Separate common causes and EPR correlations—a no-go result

Tomasz Placek and Leszek Wroński

Jagiellonian University
Kraków, Poland

Abstract

One diagnosis of Bell's theorem is that its premise of Outcome Independence is unreasonably strong, as it postulates one common screener system that purports to explain all the correlations involved. This poses a challenge of constructing a model for quantum correlations that is local, non-conspiratorial, and has many separate screener systems rather than one common screener system. In particular, the assumptions of such models should not entail Bell's inequalities. We prove that the models described do not exist, and hence, the diagnosis above is incorrect.

1 Introduction

Bell's (1964) theorem derives a testable probabilistic inequality from the assumption that quantum mechanics can be completed by states more informative than quantum states and that these "hidden" states satisfy some intuitive assumptions.[1] Since the inequality is violated by quantum mechanical predictions, and over the years a consensus has grown that it is also

[1]Acknowledgment: We are very much indebted to Gábor Hofer-Szabó since an early draft of his 2011 paper sparked our interest in models with separate screener systems for quantum correlations; he later suggested a substantial simplification of our construction. We also thank the audiences to which we read earlier versions of this paper: TP to the audience of his lunch talk at the CPS of the University of Pittsburgh and LW to the audience of *his* lunch talk at the Theoretical Philosophy Unit of Universiteit Utrecht,

empirically violated, at least one of the premises of the derivation must be false. Accordingly, Bell's paper poses a challenge of explaining the theorem, that is, arguing which of its premises is false and why. It is this project that we want to contribute to in this paper.

In this paper we will focus on a later and more advanced version of Bell's theorem that assumes a probabilistic working of the hidden states in bringing about outcomes of measurements. Two premises of the derivation draw on the idea of locality, which says that an event can be influenced by a remote event only by a mediation of neighboring (local) events. Since in the setup relevant to Bell's theorems, outcomes registered in one wing of the experiment are space-like separated from the selection of settings made in the other wing of the experiment, this idea leads to two independence conditions, typically called "Parameter (or Context) Independence" (PI) and "Outcome Independence" (OI). PI says that results at a nearby measurement apparatus are independent from settings selected at a remote measurement apparatus. OI requires that outcomes registered at a nearby apparatus are independent from outcomes registered at the remote apparatus. The remaining premise of the derivation, called "No Conspiracy" (NOCONS) postulates that selections of measurement settings are free, that is, independent from hidden states.

Following a large part of literature, by "independent" we will mean here "probabilistically independent". Accordingly, the three conditions of PI, OI, and NOCONS are given here this reading: (PI) Given each hidden state, results at a nearby measurement apparatus are probabilistically independent from settings selected at a remote measurement apparatus. (OI) Given each hidden state, outcomes registered at a nearby apparatus are probabilistically independent from outcomes registered at the remote apparatus. Finally, (NOCONS) selections of measurement settings are probabilistically independent from the hidden states.

A diagnosis of Bell's theorem that we will here analyse (and argue against) stems from observing a certain subtlety in Outcome Independence (the observation was first made by Belnap and Szabó (1996)). This assumption bears an affinity to what is known as Reichenbach's (1956) screening-off condition, which concerns a pair of correlated events and a third event "screening off" one of the events belonging to the pair from the other (for the rigorous

as well as to the audience of his talk at Ghent University. Our thanks go also to Alex Malpass for checking the English. Our work has been supported by MNiSW research grant 668/N-RNP-ESF/2010/0.

formulation see below). The difference that the above researchers noticed is that in the context of Bell's theorem, Outcome Independence posits a single screening factor for *many* pairs of correlated results, and hence seems to be unreasonably strong.

This "single vs. many" dialectics motivates a project of deriving Bell's inequalities from a weaker set of premises, with Outcome Independence being replaced by its more modest relative. A hope was that with the new premise, Bell's inequalities could not be derived, which would put blame for the derivation on the "old" Outcome Independence.

Our results definitely shatter this hope, since we show that the class of new models for Bell's correlations is not more general than the class of old models: given that there is a former model, there is also a latter model. This entails that the assumptions of new models satisfy Bell's inequality, as do the assumptions of the old ones.

Our paper is organized as follows: The next section sketches the background of the project we criticize. Section 3 gives formal definitions pertaining to the distinction we alluded to. With these definitions in hand, in Section 4 we offer a survey of earlier results. The main Section 5 contains our results, and is followed by a final section stating our conclusions.

2 Background: from Bell's local causality to separate systems of screeners

Bell proved a stochastic version of his theorem from a premise he called "local causality".[2] As he explained (1975), the underlying idea is that if 1 and 2 are space-like separated regions, then events occurring in 1 should not be causes of events occurring in region 2. Such events could be correlated, he acknowledged, as they might have a common cause. Moreover, they may remain to be correlated, even if the probability is conditioned on a specification Λ of the state of the events' common past, i.e., $P(A|\Lambda B) \neq P(A|\Lambda)$, where A and B stand for events occurring in regions 1 and 2, respectively. However, he claims that "in the particular case that Λ contains already a complete specification of beables in the overlap of the two light cones, supplementary information from region 2 could reasonably be expected to be redundant [for

[2]The stochastic version assumed probabilistic working of hidden states; there were other premises of the proof.

probabilities of events in region 1]", which he takes for justification of this screening-off formula:[3]

$$P(A|\Lambda B) = P(A|\Lambda)^4.$$

Accordingly, Λ represents here a full specification of the state in the common past and the backward line-cone of A. The formula is tacitly universally quantified, that is, it should read "for every possible state in region ..., if Λ is its full specification, then ...".

Almost two decades earlier, Reichenbach (1956, p. 159) hit upon a similar idea, while attempting to analyze the arrow of time in terms of causal forks:

> In order to explain the coincidence of A and B, which has a probability exceeding that of chance coincidence, we assume that there exists a common cause C. [...] We will now introduce the assumption that the fork ACB satisfies the following relations:
>
> $$P(AB|C) = P(A|C)P(B|C) \quad P(AB|\neg C) = P(A|\neg C)P(B|\neg C)$$
> $$P(A|C) > P(A|\neg C) \qquad P(B|C) > P(B|\neg C).$$

The two formulas on the top are called the (positive and negative) screening-off conditions, whereas the two at the bottom are known as the conditions of positive statistical relevance. It is easy to note the same motivation behind both Bell's causal locality and Reichenbach's screening-off condition. Since $P(AB|C) = P(A|C)P(B|C)$ is equivalent to $P(A|BC) = P(A|C)$ if $P(C) \neq 0$, the two concepts are formally similar as well, though the former allows for any number of factors ("screeners") to be conditioned upon, whereas the latter is dichotomous, since it admits as screeners an event and its negation only.

In the 1970's and 1980's Reichenbach's project was continued by W. Salmon. It was most likely van van Fraassen (1982) who first saw the connection between Bell's local causality and the screening-off condition. The screening-off condition, generalized to any number of screeners and applied to pairs of outcomes, with screeners identified with hidden states, is just Outcome Independence. But this condition—taken together with two more premises—entails Bell's inequalities, which are both violated by quantum mechanics

[3] At this point in his paper Bell changes notation, introducing variables for states in each event's backward light-cone. For a recent analysis of Bell's local causality, cf. Norsen (2006) and Seevinck and Uffink (2011).

[4] We frequently omit the "$\cap$" sign between names of events.

and most likely experimentally falsified. Since the two other premises look intuitive, a popular diagnosis was to reject OI. This means, however, to reject the screening-off condition, generalized from dichotomous screeners to any number of screeners. But since the screening-off condition (so generalized) appears to be a mathematical tautology,[5] how could it be empirically falsified?

A diagnosis that seems to resolve the conflict came from Belnap and Szabó's (1996) distinction between common causes and *common* common causes. Observe that in Reichenbach's approach one posits a system of screeners, i.e., C and $\neg C$, for a *single* pair of events. In contrast, in the context of Bell's theorem, one envisages a large number of correlated pairs of results, produced in mutually exclusive measurements (i.e., represented by non-commuting observables). Bell's local causality postulates a single set of screeners (full specifications of states in a relevant region) for all these correlations. The set might be arbitrarily large, but, importantly, each element of it is supposed to apply to *all* correlated pairs, making the events independent, conditional on each screener. In the recent terminology of Hofer-Szabó (2008), standard Bell-type theorems assume a *common* screener system, i.e, every element of this system pertains to all correlations under consideration—in contrast with *separate* screener systems.

With this distinction at hand, it is tempting to believe that, while the screening-off condition (as applicable to a single correlation) is correct, what lands us in trouble in the context of Bell's theorem is its extension which requires a common system of screeners for all the correlations. To put it differently, moving from a common screener system to separate screener systems relaxes Outcome Independence and Parameter Independence. The usual OI postulates that every correlation considered is screened off by every factor from the common screener system. The modified condition, call it OI', requires that every correlation is screened off by every factor from the screener system *for this correlation* only. A similar change affects Parameter Independence.

To justify or reject this belief, starting from work reported in Szabó (2000), researchers have attempted to construct models of Bell-type correla-

[5] Especially since the following simple fact holds: let $\langle \Omega, \mathcal{F}, P \rangle$ be a probability space. Let $\mathcal{G} = \big\{ \{A_i, B_i\} \big\}_{i \in I} \subseteq \mathcal{F}^2$ be a finite family of pairs of correlated events in $\mathcal{F}$. Then there exists a partition $\mathcal{C}$ of Ω such that for any $C \in \mathcal{C}$ and for any $i \in I$, $P(A_i B_i | C) = P(A_i|C)P(B_i|C)$. To construct this partition simply take all Boolean combinations of all correlated events, throwing out the empty ones should they arise.

tions that would assume the existence of separate screener systems, satisfy the weakened premises of Parameter Independence (PI') and Outcome Independence (OI'), meet the No Conspiracy requirement, and would *not* be committed to Bell-type inequalities.[6] [7] This paper present no go results for this project. We will prove that every model with separate screener systems for Bell-type correlations that assumes PI', OI' and NOCONS is committed to Bell's inequalities. Accordingly, such models do not exist for probabilities violating Bell's inequalities.

3 Screener systems: formal definitions

In the late 1990's the "Budapest school" of M. Rédei, L. Szabó, G. Hofer-Szabó, B. Gyenis and others, building upon the "*common* common causes vs. common causes" distinction, launched two projects[8]—to be briefly stated as below:

1. Is Reichenbach's common cause principle, or its generalization in form of some common common cause principle tenable?

2. Are there models for Bell's correlations which are local, non-conspiratorial, have separate screener systems for each correlation, and are not committed to Bell-type inequalities?

The projects are different: the first must pertain to the (generalizations of) positive statistical relevance conditions, which are not required by the models of Bell's correlations. In turn, Bell's theorem brings in the issues of locality and no conspiracy which are not present in the discussions of common causes. Despite these differences, the method of handling these questions is the same in the Budapest school, and it boils down to asking if probability spaces respecting certain probabilistic constraints exist.

We are here concerned with the second question and thus the models we discuss are probability spaces, constrained by some conditions which are supposed to capture the spatiotemporal aspect inherent in locality as well

[6] In some papers following Szabó's work these conditions are called Locality, No Conspiracy, and Screening-off, resp.

[7] Szabó paper reports on his computer simulations aimed to construct such models.

[8] Cf. Hofer-Szabó et al. (1999) and Szabó (2000).

as the modal aspects inherent in the conditions of no conspiracy and non-commutativity of quantum observables. We will introduce these constraints in turn. First let us recall the definition of a (classical) probability space.

Definition 1 (probability space) *A probability space is a triple* $\langle \Omega, \mathcal{F}, P \rangle$ *such that:*

- Ω *is a non-empty set (sometimes called 'sample space');*
- $\mathcal{F}$ *is a* σ*-algebra of subsets of* Ω *(sometimes called 'algebra of events');*
- P *is a function from* $\mathcal{F}$ *to* $[0,1] \subseteq \mathbb{R}$ *such that*
  - $P(\Omega) = 1$*;*
  - P *is countably additive: for a countable family* $\mathcal{G}$ *of pairwise disjoint elements of* $\mathcal{F}$*,* $P(\cup \mathcal{G}) = \sum_{A \in \mathcal{G}} P(A)$*.*

P *is called the probability function (or measure).*

During our argument we will start with a probability space modelling the Bell-Aspect experiment. Then we will construct a chain of transformations of the space, such that the probabilities of the events representing the measurement results and detector settings are preserved under them and the "fine-grained" space we end up with has interesting properties regarding screening-off (see lemma 8, p. 15).

Now let us introduce the concept of a screener system.

Definition 2 (screener system) *Let* $\langle \Omega, \mathcal{F}, P \rangle$ *be a probability space and* $A, B \in \mathcal{F}$*. A partition* $\{C_i\}_{i \in I}$ *of* Ω *satisfying for any* $i \in I$

$$P(AB|C_i) = P(A|C_i)P(B|C_i) \tag{1}$$

is called a screener system for $\langle A, B \rangle$*.*[9]

[9] This is in essence definition 5 of a screener-off system of Hofer-Szabó (2008); in contrast to his definition, the screener system is defined here for a pair of events, regardless of whether or not they are correlated. In the sequel we frequently omit the brackets when speaking about correlated pairs of events.

Since the sum of the probabilities of the elements of a screener system equals 1, only countably many of them can be positive (see e.g. Theorem 10.2 in Billingsley (1995)). And so, while screener systems may be infinite, they are at most countably infinite, otherwise some conditional probabilities would not be defined.

A straightforward calculation proves the following fact:

Fact 3 *If $\{C_i\}_{i\in I}$ is a screener system for $\langle A, B\rangle$, then it is also a screener system for each of the following pairs: $\langle A, B^\perp\rangle$, $\langle A^\perp, B\rangle$, and $\langle A^\perp, B^\perp\rangle$, where $X^\perp = \Omega \setminus X$.*

This fact notwithstanding, the above definition leaves open how it should be applied to many pairs of events, if these pairs are not algebraic combinations of one another, as displayed above. As an example, consider two pairs, $\langle A, B\rangle$ and $\langle D, E\rangle$ such that $\langle D, E\rangle$ is identical to neither of these pairs: $\langle A, B^\perp\rangle$, $\langle A^\perp, B\rangle$, $\langle A^\perp, B^\perp\rangle$ $(A, B, D, E \in \mathcal{F})$. We may then postulate two separate systems of screeners, $\{C^k_{AB}\}_{k<K(AB)}$ [10] and $\{C^{k'}_{DE}\}_{k'<K(DE)}$, one for $\langle A, B\rangle$ and the other for $\langle D, E\rangle$, that is: for every $k < K(AB)$ and $k' < K(DE)$,

$$P(AB|C^k_{AB}) = P(A|C^k_{AB})P(B|C^k_{AB}) \text{ and } P(DE|C^{k'}_{DE}) = P(D|C^{k'}_{DE})P(E|C^{k'}_{DE}).$$

Alternatively we may postulate a single common screener system $\{C_i\}_{i\in I}$ for the two pairs satisfying, for every $i \in I$:

$$P(AB|C_i) = P(A|C_i)P(B|C_i) \text{ and } P(DE|C_i) = P(D|C_i)P(E|C_i).$$

To rigorously introduce the weakened versions of Outcome Independence (OI') and of Parameter Independence (PI'), and No Conspiracy (NOCONS), let us recall the setup of Bell's theorem. A source emits pairs of objects, and the members of each pair travel in separate "wings" of the experiment towards remote detectors. For each emission, in the left wing it is possible to choose one of the two settings, a_1, a_2, of the left measuring device, and

[10] From now on, for two events X and Y, $K(XY)$ is a natural number being the size of the screener systems for these two events. We say "the" screener system even though of course many different screener systems may exist for some given events, but one particular will always be intended by the context. If we allow for an infinite screener system, we should understand by $K(XY)$ an index set of cardinality equal to the cardinality of the screener system and write $\alpha \in K(XY)$ rather than $k < K(XY)$. If $X = A_i$ and $Y = B_j$ we will use the expression $K(ij)$.

in the right wing—one of the two settings, b_3, b_4, of the right measuring device. Given that the setting selected on the left is a_i, one of the two results, A_i^+ or A_i^- occurs, and given that the setting selected on the right is b_j, one of the two results, B_j^+ or B_j^- occurs. Some pairs of remote results like A_i^m, B_j^n ($m, n \in \{+, -\}$) are correlated; in the spirit of local causality, we assume "hidden states" (complete states, or, at least, states more complete than the quantum mechanical states), which are supposed to remove the correlations, were the probabilities conditioned on each such a state. By the Bell-Aspect correlations we will understand 16 pairs of the form A_i^m, B_j^n, with their probabilities agreeing with quantum mechanical predictions. Since these 16 pairs can be seen as four groups of correlated pairs connected in the similar way as the pairs featured in Fact 3, by that fact it would be superfuous to consider more than 4 screener systems for them—therefore we posit one screener system for every pair of detector settings.

We will model the experiment in a single classical probability space[11] $\langle \Omega, \mathcal{F}, P \rangle$, which of course means that (the representations of) a_1, a_2, b_3, b_4, A_i^m, B_j^n and the hidden states belong to $\mathcal{F}$. These events should satisfy the following natural conditions:

$$A_i^m \subseteq a_i, \qquad B_j^n \subseteq b_j, \qquad a_2 = \Omega \setminus a_1, \qquad b_4 = \Omega \setminus b_3, \quad (2)$$
$$A_i^- \cup A_i^+ = a_i, \quad B_j^- \cup B_j^+ = b_j \quad \text{for } i = 1, 2; j = 3, 4; m, n \in \{+, -\}. \quad (3)$$

Note that this already incorporates some modal claims, e.g., that the result A_i^m must occur in the measurement of a_i (not of a_j), and that A_i^m cannot occur together with A_j^m if $i \neq j$. Notice also that in the single space approach we are using here the Bell-Aspect correlations are *conditional* correlations, for example:

$$P(A_1^+ B_3^+ \mid a_1 b_3) > P(A_1^+ \mid a_1 b_3) P(B_3^+ \mid a_1 b_3),$$

and so we will say that an event C screens off such a correlation between A_1^+ and B_3^+ whenever

$$P(A_1^+ B_3^+ \mid a_1 b_3 C) = P(A_1^+ \mid a_1 b_3 C) P(B_3^+ \mid a_1 b_3 C).$$

[11] We choose the single space rather then the many-space approach not because we prefer it (in fact we do not), but because it is employed in the majority of the literature on the subject of the connections between separate- and common common causes (or screener systems) and the Bell inequalities.

Note that due to our just introduced conditions (3) and fact (3), C will also screen off A_1^+ from B_3^-, and so on for other results under the same detector settings.

The conditions PI', OI', and NOCONS are expressed as follows:

$$
\begin{aligned}
P(A_i^m \mid a_i b_j C_{ij}^k) &= P(A_i^m \mid a_i b_{j'} C_{ij}^k) && \text{PI'} \\
P(B_j^n \mid a_i b_j C_{ij}^k) &= P(B_j^n \mid a_{i'} b_j C_{ij}^k) && \text{PI'} \\
P(A_i^m B_j^n \mid a_i b_j C_{ij}^k) &= P(A_i^m \mid a_i b_j C_{ij}^k) P(B_j^n \mid a_i b_j C_{ij}^k) && \text{OI'} \\
P(a_i b_j \mathfrak{A}) &= P(a_i b_j) P(\mathfrak{A}) && \text{NOCONS}
\end{aligned}
$$

where $\mathfrak{A}$ is any algebraic combination of the elements of four partitions $\{C_{ij}^k\}$ $(i = 1, 2, j = 3, 4)$ and all formulas are quantified for all $i \in \{1, 2\}$, $j \in \{3, 4\}$, $m, n \in \{+, -\}$ and $k < K(ij)$. We do *not* require a screener system for one correlation to satisfy Parameter Independence (and Outcome Independence) with respect to another correlation. (In Hofer-Szabò's papers, PI' and OI' are called, respectively, Locality and Screening-off.)

Notice that PI' can be equivalently phrased as $P(A_i^m \mid a_i b_j C_{ij}^k) = P(A_i^m \mid a_i C_{ij}^k)$ (similarly for other settings); the setting of the "remote" detector is to be irrelevant for the probability of a given result at the "nearby" detector.

Finally, here are our main definition of models for Bell-Aspect correlations:

Definition 4 ***(models with separate or common screener system(s))*** *Consider a probability space $\langle \Omega, \mathcal{F}, P \rangle$ which contains events a_i (for $i \in \{1, 2\}$) and b_j (for $j \in \{3, 4\}$) corresponding to detector settings and events A_i^m and B_j^n (for i's and j's as before and $m, n \in \{+, -\}$) corresponding to measurement results under the appropriate settings. Suppose that the model exhibits Bell-Aspect correlations.*

*The probability space is a local non-conspiratorial model with **separate screener systems** (or a **separate-ss model**) for the Bell-Aspect correlations if there exist four partitions of Ω consisting of elements of $\mathcal{F}$, $\{C_{ij}^k\}_{k<K(ij)}$, one for each pair of detector settings, such that for each such a pair a_i,b_j the partition $\{C_{ij}^k\}_{k<K(ij)}$ meets the conditions of Outcome Independence (OI'), Parameter Independence (PI') and No Conspiracy (NOCONS) with regard to the correlations arising at the detector settings a_i and b_j.*

*The probability space is a local non-conspiratorial model with **a common screener system** (or a **common-ss model**) for the Bell-Aspect correlations* *if there exists a single partition of Ω consisting of elements of $\mathcal{F}$, $\{C^k\}$, which meets the conditions of PI', OI', and NOCONS with regard to all of those correlations.*

We call such models "local" since both OI' and PI' are motivated by locality.

4 A survey of earlier results and the issue of reducibility

A first attempt at a local non-conspiratorial separate-ss model for Bell-Aspect correlations was a construction of Szabó (2000). However, it turned out that the model violates NOCONS with respect to intersections of screeners from (separate) screener systems (but satisfies NOCONS with respect to each screener). A significant development was a local non-conspiratorial separate-ss model of Grasshoff et al. (2005) et al for Bell-Aspect correlations produced in a setup with parallel settings: in this model Bell-Clauser-Horne inequalities are derivable. However, as Hofer-Szabó (2008) showed, Grasshoff et al.'s model is reducible to a model with a common screener system. All one needs to do to create a common screener system out of separate screener systems is to take intersections of elements of all separate screener systems, that is, sets like $C^{\alpha}_{13} \cap C^{\beta}_{23} \cap C^{\gamma}_{14} \cap C^{\delta}_{24}$, i.e., intersections of elements of all the screener systems. This, since Grasshoff's model is a common-ss model, the Bell-type inequalities are derivable.

This result cast a new light on the project of constructing separate-ss models for Bell-Aspect correlations: such models should not be reducible to common-ss models. But what does this new desideratum involve? If C_{13}, C_{23}, C_{14}, C_{24} are partitions of the sample space Ω, the set of intersections of their elements, i.e.,

$$C = \left\{C^{\alpha}_{13} \cap C^{\beta}_{23} \cap C^{\gamma}_{14} \cap C^{\delta}_{24} \mid \alpha < K(13), \beta < K(23), \gamma < K(14), \delta < K(24)\right\}$$

is also a partition of Ω. Note also that if a separate-ss model satisfies NOCONS with respect to every screener system $\{C_{ij}\}$ $(i = 1, 2, j = 3, 4)$, it satisfies NOCONS with respect to the common-ss system C. Thus, the only way that a separate-ss model may stop to reduce to a common-ss model is if PI' or OI' fails with respect to an intersection like the one above. This points to an interesting feature of non-reducible separate-ss models that goes against

Bell's intuitions. To recall, Bell believed that the correlation between space-like related events should disappear if a *complete* specification of a state in a relevant region is taken into account (cf. Section 2). Clearly, an intersection of elements of screener systems corresponds to a more complete state description than a single element of a screener system. Accordingly, non-reducibility entails that less complete states meet PI' and OI', whereas more complete states lose one of these properties. Now, our result (that non-reducible local non-conspiratorial separate-ss models for Bell-Aspect correlations do not exist) offers an ironical support for Bell's idea: the most complete state descriptions available in a model with separate screener systems make the correlations disappear. The irony is that the assumptions of the model entail Bell's inequalities.

5 Our results

Our central theorem, stated informally, says that if there is a local non-conspiratorial separate-ss model for some Bell-Aspect correlations, then there is a local non-conspiratorial *common*-ss model for the same correlations. Our argument will proceed in three steps. Let us explain what roles these steps play.

(1) The algebra $\mathcal{F}$ of a separate-ss model may be arbitrarily large and, in particular, may have no atoms. Our first move is to carve from $\mathcal{F}$ a smaller algebra $\mathcal{F}'$, which has atoms of the form $A_i^m \cap B_j^n \cap C_{13}^{\alpha} \cap C_{23}^{\beta} \cap C_{14}^{\gamma} \cap C_{24}^{\delta}$ and whose every element is a union of some of its atoms (i.e., $\mathcal{F}'$ is atomistic.[12]) Now, our Lemma 5 says that if $\langle \Omega, \mathcal{F}, P \rangle$ is a local non-conspiratorial separate-ss model for Bell-Aspect correlations, then so is $\langle \Omega, \mathcal{F}', P' \rangle$, where $\mathcal{F}'$ is the atomistic algebra carved from $\mathcal{F}$ and $P' = P_{\mathcal{F}'}$.

(2) $\mathcal{F}'$ may have atoms of probability zero (in such cases P' is called an "unfaithful measure"). Our next move is to construct a probability space with a faithful probability measure by removing from $\mathcal{F}'$ all atoms with probability zero; the new algebra and measure are labelled $\mathcal{F}'^{\mathfrak{F}}$ and $P'^{\mathfrak{F}}$, respectively. Our Lemma 7 then says that if $\langle \Omega', \mathcal{F}', P' \rangle$ is a local non-conspiratorial separate-ss model for some Bell-Aspect correlations with $\mathcal{F}'$ atomistic, then so is the probability space $\langle \Omega'^{\mathfrak{F}}, \mathcal{F}'^{\mathfrak{F}}, P'^{\mathfrak{F}} \rangle$ obtained from $\langle \Omega', \mathcal{F}', P' \rangle$, with the faith-

[12] Recall: an algebra of sets is *atomic* if for every non-minimal element p there exists an atom a such that $a \subseteq p$; an algebra of sets is *atomistic* if it is atomic and such that every non-minimal element is a union of atoms.

ful measure $P'^{\mathfrak{F}}$ assigning to every "new" event the measure of its "original version" given by P.

(3) In the final step we transform the space we constructed in the second step into a local non-conspiratorial common-ss model for the correlations. Our Lemma 8 says that if $\langle \Omega, \mathcal{F}, P\rangle$ is a local non-conspiratorial separate-ss model for Bell-Aspect correlations with $\mathcal{F}$ atomistic and having atoms of a specific form, while P is a faithful measure on $\mathcal{F}$, then there is a local non-conspiratorial common-ss model $\langle \Omega^*, \mathcal{F}^*, P^*\rangle$ for the same Bell-Aspect correlations.

In sum, the above steps provide a recipe how to transform any local non-conspiratorial separate-ss model for Bell-Aspect correlations into a local non-conspiratorial *common*-ss model for the same correlations. The construction guarantees that the new model contains images of "observational events", i.e., outcomes and settings, of the first model, with probabilities of these events and their images being equal. Thus, if in the new model the Bell inequalities hold referring to events which are images of events from the original model, then the Bell inequalities also hold in the original model. We already know from the literature that the Bell inequalities hold in any local non-conspiratorial common-ss model (since it satisfies NOCONS and the "old" OI and PI), and so we establish that they hold in any local non-conspiratorial separate-ss model too.

Since screener systems investigated in the literature are typically finite, our proofs assume a finite number of screeners in each screener system. However, our theorems below remain correct for infinitely large screener systems (we have already remarked that they can be at most countably infinite). In some footnotes and a remark at the end of section 5.4 we indicate how to modify our proofs for the general case.

5.1 To an atomistic algebra of events

A separate-ss model $\langle \Omega, \mathcal{F}, P\rangle$ for Bell-Aspect correlations might be algebraically very large; without any loss of generality, we may consider its "pruned" cousin, in which a (new) algebra $\mathcal{F}'$ will be atomistic with atoms being the nonempty elements of the following set:[13]

[13] If we allow for infinite screener systems, we should write $\alpha \in K(13)$, etc., as suggested in footnote 10.

$$\mathcal{A} := \big\{A_i^m \cap B_j^n \cap C_{13}^{\alpha} \cap C_{14}^{\beta} \cap C_{23}^{\gamma} \cap C_{24}^{\delta} \mid m, n \in \{+,-\}, i \in \{1,3\}, \\ j \in \{2,4\}, \alpha < K(13), \beta < K(14), \gamma < K(23) \text{ and } \delta < K(24)\big\} \tag{4}$$

Lemma 5 *Let $\langle \Omega, \mathcal{F}, P\rangle$ be a local non-conspiratorial separate-ss model for Bell-Aspect correlations. Then the probability space $\langle \Omega, \mathcal{F}', P'\rangle$ also is a local non-conspiratorial separate-ss model for (the same) Bell-Aspect correlations, where*

- *$X \in \mathcal{F}'$ iff for some $A \subseteq \mathcal{A}$ of Eq. 4: $\cup A = X$;*
- *$P' = P_{|\mathcal{F}'}$.*

Proof: Immediate. Notice that the first item above says in effect that $\mathcal{F}'$ is atomistic.

5.2 From unfaithful to faithful

The measure P of a local non-conspiratorial separate-ss model $\langle \Omega, \mathcal{F}, P\rangle$ need not be faithful, that is, it may be that for some nonempty $X \in \mathcal{F}$: $P(X) = 0$. Our final construction, however, requires probability spaces with faithful measures. The algebra of events of the model whose existence is guaranteed by lemma 5 is atomistic and has countably many atoms (since all four screener systems involved are countable). In such a case there is a simple procedure of arriving at another probability space which will also be a local non-conspiratorial separate-ss model for the same correlations, but whose measure will be faithful.

Suppose $\langle \Omega, \mathcal{F}, P\rangle$ is a probability space, with $\mathcal{F}$ atomistic and having countably many atoms. Let $\mathcal{A}$ be the set of atoms of $\mathcal{F}$, and let $\mathcal{A}^+$ be the set of atoms of $\mathcal{F}$ whose probability is greater than 0. Since $\mathcal{A}$ is countable, $\mathcal{A}^+$ is not empty. Atomicity means that for any $E \in \mathcal{F}$ there exists exactly one set $\mathcal{A}_E \subseteq \mathcal{A}$ such that $E = \cup \mathcal{A}_E$. Consider a function f with domain $\mathcal{F}$ defined in the following way: for $E \in \mathcal{F}$, $f(E) = \cup(\mathcal{A}_E \cap A^+)$. In effect, the function f "strips down" events (which are unions of atoms, due to $\mathcal{F}$ being atomistic) of their zero-measure parts. The algebra of events of the new space, $\mathcal{F}^{\mathfrak{F}}$, will be simply the image of $\mathcal{F}$ through the function f: $\mathcal{F}^{\mathfrak{F}} = f(\mathcal{F})$. The probability function $P^{\mathfrak{F}}$ assigns to all events $f(E)$ the measure of E in the original space; the important difference is that if $E \in \mathcal{F}$ is a nonempty measure zero event, then $f(E) = \emptyset$, which ensures that $P^{\mathfrak{F}}$ is faithful.

Definition 6 (faithfulisation) *Let $\mathcal{S} = \langle \Omega, \mathcal{F}, P \rangle$ be a probability space with $\mathcal{F}$ atomistic and having countably many atoms. Let $\mathcal{A}$ be the set of atoms of $\mathcal{F}$, and let $\mathcal{A}^+$ be the set of atoms of $\mathcal{F}$ whose probability is greater than* 0. *For any $E \in \mathcal{F}$, let $\mathcal{A}_E$ be the subset of $\mathcal{A}$ such that $E = \cup \mathcal{A}_E$. Consider a function $f : \mathcal{F} \to \Omega$: $f(E) = \cup(\mathcal{A}_E \cap A^+)$.*

The faithfulisation of $\mathcal{S}$ is a triple $\langle \Omega^{\mathfrak{F}}, \mathcal{F}^{\mathfrak{F}}, P^{\mathfrak{F}} \rangle$, where:

- $\mathcal{F}^{\mathfrak{F}} = f(\mathcal{F})$;
- $P^{\mathfrak{F}}\big(f(E)\big) = P(E)$;
- $\Omega^{\mathfrak{F}} = f(\Omega)$.

We leave checking the following simple lemma to the reader:

Lemma 7 *If $\langle \Omega, \mathcal{F}, P \rangle$ is a probability space such that $\mathcal{F}$ is atomistic and has countably many atoms, then its faithfulisation $\langle \Omega^{\mathfrak{F}}, \mathcal{F}^{\mathfrak{F}}, P^{\mathfrak{F}} \rangle$ is a probability space as well. Moreover, $P^{\mathfrak{F}}$ is faithful. If $\langle \Omega, \mathcal{F}, P \rangle$ is a local, non-conspiratorial separate-ss model for Bell-Aspect correlations, then its faithfulisation also is a local, non-conspiratorial separate-ss model for the same correlations.*

5.3 From separate-ss models to common-ss models

Lemma 8 *Let $\langle \Omega, \mathcal{F}, P \rangle$ be a local non-conspiratorial separate-ss model for Bell-Aspect correlations with faithful measure P and $\mathcal{F}$ atomistic and having the countable set $\mathcal{A}'$ of atoms such that*

$$\begin{aligned} \mathcal{A}' \subseteq \big\{ A_i^m \cap B_j^n \cap C_{13}^{\alpha} \cap C_{14}^{\beta} \cap C_{23}^{\gamma} \cap C_{24}^{\delta} \mid m, n \in \{+,-\}, i \in \{1,3\}, \\ j \in \{2,4\}, \alpha < K(13), \beta < K(14), \gamma < K(23) \text{ and } \delta < K(24) \big\} \end{aligned} \tag{5}$$

Then there is a local non-conspiratorial common-ss model $\langle \Omega', \mathcal{F}', P' \rangle$ for the same Bell Aspect correlations such that

- *$\mathcal{F}$ is embedded into $\mathcal{F}'$ by means of φ ...*
- *... such that $P(A_i^m \cap B_j^n \cap a_i \cap b_j) = P'(\varphi(A_i^m \cap B_j^n \cap a_i \cap b_j))$;*
- *P' is faithful.*

Proof: Let us first define the following four functions:

$$\begin{aligned}
&\wedge(A_i^m, B_j^n) := A_i^m \cap B_j^n, \quad L(A_i^m, B_j^n) := A_i^m \cap (b_j \setminus B_j^n),\\
&R(A_i^m, B_j^n) := (a_i \setminus A_i^m) \cap B_j^n, \text{ and } \emptyset(A_i^m, B_j^n) := (a_i \setminus A_i^m) \cap (b_j \setminus B_j^n) \\
&\text{where } i \in \{1,2\}; j \in \{3,4\}; m,n \in \{+,-\}.
\end{aligned} \tag{6}$$

It will also be convenient to gather the names of the four functions (we will omit any quoting devices) into the set $I := \{\wedge, L, R, \emptyset\}$.

The task is now to construct the probability space $\langle \Omega', \mathcal{F}', P' \rangle$. Let Ω' be an infinite set. $\mathcal{F}'$ will be an atomistic algebra of subsets of Ω' such that the cardinality K of the set of atoms of $\mathcal{F}'$ equals the cardinality of $\mathcal{A}'$. We will refer to the atoms of $\mathcal{F}'$ by labels of the following sort:

$$\begin{aligned}
{}_{ij}a^{\alpha\beta\gamma\delta}_{xyzt}, \quad &\text{where } i \in \{1,2\}, j \in \{3,4\}, \alpha < K(13), \beta < K(14),\\
&\gamma < K(23), \delta < K(24), x,y,z,t \in I.
\end{aligned} \tag{7}$$

However, not all labels of this sort will denote atoms of $\mathcal{F}'$. The sole class of exceptions are the labels ${}_{ij}a^{\alpha\beta\gamma\delta}_{xyzt}$ for which $A_i^m \cap B_j^n \cap C_{13}^{\alpha} \cap C_{14}^{\beta} \cap C_{23}^{\gamma} \cap C_{24}^{\delta} \notin \mathcal{A}'$. (In other words, $A_i^m \cap B_j^n \cap C_{13}^{\alpha} \cap C_{14}^{\beta} \cap C_{23}^{\gamma} \cap C_{24}^{\delta} = \emptyset \in \mathcal{F}$.)[14] Such labels will refer to $\emptyset \in \mathcal{F}'$.

It is clear from the fact that screener systems are countable that $\mathcal{F}'$ will have a countable set of atoms. If we look at cases in which there only are finite screener systems (which is our default option), so that the numbers $K(ij)$ are well-defined, the number K of atoms of $\mathcal{F}'$ is finite as well. In this case K equals the cardinality of the set $\mathcal{A}$ of Eq. (4) minus the number of those elements of $\mathcal{A}$ that had probability zero before the faithfulisation. Accordingly, $K \leqslant 1024 \cdot K(13) \cdot K(14) \cdot K(23) \cdot K(24)$ (there are $4^5 = 1024$ combinations of detector settings and possible outcomes at all settings).

The measure P' is determined by assigning the following measure to the atoms of $\mathcal{F}'$:

$$\begin{aligned}
P'({}_{ij}a^{\alpha\beta\gamma\delta}_{xyzt}) = &P(C_{13}^{\alpha} \cap C_{14}^{\beta} \cap C_{23}^{\gamma} \cap C_{24}^{\delta}) P(a_i b_j) P\big(x(A_1^+, B_3^+) \mid a_1 b_3\big) \cdot \\
&P\big(y(A_1^+, B_4^+) \mid a_1 b_4\big) P\big(z(A_2^+, B_3^+) \mid a_2 b_3\big) P\big(t(A_2^+, B_4^+) \mid a_2 b_4\big).
\end{aligned} \tag{8}$$

Since $\mathcal{F}'$ is atomistic, Eq. 8 determines the measure on all its elements. It is easy to check that P' is faithful (if for some label ${}_{ij}a^{\alpha\beta\gamma\delta}_{xyzt}$ $P'({}_{ij}a^{\alpha\beta\gamma\delta}_{xyzt}) = 0$, then the label does not denote any atom of $\mathcal{F}'$, but rather the empty set).

[14] This may happen e.g. when the correlation between A_i^m and B_j^n is perfect, i.e., when $P(A_i^m B_j^n \mid a_i b_j) = 1$. Even though such a case is experimentally unrealisable, we cater for it for more generality.

The embedding of $\mathcal{F}$ into $\mathcal{F}'$ is the unique embedding φ which acts as follows on the atoms of $\mathcal{F}$:

$$\begin{aligned}
&\text{for any } x \in I, \text{ if } C_{13}^{\alpha}\cap C_{14}^{\beta}\cap C_{23}^{\gamma}\cap C_{24}^{\delta}\cap x(A_1^+,B_3^+) \neq \emptyset, \text{ then}\\
&\qquad \varphi\big(C_{13}^{\alpha}\cap C_{14}^{\beta}\cap C_{23}^{\gamma}\cap C_{24}^{\delta}\cap x(A_1^+,B_3^+)\big) = \bigcup_{y,z,t\in I} {}_{13}a_{xyzt}^{\alpha\beta\gamma\delta};\\
&\text{for any } y \in I, \text{ if } C_{13}^{\alpha}\cap C_{14}^{\beta}\cap C_{23}^{\gamma}\cap C_{24}^{\delta}\cap y(A_1^+,B_4^+) \neq \emptyset, \text{ then}\\
&\qquad \varphi\big(C_{13}^{\alpha}\cap C_{14}^{\beta}\cap C_{23}^{\gamma}\cap C_{24}^{\delta}\cap y(A_1^+,B_4^+)\big) = \bigcup_{x,z,t\in I} {}_{14}a_{xyzt}^{\alpha\beta\gamma\delta};\\
&\text{for any } z \in I, \text{ if } C_{13}^{\alpha}\cap C_{14}^{\beta}\cap C_{23}^{\gamma}\cap C_{24}^{\delta}\cap z(A_2^+,B_3^+) \neq \emptyset, \text{ then}\\
&\qquad \varphi\big(C_{13}^{\alpha}\cap C_{14}^{\beta}\cap C_{23}^{\gamma}\cap C_{24}^{\delta}\cap z(A_2^+,B_3^+)\big) = \bigcup_{x,y,t\in I} {}_{23}a_{xyzt}^{\alpha\beta\gamma\delta};\\
&\text{for any } t \in I, \text{ if } C_{13}^{\alpha}\cap C_{14}^{\beta}\cap C_{23}^{\gamma}\cap C_{24}^{\delta}\cap t(A_2^+,B_4^+) \neq \emptyset, \text{ then}\\
&\qquad \varphi\big(C_{13}^{\alpha}\cap C_{14}^{\beta}\cap C_{23}^{\gamma}\cap C_{24}^{\delta}\cap t(A_2^+,B_4^+)\big) = \bigcup_{x,y,z\in I} {}_{24}a_{xyzt}^{\alpha\beta\gamma\delta}.
\end{aligned} \tag{9}$$

Next, the measure P on elements $A_i^m\cap B_j^n\cap a_i\cap b_j \in \mathcal{F}$ agrees with the measure P' on the φ-images of these elements, and so, by the conditions from (3) (p. 9), P agrees with P' also on the (images of) elements of the form $a_i\cap b_j$. This is crucial since these events appear in the Bell inequalities.

As an example, let us calculate:

$$\begin{aligned}
P\big(A_2^+\cap B_4^+\cap a_2\cap b_4\big) = P(a_2b_4)P\big(\wedge(A_2^+,B_4^+) \mid a_2b_4\big) = \\
\sum_{\alpha\beta\gamma\delta xyz} P'\big({}_{24}a_{xyz\wedge}^{\alpha\beta\gamma\delta}\big) = P'\big(\cup_{\alpha\beta\gamma\delta xyz}\ {}_{24}a_{xyz\wedge}^{\alpha\beta\gamma\delta}\big) = \\
P'\big(\varphi(A_2^+\cap B_4^+\cap a_2\cap b_4)\big).
\end{aligned} \tag{10}$$

We claim now that the set S below is a common screener system for the correlations and each element of S satisfies OI', PI', and NOCONS:

$$\begin{aligned}
S :=\{S_{xyzt}^{\alpha\beta\gamma\delta} \mid \alpha < K(13), \beta < K(14), \gamma < K(23), \delta < K(24), x,y,z,t \in I\}\\
\text{where } S_{xyzt}^{\alpha\beta\gamma\delta} := {}_{13}a_{xyzt}^{\alpha\beta\gamma\delta} \cup {}_{14}a_{xyzt}^{\alpha\beta\gamma\delta} \cup {}_{23}a_{xyzt}^{\alpha\beta\gamma\delta} \cup {}_{24}a_{xyzt}^{\alpha\beta\gamma\delta}.
\end{aligned} \tag{11}$$

We need to show now that (1) S is a partition of $\mathcal{F}'$, and that every $S_{xyzt}^{\alpha\beta\gamma\delta}$ satisfies (2) NOCONS, (3) OI', and (4) PI' with respect to every pair of settings. We use the tilde sign to refer to the image of an element X of $\mathcal{F}$ by the embedding φ: $\tilde{X} = \varphi(X)$.

Ad. 1. A glimpse at Eq. 11 shows that S is a partition of $\mathcal{F}'$.

Ad. 2. We show that $P'(\mathfrak{A} \mid \tilde{a}_{i'}\tilde{b}_{j'}) = P'(\mathfrak{A} \mid \tilde{a}_{i''}\tilde{b}_{j''})$ where $\mathfrak{A}$ is an arbitrary Boolean combination of elements of S. Since elements of S have empty intersection, it is enough to show that NOCONS is satisfied with respect to every $S^{\alpha\beta\gamma\delta}_{xyzt} \in S$. Note now that $\tilde{a}_{i'} \cap \tilde{b}_{j'} \cap S^{\alpha\beta\gamma\delta}_{xyzt} = {}_{i'j'}a^{\alpha\beta\gamma\delta}_{xyzt}$. Thus we calculate (the crucial thing is that $P'(\tilde{a}_{i'}\tilde{b}_{j'}) = P(a_{i'}b_{j'})$):

$$
\begin{aligned}
&P'(S^{\alpha\beta\gamma\delta}_{xyzt} \mid \tilde{a}_{i'}\tilde{b}_{j'}) = P'({}_{i'j'}a^{\alpha\beta\gamma\delta}_{xyzt})/P'(\tilde{a}_{i'}\tilde{b}_{j'}) = \\
&\Big(P(C^{\alpha}_{13} \cap C^{\beta}_{14} \cap C^{\gamma}_{23} \cap C^{\delta}_{24}) \cdot P(a_{i'}b_{j'}) \cdot P\big(x(A^+_1, B^+_3) \mid a_1 b_3\big) \cdot P\big(y(A^+_1, B^+_4) \mid a_1 b_4\big) \cdot \\
&P\big(z(A^+_2, B^+_3) \mid a_2 b_3\big) \cdot P\big(t(A^+_2, B^+_4) \mid a_2 b_4\big)\Big)/P(a_{i'}b_{j'}) = \\
&P'(S^{\alpha\beta\gamma\delta}_{xyzt} \mid \tilde{a}_{i''}\tilde{b}_{j''}) \text{ for any choice of } i', j', i'', \text{ and } j''.
\end{aligned} \tag{12}
$$

Ad. 3. Let us note these identities, useful in our proof of OI' and PI':

$$
\tilde{A}^m_1 \cap \tilde{B}^n_3 \cap \tilde{a}_1 \cap \tilde{b}_3 \cap S^{\alpha\beta\gamma\delta}_{xyzt} = \begin{cases} {}_{13}a^{\alpha\beta\gamma\delta}_{xyzt} & \text{iff } A^m_1 \cap B^n_3 = x(A^+_1, B^+_3) \\ \emptyset & \text{otherwise} \end{cases} \tag{13}
$$

$$
\tilde{a}_1 \cap \tilde{b}_3 \cap S^{\alpha\beta\gamma\delta}_{xyzt} = {}_{13}a^{\alpha\beta\gamma\delta}_{xyzt} \tag{14}
$$

$$
\tilde{A}^m_1 \cap \tilde{a}_1 \cap \tilde{b}_3 \cap S^{\alpha\beta\gamma\delta}_{xyzt} = \begin{cases} {}_{13}a^{\alpha\beta\gamma\delta}_{xyzt} & \text{iff } x \in \{x', x''\} \text{ and } A^m_1 = x'(A^+_1, B^+_3) \cup x''(A^+_1, B^+_3) \\ \emptyset & \text{otherwise} \end{cases} \tag{15}
$$

$$
\tilde{B}^n_3 \cap \tilde{a}_1 \cap \tilde{b}_3 \cap S^{\alpha\beta\gamma\delta}_{xyzt} = \begin{cases} {}_{13}a^{\alpha\beta\gamma\delta}_{xyzt} & \text{iff } x \in \{x', x''\} \text{ and } B^n_3 = x'(A^+_1, B^+_3) \cup x''(A^+_1, B^+_3) \\ \emptyset & \text{otherwise} \end{cases} \tag{16}
$$

To prove OI', consider $P'(\tilde{A}^m_1\tilde{B}^n_3 \mid \tilde{a}_1\tilde{b}_3 S^{\alpha\beta\gamma\delta}_{xyzt})$. Since measure P' is faithful, all probabilities occurring in denominators are non-zero, and hence the appropriate conditional probabilities are defined. If x satisfies: (†) $A_1^m \cap B_3^n = x(A_1^+, B_3^+)$, then the above probability equals 1, as indicated by Eqs. 13 and 14. In this case x belongs to the set $\{x', x''\}$ of Eq. 15 and to the set $\{x', x''\}$ of Eq. 16, so that each $P'(\tilde{A}_1^m \mid \tilde{a}_1 \tilde{b}_3 S_{xyzt}^{\alpha\beta\gamma\delta})$, $P'(\tilde{B}_3^n \mid \tilde{a}_1 \tilde{b}_3 S_{xyzt}^{\alpha\beta\gamma\delta})$ equals 1. On the other hand, if x does not satisfy (†), then it does not belong to the set $\{x', x''\}$ of Eq. 15 or it does not belong to the set $\{x', x''\}$ of Eq. 16, and hence both $P'(\tilde{A}_1^m \tilde{B}_3^n \mid \tilde{a}_1 \tilde{b}_3 S_{xyzt}^{\alpha\beta\gamma\delta})$ and $P'(\tilde{A}_1^m \mid \tilde{a}_1 \tilde{b}_3 S_{xyzt}^{\alpha\beta\gamma\delta}) \cdot P'(\tilde{B}_3^n \mid \tilde{a}_1 \tilde{b}_3 S_{xyzt}^{\alpha\beta\gamma\delta})$ are zero.

Ad. 4. To see that PI' is satisfied as well, let us check that the following is true:

$$P'(\tilde{A}_1^m \mid \tilde{a}_1 \tilde{b}_3 S_{xyzt}^{\alpha\beta\gamma\delta}) = P'(\tilde{A}_1^m \mid \tilde{a}_1 \tilde{b}_4 S_{xyzt}^{\alpha\beta\gamma\delta})$$

Faithfulness of P' and Eq. 14 show that these probabilities are well-defined. If (†) $x \in \{x', x''\}$ such that $A_1^m = x'(A_1^+, B_3^+) \cup x''(A_1^+, B_3^+)$, then the LHS above equals 1. But then $A_1^m = x'(A_1^+, B_4^+) \cup x''(A_1^+, B_4^+)$ as well, so RHS is also 1. On the other hand, if x does not satisfy (†), LHS and RHS are zero. The argument for other detector settings is analogous. □

5.4 The central theorems

By combining the chain of facts, lemmas, and theorems above, we obtain the two central theorems of this paper:

Theorem 9 *If there is a local non-conspiratorial separate-ss model for some Bell-Aspect correlations, then there is a local non-conspiratorial common-ss model for the same correlations.*

PROOF: Let $\langle \Omega, \mathcal{F}, P \rangle$ be a local non-conspiratorial separate-ss model for some Bell-Aspect correlations. By Lemma 5, there exists a local non-conspiratorial separate-ss model for the same correlations whose algebra of events is atomistic and whose atoms have a specific form. Lemma 7 guarantees that in such a case there exists a local non-conspiratorial separate-ss model for these correlations whose measure is faithful. Finally, by Lemma 8, there is then a local non-conspiratorial common-ss model for these correlations. □

Theorem 10 *Bell's inequalities are derivable in every local non-conspiratorial separate-ss model for Bell-Aspect correlations.*

Proof: Let $\langle \Omega, \mathcal{F}, P \rangle$ be a local non-conspiratorial separate-ss model for some Bell-Aspect correlations. Bell's inequalities, if derivable in this model, are expressed in terms of 'surface' probabilities $P(A_i^m B_j^n \mid a_i b_j), P(A_i^m \mid a_i)$ and $P(B_j^n \mid b_j)$, which (given the assumptions in (3), p. 9), are definable in terms of probabilities (†) $P(A_i^m B_j^n a_i b_j) = P(A_i^m B_j^n)$. By Lemma 5, there exists a local non-conspiratorial separate-ss model $\langle \Omega, \mathcal{F}', P' \rangle$ for the same correlations whose atoms have a specific form. The construction guarantees that ($\star$) $P'(A_i^m B_j^n) = P(A_i^m B_j^n)$. Next, by Lemma 7 there is a faithfulisation $\langle \Omega^{\mathfrak{F}}, \mathcal{F}^{\mathfrak{F}}, P^{\mathfrak{F}} \rangle$ of $\langle \Omega, \mathcal{F}', P' \rangle$, which is a local, non-conspiratorial separate-ss model for the same Bell-Aspect correlations. If for $A_i^m \cap B_j^n \in \mathcal{F}'$: $P'(A_i^m \cap B_j^n) = 0$, then $A_i^m \cap B_j^n$ is identified with $\emptyset \in \mathcal{F}^{\mathfrak{F}}$, and hence (‡1) $P^{\mathfrak{F}}(A_i^m \cap B_j^n) = 0 = P'(A_i^m \cap B_j^n)$. And, if $P'(A_i^m \cap B_j^n) \neq 0$, the faithfulisation again leaves the probabilities intact, that is (‡2) $P^{\mathfrak{F}}(A_i^m \cap B_j^n) = P'(A_i^m \cap B_j^n)$, Further by Lemma 8 there is a local non-conspiratorial common-ss model $\langle \Omega^*, \mathcal{F}^*, P^* \rangle$ for the same Bell Aspect correlations such that $\mathcal{F}^{\mathfrak{F}}$ is embedded into $\mathcal{F}^*$ and (†) $P^{\mathfrak{F}}(A_i^m \cap B_j^n \cap a_i \cap b_j) = P^*(\tilde{A}_i^m \cap \tilde{B}_j^n \cap \tilde{a}_i \cap \tilde{b}_j))$, where a tilde indicates images of elements of $\mathcal{F}^{\mathfrak{F}}$ by the embedding. Now, by combining Eqs. ($\star$), (‡1), (‡2), and (†), we get $P(A_i^m \cap B_j^n) = P^*(\tilde{A}_i^m \cap \tilde{B}_j^n)$. Finally, since $\langle \Omega^*, \mathcal{F}^*, P^* \rangle$ is is a local non-conspiratorial common-ss model for Bell-Aspect correlations, Bell's inequalities are derivable in it, and they are defined in terms of probabilities $P^*(\tilde{A}_i^m \cap \tilde{B}_j^n)$. Combining the last two facts, Bell's inequalities are derivable in the model $\langle \Omega, \mathcal{F}, P \rangle$ we started with, which is a local non-conspiratorial separate-ss model for Bell-Aspect correlations. □

6 Conclusions

The essence of the project we have analysed is the distinction between (many) separate screener systems and (single) common screener system. Importantly, the condition of Outcome Independence present in the usual derivations of Bell's theorem pertains to a common screener system since it requires that a posited set of hidden states forms a common screener system for all correlated pairs of results. Could one block the derivation of Bell's inequalities by assuming that the posited hidden states form (many) separate screener systems rather than a common screener system? The success of this programme would afford a nice diagnosis of Bell's theorem by showing that the original Outcome Independence is too strong.

Our proofs say "no" to the last question, which means that the purported diagnosis above is incorrect: If for some Bell-Aspect correlations there is a local non-conspiratorial model with separate screener systems, then there is a local non-conspiratorial model with common screener system for the same correlations. And, since any local non-conspiratorial model with a common screener system satisfies Bell's inequalities, the new models (i.e., those with separate screener systems) satisfy these inequalities as well. Accordingly, there is no local non-conspiratorial model with separate screener systems for those Bell-Aspect correlations that violate Bell's inequalities.

The distinction "common screener system vs. separate screener systems" does not explain Bell's theorem.

References

Bell, J. (1964). On the Einstein-Podolski-Rosen paradox. *Physics*, 1:195–200. reprinted in his *Speakable and unspeakable in quantum mechanics*, Cambridge UP, 1987.

Bell, J. (1975). The theory of local beables. Technical report, CERN: TH-2053. Reprinted in his *Speakable and unspeakable in quantum mechanics*, Cambridge UP, 1987.

Belnap, N. and Szabó, L. (1996). Branching space-time analysis of the GHZ theorem. *Foundations of Physics*, 26(8):982–1002.

Billingsley, P. (1995). *Probability and Measure*, 3rd edition. John Wiley & Sons, Inc.

Grasshoff, G., Portmann, S., and Wüthrich, A. (2005). Minimal assumption derivation of a Bell-type inequality. *British Journal for the Philosophy of Science*, 56:663–680. preprint: quant-ph/0312176.

Hofer-Szabó, G. (2008). Separate- versus common -common-cause-type derivations of the Bell inequalities. *Synthese*, 163(2):199–215.

Hofer-Szabó, G. (2011). Separate common causal explanation and the Bell inequalities. *Submitted.*

Hofer-Szabó, G., Rédei, M., and Szabó, L. (1999). On Reichenbach's common cause principle and Reichenbach's notion of common cause. *British Journal for the Philosophy of Science*, 50:377–399.

Norsen, T. (2006). J.S. Bell's Concept of Local Causality. *Arxiv preprint quant-ph/0601205*.

Reichenbach, H. (1956). *The Direction of Time*. University of California Press, Berkeley, CA.

Seevinck, M. and Uffink, J. (2011). Not Throwing out the Baby with the Bathwater: Bell's Condition of Local Causality Mathematically 'Sharp and Clean'. In Dieks, D., Gonzalez, W., Hartmann, S., Uebel, T., and Weber, M., editors, *Explanation, Prediction, and Confirmation: New Trends and Old Ones Reconsidered*, pages 425–450. Springer, Dordrecht.

Szabó, L. E. (2000). Attempt to resolve the EPR-Bell paradox via Reichenbach's concept of common cause. *International Journal of Theoretical Physics*, 39:901–911.

van Fraassen, B. (1982). The Charybdis of realism: Implications of Bell's inequality. *Synthese*, 52:25–38. Reprinted in Cushing, J. T. and McMullin, E., editors (1989). *Philosophical Consequences of Quantum Theory: Reflections on Bell's Theorem*. University of Notre Dame Press, Notre Dame, IN.