Persistence and Change in Minkowski Spacetime

(Claudio Calosi)

University of Urbino, Department of Foundations of Science

claudio.calosi@uniurb.it

1 Introduction

There are famously two main alternative metaphysics of persistence, namely Three and Four-Dimensionalism.[1] 3D, or endurantism, roughly maintains that material objects are three-dimensional wholes that are multi-located at different temporally unextended regions of spacetime and persist through time by being wholly present at each time of their existence. 4D, or perdurantism, maintains roughly that material objects are four-dimensional wholes that are singly located at a temporally extended region of spacetime and persist through time by having a different temporal part at each time of their existence. Both those metaphysics of persistence yield a particular solution to the so called puzzle of change. In this paper I argue that endurantist solutions to the puzzle face insurmountable difficulties when framed within the context of Minkowski spacetime.[2] This result could be taken either to favor 4D or as a challenge for 3D to give a new account of change in a relativistic setting. Here's a plan of the paper. In section 2 I will sketch a formal theory of both parthood and location using two primitive notions, that of parthood itself and that of exact location. This will lead to a rigorous formulation of controversial metaphysical thesis in section 3. In section 4 I will give an argument, which I label the Location Argument to show that, in a relativistic context, three-dimensional objects are exactly multi-located at different spacetime regions that typically overlap each other. In section 5 I will briefly rehearse the so called puzzle of change and the typical endurantist solutions to it. Then in section 6 I will present my main argument, which I label the Relativistic Argument from Change. The core of the Relativistic Argument from Change is that the three-dimensionalist solutions presented in section 5fail, due to the fact that different exact locations of a three-dimensional object overlap each other. This argument depends crucially on the geometric structure of Minkowski spacetime. In the concluding section I consider different ways to resist my argument and discuss them.

2 Parthood and Location

It is clear from the introduction that at least two different ways of formulating the disagreement between 3D and 4D are possible. The first one is in terms of the location of material objects, the other one is in terms of their mereological structure.[3] Attempts to define clearly the notion of temporal part have proved extremely difficult.[4] The basic idea to make use of the notion of location to formulate precisely controversial metaphysical theories about persistence without appealing to the allegedly obscure notion of temporal part goes at least back to Van Inwagen (1990). The driving intuition behind this strategy is both simple and powerful. Suppose we can grasp an informal notion of exact location along these terms.[5] An object x is exactly located at region R if it exactly *fits into* this region, i.e the object x and the region R have the same shape, size and so on. Thus for example a square region cannot be the exact location of a

1. 3D and 4D from now on respectively.
2. From now on I will take Minkowski spacetime to be an example of a metric affine space with signature (1, n-1), where n = 4. See Malament (2007) and Sartori (1996) for details.
3. The relationship between locational and mereological persistence are very interesting, yet they can not be explored here. See Gilmore (2007).
4. See Sider (2001) and reference therein.
5. See Gilmore (2007).

circle or my office cannot be the exact location of Mount Everest. If we can grasp such a notion it seems that a vivid difference between 3D and 4D can be captured in those terms. Since three-dimensional objects and four-dimensional objects have different shapes and sizes they will have different regions as their exact locations.

Now the problem at hand is to propose formal theories of location that try to regiment different locational notions. The relation of exact location can be taken as primitive or defined via other sets of primitives. In what follows I will make use of just two primitive notions, the mereological notion of parthood and the locational notion of exact location. I will write

(2.1) (*Parthood*) $x < y$

(2.2) (*Exact Location*) ExL (x,R)

That are read "x is part of y" and "x is exactly located at region R" respectively. From (2.1) is possible to define[6] further mereological notions such as proper parthood and overlap via

(2.3) (*Proper Parthood*)[7] $x << y =_{\text{df}} x < y \wedge x \neq y$

(2.4) (*Overlap*) O (x,y) $=_{\text{df}} (\exists z)\ (z < x \wedge z < y)$

Informally two things overlap if they share a part. We regiment these notions with these axioms

(2.5) (*Reflexivity*) $x < x$

(2.6) (*Antisymmetry*) $(x < y \wedge y < x) \rightarrow x = y$

(2.7) (*Transitivity*) $x < y \wedge y < z \rightarrow x < z$

(2.8) (*Weak Supplementation*) $x << y \rightarrow (\exists z)\ (z << y \wedge \sim \mathrm{O}\ (x,z))$

The mereological theory comprising axioms (2.5) – (2.8) is known as Minimal Mereology.[8] It will do double duty as a theory of parthood both for material objects and spacetime regions. It is now possible to define the locative notions of weak location, overfilling and underfilling, as

(2.9) (*Weak Location*) WL $(x,R) =_{\text{df}} (\exists R_1)$ (ExL $(x,R_1) \wedge$ O (R,R_1))

(2.10) (*Overfilling*) OvF $(x, R) =_{\text{df}} (\exists R_1)$ (ExL $(x,R_1) \wedge R < R_1$)

(2.11) (*Underfilling*) UnF $(x,R) =_{\text{df}} (\exists R_1)$ (ExL $(x,R_1) \wedge R_1 < R$)

Gilmore (2007) proposes different constraints to regiment such notions:

(2.12) Suppose ExL(x,R) holds. Then x has the same geometrical properties of R and stands in the same geometrical relations as R

Gilmore calls *Bifurcation* the thesis according to which the shapes of material objects can be reduced to the shapes of spacetime regions via the schema " x has shape S = x is exactly located at an S-shaped speactime region". Condition (2.12) is the denial of the Bifurcation thesis.

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6 First order logic with identity is presupposed, thought identity could be introduced by definition. Formulas, unless otherwise noted are to be taken as universally closed.

7 Another definition is found in literature, namely x is a proper part of y = x is part of y and y is not part of x. Given anti-symmetry it turns out that the definitions are equivalent.

8 See Simons (1987) and Varzi (2009) for details.

(2.13) It is not impossible for an object x to be exactly multi-located, to be exactly located at two different, even non overlapping, regions R_1 and R_2

This condition simply says that multi-location is possible.[9]

(2.14) Suppose the following hold: ExL(x,R_1), ExL(x,R_2) and $R_1 \neq R_2$. Then it does not follow that ExL(x, $R_1 \cup R_2$)[10]

Condition (2.14) informally says that a multi-located object can fail to be exactly located at the union of its exact locations. This should not come as a surprise. It in fact fallows straightforwardly from (2.12). Suppose x is exactly multi-located at two different spherical regions. Then the sum of those regions is itself not spherical and x cannot possibly be exactly located there.

(2.15) Suppose ExL(x,R) holds. Then it is possible that x, even if when mereologically simple, fails to be exactly located at any other region R_1 that is distinct from R, in particular at any subregion of R

The argument for (2.15) parallel the one in favor of (2.14). And, finally

(2.16) Suppose the following hold. ExL(x,R_1), ExL(x,R_2) and $R_1 \neq R_2$. Then it is possible for x to have a different mereological composition at R_1 and R_2

Condition (2.16) is supposed to capture the very strong pre-analytical intuition that a certain thing can gain and lose parts while remaining identical to itself. It is worth noting that this condition has been challenged by the advocates of the so called mereological essentialism, most notably Chisolm (1973).

The one I just sketched out can be considered a multi-location theory of location. With this in hand it is possible to give a rigorous formulation of the controversial metaphysical thesis of 3D and 4D. I now turn on to such a formulation.

3 Metaphysics of Persistence in Minkowski Spacetime

Recall the pre-analytical intuitions we are supposed to vindicate. A three-dimensional object is a persisting object that is multiply located at different temporally unextended spacetime regions while a four-dimensional object is a persisting object that is singly located at a temporally extended one. Thus, it seems that we do need some formal counterparts for such informal notions as persisting object and temporal extension. Let's start from the latter.[11] Suppose a proper subregion R of Minkwoski spacetime contains two distinct points p, q such that the vector **pq**[12] is a future directed causal[13] vector. Then an event e (q) located[14] at q will always be later than an event e (p) located at p. Hence the subregion R

[9] This is the condition Barker and Dowe (2003) explicitly denies. See also Parsons (2006).

[10] The set-theoretic notion of union is used here and in other places instead of the notion of mereological sum. This is just for sake of clarity. The reader is probably more familiar with the set theoretical notion. However everywhere in the arguments it could be replaced by the notion of sum or fusion defined only in terms of the mereological notions introduced.

[11] This construction is specific for relativistic spacetimes. In fact the notion of temporal precedence can be grounded in the mathematical structure of inner product for such spacetimes. If spacetime is Newtonian you will have to take as primitive the notion of absolute precedence and then go define the notion of an achronal region via that notion.

[12] As a conventional device vectors are written in bold characters.

[13] A vector is causal if it is either timlike or null like.

[14] There is no need to be formal here.

should count as temporally extended. This suggests an elegant definition of an achronal region grounded in the metrical structure of Minkowski spacetime[15]

(3.1) (*Achronal*) Achr (R) $=_{\text{df}}$ ($\forall$p) ($\forall$q) (p$\in R \wedge$ q$\in R$) $\rightarrow$ **<pq, pq>** < 0

An achronal region is a region such that every two points in it are connected by a spacelike vector.

Objects persist trough time. That much seems uncontroversial. The controversy is about how they do so. Lewis (1986) defines the uncontroversial notion of persisting object along the following lines. A persisting object is an object that exists at two different times, no matter how it does so. To give a rigorous formulation let me introduce the definition of a path of an object. The path of an object comprises its entire spatiotemporal career:

(3.2) (*Path*) Path (x) $=_{\text{df}} \cup_{\text{i}}$ (R_i) ExL (x,R_i)[16]

Thus, the path of an object is just the union of its exact locations. If x is singly located, for example at R, we will simply have Path (x) = R. A persisting object is then an object such that its path is not achronal:

(3.3) (*Persisting*) Pers (x) $=_{\text{df}}$ ~ Achr (Path (x))

Next we define a three-dimensional object via

(3.4) (*Three-Dimensional Object*) 3D (x) $=_{\text{df}}$ Pers (x) $\wedge$ ($\forall$R) (ExL (x,R) $\rightarrow$ Achr (R))

There are several things to note about definition (3.4). First of all the definition informally says that a three-dimensional object is a persisting object that is exactly located at spacetime subregions that are achronal, i.e. allegedly temporally unextended. This was however just one part of those pre-analytical intuitions about three-dimensional objects we were supposed to capture. The other one was that three-dimensional objects are multi-located. This worry can be set aside. It follows from (3.1) – (3.4) that a three-dimensional object is exactly multi-located:

(3.5) 3D (x) $\rightarrow$ ($\exists R_1$) ($\exists R_2$) ($R_1 \neq R_2 \wedge$ ExL (x,R_1) $\wedge$ ExL (x,R_2))

Proof. Suppose it does not hold. Then x is a 3D object but we have that $R_1 = R_2$. By definition of a 3D object it follows from this definition that R_1 is achronal, i.e. Achr (R_1) holds. Suppose for sake of argument that x is just located at R_1 and R_2. Then Path (x) = $R_1 \cup R_2 = R_1 = R_2$. But then by the argument above from Achr (R_1) follows Achr (Path (x)). Thus x does not qualify as a persisting object, and *a-fortiori*, does not qualify as a 3D object. And this contradicts our assumption, so (3.5) holds.

It moreover follows that the exact locations of an object are proper subregions of its path:

(3.6) 3D (x) $\rightarrow$ (ExL (x, R) $\rightarrow$ R << Path (x))

Proof. R < Path (x) follows by definition of Path (x) in (3.2). Then, from minimal mereology either R = Path (x) or R << Path (x). In the first case since x is a persisting object ~ Achr (R) will follow. But this will imply that x is not a 3D object according to definition (3.4) for R is supposed to be one of its exact locations. Then it has to be the second case, which is exactly what (3.6) claims.

There is a crucial thing to note. From (3.4) and (3.6) we know that the exact locations of a 3D object x are achronal proper subregions of Path (x) but we *do not know which* achronal proper subregions of Path (x)

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15 If this notion of achronality vindicates our pre-analytical intuitions about temporal extension is an interesting question, but one that I will leave it aside. I will take it for granted that it could. Personally I am skeptical about it. Note however that this will cause problems only for three-dimensionalism.

16 The index i ranges over spacetime subregions.

they are. In other words, (3.4) and (3.6) give us properties of those subregions but are not enough to single them out among many. The Location Argument in the next section is a tentative answer to such a problem.

Given these results 3D is just the universal claim that all persisting material objects MO are three-dimensional objects:

(3.7) (*Three-Dimensionalism*) $3D =_{df} MO(x) \rightarrow (Pers(x) \rightarrow 3D(x))$

On the other hand four-dimensional objects are supposed to be singly exactly located at an unique temporally extended spacetime region. That region cannot but be Path(x), which suggests:

(3.8) (*Four-Dimensional Object*) $4D(x) =_{df} Pers(x) \wedge ExL(x, Path(x))$

And

(3.9) (*Four-Dimensionalism*) $4D =_{df} MO(x) \rightarrow (Pers(x) \rightarrow 4D(x))$

All this gives us a rigorous formulation of controversial metaphysical thesis in the simplest relativistic setting.

4 The Location Argument[17]

As I have already pointed out in the previous section the definitions I have offered of three and four-dimensional objects leave some important questions unanswered. In particular they leave unanswered what Gilmore (2007), following Smart (1972) and Rea (1998), calls the Location Question. Here is a formulation:

(4.1) (*Location Question*). Let x be a persisting object. Which subregions of Path (x) is x exactly located at?

It is precisely in answering this question that it is possible to appreciate a structural difference in the definitions of three and four-dimensional objects. The first definition, that of a 3D object, gives us only properties of the exact locations of an object, while the latter does not only give us properties of the unique exact location, but it is also able to single it out. As a consequence 4D has a straightforward and very simple answer to (4.1), namely:

(4.2) (*Path Principle*) Let x be a 4D object. Then x is exactly located at Path (x) which is the only improper subregion of itself

The Path principle follows trivially from definition (3.8). Things are different for three-dimensionalists. They will not be able to single out which proper subregions of Path (x) are to be counted as x's exact locations simply by looking at the definition of a three-dimensional object. They need a substantive argument to answer the Location Question. Gilmore (2008) divides the possible three-dimensionalist answers into *Overlap* and *No Overlap* answers. Overlap answers hold that typically a 3D object x will be exactly located at different proper subregions of Path (x) that overlap each other. No Overlap answers deny such a claim. To discuss them I first need to introduce the notion of a foliation. A foliation F of a spacetime region R is a set of subregions of R such that i) each subregion R_1 of R in F is achronal, ii) each subregion in F is a maximal subregion, i.e. if R_1 is in F than there is no achronal region R_2 such that $R_1 << R_2$ and R and, iii) each point in R belongs to just one maximal achronal subregion R_i. It is now possible to consider different No Overlap answers. Probably the first that comes to mind is given by the Rest Frame Principle:

[17] My discussion here follows closely the excellent Gilmore (2007) which I consider the best paper on the subject.

> (4.3) (*Rest Frame Principle*) Let x be a 3D object. Then x exactly occupies only those subregions that are given by its rest frame foliation. Suppose for sake of simplicity that x is not accelerating. Then, given the geometry of Minkwoski spacetime it follows that x is exactly located just at those subregions of Path (x) that are orthogonal with Path (x)

The main problem with the rest Frame Principle is that it will give raise to implausible consequences. Consider a spatially extended object[18] x whose endpoints move along the timelike lines spanned by the timelike vector $\mathbf{u}$. Then, given (4.3), the exact locations of x will be subregions of the spacelike submanifold orthogonal to $\mathbf{u}$, usually indicated as $u^{\perp}$. Now consider y, one of x's spatially extended proper parts and suppose y is moving with a certain velocity $v \neq 0$ with respect to x. Suppose its endpoints move along the timelike lines spanned by the timelike vector $\mathbf{v}$ such that $\mathbf{v} \neq a\,\mathbf{u}$ for any real number a. Then, given (4.3) again, the exact locations of y will be subregions of $\mathbf{v}^{\perp}$. But for any spacetime point p in Path (x) the subregion of $u^{\perp}$ passing through p does not contain the subregions of $v^{\perp}$ passing through that point. This means that the exact locations of y will not be subregions of the exact locations of x. In other words y will be *exactly located outside* x, though it is one of its proper parts. Here is another way to put the same point. The Rest Frame Principle entails that 3D objects constitute a counterexample to the very plausible locational principle of expansivity, according to which an object cannot fail to be located where its parts are. This seems enough to conclude that (4.3) is not an adequate answer to the Location Question in a relativistic setting.

Then a three-dimensionalist might try:

> (4.4) (*Top-Down Principle*) Let x be a 3D object and let F be a physically preferred foliation of the entire Minkowski spacetime. Then x exactly occupies just those subregions of Path (x) that are given by the preferred foliation F

This answer is easily dispensed with in the context of Minkoski spacetime.[19] Minkowski spacetime simply does not admit such a preferred foliation[20], so the Top Down Principle is a non starter.

The last plausible No Overlap answer is the Bottom Up Principle. Suppose we define a foliation F according to the following procedure. Take the first slice of Path (x). Identify the simple particles that compose x at that slice. Then attach a natural clock to each of those particles. Call a Particle Synchrony Slice a sum of those exact locations of each particle that compose x in which the natural clocks are synchronized, i.e. indicate the same number. A three-dimensionalist could go on to argue for:

> (4.5) (*The Bottom Up Principle*) Let x be a 3D object. Then it exactly occupies those regions that are given by the Particle Synchrony Slices of Path (x)

The problem of this answer is that natural clocks, such as the ones attached to the particles composing x, read proper time in Minkowski spacetime. Let x be composed, for sake of simplicity, by two simple particles y and z. They move along timelike lines P and Q spanned by the timelike vectors $\mathbf{u}$ and $\mathbf{v}$ respectively such that $\mathbf{u} \neq a\,\mathbf{v}$ for any real number a. Suppose to indicate with $p_{1\ldots n}$ and $q_{1\ldots n}$ the points on P and Q respectively that are the exact locations of y and z. Then the First Particle Synchrony Slice of x will be given by $p_1 \cup q_1$, the Second Particle Synchrony slice by $p_2 \cup q_2$, and so on till the n-th, given by

18 The argument presupposes that both composite objects exist, and that they are spatially extended.

19 The issue of the tenability of the Top Down principle in general relativistic spacetime is more complex. Some general relativistic spacetimes seem to admit the possibility of a physically preferred foliation after all. However the principle will yield highly counterintuitive results when shapes of locational 3D objects are taken onto account. Moreover it will affect the modal status of three-dimensionalism. Since not every general relativistic spacetime admits such a foliation, it will follow that three-dimensionalsim is contingently true if true at all only for those models that admit such a preferred foliation. See Gilmore (2007) for a more detailed discussion.

20 It follows from the fact that given any timelike line L there is always an isometry of Minkowski spacetime that maps L into any other timelike line L_1.

$p_n \cup q_n$. But as I have mentioned natural clocks read proper time. This means that the elapsed time between two spacetime points is just given by the length of the timelike vector that connects the two points. Since the natural clocks have to be synchronized we will have that $|\mathbf{p_1p_2}| = |\mathbf{q_1q_2}|$, $|\mathbf{p_1p_3}| = |\mathbf{q_1q_3}|$ and so on till $|\mathbf{p_1p_n}| = |\mathbf{q_1q_n}|$. But here is the problem. Given this there is going to be a spacetime point m such that $|\mathbf{p_1p_m}| = |\mathbf{q_1q_m}|$ and that $\mathbf{p_mq_m}$ is timelike. Now, $p_m \cup q_m$ counts as a particle synchrony slice of *x*, and thus, according to (4.5), as an exact location of *x*. But this is not an achronal region and so, by definition of a three-dimensional object *x* could not possibly be exactly located there. It follows that also the Bottom-Up Principle is not able to give a viable answer to the Location Question in Minkowski spacetime.

Principles (4.3) – (4.5) are surely not the only possible No Overlap answers to the Location question. They seemed however the most promising ones. Thus, it is possible to conclude safely that the answer to (4.1), whatever that might be, is an Overlap answer.[21] The simplest of such answers is given by the Every Slice Principle:

> (4.6) (*Every Slice Principle*) Let *x* be a 3D object. Then *x* is exactly located at each and every achronal slice of Path (*x*)

Note however that the following arguments of mine do not make use of the full force of (4.6). They depend just on the much weaker assumption of overlap among the exact locations of a 3D object.

5 The Puzzle of Change

In this section I will briefly rehearse the puzzle of change[22] and the typical endurantist solutions to it. In a nutshell a solution to the puzzle of change requires an explanation of how one and the same object can instantiate different incompatible properties. More in particular Kurtz and Haslanger (2006) argues that the following pre-analytical intuitions abut change are so firmly rooted that they seem not negotiable:

> (5.1) (*Consistency*) No material object *x* can have incompatible properties

> (5.2) (*Change*) Change involves incompatible properties

And

> (5.3) (*Persistence*) Objects persist trough change

Yet, they seem to entail a contradiction. The very same flower seems in fact to instantiate the incompatible properties of being in bloom and being withered. Three-dimensionalism solves the puzzle of change holding that spatiotemporal[23] facts mediate somehow the instantiation of incompatible properties. This general strategy is implemented differently in different authors, for example in Mellor (1983), Van Inwagen (1990) and Hinchliff (1996). I do not want to enter the details of those applications here. The argument I am giving against these solutions in the relativistic setting is rather general and it applies to all of them. So I will introduce the following notation

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[21] It is probable for example that Gibson and Pooley (2006) maintains that there is no simple answer to the Location Question and that the true, very complex answer, is an Overlap Answer.

[22] It is not my aim to give an exhaustive characterization of the puzzle. See Kurtz and Haslanger (2006) and references therein.

[23] In the classic formulation of the arguments these are just temporal fact. However in spacetime physics time has to be modeled as an aspect of a more fundamental four-dimensional entity, namely spacetime. To consider seriously the spacetime conception of time I endorse what Sattig (2006) calls temporal supervenience. It is not possible to enter details here. As a rough approximation the thesis of temporal supervenience maintains that temporal facts do supervene on spatiotemporal facts. In other words, it is not possible to change temporal facts without changing spatiotemporal facts. For a much more detailed discussion see Sattig (2006).

(5.10) (*Property F at region R*) F @ R_1 (x) = object x has property F-at-region-R_1

Now, the instantiation of the incompatible properties is somehow modified by spatiotemporal facts and so no contradiction arises. In fact an object can instantiate both F @ R_1 and not F @ R_2, provided that $R_1 \neq R_2$, without violating (5.1). That is, three-dimensionalist solutions to the puzzle of change imply that

(5.11) (*Three-dimensionalist Solutions*) F @ R_1 (x) $\wedge$ ~ F @ R_2 (x) $\rightarrow R_1 \neq R_2$

In the next section I will argue that this strategy is at best deeply problematic in Minkowski spacetime.

6 The Relativistic Argument from Change

Let x be a 3D object, and suppose that the following hold:

(6.1) ExL (x,R_1)

(6.2) ExL (x,R_2)

(6.3) $R_1 \neq R_2$

(6.4) O (R_1, R_2)

Note that they are all plausible assumptions: the first three stem directly from the definition of a 3D object, the last one is guaranteed by the location argument in section 4. Then:

(6.5) ($\exists R$) ($R < R_1 \wedge R < R_2$)

Proof. It follows trivially from (6.4) and definition of Overlap from minimal mereology. But

(6.6) $R << R_1$

Proof. $R < R_1$ by (6.5). So, either $R = R_1$ or R << R_1. If $R = R_1$ then $R_1 < R_2$ by substitution in (6.5) again. But $R_1 \neq R_2$. It will follow that $R_1 << R_2$. But an object cannot be exactly located at a region and at one of its proper subregions without having changed its shape. And x did not. It will then follow that ~ ExL(x,R_1), since ExL(x,R_2). But this contradicts our assumptions. And

(6.7) $R << R_2$

Proof. Just repeat the previous argument changing R_1 and R_2.

Let me introduce two particular properties, namely being uniformly F and being uniformly not F.[24] I will write UnF (x) and Un ~F (x) for x is uniformly F and x is uniformly not F respectively. Let me give a more formal characterization of such properties. I say that an object x is uniformly F-at-R if it is F at every proper subregion of R. Since x overfills all subregions of its exact location, x is uniformly F at R_x, where R_x is one of x's exact locations, iff it is F at every subregion of R_x. Put more formally:

(6.8) UnF @ R_x (x) $=_{\text{df}}$ ($R << R_x$) $\rightarrow$ F @ R (x)

Assume now

(6.9) UnF @ R_1 (x)

(6.10) Un~F @ R_2 (x)

[24] Think for example to the following property, being uniformly red.

These claims are supposedly grounded on the possibility of change. They say that x is uniformly F at R_1 and then it changes, say its color for example, to being uniformly not F at R_2. But now

(6.11) UnF @ R_1 (x) → F @ R (x)

Proof. By definition (6.8) if x is uniformly F at R_1 it is F *simpliciter* at every proper subregion of R_1. But R is one of these proper subregions by (6.6). So (6.11) holds. Moreover, by the same argument

(6.12) Un~F @ R_2 (x) → ~ F @ R (x)

But now we end up with

(6.13) F@ R (x)

Proof. By *modus ponens*, from (6.9) and (6.11), and

(6.14) ~ F @ R (x)

Proof. By *modus ponens*, from (6.10) and (6.12).

And this is terrible news for the endurantist. For we have seen in (5.5) that three-dimensionalist solutions to the puzzle of change entail that if x has incompatible properties then it has them at different spacetime regions. And this is not the case here, given (6.13) and (6.14). It follows that there are cases in a relativistic setting in which endurantist solutions to the puzzle of change do not work. Note that this argument crucially depends upon the geometric structure of Minkowski spacetime, in particular from the fact that the exact locations of a three-dimensional object do overlap each other. I now turn to a final discussion about different ways to resist the argument and to considerations about its extent.

7 Conclusion

Let me start by saying that, as it is the case with every philosophical argument, there are several ways to resist it. I will discuss some of them in order of increasing strength, or what struck me as increasing strength. Recently Rychter[25] has argued that there is no puzzle of change after all. If this is the case then any purpose of any metaphysics of persistence is independent of its suitability to solve it. It is not possible to reply to such an argument here. If valid the argument in section 6 could be still read in conditional form. For the many three-dimensionalists that still hold that there is a certain puzzle in the vicinity of the one presented in section 5, the relativistic argument from change is still a threat.

Another way is to point out that the argument presupposed implicitly that the spacetime subregions that are x's exact locations are ontologically on a par. While this is fairly uncontroversial in the case of space it is far less uncontroversial in the case of time. There is a famous metaphysics of time, namely Presentism, that exactly denies such a claim. There are two things to notice. The first one is the tenability of Presentism in a relativistic setting is very controversial. But never mind that. This is not really a reply on behalf of the three-dimensionalist after all. It is widely held that presentism *per se* constitute a possible solution to the puzzle of change. If you had presentism you would have not proposed the problematic solution of section 5 in the first place. The following replies are more to the point.

Thus the problematic claims (6.13) and (6.14) do not hold. Again, two things can be said in reply. First of all, this commits the endurantist to a lot of metaphysical works on instantiation of properties. Second of all, suppose that a three-dimensionalist believe, as many do, in purely spatial parts. Then it could be argued that x has a proper part z that is exactly located at R and instantiate the incompatible properties F @ R and ~ F @ R. Thus the argument still stands. Naturally a three-dimensionalist that believe that

[25] See Rychter (2009).

material objects are spatially extended simple would be unaffected by my argument. I grant that. However I do not believe that this was what any endurantist had originally in mind when formulating her metaphysics of persistence[26].

The most promising strategy seem to me the one proposed by Gilmore[27]. It is to claim that the case presented in section 6 is nothing like an ordinary case of change, given that R_1 and R_2 overlap. And so something more than the simple fact of the possibility of change is needed to ground the problematic claims (6.13) and (6.14). As far as I can see there are two possibilities here: either change in the case of overlapping regions is possible or it is not. If it is the latter then my argument is misdirected. If it is the former then my argument is a threat. But we do seem to have cases of change at overlapping regions in a relativistic context. After all that is all length contraction is all about, having different length at different overlapping spacetime regions.

No doubt there are many other ways to resist the argument. However I will conclude pointing out two different things which cannot be fully addressed here. The first one is that the argument can be read not as a direct argument against 3D, but rather as a challenge for three-dimensionalists to come up with a different solutions for the puzzle of change that is suitable in Minkowski spacetime. When passing to a relativistic setting we did have to make all sorts of change and this one is maybe just one of them. The last one is that the argument can also be endorsed but three-dimensionalist. They will simply go on to argue that it does not cut simply against 3D but against every metaphysics of persistence, 4D included. I do not know whether this *tu quoque* argument is successful. But it seems to me to give raise to interesting questions. It raises the problem of instantiation of properties by four-dimensional objects. And this in turns calls into question the possibility of drawing an unappreciated distinction between locational persistence and mereological persistence, or so I believe. These are all serious questions, and they'll have to wait for another time.

Acknowledgments. I am very much indebted to Achille Varzi, Cody Gilmore, Yuri Balashov, Andrea Bottani, Vincenzo Fano and Mauro Dorato for helpful and detailed comments and critiques on previous drafts of the paper. Also I would like to thank the audiences of the SILFS conference in Bergamo and of the Metaphysics and Philosophy of Science conference in Toronto for fruitful discussions.

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26 But see Parsons (2006).

27 Private conversation.

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