other hand, if the coin lands Tails, Beauty will be awakened twice: on Monday and on Tuesday. After each waking, she will be put into sleep again and will forget that waking. Furthermore, once awakened, Beauty will have no idea of whether it is Monday or Tuesday. On awakening on Monday, what should then be Beauty's credence that the coin did land Heads?

At this step, one obvious first answer (I) goes as follows: since the coin is fair, the initial probability that the coin lands Head is $1/2$. But during the course of the experiment, Sleeping Beauty does not get any novel information. Hence, the probability of Heads still remains $1/2$.

By contrast, an alternative reasoning (II) runs as follows. Suppose the experiment is repeated many times, say, to fix ideas, 1000 times. Then there will be approximately 500 Heads-wakings on Monday, 500 Tails-wakings on Monday and 500 Tails-wakings on Tuesday. Hence, the reasoning goes, the probability of Heads equals $500/1500 = 1/3$.

The argument for $1/2$ and the argument for $1/3$ yield conflicting conclusions. The Sleeping Beauty problem is usually presented accordingly as a problem arising from conflicting conclusions resulting

from the two above-mentioned competing lines of reasoning aiming at assigning the probability of Heads once Beauty is awakened. I shall argue, however, that this statement of the Sleeping Beauty problem is restrictive and that we need to envisage the issue from a wider perspective. For present purposes, the Sleeping Beauty problem is the issue of calculating properly (i) the probability of Heads (resp. Tails) once Beauty is awakened; (ii) the probability of waking on Monday (resp. Tuesday); and (iii) the probability of Heads (resp. Tails) on waking on Monday. From the halfer perspective, the probability of waking on Monday equals $3/4$, and the probability of waking on Tuesday is $1/4$. By contrast, from the thirder's perspective, the probability of waking on Monday equals $2/3$ and the probability of waking on Tuesday is $1/3$.

But the argument for $1/2$ and for $1/3$ also have their own account of conditional probabilities. To begin with, the probability of Heads on waking on Tuesday is not a subject of disagreement, for it equals 0 in both accounts. The same goes for the probability of Tails on waking on Tuesday, since it equals 1 from the halfer's or from the thirder's viewpoint. But agreement stops when one considers the probability of Heads on waking on Monday. For it equals $2/3$ from a halfer's perspective. However, from a thirder's perspective, it amounts to $1/2$. On the other hand, the probability of Tails on waking on Monday is $1/3$ from a halfer standpoint, and $1/2$ for a thirder.

3. The urn analogy

In what follows, I shall present an ontological solution to the Sleeping Beauty problem, which rests basically on the hyper-entanglement urn* experiment. A specific feature of this account is that it incorporates insights from the halfer and thirder standpoints, a line of resolution initiated by Nick Bostrom (2007) that has recently inspired some new contributions (Groisman 2008, Delabre 2008)¹.

The argument for $1/3$ and the argument for $1/2$ rest basically on an urn analogy. This analogy is made explicit in the argument for $1/3$ but is less transparent in the argument for $1/2$. The argument for $1/3$, to begin with, is based on an urn analogy which associates the situation related to the Sleeping Beauty experiment with an urn that contains, in the long run (assuming that the experiment is repeated, say, 1000 times), 500 red balls (Heads-wakings on Monday), 500 red balls (Tails-wakings on Monday) and 500 green balls (Tails-wakings on Tuesday), i.e. 1000 red balls and 500 green balls in total. In this context, the probability of Heads upon awakening is determined by the ratio of the number of Heads-wakings to the total number of wakings. Hence, $P(\text{Heads}) = 500/1500 = 1/3$. The balls in the urn are normal ones and for present purposes, it is worth calling this sort of urn a "standard urn".

On the other hand, the argument for $1/2$ is also based on an urn analogy, albeit less transparently. The main halfer proponent grounds his reasoning on calculations (Lewis 2001), but for the sake of clarity, it is worth rendering the underlying associated analogy more apparent. For this purpose, let us recall how the calculation of the probability of drawing a red ball is handled by the argument for $1/2$. If the coin lands Heads then the probability of drawing a red ball is 1, and if the coin lands Tails then this latter probability equals $1/2$. We get then accordingly the probability of drawing a red ball (Monday-waking): $P(R) = 1 \times 1/2 + 1/2 \times 1/2 = 3/4$. By contrast, if the coin lands Tails, we calculate as follows the probability of drawing a green ball (Tuesday-waking): $P(G) = 0 \times 1/2 + 1/2 \times 1/2 = 1/4$. To sum up, according to the argument for $1/3$: $P(R) = 3/4$ and $P(G) = 1/4$. For the sake of comparison, it is worth transposing this reasoning in terms of an urn analogy. Suppose then that the Sleeping Beauty experiment is iterated. It appears then that the argument for $1/2$ is based on an analogy with a standard urn that contains $3/4$ of red balls and $1/4$ of green ones. These balls are also normal ones and the analogy underlying the argument for $1/2$ is also with a "standard urn". Now assuming as above that the experiment is repeated 1000 times, we get accordingly an urns that contains 500 red balls (Heads-wakings on Monday), 250 red balls (Tails-wakings on Monday) and 250 green balls (Tails-wakings on Tuesday), i.e. 750 red balls and 250 green balls in total. Such content of the urn results directly from Lewis' calculation. However, as it stands, this analogy would arguably be a poor argument in favour of the halfer's viewpoint. But at this step, we should pause and consider that Lewis' argument for $1/2$ did not rely on this urn analogy, though the latter is a consequence of Lewis' calculation. We shall now turn to the issue of whether the standard urn is the correct analogy for the Sleeping Beauty experiment.

¹ Bostrom opens the path to a third way out to the Sleeping Beauty problem: "At any rate, one might hope that having a third contender for how Beauty should reason will help stimulate new ideas in the study of self-location". In his account, Bostrom sides with the halfer on $P(\text{Heads})$ and with the thirder on conditional probabilities, but his treatment has some counter-intuitive consequences on conditional probabilities.

In effect, it turns out that the argument for $1/3$ and the argument for $1/2$ are based on an analogy with a standard urn. But at this stage, a question arises: is the analogy with the standard urn well-suited to the Sleeping Beauty experiment? In other terms, isn't another urn model best suited? In the present context, this alternative can be formulated more accurately as follows: isn't the situation inherent to the Sleeping Beauty experiment better put in analogy with the *hyper-entanglement urn*, rather than with the *standard urn*? I shall argue, however, that the analogy with the standard urn is mistaken, for it fails to incorporate an essential feature of the experiment, namely the fact that Monday-Tails wakings are indissociable from Tuesday-Tails wakings. For in the Tails case, Beauty cannot wake up on Monday without also waking up on Tuesday and reciprocally, she cannot wake up on Tuesday without also waking up on Monday.

When one reasons with the standard urn, one feels intuitively entitled to add red-Heads (Heads-wakings on Monday), red-Tails (Tails-wakings on Monday) and green-Tails (Tails-wakings on Tuesday) balls to compute frequencies. But red-Heads and red-Tails balls appear to be objects of a fundamentally different nature in the present context. In effect, red-Heads balls are in all respects similar to our familiar objects, and can be considered properly as single objects. By contrast, it turns out that red-Tails balls are quite indissociable from green-Tails balls. For we cannot draw a red-Tails ball without picking up the associated green-Tails ball. And conversely, we cannot draw a green-Tails ball without picking up the associated red-Tails ball. In this sense, red-Tails balls and the associated green-Tails balls do not behave as our familiar objects, but are much similar to entangled quantum objects. For Monday-Tails wakings are indissociable from Tuesday-Tails wakings. On Tails, Beauty cannot be awakened on Monday (resp. Tuesday) without being also awakened on Tuesday (resp. Monday). From this viewpoint, it is mistaken to consider red-Tails and green-Tails balls as separate objects. The correct intuition, I shall argue, is that the red-Tails and the associated green-Tails ball can be assimilated to a pair of hyper-entangled balls and constitute but one single object. In this context, red-Tails and green-Tails balls are best seen intuitively as constituents and mere parts of one single object. In other words, red-Heads balls and, on the other hand, red-Tails and green-Tails balls, cannot be considered as objects of the same type for probability purposes. And this situation justifies the fact that one is not entitled to add unrestrictedly red-Heads, red-Tails and green-Tails balls to compute probability frequencies. For in this case, one adds objects of intrinsically different types, i.e. one single object with the mere part of another single object.

Given what precedes, the correct analogy, I contend, is with an hyper-entanglement urn rather than with a normal urn. As will become clearer later, this new analogy incorporates the strengths of both above-mentioned analogies with the standard urn. And we shall now consider the Sleeping Beauty problem in light of this new perspective.

4. Consequences of the analogy with the hyper-entanglement urn

At this step, it is worth drawing the consequences of the analogy with the hyper-entanglement urn, that notably result from the ontological properties of the balls. Now the key point proves to be the following one. Recall that nothing seemingly distinguishes normal balls from hyper-entangled ones within the hyper-entanglement urn. And among the red balls, half are normal ones, but the other half is composed of red balls that are each hyper-entangled with a different green ball. If one considers the behaviour of the balls, it turns out that normal balls behave as usual. But hyper-entangled ones do behave differently, with regard to statistics. Suppose I add the red ball of an hyper-entangled pair into the hyper-entanglement urn. Then I also add instantly in the urn its associated green ball. Suppose, conversely, that I remove the red ball of an hyper-entangled pair from the urn. Then I also remove instantly its associated green ball.

At this step, we are led to the core issue of calculating properly the probability of drawing a red ball from the hyper-entanglement urn. Let us pause for a moment and forget temporarily the fact that, according to its classical formulation, the Sleeping Beauty problem arises from conflicting conclusions resulting from the argument for $1/3$ and the argument for $1/2$ on calculating the probability of Heads once Beauty is awakened. This could be a red herring. For as we did see it before, the problem also arises from the calculation of the probability of waking on Monday (drawing a red ball), since conflicting conclusions also result from the two competing lines of reasoning. In effect, Elga argues for $2/3$ and Lewis for $3/4$. Hence, the Sleeping Beauty problem could also have been formulated alternatively as follows: once awakened, what probability should Beauty assign to her waking on

Monday? In the present context, this is tantamount to the probability of drawing a red ball from the hyper-entanglement urn.

What is then the response of the present account, based on the analogy with the hyper-entanglement urn, to the issue of calculating the probability of drawing a red ball? In the present context, “drawing a red ball” turns out to be somewhat ambiguous. For according to the ontological properties of the balls within the hyper-entanglement urn, one can consider red balls either from the viewpoint of colourness, or from the standpoint of object-ness². Hence, in the present context, “drawing a red ball” can be interpreted in two different ways: either (i) “drawing a red ball-as-colour”; or (ii) “drawing a red ball-as-object”. Now disambiguating the notion of drawing a red ball, we should distinguish accordingly between two different questions. First, (i) what is the probability of drawing a red ball-as-colour (Monday-waking-as-time-segment)? Let us denote by $P(R\uparrow)$ the latter probability. Second, (ii) what is the probability of drawing a red ball-as-object (Monday-waking-as-object)? Let us denote it by $P(R\rightarrow)$. This distinction makes sense in the present context, since it results from the properties of the hyper-entangled balls. In particular, this richer semantics results from the case where one draws a green ball of an hyper-entangled pair from the urn. For in the latter case, this green ball is not a red one, but it occurs that one also picks up a red ball, since the associated red ball is withdrawn simultaneously.

Suppose, on the one hand, that we focus on the colour of the balls, and that we consider the probability $P(R\uparrow)$ of drawing a red ball-as-colour. It occurs now that there are 2/3 of red balls-as-colour and 1/3 of green balls-as-colour in the urn. Accordingly, the probability $P(R\uparrow)$ of drawing a red ball-as-colour equals 2/3. On the other hand, the probability $P(G\uparrow)$ of drawing a green ball-as-colour equals 1/3.

Assume, on the other hand, that we focus on balls as objects, considering that one pair of hyper-entangled balls behaves as one single object. Now we are concerned with the probability $P(R\rightarrow)$ of drawing a red ball-as-object. On Heads, the probability of drawing a red ball-as-object is 1. On Tails, we can either draw the red or the green ball of an hyper-entangled pair. But it should be pointed out that if we draw on Tails the green ball of an hyper-entangled pair, we also pick up instantly the associated red ball. Hence, the probability of drawing a red ball on Tails is also 1. Thus, $P(R\rightarrow) = 1 \times 1/2 + 1 \times 1/2 = 1$. Conversely, what is the probability $P(G\rightarrow)$ of drawing a green ball-as-object (a waking on Tuesday)? The probability of drawing a green ball-as-object is 0 in the Heads case, and 1 in the Tails case. For in the latter case, we either draw the green or the red ball of an hyper-entangled pair. But even if we draw the red ball of the hyper-entangled pair, we draw then instantly its associated green ball. Hence, $P(G\rightarrow) = 0 \times 1/2 + 1 \times 1/2 = 1/2$. To sum up: $P(R\rightarrow) = 1$ and $P(G\rightarrow) = 1/2$. The probability of drawing a red ball-as-object (a waking on Monday) is then 1, and the probability of drawing a green ball-as-object (a waking on Tuesday) is 1/2. Now it turns out that $P(R\rightarrow) + P(G\rightarrow) = 1 + 1/2 = 1.5$. In the present account, this results from the fact that drawing a red ball and drawing a green ball – in general – are *not exclusive events*. And – in particular – drawing a red ball-as-object and drawing a green ball-as-object from an hyper-entangled pair are not exclusive events for probability purposes. For we cannot draw the red-Tails (resp. green-Tails) ball without drawing the associated green-Tails (resp. red-Tails) ball.

To sum up now. It turns out that the probability $P(R\uparrow)$ of drawing a red ball-as-colour (Monday-waking-as-time-segment) equals 2/3. And the probability $P(G\uparrow)$ of drawing a green ball-as-colour (Tuesday-waking-as-time-segment) equals 1/3. On the other hand, the probability $P(R\rightarrow)$ of drawing a red ball-as-object (Monday-waking-as-object) equals 1; and the probability $P(G\rightarrow)$ of drawing a green ball-as-object (Tuesday-waking-as-object) equals 1/2.

² This issue relates to the identity of indiscernibles and is notably hinted at by Max Black (1952, p. 156) who describes a universe composed of two identical spheres: “Isn't it logically possible that the universe should have contained nothing but two exactly similar spheres? We might suppose that each was made of chemically pure iron, had a diameter of one mile, that they had the same temperature, colour, and so on, and that nothing else existed. Then every quality and relational characteristic of the one would also be a property of the other.” In the present context, it should be pointed out that the colours of the hyper-entangled balls are anti-correlated. John Leslie (2001, p. 153) also raises a similar issue with his paradox of the balls: “Here is a yet greater paradox for Identity of Indiscernibles to swallow. Try to picture a cosmos consisting just of three qualitatively identical spheres in a straight line, the two outer ones precisely equidistant from the one at the centre. Aren't there plain differences here? The central sphere must be nearer to the outer spheres than these are to each other. Identity of Indiscernibles shudders at the symmetry of the situation, however. It holds that the so-called two outer spheres must really be only a single sphere. And this single sphere, which now has all the same qualities as its sole surviving partner, must really be identical to it. There is actually just one sphere!”

At this step, we are led to the issue of calculating properly the number of balls present in the urn. Now we should distinguish, just as before, according to whether one considers balls-as-colour or balls-as-object. Suppose then that we focus on the colour of the balls. Then we have grounds to consider that there are in total $2/3$ of red balls and $1/3$ of green balls in the hyper-entanglement urn, i.e. 1000 red ones and 500 green ones. This conforms with the calculation that results from the thirder's standpoint. Suppose, that we rather focus on balls as single objects. Things go then differently. For we can consider first that there are 1000 balls as objects in the urn, i.e. 500 (red) normal ones and 500 hyper-entangled ones. Now suppose that the 500 (red) normal balls are removed from the urn. Now there only remain hyper-entangled balls within the urn. Suppose then that we pick up one by one the remaining balls from the urn, by removing alternatively one red ball and one green ball from the urn. Now it turns out that we can draw 250 red ones and 250 green ones from the urn. For once we draw a red ball from the urn, its associated green ball is also withdrawn. And conversely, when we pick up a green ball from the urn, its associated red ball is also withdrawn. Hence, inasmuch as we consider balls as objects, there are in total 750 red ones and 250 green ones in the urn. At this step, it should be noticed that this corresponds accurately to the composition of the urn which is associated with Lewis' halfer calculation. But this now makes sense, as far as the analogy with the hyper-entanglement urn is concerned. The above-mentioned analogy with the urn associated with Lewis' halfer calculation was a poor argument inasmuch as the urn was a standard one, but things go differently when one now considers the analogy with the hyper-entanglement urn.

5. A two-sided account

From the above, it results that the line of reasoning which is associated with the balls-as-colour standpoint corresponds to the thirder's reasoning. And conversely, the line of thought which is associated with the balls-as-object viewpoint echoes the halfer's reasoning. Hence, the balls-as-colour/balls-as-object dichotomy parallels the thirder/halfer opposition. Grounded though they are on an unsuited analogy with the standard urn, the argument for $1/3$ and the argument for $1/2$ do have, however, their own strengths. In particular, the analogy with the urn in the argument for $1/3$ does justice to the fact that the Sleeping Beauty experiment entails that $2/3$ of Monday-wakings will occur in the long run. On the other hand, the analogy with the urn in the argument for $1/2$ handles adequately the fact that one Heads-waking is put on a par with two Tails-wakings. In the present context however, these two analogies turn out to be *one-sided* and fail to handle adequately the probability notion of drawing a red ball (waking on Monday). But in the present context, the probability $P(R\uparrow)$ of drawing a red ball-as-colour corresponds to the thirder's insight. And the probability $P(R\rightarrow)$ of drawing a red ball-as-object corresponds to the halfer's line of thought. At this step, it turns out that the present account is *two-sided*, since it incorporates insights from the argument for $1/3$ and from the argument for $1/2$.

Finally, it turns out that the standard urn which is usually used to model the Sleeping Beauty problem does not allow for two possible interpretations of the probability of drawing a red ball. Rather, in the standard urn model, the two interpretations are exclusive of one another and this yields the classical contradiction between the argument for $1/3$ and the argument for $1/2$. But as we did see it, with the hyper-entanglement urn model, this contradiction dissolves, since two different interpretations of the probability of drawing a red ball (waking on Monday) are now allowed, yielding then two different calculations. In the latter model, these probabilities are no more exclusive of one another and the contradiction dissolves into complementarity.

Now what precedes casts new light on the argument for $1/3$ and the argument for $1/2$. For given that the Sleeping Beauty experiment, is modelled with a standard urn, both accounts lack the ability to express the difference between the probability $P(R\uparrow)$ of drawing a red ball-as-colour (a Monday-waking-as-time-segment) and the probability $P(R\rightarrow)$ of drawing a red ball-as-object (a Monday-waking-as-object), for it does not make sense with the standard urn. Consequently, there is a failure to express this difference with the standard urn analogy, when considering drawing a red ball. But such distinction makes sense with the analogy with the hyper-entanglement urn. For in the resulting richer ontology, the distinction between $P(R\uparrow)$ and $P(R\rightarrow)$ yields two different results: $P(R\uparrow) = 2/3$ and $P(R\rightarrow) = 1$.

At this step, it is worth considering in more depth the balls-as-colour/balls-as-object opposition, that parallels the thirder/halfer contradiction. It should be pointed out that “drawing a red ball-as-colour” is associated with an indexical (“this ball is red”), somewhat internal standpoint, that corresponds to the thirder's insight. Typically, the thirder's viewpoint considers things from the inside, grounding the calculation on the indexicality of Beauty's present waking. On the other hand, “drawing a red ball-as-object” can be associated with a non-indexical (“the ball is red”), external viewpoint. This corresponds to the halfer's standpoint, which can be viewed as more general and external.

At this step, it is worth recalling the diagnosis of the Sleeping Beauty problem put forth by Berry Groisman (2008). Groisman attributes the two conflicting responses to the probability of Heads to an ambiguity in the protocol of the Sleeping Beauty experiment. He argues that the argument for 1/2 is an adequate response to the probability of Heads on awakening, under the setup of coin tossing. On the other hand, he considers that the argument for 1/3 is an accurate answer to the latter probability, under the setup of picking up a ball from the urn. Groisman also considers that putting a ball in the box and picking up a ball out from the box are two different events, that lead therefore to two different probabilities. Roughly speaking, Groisman's “coin tossing/picking up a ball” distinction parallels the present balls-as-colour/balls-as-object dichotomy. However, in the present account, putting a ball in the urn is no different from picking up a ball from the urn. For if we put in the urn a red ball of an hyper-entangled pair, we also immediately put in the urn its associated green ball. Rather, from the present standpoint, drawing (resp. putting in the urn) a red ball-as-colour from the urn is probabilistically different from picking up a red ball-as-object. The present account and Groisman's analysis share the same overall direction, although the details of our motivations are significantly different.

As we did see it, the calculation of the probability of drawing a red ball (waking on Monday) is the core issue in the Sleeping Beauty problem. But what is now the response of the present account on conditional probabilities and on the probability of Heads upon awakening? Let us begin with the conditional probability of Heads on a Monday-waking. Recall first how the calculation goes on the two concurrent lines of reasoning. To begin with, the probability $P(\text{Heads}|\text{G})$ of Heads on drawing a green ball is not a subject of disagreement for halfers and thirders, since it equals 0 on both accounts. The same goes for the probability $P(\text{Tails}|\text{G})$ of Tails on drawing a green ball, since it equals 1 from the halfer's or the thirder's viewpoint. But agreement stops when one considers the probability $P(\text{Heads}|\text{R})$ of Heads on drawing a red ball. For $P(\text{Heads}|\text{R}) = 1/2$ from the thirder's perspective and $P(\text{Heads}|\text{R}) = 2/3$ from the halfer's viewpoint. On the other hand, the probability $P(\text{Tails}|\text{R})$ of Tails on drawing a red ball is 1/2 for a thirder and 1/3 for a halfer.

Now the response of the present account to the calculation of the conditional probability of Heads on drawing a red ball (waking on Monday) parallels the answer made to the issue of determining the probability of drawing a red ball. In the present account, $P(\text{Heads}|\text{G}) = 0$ and $P(\text{Tails}|\text{G}) = 1$, as usual. But we need to disambiguate how we interpret “drawing a red ball” by distinguishing between $P(\text{Heads}|\text{R}\uparrow)$ and $P(\text{Heads}|\text{R}\rightarrow)$, to go any further. For $P(\text{Heads}|\text{R}\uparrow)$ is the probability of Heads on drawing a red ball-as-colour. And $P(\text{Heads}|\text{R}\rightarrow)$ is the probability of Heads on drawing a red ball-as-object. $P(\text{Heads}|\text{R}\uparrow)$ is calculated in the same way as in the thirder's account. Now we get accordingly: $P(\text{Heads}|\text{R}\uparrow) = 1/2$. On the other hand, $P(\text{Heads}|\text{R}\rightarrow)$ is computed in the same way as from the halfer's perspective, and we get accordingly: $P(\text{Heads}|\text{R}\rightarrow) = [P(\text{Heads}) \times P(\text{R}\rightarrow|\text{Heads})] / P(\text{R}\rightarrow) = [1/2 \times 1] / 1 = 1/2$.

Now the same goes for the probability of Heads upon awakening. For there are two different responses in the present account, depending on whether one considers $P(\text{R}\uparrow)$ or $P(\text{R}\rightarrow)$. If one considers balls-as-colour, the probability of Heads upon awakening is calculated in the same way as in the argument for 1/3, and we get accordingly: $P(\text{Heads}\uparrow) = 1/3$ and $P(\text{Tails}\uparrow) = 2/3$. On the other hand, if one is concerned with balls-as-object, it ensues, in the same way as with the halfer's account, that there is no shift in the prior probability of Heads. As Lewis puts it, Beauty's awakening does not add any novel information. It follows accordingly that the probability $P(\text{Heads}\rightarrow)$ of Heads (resp. Tails) on awakening still remains 1/2.

Finally, the above results are summarised in the following table:

	<i>halfer</i>	<i>thirder</i>	<i>present account</i>
$P(\text{Heads}\uparrow)$		1/3	1/3
$P(\text{Tails}\uparrow)$		2/3	2/3
$P(\text{Heads}\rightarrow)$	1/2		1/2
$P(\text{Tails}\rightarrow)$	1/2		1/2
$P(\text{drawing a red ball-as-colour}) \equiv P(R\uparrow)$		2/3	2/3
$P(\text{drawing a green ball-as-object}) \equiv P(G\uparrow)$		1/3	1/3
$P(\text{drawing a red ball-as-object}) \equiv P(R\rightarrow)$	3/4		1
$P(\text{drawing a green ball-as-object}) \equiv P(G\rightarrow)$	1/4		1/2
$P(\text{Heads} \text{drawing a red ball-as-colour}) \equiv P(\text{Heads} R\uparrow)$		1/2	1/2
$P(\text{Tails} \text{drawing a red ball-as-colour}) \equiv P(\text{Tails} R\uparrow)$		1/2	1/2
$P(\text{Heads} \text{drawing a red ball-as-object}) \equiv P(\text{Heads} R\rightarrow)$	2/3		1/2
$P(\text{Tails} \text{drawing a red ball-as-object}) \equiv P(\text{Tails} R\rightarrow)$	1/3		1/2

6. Handling the variations of the Sleeping Beauty problem

From the above, it follows that the present treatment of the Sleeping Beauty problem, is capable of handling several variations of the original problem which have recently flourished in the literature. For the above solution to the Sleeping Beauty problem applies straightforwardly, I shall argue, to these variations of the original experiment. Let us consider, to begin with, a variation were on Heads, Sleeping Beauty is not awakened on Monday but instead on Tuesday. This is modelled with an hyper-entanglement urn* that receives one normal green ball (instead of a red one in the original experiment) in the Heads case.

Let us suppose, second, that Sleeping Beauty is awakened two times on Monday in the Tails case (instead of being awakened on both Monday and Tuesday). This is then modelled with an hyper-entanglement urn* that receives one pair of hyper-entangled balls which are composed of two red balls in the Tails case (instead of a pair of hyper-entangled balls composed of a red and a green ball in the original experiment).

Let us imagine, third, that Beauty is awakened two times – on Monday and Tuesday – in the Heads case, and three times – on Monday, Tuesday and Wednesday – in the Tails case. This is then modelled with an hyper-entanglement urn* that receives one pair of hyper-entangled balls composed of one red ball and one green ball in the Heads case; and in the Tails case, the hyper-entanglement urn* is filled with one triplet of hyper-entangled balls, composed of a red, a green and a blue ball.

Finally, the lesson of the Sleeping Beauty Problem appear to be the following: our current and familiar objects or concepts such as balls, wakings, etc. should not be considered as the sole relevant classes of objects for probability purposes. We should bear in mind that according to an unformalised axiom of probability theory, a given situation is classically modelled with the help of urns, dices, balls, etc. But the rules that allow for these simplifications lack an explicit formulation. However in certain situations, in order to reason properly, it is also necessary to take into account somewhat unfamiliar objects whose constituents are pairs of indissociable balls or of mutually inseparable wakings, etc. This lesson was anticipated by Nelson Goodman, who pointed out in *Ways of Worldmaking* that some objects which are prima facie completely different from our familiar objects also deserve consideration: “we do not welcome molecules or concreta as elements of our everyday world, or combine tomatoes and triangles and typewriters and tyrants and tornadoes into a single kind”.³ As we did see it, in some cases, we cannot add unrestrictedly an object of the Heads-world with an object of

³ Goodman (1978, p. 21).

the Tails-world. For despite the appearances, objects of the Heads-world may have ontologically different properties from objects of the Tails-world. And the status of our probabilistic paradigm object, namely a ball, appears to be world-relative, since it can be a whole in the Heads-world and a part in the Tails-world. Once this goodmanian step accomplished, we should be less vulnerable to certain subtle cognitive traps in probabilistic reasoning.

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