Independent Repeated Sleeping Beauty

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In this paper, I shall provide a simple argument for the thirder analysis of the usual version of Sleeping Beauty[4] and the hslfer analysis of a bare-bones version of the scenario. This argument depends upon a calculation of relative frequencies when the Sleeping Beauty experiment is repeated, but it is crucial that we be dealing with independent repetitions of the same experiment. The versions of repeated Sleeping Beauty discussed in the literature(for example[4, 1, 2]) violate the independence requirement.

Sleeping Beauty[4] is the subject of an experiment. As a consequence of the experiment, she is in a certain subjective psychological state S when she wakes up on Monday and if a certain fair coin lands tails she is also in that same state upon awakening on Tuesday. No one other than Beauty is ever in state S and Beauty is never in state S except on one particular Monday if the coin lands heads and one particular Monday and Tuesday if the coin lands tails. Whenever she notices that she is in state S, Sleeping Beauty will want to estimate the probability that the coin landed heads.

Beauty is assumed to be perfectly rational and to have perfect knowledge of the protocol of the experiment. The question is what should be her estimate for the probability that the coin landed heads. The two most popular answers are one half[6] and one third[4].

The correct answer might depend on the details of the experimental protocol and different versions of the Sleeping Beauty story have been discussed in the literature. It should not matter if instead of being in the same exact state S on Monday and Tuesday if the coin landed tails and the same exact state S on Monday regardless of how the coin landed, she simply has no relevant information available that could give her any clue as to whether it is Monday and the coin landed heads or it is Monday and the coin landed tails or it is Tuesday and the coin landed tails[3, 7]. The weather might be different on Monday and Tuesday but that is useless information.

It might matter as Dorr points out[3] how long Beauty lives and how many other people exist in the universe and how long they live. In the basic version of the scenario, Beauty is the only person who has ever lived and she is only alive and conscious for one short moment on Monday if the coin landed heads or two short moments, one on Monday and one on Tuesday, if the coin landed tails. For the basic version of the scenario, the halfer answer makes sense. In what I shall call the standard version of the scenario, how the coin lands does not affect the total amount of conscious, at least minimally rational human life in the universe, and Dorr would prefer the thirder answer. If how the coin lands has only minimal effect on the total amount of conscious, at least minimally rational human life in the universe, for example, because Beauty is asleep all of Tuesday if and only if the coin lands tails but she lives a long life, then the thirder answer might be considered approximately correct.

In the generalized Sleeping Beauty scenario, Beauty is in state S for M moments if the coin lands heads and N moments if the coin lands tails. It is intuitively clear that Beauty's estimate for the probability that the coin landed heads should not be affected by the value of M as long as $\frac{N}{M} = 2$. But the question still remains whether the halfer or thirder analysis is the correct analysis.

One might try to answer the question by computing relative frequencies when the Sleeping Beauty experiment is repeated n times where n is very large. Thus one might have n different coins tossed in n successive weeks. In any week in which the coin lands heads, Beauty is in state S for one moment but she is in the state S for two moments if the coin landed tails. In the basic version of the scenario, Beauty is the only conscious life in the universe and she is only conscious on days when she wakes up in state S. In the standard version of the scenario, how the n coins land does not affect the total amount of conscious, at least minimally rational human life in the universe. In both versions, whenever she is in state S, Beauty needs to estimate the probability that the coin tossed during the current week landed heads.

In either version of the repeated Beauty scenario, if n is large enough, it is almost certain that the coin will land heads in approximately half the weeks and thus there will be approximately twice as many S and tails moments as S and heads moments. This seems to justify a thirder analysis.

The problem is that the n different experiments are not independent[5]. If i and j are different integers between 1 and n, then whether the current moment is part of Monday of week i, part of Tuesday of week i or neither is not independent of whether the current moment is part of Monday of week j, part of Tuesday of week j or neither. Another reason to believe that we may not be able to transfer our analysis from this repeated Sleeping Beauty scenario to the unrepeated scenario is that the effect of a specific coin landing tails rather than heads in repeated Sleeping Beauty is to multiply the total number of moments during which Beauty is in state S by approximately $\frac{1.5n+.5}{1.5n-.5}$ if n is large. But in unrepeated Sleeping Beauty the effect is to multiply the total number of times when Beauty is in state S by 2.

However, we can construct an independent repeated Sleeping Beauty scenario whose analysis can be transferred to the unrepeated case. Our repeated experiment will last 2^n days if we are repeating basic Sleeping Beauty and M^n days if we are repeating standard Sleeping Beauty. Here M should be a large positive integer. In the basic version, we will represent a day as a sequence of bits of length n; a bit can have the value 0 (Monday) or 1 (Tuesday). In the standard version, we will represent a day as a sequence of length n of non-negative integers. Each element of the sequence can be either 0 (Monday), 1 (Tuesday), or some number greater than 1 but less than M (some other day). In both versions of our scenario, Beauty's relevant psychological state will consist of a sequence of n substates. For each integer between 1 and n, there is a special substate S_i. This special substate is a special possible value for the ith member of the sequence of state representing Beauty's psychological state.

In both versions of the scenario, Beauty's psychological state is affected by n independent tosses of fair coins. Beauty is in substate S_i upon awakening on day x if and only if either the ith element of the sequence x is equal to 0 or the ith coin landed tails and the ith element of the sequence x is equal to 1. In the basic version of the scenario, Beauty is unconsious except on days when she wakes up in state $(S_1, S_2, S_3, \ldots, S_n)$. In the standard version of the scenario, Beauty is conscious on all M^n days. For any i, whenever Beauty is in state S_i, she has to estimate the probability that the ith coin landed heads.

If n is large, then for both versions of the scenario, it is almost certain that approximately half the coins landed heads. In the basic version of the scenario, we could pick any day x during which Beauty is conscious and notice that for all $1 \leq i \leq n$, Beauty wakes up in substate S_i and approximately half of the i are such that the coin landed heads.

In the standard version of the scenario, we examine a typical day represented by the sequence $x = (x_1, x_2, x_3, \ldots, x_n)$. If n is large enough approximately $\frac{n}{M}$ of the x_i will be equal to 0 and for these i, Beauty will awa in state S_i. Of the i such that $x_i = 0$, approximately half are such that the ith coin landed heads and half are such that it landed tails. But whenever $x_i = 0$, Beauty awakens in substate S_i. Thus we have approximately $\frac{n}{2M}$ indexes i such that Beauty is in state S_i with the ith coin landing heads and $x_i = 0$ (heads and Monday) and approximately $\frac{n}{2M}$ indexes that are tails and Monday. The only other way Beauty can be in state S_i is if i is a tails and Tuesday index and there should be approximately $\frac{n}{2m}$ of these indexes. Thus unless x is a very atypical day, the number of i such that Beauty is in substate S_i and the ith coin landed tails is twice the number such that Beauty is in state S_i and the coin landed tails. This seems to justify the thirder analysis of unrepeates Sleeping Beauty and if M is much larger than 2, the calculation would not be very different if we specified in the standard scenario that Beauty is unconscious on any day x if for any i, $x_i = 1$ and the ith coin landed heads. (Beauty is unconscious on Tuesday when the coin lands heads.) Thus our analysis also applies to a repetition of the $(n = 1)$ scenario where Beauty does not awaken on Tuesday if the coin landed heads and Beauty lives a long life, much longer than two days.

I believe it should be clear why we are allowed to transfer our results from independent repeated Sleeping Beauty to unrepeated Sleeping Beauty. Independent repeated Sleeping Beauty really does involve independent repetitions of unrepeated Sleeping Beauty. So we can (approximately) equate probabilities in the unrepeated scenario to relative frequencies in the repeated version.

We really are repeating the same experiment. The only information that Beauty has that is relevant to figuring out whether the ith coin has landed heads or tails is the ith element of the sequence that represents her psychological state. This follows from the independence of the different experiments. If i and j are two positive integers between 1 and n, whatever estimate Beauty has for the

ith coin landing heads given that she is in substate S_i should be the same as her estimate for the jth coin landing heads given that she is in substate S_j. But if we ignore additional irrelevant information Beauty might have, Beauty's problem of estimating whether the ith coin landed heads in repeated Sleeping Beauty is the same as her problem of estimating whether the one and only relevant coin landed heads in unrepeated Sleeping Beauty. So we really are repeating unrepeated Sleeping Beauty and the different experiments really are independent.

References

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