Part III Essay

Pilot Wave Theory and Quantum Fields

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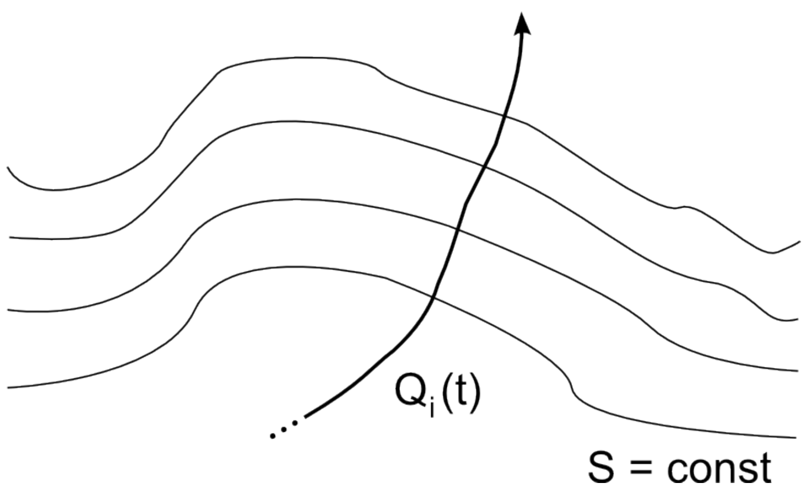

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Abstract

Assignment: One approach to solving the measurement problem in quantum theory proposes that a certain quantity Q is 'preferred' in that a quantum system always has a definite value for it. So Q needs to be chosen so that:

1. its definite values appropriately explain the definiteness of the macro-realm, and this will presumably involve equations of motion for the values that mesh suitably with the quantum state's unitary evolution;
2. its definite values do not violate various no-go theorems such as the Kochen-Specker theorem.

The best-developed example is the pilot-wave approach of de Broglie and Bohm [28], [12]. This approach can be adapted to field theories: indeed, Bohm's original paper [9] gave a pilot-wave model of the electromagnetic field. In general, the approach faces difficulties in constructing models that are relativistic in a more than phenomenological sense. But recently there has been considerable progress, and clarification of the various options: both for particle ontologies [15] and for field ontologies [36]. The purpose of the essay will be to review these developments.

Summary: Pilot wave theory (PWT), also called de Broglie-Bohm theory or Bohmian Mechanics, is a deterministic nonlocal 'hidden variables' quantum theory without fundamental uncertainty. It is in agreement with all experimental facts about nonrelativistic quantum mechanics (QM) and furthermore *explains* its mathematical structure. But in general, PWT describes a nonequilibrium state, admitting new physics beyond standard QM.

This essay is concerned with the problem how to generalise the PWT approach to quantum field theories (QFTs). First, we briefly state the formulation of nonrelativistic PWT and review its major results. We work out the parts of its structure that it shares with the QFT case. Next, we come to the main part: We show how PW QFTs can be constructed both for field and particle ontologies. In this context, we discuss some of the existing models as well as general issues: most importantly the status of Lorentz invariance in the context of quantum nonlocality. The essay concludes with a more speculative outlook in which the potential of PWT for open QFT questions as well as quantum nonequilibrium physics are considered.

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1 Introduction

Nonrelativistic pilot wave theory (PWT) is a deterministic quantum theory of point particles moving according to a nonclassical (nonlocal) continuous law of motion. As such, it recovers the standard textbook formulation of quantum mechanics (QM) as its statistical equilibrium theory and disposes of the usual quantum paradoxes. Historically, PWT was mainly developed by de Broglie [49] and Bohm [8], [9]. Amongst other successes, it led Bell to refute von Neumann's flawed theorem [5] and to his famous inequalities [4], indicating (together with their experimental verification) that every realistic quantum theory has to be nonlocal.

While the nonrelativistic case is well developed (see e.g. [12], [28] and [22]), although not well known, approaches to PW quantum field theory (QFT) have not yet reached the maturity attained by the nonrelativistic theory. This is mainly due to questions which are not explicit in standard QFT (see e.g. [33]) because of the latter's physical vagueness [1, p. 179]. Put plainly, it is not clear what standard QFT is about. It could describe fields [36], particles[2] [1], [15], [41] or even both, depending on the particular case [37]. Besides, because of the explicit nonlocality of PWT, the problem how quantum theory and Special Relativity (SR) combine is immediately brought into the focus of attention.

In this essay, we give a short exposition of nonrelativistic PWT, presenting its major results. We discuss which structures apply also to the QFT case. Next, we show how candidates for possible PW QFTs can be constructed. At this stage, we briefly comment on questions of gauge theory and quantisation. Some existent models both for field and particle ontologies are presented and discussed. Furthermore, we review different results concerning the status of Lorentz invariance in PWT. Finally we come to a general discussion, ending with a more speculative discussion about QFT questions on which PWT might shed new light, and also about PW nonequilibrium.

This section concludes with (a) a list of special vocabulary used, and (b) a collection of possible criteria for a scientific theory.

Vocabulary: See table 1

Criteria for a scientific theory: These are only suggestions. The reader is invited to modify or supplement the following list:

1. Agreement with experiments, testable in principle;
2. Simplicity (Ockham's razor[3]) and completeness of explanation;
3. Precisely defined quantities, sound internal logic and no paradoxes;
4. Locality (*if possible!*);

[2] We will try to use the word particle in its clear meaning of a point-like object, if not otherwise indicated.

[3] This criterion is often misunderstood: the best explanation in its sense is not necessarily the simplest, but the one with the fewest and most logical premises, *that can still give a complete explanation* of the phenomenon in question. Historically, it was invented to oppose superstition.

ontology	philosophical study of the nature of being
beable	physical property that exists independently of an observer (Bell's proposed replacement for the physically ambiguous notion of an *observable* [6])
hidden variables	historical expression for beables besides the QM wave function
standard QM	Copenhagen interpretation and its modern reformulations
projection postulate	postulate of standard QM that immediately after performing an experiment the wave function discontinuously collapses to an eigenfunction of the operator that represents the experimental situation
measurement problem	problem of how (of if) wave function collapse occurs
particle	point-like object

Table 1: Definition of most important special vocabulary

5. Implies previous successful theories in appropriate limits;

6. Determinism;

7. Robustness and compatibility with future theories: a small variation in the formalism (e.g. an arbitrarily small addition of a term to the governing equations) should not be incompatible with the principles of the theory (bearing in mind the previous points).

Remark concerning 1: One should be careful not to fall for extreme positivism: in the common situation in which two different theories agree with experiment, the other criteria should be given priority.

2 Nonrelativistic pilot wave theory

We review the mathematical formulation of nonrelativisitic PWT, mainly following Dürr and Teufel [22]. Since our essay topic is PW QFT, the argumentation will be brief. We therefore refer the interested reader to a graduate course in PWT by Towler [39], an entry in the Stanford Encyclopedia of Philosophy by Goldstein [24] as well as the books by Dürr and Teufel [22] (from a recent perspective), by Bohm and Hiley [12] and by Holland [28]. The latter two use a second order 'Newtonian' formulation of PWT which is considered not to be adequate by many authors, but the books are useful in many other respects. The most important historical references are the papers of de Broglie (see [49] for an English translation) and Bohm [8], [9].

2.1 Formulation

Nonrelativistic pilot wave theory is a theory of N point-like particles with masses $m_1, ..., m_N$ and (actual) positions $\mathbf{Q}_1, ..., \mathbf{Q}_N \in \mathbb{R}^3$. These are the beables of the theory. Let $\mathbf{Q} = (\mathbf{Q}_1, ..., \mathbf{Q}_N)$ be the configuration of the system in configuration space $\mathcal{Q} = \mathbb{R}^{3N}$. The physical idea of PWT (responsible for the name "pilot wave") is that the particles are guided by a wave function

$$\begin{aligned} \psi : \mathbb{R}^{3N} \times \mathbb{R} &\rightarrow \mathbb{C}, \\ (\mathbf{q}, t) &\mapsto \psi(\mathbf{q}, t). \end{aligned}$$

This means, the k-th particle is supposed to follow a trajectory defined by the so-called *guidance equation*:

$$\frac{\mathrm{d}\mathbf{Q}_k}{\mathrm{d}t} = \frac{\hbar}{m_k} \operatorname{Im} \left(\frac{\nabla_k \psi}{\psi} \right) (\mathbf{Q}, t). \tag{1}$$

Note the difference between $\mathbf{q}$ (an arbitrary point in configuration space) and $\mathbf{Q}$ (the configuration the system actually has)! The above equation implies the important feature of *explicit nonlocality*: The time dependence of $\mathbf{Q}_k$ is influenced by the simultaneous positions $\mathbf{Q}_i$ also with $i \neq k$.
Finally, the wave function is taken to satisfy the N-particle Schrödinger equation:

$$i\hbar \frac{\partial \psi}{\partial t}(\mathbf{q}, t) = H\psi(\mathbf{q}, t), \tag{2}$$

where H is the Hamiltonian of the system. Eqs. (1) and (2) govern a unique continuous and deterministic behavior[4]. Since the wave function is a field on configuration space rather than physical space, the question of its role arises. Because of eq. (1), it influences the behavior of trajectories which are regarded as physical. Must the wave function therefore be ontological, too? The author's opinion is that it might rather *encode the law of motion*.

Some remarks on the physical meaning of the defining equations and their motivation are appropriate. The meaning of the guidance equation can be seen if one decomposes $\psi = |\psi| e^{iS}$ in polar form and inserts this into eq. (1):

$$\frac{\mathrm{d}\mathbf{Q}_k}{\mathrm{d}t} = \frac{1}{m_k} \left. \nabla_k S \right|_{\mathbf{q}_i = \mathbf{Q}_i}. \tag{3}$$

This can be phrased as follows (de Broglie, see [49, p. 40]):
"At each point of its trajectory, a free moving body follows in a uniform motion the ray of its phase wave, that is (...), the normal to the surfaces of equal phase."
In short, de Broglie postulated that classical mechanics is to quantum mechanics what ray optics is to wave optics. For the case of a single particle, this is shown in fig. 1. Further illustrations, e.g. simulations of scattering processes, can be found in reference [25].
Concerning motivation, it is shown in [22, ch. 8.1] that one arrives at eqs. (1) and (2) if one assumes a simple and minimal theory of point particles, Galilean invariance of the velocity field, time-reversal invariance, as well as agreement with classical mechanics (in Hamilton-Jacobi formulation) in a suitable limit (elaborated further in [22, ch. 9.4]).

[4] The existence and uniqueness of solutions for eq. (1) are not entirely trivial because of the possibility of nodes of ψ (see [22, p. 153] and references therein).

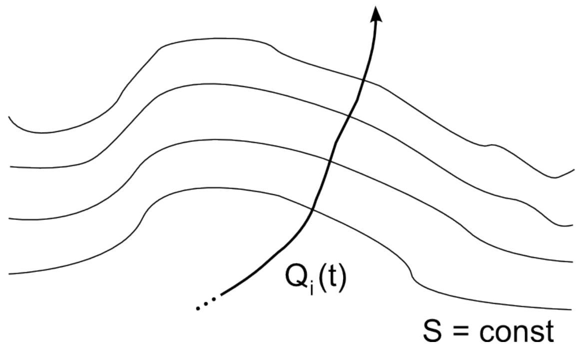

Figure 1: Illustration of the guidance equation (3) in the case of a single particle

2.2 Major results

In the previous section we made only two postulates:[5]

1. Existence of a wave function $\psi \in \mathcal{Q}$ satisfying the Schrödinger equation;

2. Existence of point particles with exact positions guided by ψ.

In particular, we did not assume the following statements (characterising standard QM):

1. The *Born rule:* $\rho = |\psi|^2$ (ρ: statistical distribution of particles);

2. The *uncertainty principle*;

3. The *projection postulate*;

4. The *correspondence principle*, i.e. a presupposition of a classical world existing on the physical scales of everyday life;

5. Necessity of describing the system in terms of *Hilbert space.*

In the following paragraphs, we show which of these statements are still appropriate in PWT and how these can be recovered as *theorems*[6]. We thereby discuss how this resolves the usual quantum paradoxes.

Origin of the Born rule: In PWT, we did not *pre*sume that $\rho = |\psi|^2$ is valid. Nevertheless, in all known experiments (if they contain large enough statistics), this simple relation seems to be the basis of the success of QM. In PWT there are two ways to recover the Born rule as a *statistical theorem:*

1. Typicality analogous to Boltzmann's theorems about classical statistical physics [22, ch. 11];

[5]The existence of positions of objects need not be seen as a genuinely new postulate but might rather be necessary to recover classical mechanics. As Dürr and Teufel put it [22, p. 150]: *"After all, classical physics is supposed to be incorporated in quantum mechanics, but when there is nothing, then there is nothing to incorporate, and hence no classical world."*

[6]For a discussion of the classical limit, we refer to [22, ch. 9.4].

2. Dynamical relaxation into equilibrium for a large class of "non-pathological" initial conditions [10], [42], [50] and [40].

These two approaches might after all not be too different since the "non-pathological" initial conditions for relaxation into equilibrium may be viewed as requiring some kind of "typicality"[7].

Remarks:

- If $\rho(t_0) = |\psi(t_0)|^2$ is true for some t_0, then $\rho(t) = |\psi(t)|^2 \; \forall t$ (*equivariance*, [22, p. 152]). In the thus defined *quantum equilibrium*, one recovers the statistical predictions of standard QM (see e.g. [12] and [28] for a variety of physical examples).
- The derivation of the Born rule essentially builds on the concepts of position and trajectories (consider point 2, sec. 1).
- In PWT, it is not excluded *a priori* that $\rho = |\psi|^2$ may be violated in very rare situations. In such a case of *quantum nonequilibrium*, predictions different from standard QM arise, yielding a whole spectrum of new physics (see sec. 4.3).

Uncertainty and the role of 'observables': Since PWT always has a precisely defined position for particles, there is no ontological uncertainty as in the Copenhagen interpretation (see point 3, sec. 1). The only "uncertainty" is a statistical relation between outcomes of certain experiments. This is easily understood: ψ has properties of a wave, and thus the statistical distribution $\rho = |\psi|^2$ in quantum equilibrium obeys the same "uncertainty" relations as do classical waves (a result of Fourier analysis).

In PWT, the Hilbert space simply arises from the set of solutions of the linear Schrödinger equation if one restricts to normalisable functions. Note that it is secondary to (but useful for) the theory. It only works if one assumes that the linearity of the Schrödinger equation is valid exactly.[8] Contrary to standard QM (and some of its interpretations), PWT does not require exact linearity as a postulate and could in principle work well without it (consider point 7, sec. 1).

Finally, what is the role of "observables" (self-adjoint operators on a Hilbert space) in PWT? Using the words of Dürr and Teufel [22, p. 228], *"(...) the operator observables of quantum mechanics are book-keeping devices for effective wave function statistics"*. Roughly speaking, one may say that such an "observable" defines a map from the initial (statistical) experimental situation to the final one. See reference [22, chs. 12, 13, 15] for mathematical details and a generalisation to positive operator-valued measures (POVMs).

An obvious but important feature resulting from this role of "observables" is *contextuality*: The outcomes of an experiment depend on the way how it is performed.

[7] A notion of "typicality" is also required for the global existence and uniqueness of trajectories. As in Newtonian mechanics, the equations alone are not enough to guarantee this.

[8] Experiments have not ruled out the possibility of small nonlinear terms that e.g. arise if one assumes the Schrödinger equation to be the lowest order of a series expansion.

Surely, this is not paradoxical if one does not insist on the view[9] that experiments just reveal pre-existing properties of the system. Rather, as Bell remarks [5, p. 166]: *"Quantum experiments are just not like that, as we learned especially from Bohr. The results have to be regarded as the joint product of 'system' and 'apparatus', the complete experimental set-up. But the misuse of the word 'measurement' makes it easy to forget this and then to expect that the 'results of measurements' should obey some simple logic in which the apparatus is not mentioned."*
This is also how one should read the well-known *Kochen-Specker theorem* (see [27] for a discussion): there exists no non-contextual model of QM with 'hidden variables'.

Solution of the measurement problem: Probably the most important problem of standard QM is the measurement problem. In section 2.1 we found that the time evolution in PWT is deterministic and always continuous. Bohm showed how this solves the measurement problem [9, 1952]:

If a particle interacts with another quantum system but with a large (thermodynamic) number of degrees of freedom, the wave function branches in configuration space with negligible overlap between the different parts. This fact is known as *decoherence.* However, decoherence alone does not solve the measurement problem[10]: In standard QM the system would still be in a macroscopic superposition (with the resulting paradox of Schrödinger's cat).

The crucial point in PWT is the following: Depending on the initial position of the N-body system, under a complicated, chaotic but continuous and deterministic time evolution, the system ends up in the support of only one of the branches (see fig. 2). It is then justified as a sensible *approximation* to replace the wave function of the system by the effective ("collapsed") wave function of the branch in which it ended up. This step is based on the fact that in PWT, the system always has an actual configuration $\mathbf{Q}$. Thus, in PWT there is only an *apparent* collapse and therefore no measurement problem (consider point 3, sec. 1). Note also that *no division of the world into "classical" or "quantum"* was required, in particular no different status for "measured objects" and "observers".

Remark: Note that the results of this section depend only on structural aspects so that similar reasoning applies to the QFT case.

2.3 Conclusions

We have seen that PWT provides a simple realistic, continuous and deterministic but explicitly nonlocal[11] account of quantum phenomena. The predictions of standard QM follow from PWT as its equilibrium statistics. PWT is independent of an ambiguous division of the world into "classical" and "quantum" parts. Concerning empirical predictions in (ubiquitous) quantum equilibrium, one should not see PWT

[9] Called "naive realism about operators" in [22, p. 244/45].

[10] Some arguments using the density matrix formalism, that claim this are flawed, because the ignorance interpretation is incorrect for improper mixtures.

[11] As Bell showed [4], the nonlocality seems inevitable if one tries to develop a realistic quantum theory.

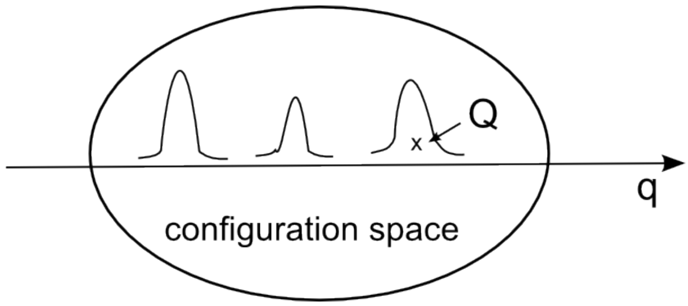

Figure 2: Illustration of the apparent collapse via decoherence

as a rival theory of standard QM but rather as an explanation for it on a microscopic basis.

Nevertheless, the theories are neither equivalent conceptually nor equivalent concerning physical predictions: PWT might give answers where standard QM is silent or impractical to apply.[12] Possible examples are quantum chaos, scattering theory, semiclassical trajectories as well as dwell and tunnelling times [24, sec. 15]. Furthermore, out of quantum equilibrium, new predictions in conflict with standard QM arise (see section 4.3). Also, PWT could be compatible with modifications such as nonlinear Schrödinger equations – should they ever become necessary. Interestingly, in the recent decades, the benefits of PWT for a numerical implementation and visualisation of quantum processes have also been realised [51], [17], [38], [34].

It is left to the reader (and his/her modified version of the criteria in section 1) to compare PWT with standard QM and different interpretations. Whatever the result should be, please note that PWT provides an explicit counter-example to various claims about the impossibility of theories with beables, or about the necessity of indeterminism and uncertainty in every quantum theory. The critical reader may also be interested in an article by Passon [32] who discusses common objections to PWT.

3 Pilot wave QFT

Sticking to his criteria for a scientific theory, the author finds it worthwhile to study PWT approaches to QFT. We are now coming into an area of "working ground" – the models discussed are not clarified fully and should be regarded as developing candidate theories. We set $\hbar = c = 1$ in the following, unless stated otherwise.

3.1 General remarks

Reconsidering nonrelativistic PWT, one may ask which features of the theory will remain general also in a QFT context. At first, aiming for a realistic and precise description, we will need beables for the theory. It turns out that (at least) two choices are possible: fields and particles. Models for these choices will be studied in sections 3.2 and 3.3, respectively.

[12] That is, in order to remain in the standard QM formalism, more and more of the system would have to be taken to be "on the quantum side".

Next, one will have a generalised version of the ψ-function, Ψ, fulfilling an appropriate wave equation. Ψ will encode the law of motion for the beables via a new guidance equation. Working in the same spirit as before, experimental predictions of standard QM should follow from analysing the quantum equilibrium statistics.

Additional criteria a PW QFT has to fulfil: The very precision of a model with beables demands new criteria to be fulfilled. Roughly speaking, the beables should reflect the empirical impression of macroscopic objects. The assumption that the beables are distributed according to the quantum equilibrium distribution is not sufficient for this to be the case [36, p. 19/20]. Additionally, *different macroscopic states have to correspond to different typical*[13] *beable configurations* [15, p. 20].
To clarify the meaning of the above requirements, consider the following aspects:

1. At the discussion of the solution of the measurement problem in PWT (sec. 2.2), we saw that outcomes of experiments get recorded[14] if the configuration of the system ends up in the support of one of the approximately disjoint branches of the wave function. We will assume that also for a general PWT the wave functions/functionals corresponding to macroscopically distinct states must not have significant overlap. This is ensured if the above statement about typical beable configurations is satisfied [15, p. 20]. Conversely, the named statement is satisfied in quantum equilibrium (and presumably also in mild nonequilibrium) if the property about the overlap of macroscopically distinct states is given.

2. The term "beable" implies a certain materialistic aspect. This is not a very strong point, as properties of objects may be characterised by subtle relations among the beables. Nevertheless, there exist situations where it is intuitively clear that the mere density of beables (e.g. particle positions) characterises macroscopic properties. An example is the difference between matter and empty space.
   Colin and Struyve developed this idea in [15, sec. 4.2], arriving at a useful (but crude) criterion to judge about the empirical adequacy of beable theories: If one can define a suitable "beable density" (e.g. the number density of particles or the mass density of massive fields), *average density fluctuations in quantum equilibrium have to be much smaller than the difference in density of materials which are distinct on an empirical basis in an experimentally accessible volume V.* This criterion will be of crucial further use and deserves its own name: (C).

For further discussion and clarification about these additional points that have to be clarified for a viable model, we refer to [36, pp. 19-20, 27-30, 34, 49-51] and [15, sec. 4.2]. Examples for the application of the additional criteria will be presented in sections 3.2.2, 3.2.3 and 3.3.3.B.

[13] For our purposes, this is taken to mean "distributed according to the quantum equilibrium distribution".
[14] This is a necessary property to be able to speak about "outcomes of experiments".

Spin: In the QFT case, we naturally have to do with objects involving spin. In PWT, the simplest possibility to model spin is to assume a multi-component wave function [5], [22, ch. 8.4]. This means that spin is not necessarily an intrinsic property of a particle. See however [28, ch. 10] for an alternative view. In the context of spin it is important to recognise that every experiment is finally only concerned with positions of objects[15], namely the pointer of an apparatus (see also figs. $3-6$ for an illustration of the Stern-Gerlach experiment which is commonly used to define spin).

Indistinguishable particles: So far, we have not discussed indistinguishable particles in PWT. One might suspect that in PWT particles might be labelled according to their trajectories and therefore are distinguishable. However, point particles with identical properties like mass and charge (regardless of their position) are *physically* indistinguishable. The label is arbitrary and carries no physical meaning. The true question is why physicists did not realise this in classical mechanics. A possible answer is that in classical mechanics one usually regards point particles as an overidealisation for objects which are actually distinguishable by properties like masses, charges or shapes[16]. Having understood this point, by topological arguments one straightforwardly arrives at wave functions which have to be either symmetric or antisymmetric [22, ch. 8.5]. Concerning terminology, we use the words "bosons" and "fermions" for objects (not necessarily particles) guided by symmetric or antisymmetric wave functions, respectively.

3.2 Field ontologies

3.2.1 How to construct a field ontology

Gauge theory is concerned with the question how to dispose of redundancy in the mathematical description of physics. Questions of gauge theory arise in the presence of constraints [18]. It is therefore a general question in field theory (and not exclusively associated with "quantisation"). For a PW field ontology it is expected to play an important role as well, since the word "beables" used in Bell's sense [6, p. 52/53] is also supposed to carry the distinction between physical and unphysical degrees of freedom. A detailed analysis how gauge theory and canonical quantisation can be used to construct PW QFTs was carried out by Struyve [36, sec. 2]. However, a general remark on quantisation seems appropriate.

Quantisation: Consider what "quantisation" means: Schematically, it is used as a map

$$Q: \text{ classical theory } \longmapsto \text{ quantum theory.}$$

But it is not clear that such a recipe should be available. Actually only Q^{-1} has to exist (see point 5, sec. 1)!
In this respect the author agrees with Tumulka [41, p. 22]:
"It is obvious that quantization as a method of obtaining quantum theories has its

[15]Ironically, these are not described by standard QM and said to be "hidden variables".
[16]This is probably why Gibb's paradox arose historically in classical statistical physics.

limitations, as one would not have guessed the existence of spin, or the Dirac equation, in this way. Even less meaning is attributed to quantization rules from the Bohmian perspective, since quantization rules focus on the operators-as-observables, but these are no longer the central objects of the theory (...)"
And he concludes: *"(...) the program of finding all covariant linear wave equations (...) is more in the Bohmian spirit than quantization."*
Indeed, Bell [3, p. 33/34] developed an idea how to construct a PW model without starting from a classical theory. While it is applicable to particle ontologies (see sec. 3.3), the author is not aware of a generalisation to field ontologies. There are possibilities how to *motivate* PW models with field ontologies starting from classical theories. Examples are mentioned in the next section.

3.2.2 Bohm's model of the electromagnetic field

The first and historically most important PW QFT model with a field ontology is Bohm's model of the electromagnetic field [9]. The following presentation is a combination of the work of Bohm and Struyve [36]:

Let $A^\mu = (A_0, \mathbf{A})$ denote the vector potential for the electromagnetic field. Choose the Coulomb gauge $\mathrm{div}\mathbf{A} = 0$. As a consequence, the remaining physical degrees of freedom (the beables) are those of the *transverse part of the vector potential* $\mathbf{A}_T$. One can use any suitable way of "deriving" a guidance equation for $\mathbf{A}_T$ (or an equivalent set of beables) as well as an equation for a wave function (or functional) guiding the beables. Examples for such a construction procedure are those: (i) of Bohm, taking the discrete Fourier components of $\mathbf{A}_T$ to define beables $\mathbf{q}_{\mathbf{k},\mu}$ and obtaining equations of motion by analogy with Hamilton-Jacobi theory, and (ii) of Struyve who uses canonical quantisation and gauge theory to formally arrive at the following equations:

1. Guidance equation:
$$\frac{\partial \mathbf{A}_T(\mathbf{x},t)}{\partial t} = \left.\frac{\delta S[\mathbf{A}_T,t]}{\delta \mathbf{A}_T(\mathbf{x})}\right|_{\mathbf{A}_T(\mathbf{x})=\mathbf{A}_T(\mathbf{x},t)}, \tag{4}$$
where S is the phase of the *wave functional* $\Psi[\mathbf{A}_T,t] = |\Psi|e^{iS}$.

2. Functional Schrödinger equation:
$$i\frac{\partial \Psi[\mathbf{A}_T,t]}{\partial t} = \frac{1}{2}\int \mathrm{d}^3x \left(-\frac{\delta}{\delta \mathbf{A}_T}\cdot\frac{\delta}{\delta \mathbf{A}_T} - \mathbf{A}_T\cdot\nabla^2\mathbf{A}_T\right)\Psi[\mathbf{A}_T,t]. \tag{5}$$

The resulting model is gauge independent [36, p. 31]. The *quantum equilibrium distribution* (cf. sec. 2.2) is given by $|\Psi[\mathbf{A}_T,t]|^2\,\mathcal{D}\mathbf{A}_T$. Here, $\mathcal{D}\mathbf{A}_T$ is a "continuum measure" which is, as in standard QFT, ill-defined due to the non-existence of an infinite-dimensional generalisation of the Lebesgue measure. This calls for a suitable regularisation, e.g. a momentum cutoff[17]. Alternatively, one has to consider eqs.

[17]Note that such a cutoff violates Lorentz invariance already at the level of the wave equation. But this violation is mostly for mathematical convenience and the cutoff is thought to be taken to infinity at the end of the calculation.

(4) and (5) as purely formal expressions which are shorthand for a very tight lattice (or Bohm's discrete Fourier components). In quantum equilibrium, the claim is that this PW model yields the same statistical predictions as standard QFT.

The electric field $\mathbf{E}$ can be obtained from $\mathbf{A}_T$ and Ψ via $\mathbf{E} = -\frac{\delta S}{\delta \mathbf{A}_T}$ and the magnetic field $\mathbf{B}$ via $B_i = \varepsilon_{ijk}\partial_j A_{T,k}$. As in the nonrelativistic part, the crucial difference from standard QM is that at all times and independent of observers, there is an actual field configuration $\mathbf{A}_T(\mathbf{x})$. One could go on to discuss experiments like the photoelectric effect or the Compton effect [9], and it is also not excluded to run a corresponding computer simulation.

What about the additional criteria for beable theories discussed in section 3.1? Struyve [36, p. 34] finds:
"Wave functionals representing macroscopically distinct states of the electromagnetic field describe distinct classical electric and magnetic fields. This implies in particular that the wave functionals give approximately disjoint magnetic field distributions. Since different magnetic fields B_i correspond to different fields A_i^T, we have that these wave functionals also give approximately disjoint distributions for the fields A_i^T."

An interesting observation about the conventional definition of "particles" in QFT (not referring to localised objects) is the following (see also [36, sec. 4.2.3]): Consider a state $|N\rangle$ describing a definite number of these "QFT-particles". The electric (or magnetic) field operator $\hat{\mathbf{E}}$ ($\hat{\mathbf{B}}$) is a linear combination of terms involving creation and annihilation operators. Thus, the result of applying such an operator to $|N\rangle$ is orthogonal to $|N\rangle$. We conclude that only states with an indefinite QFT-particle number yield a non-vanishing expectation value of $\hat{\mathbf{E}}$ ($\hat{\mathbf{B}}$). Since one finds electromagnetic fields with non-vanishing expectation values in experiments, it follows that the corresponding states have an indefinite QFT-particle number. What does this mean?

For PW QFT (and possibly every realistic QFT), the QFT-particle number seems to be meaningless for ontology. If the QFT-particle number is just a certain basis-dependent characterisation of wave functions/functionals resulting from the use of perturbation theory (see the discussion in [30, ch. 9]), it poses no problem to superpose states with a different "particle" number. If, on the contrary, the "particles" were to refer to point-like spots seen on detector screens, such a superposition would seem questionable, to say the least.

In his article Struyve also discusses how Bohm's model could be generalised to other bosonic fields, as well as to a PW Standard Model [36, sec. 8]. While the first point faces no particular difficulties, the main problems for the latter seem to be the following:

1. Description of fermions by a field ontology (see next section and discussion)

2. Dealing with nonlinear constraints in non-Abelian gauge theories (such as Yang-Mills theories). For technical reasons it is then difficult to identify gauge invariant variables (candidate beables).

3.2.3 Failure of fermionic field ontologies

So far, there have been two suggestions for fermionic field ontologies:

1. Holland's model with Euler angles in momentum space as beables [28, ch. 10.6.2]
2. Valentini's model with Grassmann-valued fields as beables [44], [36, sec. 9.2]

However, Struyve has shown that both models suffer from different problems [36]:

1. In order to discuss criterion (C) from sec. 3.1, Struyve considered a similar model with Euler angles in position space as beables.[18] Suggesting a suitable "beable density", Struyve found that even for optimistically chosen numbers, a cubic region of space filled with matter should have a length of about 10^{-5} m, in order to be distinguishable from empty space. This clearly stands in conflict with our empirical impression of the world.
2. For Valentini's model there exists an even more serious problem: The wave functional Ψ operating on Grassmann fields is Grassmann-valued, too. Thus, one has no basis for adopting a guidance equation. Furthermore, it is unclear which measure should be considered for the quantum equilibrium distribution. In conclusion, the model seems to be only a formal construction.[19]

3.2.4 Minimalist picture

Inspired by the problems of fermionic field ontologies described above, Struyve and Westman suggested a radically minimalistic model (here summarised following [36, sec. 10] and [37, sec. 3.4]): Considering the example of quantum electrodynamics (QED) they only introduced beables for the bosonic degrees of freedom while the fermionic are taken to be only an *appearance* via the bosonic degrees of freedom. This means, the field configuration can in some cases behave as if matter was present (see also fig. 4). For the bosonic parts, the model strongly resembles Bohm's model of the free electromagnetic field (see sec. 3.2.2). More concretely, the beables are those of a transverse vector potential $\mathbf{A}_T(\mathbf{x})$, and a hint of the fermionic degrees of freedom is reflected in an additional label f of the wave functional $\Psi_f[\mathbf{A}_T, t]$. Ψ_f is taken to satisfy an appropriate functional wave equation involving the Hamiltonian of scalar QED. The guidance equation is similar to eq. (4):

$$\frac{\partial \mathbf{A}_T(\mathbf{x},t)}{\partial t} = \mathrm{Im} \left. \frac{\sum_f \Psi_f^*[\mathbf{A}_T,t] \frac{\delta}{\delta \mathbf{A}_T(\mathbf{x})} \Psi_f[\mathbf{A}_T,t]}{\sum_f |\Psi_f[\mathbf{A}_T,t]|^2} \right|_{\mathbf{A}_T(\mathbf{x})=\mathbf{A}_T(\mathbf{x},t)} . \tag{6}$$

The quantum equilibrium density is given by $\sum_f |\Psi_f[\mathbf{A}_T,t]|^2$ as usual.

Struyve and Westman argued that this model is capable of passing the additional criteria for beable theories in sec. 3.1. Furthermore, they pointed out the possibility to introduce additional beables via local expectation values of operators such as the

[18] For Holland's original model, the ontology in physical space is not specified.

[19] From this perspective it is interesting that in standard QFT (i.e. without a clear ontology) one can use Grassmann fields to calculate fermion statistics [33, sec. 9.5].

charge or mass density operators. The thus defined beables would behave similarly to the mass density in objective collapse theories.

Considering the proposal of Westman and Struyve, one is immediately led to the question whether such an indirect model can be appropriate. Having in mind the discussion of the simplicity criterion in sec. 1, one may ask if it gives a complete explanation of fermionic degrees of freedom at all. The author therefore agrees with Struyve when he admits [37, p. 7]: *"Nevertheless such a model seems rather far removed from our everyday experience of the world and probably takes minimalism too far."*
Besides, the model only benefits from simplicity as long as no further beables are introduced. The way this is done via operator observables was already questioned in the discussion of quantisation (see sec. 3.2.1). The resulting *objective* collapse furthermore seems to violate the spirit of nonrelativistic PWT where there exists only an *apparent* collapse in the measurement problem (see sec. 2.2).
The question how fermionic degrees of freedom can be described adequately in PWT therefore remains and will be one of the central subjects of the next section.

3.3 Particle ontologies

The choice of fields as beables is far from unique. Historically, the first PW QFT model with a particle ontology was developed by Bell [1] who was motivated to develop a precise alternative to the standard QFT description of fermions (which he found too vague for a professional physicist).

In contrast to the present situation for field ontologies, particle ontologies seem to be capable of describing both fermions and bosons. Contrary to what Bohm, Hiley and Kaloyerou thought [11, sec. 5.2], it seems for example to be possible to introduce trajectories for photons [41, p. 23], [23].

In the following, we present a method for constructing guidance equations for particle ontologies. Subsequently, we briefly review Bell's prototype model and sketch two possible continuum generalisations: a model of particles and antiparticles with stochastic jumps by Dürr *et al.* [20], [21] and a deterministic Dirac sea model by Colin and Struyve [15].

3.3.1 How to construct a particle ontology

In section 3.2.1, we argued that quantisation should be avoided to motivate a PW model and briefly mentioned an alternative method by Bell [3, p. 33/34]. It can be applied to particle ontologies with positions $\mathbf{Q}_k,\ k = 1, ..., N$ as follows:

1. Find all covariant linear wave equations for a wave function $\Psi(\mathbf{q}_1, ..., \mathbf{q}_N)$.
2. Analyse the equations if they have conserved currents $\mathbf{j}_k$ and density ρ and pick a suitable one to define the theory.
3. Define guidance equations by $\frac{\mathrm{d}\mathbf{Q}_k}{\mathrm{d}t} = \frac{\mathbf{j}_k}{\rho}$ (cf. eq. (1)).

Examples for this method can be found in the following sections.

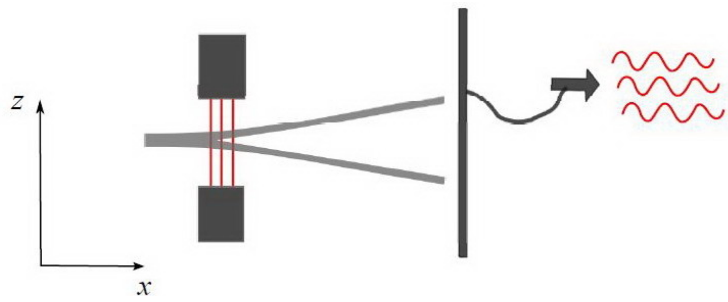

Figure 3: Schematical setup for a Stern-Gerlach experiment. The straight lines stand for the magnetic field, the wavy lines for the electromagnetic radiation around the pointer and black areas/dots represent matter (picture credit: [37]).

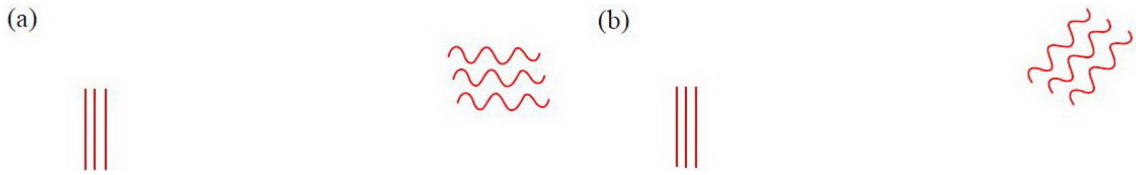

Figure 4: Stern-Gerlach experiment in the minimalist model (a) before and (b) after the experiment (picture credit: [37]). Radiation may in certain situations behave as if it had been scattered off or emitted by the pointer.

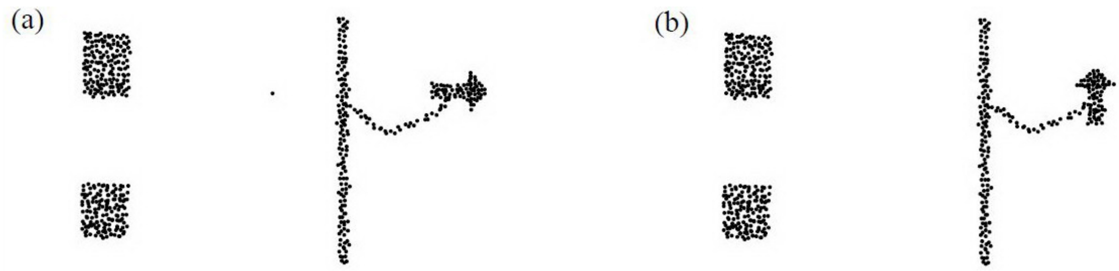

Figure 5: Stern-Gerlach experiment in the models of Bell and Dürr *et al.* (picture credit: [37]). The outcome of the spin experiment is recorded in the configuration of the pointer.

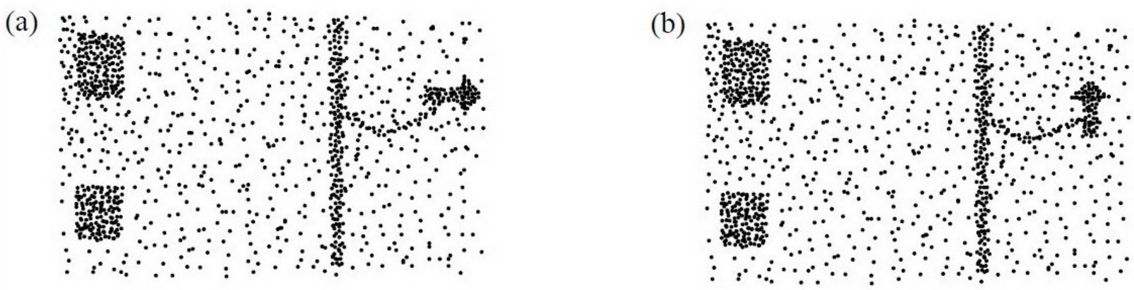

Figure 6: Stern-Gerlach experiment in the Dirac sea model of Colin and Struyve, with a momentum cutoff rendering the number of particles in every volume finite (picture credit: [37])

Remark: Although this procedure does not use a classical theory to arrive at a quantum theory, it can also be seen only as a motivation for the final equations, not as a proper "derivation". Ultimately, the criteria in sec. 1 have to be consulted.

3.3.2 Bell's lattice model

In [1], Bell considered a spatial lattice with points labelled by $l = 1, ..., L$. At each point, he defined fermion number operators $\psi^\dagger(l)\psi(l)$ with eigenvalues $F(l) = 1, 2, ..., 4N$ where N is the number of Dirac fields. The complete specification of the system is taken to be $(|t\rangle, n(t))$ where $|t\rangle$ is a (nonlocal) state[20] satisfying the Schrödinger equation and $n(t) = (F(1), ..., F(L))$ is the list of fermion numbers at the lattice points (which is a local beable). Figure 5 illustrates the impression resulting from the model.

Bell prescribed a *stochastic* development for the fermion number configuration: In a time interval $\mathrm{d}t$ he assumed that there are m jumps to configuration n with probability $\mathrm{d}t\, T_{nm}$, where:

$$T_{nm} = J_{nm}/D_m \cdot \Theta(J_{nm}), \tag{7}$$

$$J_{nm} = \sum_{pq} 2\,\mathrm{Re}\, \langle t|nq\rangle\langle nq| - iH|mp\rangle\langle mp|t\rangle \qquad \text{(current)}, \tag{8}$$

$$D_m = \sum_{q} |\langle mq|t\rangle|^2 \qquad \text{(density)}, \tag{9}$$

where $\Theta(x)$ is the step function. From the equations above, the time evolution of a probability distribution P_n over configurations n is found to be:

$$\frac{\mathrm{d}}{\mathrm{d}t} P_n = \sum_m (T_{nm} P_m - T_{mn} P_n) \tag{10}$$

As a consequence of Schrödinger's equation for $|t\rangle$, D_n satisfies the same equation as P_n, and it follows that if at an initial time $P_n(0) = D_n(0)$, then $P_n(t) = D_n(t)\, \forall t$. That is, we obtain the equivariance property. Like in nonrelativistic PWT, the model is thus capable of obtaining the standard predictions as its equilibrium statistics. Bell, however, sees the stochastic element as unwanted: the reversibility of the Schrödinger equation speaks against a stochastic law. He suspects that this stochastic element might disappear in the continuum limit. Note also that the model heavily relies on a special status of time compared to space (see sec. 3.4).

3.3.3 Continuum generalisations

3.3.3.A Model of Dürr et al. The model of Dürr *et al.* (here summarised following [20] and [41, sec. 3.1.2]) ascribes positions to both particles and antiparticles, the beables of the model (see also fig. 5). It can be applied to either bosons or fermions. The motion along trajectories is deterministic but interrupted by stochastic jumps corresponding to creation and annihilation events.

[20] Contrary to ψ in nonrelativistic PWT, $|t\rangle$ *definitely* has beable status in Bell's model: *"Otherwise its appearance in the transition probablities would be quite unintelligible"* [1, p. 177].

For a more precise formulation, consider particles and antiparticles (as point-like objects). Each species is described by a position beable Q_t in configuration space $\mathcal{Q} = \bigcup_{n=0}^{\infty} \mathcal{Q}^{[n]}$ where $\mathcal{Q}^{[n]} = \mathbb{R}^{3n}/S_n$ and S_n is the group of permutations of n objects. Q_t is guided by a wave function Ψ_t in Fock space. Ψ_t may be a superposition of states with a different "QFT-particle" number (used as in sec. 3.2.2). This is fine, since the QFT-particle number is seen only as a way to characterise states in Fock space. The actual particle number $N(t)$ is given by the number of entries in Q_t[21]. The equations governing the deterministic part of the evolution of the system are:

1. The wave equation:
$$i\hbar \frac{\mathrm{d}\Psi_t}{\mathrm{d}t} = H\Psi_t, \tag{11}$$
where $H = H_0 + H_I$ is the Hamiltonian of the system, composed of a free and an interaction part.

2. The guidance equation:
$$\frac{\mathrm{d}Q_t}{\mathrm{d}t} = \mathrm{Re}\, \frac{\Psi_t^*(Q_t)(\dot{\hat{q}}\Psi_t)(Q_t)}{\Psi_t^*(Q_t)\Psi_t(Q_t)}, \tag{12}$$
where $\hat{q}$ is the "$\mathcal{Q}$-valued position operator"[22] in the Heisenberg picture. Note that for the Schrödinger Hamiltonian, eq. (12) reduces to the familiar guidance equation (1) of nonrelativistic PWT.

Finally, the stochastic (jump) part of the evolution is given by a rate of $\sigma^{\Psi}(\mathrm{d}q|Q_t)\,\mathrm{d}t$ jumps to the infinitesimal element $\mathrm{d}q$ in $\mathcal{Q}$ in time $\mathrm{d}t$, defined by:

$$\sigma^{\Psi}(\mathrm{d}q|q') = \frac{2}{\hbar}\, \frac{[\mathrm{Im}\,\Psi^*(q)\langle q|H_I|q'\rangle\Psi(q')]^+}{\Psi^*(q')\Psi(q')}\,\mathrm{d}q, \tag{13}$$

where $x^+ = \max\{x, 0\}$. Equations (12) and (13) together define a *Markov process* (which is analysed further in reference [21]).

Some comments seem appropriate. By construction, the model has the equivariance property and is thus capable of reproducing the standard QFT predictions as equilibrium statistics, while maintaining most benefits of nonrelativistic PWT. Furthermore (setting aside the question of Lorentz invariance until sec. 3.4), one may criticise the stochastic element (which, we recall, Bell hoped would disappear in the continuum limit). In effect, the creation/annihilation events cannot be explained further. Also, the existence of antiparticles is an input to the model. Concluding, the model is an interesting (and quite universal) suggestion how a particle ontology of PWT can be extended to the QFT case; but the stochastic element makes it look rather provisional – at least according to the spirit of the PWT approach. Of course, standard QFT is an entirely statistical theory – not just concerning particle creation/annihilation.

[21] Note that $Q_t \in \mathcal{Q}^{[N(t)]}$ for fixed t.

[22] See reference [20] for a detailed explanation of the term.

3.3.3.B Dirac sea model of Colin and Struyve In Colin's and Struyve's model [15], the Dirac sea picture for QFT is taken seriously: Position beables are associated with fermions in both negative energy states (those in the Dirac sea) as well as positive energy states of the many-body Dirac equation (see fig. 6). Colin and Struyve define the *fermion number* F_d as the number of positive energy particles plus the number of negative energy particles. The idea is that in the regime in which the fermion number is superselected,[23] one can restrict the fermionic Fock space to a N-particle Hilbert space where N is fixed for all times. As usual in the Dirac sea picture, antiparticles are interpreted as holes moving in the filled Dirac sea. One thus arrives at a deterministic PW QFT which has the possibility of explaining particle creation/annihilation.

For a more precise presentation, consider the Hamiltonian

$$H = H^B + H_0^F + H_I, \tag{14}$$

where H^B is the bosonic part of the Hamiltonian, H_0^F the free fermionic part and H_I includes the interactions. In the following we focus on the fermionic part; (this will be justified shortly). The construction of the PW model goes as follows [15, p. 6]: we start from a *wave equation*:

$$i\hbar \frac{\mathrm{d}|\psi(t)\rangle}{\mathrm{d}t} = H|\psi(t)\rangle. \tag{15}$$

Suppose, we have a POVM (the generalisation of an operator observable) $P(\mathrm{d}^3x_1...\mathrm{d}^3x_N)$ for position.[24] This determines a probability density $\rho^\psi(\mathbf{x}_1...\mathbf{x}_N)$ via:

$$\rho^\psi(\mathbf{x}_1...\mathbf{x}_N)\, \mathrm{d}^3x_1...\mathrm{d}^3x_N = \langle\psi(t)|P(\mathrm{d}^3x_1...\mathrm{d}^3x_N)|\psi(t)\rangle. \tag{16}$$

From the wave equation (15), we obtain the following time evolution of ρ^ψ:

$$\frac{\partial \rho^\psi}{\partial t}\mathrm{d}^3x_1...\mathrm{d}^3x_N + \langle\psi(t)|i\hbar[P(\mathrm{d}^3x_1...\mathrm{d}^3x_N), H]|\psi(t)\rangle = 0. \tag{17}$$

If one can find vector fields $\mathbf{v}_k^\psi$ so that the second term in eq. (17) becomes

$$\langle\psi(t)|i\hbar[P(\mathrm{d}^3x_1...\mathrm{d}^3x_N), H]|\psi(t)\rangle = \sum_{k=1}^{N} \nabla_k \cdot (\mathbf{v}_k^\psi\, \rho^\psi)\mathrm{d}^3x_1...\mathrm{d}^3x_N, \tag{18}$$

then one can use $\mathbf{v}_k^\psi$ to define *guidance equations*:

$$\frac{\mathrm{d}\mathbf{x}_k}{\mathrm{d}t} = \mathbf{v}_k^\psi, \tag{19}$$

where $\mathbf{x}_k$ are the positions of the fermions both in and above the Dirac sea. The *equilibrium density* is given by ρ^ψ, and an equivariance property ensures that one

[23]Note that the Standard Model predicts a violation of fermion number superselection as a non-perturbative effect e.g. in weak processes. Tests of this have so far not been accessible to experiment [15, sec. 1].

[24]A familiar example is $P(\mathrm{d}^3x_1...\mathrm{d}^3x_N) = |\mathbf{x}_1...\mathbf{x}_N\rangle\langle\mathbf{x}_1...\mathbf{x}_N|\, \mathrm{d}^3x_1...\mathrm{d}^3x_N$ from nonrelativistic quantum theory.

recovers the predictions of standard QFT as equilibrium statistics. Note also that from eq. (18) only the parts of H that do not commute with $P(\mathrm{d}^3x_1...\mathrm{d}^3x_N)$ contribute to the guidance equation. In [15], it is shown that only H_0^F contributes which justifies our considering the fermionic part of the theory separately.

While this defines a model capable of reproducing standard QFT predictions, the precision of a model with position beables demands other criteria to be fulfilled. Most importantly, since the Dirac sea involves a potentially infinite number of particles in each region of space, one may ask if one can distinguish macroscopic matter distributions from the Dirac sea "vacuum" (criterion (C), sec. 3.1). The threat is that density fluctuations might make this impossible. Colin and Struyve showed that this question can be answered if one chooses a regularisation scheme which at the same time makes the number of particles in every volume finite.

The need for regularisation arises because of (a) the usual field theoretical divergencies and (b) the non-existence of a generalisation of the Lebesgue measure to an infinite-dimensional space which would be needed for well-defined probabilities [15, sec. 3.6]. Both problems can be circumvented if one assumes a finite volume V with suitable boundary conditions and introduces a momentum cutoff Λ, keeping space continuous. The disadvantages of this regularisation are that the commutation relations are altered and that an additional term appears in the guidance equation [15, sec. 4], which Colin and Struyve write as:

$$\frac{\mathrm{d}\mathbf{x}_k}{\mathrm{d}t} = \mathbf{v}_k^{\psi} + \tilde{\mathbf{v}}_k^{\psi}. \tag{20}$$

The hope seems to be that a sufficiently large natural momentum cutoff Λ exists so that the additional term $\tilde{\mathbf{v}}_k^{\psi}$ can be neglected. Note that the cutoff breaks Lorentz invariance already on the level of the wave equation.

We now come to the discussion of criterion (C). The crucial point is that because of the above regularisation, there only exists a finite number of modes in V which is cutoff-dependent. Since the fermion number is fixed, the fermion number density operator can be written as:

$$F_d(\mathbf{x}) = \int \sum_{i=1}^{N} \delta(\mathbf{x} - \mathbf{x}_i) P(\mathrm{d}^3x_1...\mathrm{d}^3x_N). \tag{21}$$

We can now introduce an operator corresponding to the number of fermions in a region B as $F_d(B) = \int_B \mathrm{d}^3x F_d(\mathbf{x})$. With these operators one can evaluate criterion (C) in detail, calculating the average density and comparing it with average fluctuations. This was done in [15, sec. 4.2], with the following results:
Neglecting interactions, the volume V of a region should satisfy $V \gg (\Lambda/\rho^2)^{3/5}$ where ρ is the particle density of ordinary matter [37, p. 9]. For a cutoff at the Planck scale ($\Lambda \sim 10^{35}\,\mathrm{m}^{-1}$) and a particle density of $\rho = 10^{30}\,\mathrm{m}^{-3}$, a spherical region of space should have radius $b \gg 10^{-6}\,\mathrm{m}$. While the lower bound of $10^{-6}\,\mathrm{m}$ is relatively large, the result should improve when interactions are taken into account [15, sec. 4.2] and can be made smaller by considering a smaller cutoff – as long as this is not excluded by experiment. One can thus say that (C) is fulfilled.

A potential problem for the model:[25] We have seen that a finite cutoff Λ is essential to distinguish matter from the Dirac sea vacuum. In essence, this builds on the property that with the cutoff a finite volume V contains only a finite number particles, so that density fluctuations do not render macroscopic objects indistinguishable from the vacuum [15, p. 20]. But how does the Dirac sea vacuum, thus defined, combine with the invariance of physical predictions under Lorentz boosts? Equivalently, why does a particle detector find a constant rate of particles when moving through the vacuum with different but constant velocities v, v'?

One might think that as the standard quantum mechanical predictions are reproduced due to the equivariance property, this question does not arise. But the standard quantum mechanical predictions have only been shown to be reproduced in [15] for $\Lambda = \infty$. This demonstration seems to be invalid since, as we noted, one cannot actually take $\Lambda \to \infty$. Thus, the model seems to stand in conflict with the predictions of standard QFT.

The author hopes not to have missed an obvious counter-argument when he claims that it is also physically implausible that the Dirac sea vacuum with finitely many particles in every volume of space should be invariant under Lorentz boosts. It could be seen as a gas of particles, and a particle detector moving in such a gas will detect a different rate of particles bouncing against it, depending on its velocity. Note that the statistical predictions for $\Lambda = \infty$ are indeed invariant under Lorentz boosts, as the mentioned rate would always be infinite. Whether the statistical violation of Lorentz invariance is observable in known experiments, is a different question – but one that calls for a separate analysis: criterion (C) yields upper bounds on the cutoff that are specific to the model of Colin and Struyve.

Concluding, *it seems that the model cannot reproduce the standard predictions about the Lorentz invariance of the "vacuum" – not even on a statistical level.* Note that this feature arises for any regularisation which makes the number of particles in every volume finite and can thereby fulfil criterion (C). This constitutes a crucial difference to the case of standard QFT where the regularisation mainly serves mathematical purposes.

3.4 The question of Lorentz invariance

J. S. Bell[26]: "[The usual quantum] *paradoxes are simply disposed of by the 1952 theory of Bohm, leaving as the question, the question of Lorentz invariance. So one of my missions in life is to get people to see that if they want to talk about the problems of quantum mechanics – the real problems of quantum mechanics – they must be talking about Lorentz invariance."*

In this section, we present the apparently essential conflict of Special Relativity (SR) with quantum nonlocality – which is manifest in PWT. This motivates developing an alternative to Einstein's view on SR, that involves absolute space and time. The issue with Lorentz invariance (as opposed to causality) is shown to be subtler, and there is no agreement on it in the PWT community. The section ends with a presentation of the two main standpoints.

[25] The author thanks W. Struyve for a discussion about this question.

[26] 1990, interview with philosopher Renee Weber.

The basic problem with Lorentz invariance is how it combines with the apparent nonlocality of quantum mechanics. PWT makes this nonlocality explicit: In the guidance equations – also for processes in QFT – happening with a speed comparable to that of light (e.g. eqs. (4), (12) and (19)) the movement of one coordinate (of a field or particle) *instantaneously* depends on the position of other coordinates throughout space, evaluated *at a fixed time*. The latter notion requires a notion of *absolute simultaneity* – which stands in clear contrast to Einstein's interpretation of SR where simultaneity is relative to a frame.[27] It seems sensible to re-examine SR, aiming for an interpretation that is more suitable for PWT.

3.4.1 An alternative view on Special Relativity

Consider the old problem of the division between *kinematics* (structure of space and time) and *dynamics* (laws of motion in space and time). It should be clear that one can usually compensate changes in the supposed structure of space and time by changes in dynamical laws (and vice versa). Thus, the question of the "true" structure of spacetime seems to be meaningless. The choice of spacetime is rather a question of *convenience*, the main considerations being criteria like internal logic, simplicity and consistency with other theories.

The following is inspired by Bell [2]: Consider a system consisting of matter which is held together by interactions travelling at a certain speed c in *one* given, arbitrary inertial frame F. Let that object gently accelerate until it moves with speed v close to c. As seen in F, the object will look contracted because the interactions need time (defined by a clock in F) to reach the bound components, thus altering the effective strength of interaction. In the moving system S, if one defines distances by comoving rods and time by the events of interactions reaching so-defined distance marks (i.e. by Einstein's operational definitions), then everything will seem normal and uncontracted/dilated – although this is a physical process associated with forces (see Bell's "spaceship paradox" in [2]). The speed of the interactions in S with these definitions will also *seem* to be c (although in F the speed would be different). Note that this was not postulated from the beginning. Note also that it would not be paradoxical if certain objects or interactions could travel at speed $c' > c$. The notion of causality depends on the definition of time.

Taking now $c =$ the speed of light, we are led to the question if it is *sensible* to accept Minkowski spacetime for our kinematics (which is equivalent to accepting Einstein's operational definitions of distance and time). This clearly leads to paradoxical situations if certain processes are allowed to happen at speed $c' > c$. Indeed, nonlocal correlations in QM seem to involve the speed $c' = \infty$[28]. But there is no need to view them as paradoxical – the price being to reject Einstein's *principle of contiguity* [29, p. 157] ("causal effects are propagated via a chain of local events").

In order to fully develop the alternative view, one can, with Valentini [48], suggest a different operational definition of time. Valentini shows that for PWT (and a gene-

[27] For precision of terminology we distinguish between (a) a *frame* (associated with a state of motion) and (b) a *foliation of spacetime* into hypersurfaces (defining "space at a given time"). Yet another distinct notion is (c) a *coordinate system* (referring to a mathematical choice of language for the description of spacetime in a frame). Note that up to choices of zeroes and of units for measurements of space and time, a coordinate system is naturally induced by an *inertial* frame.

[28] See [13] for a discussion of the hypothetical speed involved in quantum correlations.

ral class of beable theories) nonlocal correlations in quantum *non*equilibrium can be used for *nonlocal instantaneous signalling*[29]. Even if one believes that quantum equilibrium is ubiquitous, nevertheless, since the justification of the Born rule goes by statistical arguments (see sec. 2.2), one should in PWT expect small, usually undetectable fluctuations around the equilibrium state which correspond to a weak noise of instantaneous disturbances.

If one accepts these arguments as a motivation to consider instantaneous signals, one can operationally define an *absolute time* by taking a clock in one inertial system (like F above) and communicating the time it shows via instantaneous signals. This notion of time is different from the time defined by moving clocks in other frames. One might be tempted to think that the inertial frame in which the absolute time coincides with the time measured by clocks constitutes a preferred frame. But as one can both change the state of motion of the clock and the coordinates in the thus defined frame, the statement has to be weakened to "there exists a preferred foliation of spacetime" (see footnote 27 for terminology).

Note that for the argument given in this subsection, it is not of primary importance that nonlocal signalling is practically feasible. It is rather the mere possibility of something happening at an infinite speed that is enough to make the change of view about SR *plausible.*[30] In any case, one arrives at an alternative interpretation of SR that seems to be required for the guidance equations of PWT.

3.4.2 Two views on Lorentz invariance in pilot wave theory

Definition: A theory is Lorentz-invariant if the Lorentz-transformed solutions of the governing equations are themselves solutions of these equations. The equations are then said to be ‘Lorentz-covariant’.

Some facts about the models presented: Indeed, some of the models above are not fundamentally Lorentz invariant. For example, the wave equation (5) in Bohm’s model of the electromagnetic field is *not manifestly* Lorentz-covariant (but it is Lorentz-covariant). More importantly, the guidance equation (4) which encodes the microscopic processes is not Lorentz-covariant.

View 1: A detailed microscopic description of QM must violate fundamental Lorentz invariance: Considering the possibility of quantum nonequilibrium, Valentini argues [43, p. 6]:
"The equilibrium distribution $P = |\Psi|^2$ has special features which are not fundamental to the underlying theory (...). Two such features are signal-locality and uncertainty (...). One also generally expects that a maximum-entropy (statistical) equilibrium state will show an especially high degree of symmetry. And indeed, the symmetry of Lorentz covariance holds only for $P = |\Psi|^2$ (in the context of the pilot wave theory of fields [11].)"

In the paper Valentini refers to, Bohm, Hiley and Kaloyerou conclude their review of Bohm’s model of the electromagnetic field with the following words [11, p.

[29] In quantum equilibrium, one recovers the usual no-signalling theorems as a statistical consequence [43].

[30] What is counted as kinematics or dynamics may anyway only be a matter of convenience.

356]:
"It follows that the non-covariant ground state in our interpretation will not be accessible to observation as long as the quantum theory in its current form is valid. And so no violations of relativity will be obtained. However, it is possible, as pointed out in the earlier paper [9], that quantum theory will fail to hold in some, as yet, unexplored domain. For example, if we extend our theory to include stochastic processes in the manner suggested earlier, there will be some relaxation time, τ, for the probability function to approach the usual one, $|\psi|^2$. Measurement in times shorter than τ might show this discrepancy and these results would not be covariant. If this should happen then relativity would evidently hold only as a statistical approximation valid for distributions close to equilibrium in the stochastic process underlying quantum mechanics."
The mentioned stochastic processes refer to a special way used by Bohm *et al.* to enforce relaxation into equilibrium in the second order formulation of PWT used in Bohm's original papers [8], [9].

However, Valentini seems to think that the violation of Lorentz invariance mentioned by Bohm *et al.* is a general feature of *every* detailed microscopic process. A similar opinion can be found in Bohm and Hiley [12, ch. 12] and Holland [28, ch. 12] where several examples showing this conflict are analysed.

Furthermore, in a different paper [45], Valentini argues that even the Galilean invariance of nonrelativistic PWT (in first order formulation) should be regarded as a fictitious symmetry, like the invariance of Newtonian mechanics under certain changes involving a constant acceleration. He sketches an Aristotelian universe with the natural state being that of rest, and with spacetime $E \times E^3$ where E is the affine real line, so E^3 is Euclidian 3-space. Valentini concludes that, in an analogous way, the Lorentz symmetry of an relativistic extension of PWT would also be merely fictitious and only a statistical consequence of the equilibrium distribution.

The latter view might be seen to find support in general results such as claimed by Hardy in [26]: Hardy argues that "realistic hidden variables theories" seem to be incompatible with fundamental Lorentz invariance, in the sense that *"if we assume realism and we assume that the 'elements of reality' corresponding to Lorentz-invariant observables are themselves Lorentz invariant, we can derive a contradiction with quantum mechanics."*

Note that from the viewpoint described in this paragraph, Lorentz invariance is an *emergent symmetry*, valid exactly in quantum equilibrium. As we argued in section 3.3.3, the Dirac sea model of Colin and Struyve seems to necessarily break Lorentz invariance also on a statistical level – carrying even more radical implications.

View 2: A fundamentally Lorentz invariant account of detailed microscopic phenomena is not excluded: To make sense of this viewpoint, one has to answer results like Hardy's first. Berndl *et al.* analysed this issue in [7, p. 4]:
"There have been a number of arguments to the effect that a Bohmian theory must involve a preferred frame of reference, and thus must violate Lorentz invariance. The most interesting such argument has been put forward by Hardy [26], who by discussing an intriguing experiment (...) claims to have shown that every realistic quantum theory must possess a preferred frame of reference, and thus that there can

be no Lorentz invariant realistic quantum theory.
However, because it rests on an unsuitable 'reality criterion' (...), Hardy's argument is wrong. There are even counterexamples to Hardy's argument (...)"

It is crucial to logically separate (a) nonlocal instantaneous effects and signals from (b) violations of Lorentz invariance. Goldstein phrases the situation as follows [24, sec. 14]:
"However – unlike nonlocality – violation of Lorentz invariance is not inevitable. It should be possible, it seems, to construct a fully Lorentz invariant theory providing a detailed description of microscopic quantum processes."

Supposing one is given a hypothetical PWT model accounting for relativistic phenomena including interactions which is mathematically consistent in the sense of global existence and uniqueness of solutions out of quantum equilibrium, Dürr[31] thinks that such a model could be both Lorentz invariant and allow for nonlocal signals. The signals could not be used to create causal loops. (This can be seen by considering the alternative view of SR described above – according to which, each signal has a definite causal order in absolute time. Butterfield stresses a similar point in the context of a general discussion of relativistic causality [14, end of sec. 7.1.3.1].)

A suggestion for a fundamentally Lorentz invariant model: First steps towards fundamentally Lorentz invariant models have been made by Dürr *et al.* with so-called "Hypersurface-Bohm-Dirac models" [19]. These describe noninteracting but entangled relativistic N-particle systems in the framework of PWT using a preferred foliation as an (arguably) additional spacetime structure. Setting aside particle creation and annihilation, this idea could be applied to particle ontologies in PW QFT. In our presentation we mainly follow a review by Tumulka [41, sec. 3.3.1].

To define a division between time and space, consider a *preferred time foliation* $\mathcal{F}$ of spacetime into spacelike hypersurfaces Σ, called *time leaves.* The purpose of $\mathcal{F}$ is to define which parts of spacetime are considered to be space at a given (absolute) time. This simultaneity structure is required by guidance equations like (12) and (19). Besides, it allows us to deal with the problem that quantum equilibrium cannot hold in all Lorentz frames [19, p. 13]. With the aid of $\mathcal{F}$, the relativistic generalisation of N-particle guidance equations is straightforward:

$$\frac{\mathrm{d}X_k^{\mu_k}}{\mathrm{d}\tau} \propto j^{\mu_1 \dots \mu_N}(X_1(\Sigma), ..., X_N(\Sigma)) \prod_{i \neq k} n_{\mu_i}(X_i(\Sigma)). \tag{22}$$

Here, $X_k(\tau)$ is the world line of the k-th particle, Σ is the time leaf containing $X_k(\tau)$, $n(x)$ is the normal vector on Σ at $x \in \Sigma$, $X_i(\Sigma)$ is the intersection point of the world line of particle i with Σ and

$$j^{\mu_1 \dots \mu_N} = \overline{\psi}(\gamma^{\mu_1} \otimes ... \otimes \gamma^{\mu_N})\psi \tag{23}$$

is the current of an N-particle Dirac equation for a wave function ψ on $\bigcup_{\Sigma \in \mathcal{F}} \Sigma^N$.

In order to arrive at a fundamentally Lorentz invariant theory, it is crucial that the time foliation $\mathcal{F}$ is itself regarded as dynamical, with a Lorentz invariant law for its time evolution. The importance of this aspect is stressed in [19, sec. IV] where

[31] D. Dürr, private communication (4/2011).

also possible suggestions for the law are made. But one should note that the nature of this hypothetical law is not yet clarified. It is also not clear to what extent the time foliation is to be counted as an additional structure: it might be generated by the wave function, a structure which is already at hand.

The possibility of a foliation of spacetime and its philosophical implications are discussed by Maudlin [29]. In [41, sec. 3.3.2], Tumulka reviews different proposals to make PWT relativistic – but they either seem to suffer from certain difficulties or have a similar tension with SR.

4 Discussion and outlook

4.1 Summary of the QFT parts

In this essay, we have seen several examples for possible PW QFTs. First, Bohm's model of the electromagnetic field was presented which seems to work formally (having "only" the usual mathematical problems of QFT). Similar models can be developed for bosonic fields [36]. While these make it possible to discuss physical effects from a realistic nonlocal perspective [9], it is not immediately clear how to find a sensible definition of "particles" in a field ontology (see also the next section). Once basic questions like these are clarified, it might be desirable to develop a PWT version of (the bosonic sector of) the Standard Model. At the moment, technical problems with nonlinear constraints in gauge theory seem to make immediate progress difficult.

Next, we reviewed Struyve's discussion of models with fermionic field ontologies which were shown to suffer from different serious problems. This motivated introducing minimalistic models. These seem to be *formally* able to reproduce the standard predictions but they give no further explanation of physical questions about all fermionic matter (viz. by somehow still containing fermionic degrees of freedom but not explicitly referring to them).

Finally, we discussed models with particle ontologies that developed from a stochastic lattice model by Bell. The first continuum generalisation by Dürr et al. seems to be appropriate for both fermions and bosons – but it includes stochastic jumps (corresponding to particle creation and annihilation). The second possible continuum generalisation by Colin and Struyve takes the Dirac sea literally and would be an interesting way to explain particle creation/annihilation for a particle ontology. But we found a discrepancy between the ambitions both to have invariance under Lorentz boosts and to be able to distinguish matter from the vacuum. The problem we found has to be answered before the model can be seriously considered.

Concerning the status of Lorentz invariance, the explicit nonlocality of PWT served as a motivation to develop an alternative view on Special Relativity involving absolute space and time. Einstein's operational definition of spacetime is implemented in this framework as the problem of rods and clocks which are distorted due to real physical effects that are present in a movement near the speed of the interactions holding the moving matter together. This change of view is put forward by the idea that in PWT quantum nonequilibrium can be considered (and in some sense has to be considered, so as to motivate the Born rule) which, according to

Valentini, would allow for instantaneous nonlocal signalling. This would provide an operational definition of absolute simultaneity. Furthermore, two conflicting views on the status of Lorentz invariance in PWT were presented:

1. A detailed microscopic account of nonlocal quantum phenomena in PWT must violate fundamental Lorentz invariance.
2. Fundamental Lorentz invariance might be achieved and could potentially be compatible with nonlocal instantaneous signals.

Both views violate the spirit of Special Relativity, as a notion of absolute simultaneity seems to be required to define guidance equations in PWT.

4.2 Discussion of the QFT parts

In the author's opinion the most important questions about PW QFTs are the following:

1. Localisation and the concept of particles and antiparticles;
2. The description of particle creation and annihilation;
3. The description of fermions;
4. Uniqueness of beables (fields, point particles, ...);
5. The status of Lorentz invariance.

It seems worthwhile to analyse these questions briefly: Field ontologies could in principle describe particles as soliton-like excitations. This seems not to be done usually (and in standard QFT for "probability solitons" only). Another question is whether such a strict concept of localisation is required[32] to explain spots on a detector screen, images of single atoms and trajectories in a bubble chamber. Since PW QFTs provide more than a probabilistic description, one may hope PWTs have some potential for a precise analysis of these questions, e.g. by running computer simulations for various experimental situations. On the other hand, there is so far no working PW QFT with a fermionic field ontology. As indicated in [1, p. 173], one possibility besides the minimalistic approach is that fermions might be composite "particles" of bosons. An example for the converse is the collective description of interacting fermions in one-dimensional Luttinger liquids.
Particle ontologies on the other hand do possess a simple and strict concept of localisation: "particles" as point-like objects. This seems to be an overidealisation – but one that is happily accepted in classical mechanics. Furthermore, particle ontologies can be applied to both fermions and bosons (e.g. in the model of Dürr *et al.*).

However, particle ontologies cannot easily explain creation and annihilation events. One either has to apply a stochastic description [20], or to imagine that the particles are not really created or annihilated at all. The latter idea is found in the Dirac sea

[32] Or even appropriate – considering e.g. the quark gluon plasma.

model of Colin and Struyve, but this seems to have its own potentially dangerous unresolved question (see sec. 3.3.3). Also questions of gravity (e.g. "dark matter" or the cosmological constant) immediately come to the mind when considering the Dirac sea.
Another approach was taken by Nikolic [31] who considered the possibility that particle detectors could behave as if a particle was annihilated (created) if it came to rest (was accelerated from rest). A related idea is to regard particles as bound states of another species of particles which is (temporarily) regarded as irreducible. But one may ask if this picture can give the impression of vacuum fluctuations expected from standard QFT. Since the ideal PW QFT is deterministic, these could only be statistical fluctuations. On the other hand, experiments in particle colliders should not prove a problem for the idea.

Generally, for both field and particle ontologies one should work towards a clarification of the concept of an "antiparticle". Possible ideas might be (speaking figuratively) particles and antiparticles as the analogue of excitations of a "membrane" in different directions (for field ontologies) – or the Dirac sea for particle ontologies[33].

We come to the question of the uniqueness of beables. Considering the above (largely not yet analysed) possibilities, one is far from answering the question if only fields or only particles can give a unique description of QFT phenomena. There might be good reasons for using particles and fields to model different aspects of QFT, say particles for fermions and fields for bosons. It might even be possible to adopt a completely different ontology, say strings. But before speculating about such questions, consider that even for theories like classical electromagnetism the ontology is not unique. While Maxwell's well-known presentation uses fields, Wheeler-Feynman absorber theory contains only particles [22, sec. 2.5]. One may take a step further and ask if the question "what nature really is" is a sensible one. The answer might not be unique. Rather, it seems to be most relevant to develop a further understanding of detailed physical (and logical) questions like those given above for PW QFT.

Concerning the question of Lorentz invariance, there seem to be three classes of opinions: Firstly, the Dirac sea model by Colin and Struyve seems to be incompatible with fundamental Lorentz invariance even on a statistical level. Secondly, one can imagine Lorentz invariance to be only an emergent symmetry of the quantum equilibrium distribution. Thirdly, it seems not to be excluded by a rigorous theorem that fundamental Lorentz invariance could be achieved – even in quantum nonequilibrium. This attitude stands in conflict with Valentini's idea of an Aristotelian spacetime. The main question seems not to be on which level Lorentz invariance is *possible*, but on which it is *appropriate*.
All viewpoints share a pronounced tension with Einstein's interpretation of Special Relativity but they violate it on different levels. A common feature is that PWT's explicit nonlocality requires a structure that defines absolute time, leading to a different understanding of causality. While there is a debate on these issues in philosophy [16], it is an open question to what extent this discussion will influence the future of physics as long as no further experimental clues become available.

[33] It is perhaps worth a thought if this asymmetric description of particles and antiparticles e.g. allows for bound states of antiparticles.

4.3 Conclusion and outlook

One should remember the primary reasons to do PW QFT: Practically all the benefits of the nonrelativistic approach (see sec. 2.2) carry over to the QFT case. One can see it as a merit that PWT makes questions explicit for which the statistical description of standard QFT usually gives only vague answers: the status of Lorentz invariance, the definition of a particle as well as particle creation and annihilation. In trying to develop an explicit deterministic and realistic example for a PW QFT, one can hope to learn more about these questions. Perhaps, PW QFT might allow for a complementary approach to quantum gravity (see e.g. [41, sec. 4] for steps in this direction and [35] for a surely controversial discussion). At present, one should not forget that PW QFT has a developing status and many detailed physical questions need to be clarified. Finally, the status of causality and Lorentz invariance in PWT leads to deep questions concerning the tension between quantum nonlocality and Special Relativity.

Quantum nonequilibrium: The possibility of quantum nonequilibrium is an interesting but perhaps experimentally inaccessible source of clarification of many of the issues discussed here. It was pioneered mainly by Valentini in the context of cosmology. If quantum nonequilibrium were present in physically extreme situations and effects such as near the hypothetical "Big Bang", this would lead, for relic cosmological particles and the hypothetical Hawking radiation around a black hole, to predictions of statistical anomalies in PWT [47], i.e. in conflict with standard QM. Speculating further, nonequilibrium of this sort might (among similarly spectacular possibilities) be used as a theoretical motivation of nonlocal signalling [43] and measurement with unlimited precision [46]. The latter point would allow us to resolve individual events in PWT whereby the question of Lorentz invariance could be attacked directly. Note that the usual impossibility proofs e.g. of nonlocal signalling and measurement with unlimited precision depend on statistical arguments valid only in quantum equilibrium. In PWT, they thus have a status similar to the second law of thermodynamics (which might be conceived to be violated microscopically, e.g. by Poincare recurrence). But like the second law of thermodynamics, quantum equilibrium would be dominant *for all practical purposes*.

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