Quasi-Truth as Truth of a Ramsey Sentence

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Abstract

I show the quasi-truth of a sentence in a partial structure to be equivalent to the truth of a specific Ramsey sentence in a structure that corresponds naturally to the partial structure. Hence quasi-truth, the core notion of the partial structures approach, can be captured in the terms of the received view on scientific theories as developed by Carnap and Hempel. I further show that a mapping is a partial homomorphism/isomorphism between two partial structures if and only if it is a homomorphism/isomorphism between their corresponding structures. It is a corollary that the partial structures approach can be expressed in first or second order model theory.

Keywords: partial structure; quasi-truth; pragmatic truth; partial truth; subtruth; partial homomorphism; partial isomorphism; model theory; expansion; Ramsey sentence; received view; logical empiricism

The partial structures approach is in the vanguard of the semantic view on scientific theories and models (da Costa and French 2000; Le Bihan 2011, n. 3, §5), and it is one of the main reasons why the received view on scientific theories as developed within logical empiricism by, for example, Carnap (1966) and Hempel (1958) is considered inferior to the semantic view (French and Ladyman 1999). I will show that the core notion of the partial structures approach, quasi-truth, can be captured very naturally within the received view.

The partial structures approach is motivated by a simple epistemological point: Most of the time, scientists do not have enough information about a domain to determine its structure with arbitrary precision. For most relations, it is at best known of *some* tuples of objects that they fall under the relation and known of *some* objects that they do not fall under it. For many if not most tuples this is unknown. Similarly, the value of a function is not know for all of its possible arguments. Partial structures are defined to take this lack of knowledge into account.

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Assume a language $\mathscr{L} = \{R_i, F_j, c_k\}_{i\in I, j\in J, k\in K}$, where R_i is an m_i-place relation symbol for every $i \in I$, F_j an n_j-place function symbol for every $j \in J$, and c_k a constant symbol for every $k \in K$.[1] While most works on partial structures in the philosophy of science (e. g., da Costa and French 1990; 2000) do not consider functions, and the foundational paper by Mikenberg et al. (1986) does not consider constants, the respective definitions can be easily combined to give

Definition 1. $\tilde{\mathfrak{A}}$ is a *partial $\mathscr{L}$-structure* if and only if

$$\tilde{\mathfrak{A}} = \langle A, \langle R_i^{\tilde{\mathfrak{A}},+}, R_i^{\tilde{\mathfrak{A}},-}, R_i^{\tilde{\mathfrak{A}},\circ} \rangle, F_j^{\tilde{\mathfrak{A}}}, c_k^{\tilde{\mathfrak{A}}} \rangle_{i\in I, j\in J, k\in K}\,, \tag{1}$$

where $\{R_i^{\tilde{\mathfrak{A}},+}, R_i^{\tilde{\mathfrak{A}},-}, R_i^{\tilde{\mathfrak{A}},\circ}\}$ is a partition of A^{m_i} for each $i \in I$, $F_j^{\tilde{\mathfrak{A}}} : C_{\tilde{\mathfrak{A}},j} \longrightarrow A$ is a function with domain $C_{\tilde{\mathfrak{A}},j} \subseteq A^{n_j}$ for each $j \in J$, and $c_k^{\tilde{\mathfrak{A}}} \in A$ for each $k \in K$.

The definition of partial structures by Mikenberg et al. (1986, def. 1) is recovered for $K = \varnothing$, the definition by da Costa and French (1990, 255f) for $J = \varnothing$.[2] Lack of knowledge is represented by non-empty sets $R_i^{\tilde{\mathfrak{A}},\circ}$ and sets $C_{\tilde{\mathfrak{A}},j} \subset A^{n_j}$, for which $F_j^{\tilde{\mathfrak{A}}}$ is a proper partial function on A^{n_j}. Constant symbols are interpreted as in a structure, and thus not used to express lack of knowledge.[3]

The core notion of the partial structures approach, quasi-truth, also takes background knowledge into account, expressed by the *primary statements*, a set $\tilde{\Pi}$ of $\mathscr{L}$-sentences (Mikenberg et al. 1986, def. 3; da Costa and French 1990, 256):

Definition 2. $\mathscr{L}$-sentence φ is *quasi-true* in partial $\mathscr{L}$-structure $\tilde{\mathfrak{A}}$ relative to $\tilde{\Pi}$ if and only if there is an $\mathscr{L}$-structure $\mathfrak{B}$ with $B = A$, $R_i^{\tilde{\mathfrak{A}},+} \subseteq R_i^{\mathfrak{B}} \subseteq A^{m_i} - R_i^{\tilde{\mathfrak{A}},-}$ for each $i \in I$, $F_j^{\mathfrak{B}}|_{C_{\tilde{\mathfrak{A}},j}} = F_j^{\tilde{\mathfrak{A}}}$ for each $j \in J$, and $c_k^{\mathfrak{B}} = c_k^{\tilde{\mathfrak{A}}}$ for each $k \in K$,[4] such that

$$\mathfrak{B} \models \{\varphi\} \cup \tilde{\Pi}\,. \tag{2}$$

Quasi-truth is also called ‘pragmatic truth’ and ‘partial truth’. One of the most important properties of quasi-truth is that incompatible sentences can be quasi-true without quasi-truth being trivial: Let $\tilde{\mathfrak{A}}$ be the partial structure

[1] I will more or less follow the model theoretic notation of Chang and Keisler (1990), so that, for example, A is the domain $\mathrm{dom}(\mathfrak{A})$ of structure $\mathfrak{A}$, and $R^{\mathfrak{A}}$ is the extension of R in $\mathfrak{A}$.

[2] While da Costa and French (1990, 255) define partial structures only for relations, their further definition of quasi truth presumes that partial structures can contain constants as well.

[3] Thus this treatment of constants cannot capture situations in which constants are unknown or not known with arbitrary precision (cf. Lutz 2011, §3.2).

[4] $\mathfrak{B}$ is called an *extension* of $\tilde{\mathfrak{A}}$, and $\tilde{\mathfrak{A}}$-*normal* iff $\mathfrak{B} \models \tilde{\Pi}$. If $\tilde{\Pi}$ is taken to contain only the penumbral connections of the language, an $\tilde{\mathfrak{A}}$-normal structure is a complete extension of an $\tilde{\mathfrak{A}}$-structure in the sense of Fine (1975, §2). Quasi-truth is then subtruth (cf. Hyde 1997). Although partial structures can thus formally be seen as giving vague denotations to a vocabulary, quasi-truth is meant as an epistemic, not a semantic concept.

$\langle A, \langle R_1^{\tilde{\mathfrak{A}},+}, R_1^{\tilde{\mathfrak{A}},-}, R_1^{\tilde{\mathfrak{A}},\circ}\rangle, c_1^{\tilde{\mathfrak{A}}}\rangle$ with $A = \{1,2,3\}$, $R_1^{\tilde{\mathfrak{A}},+} = \{1\}$, $R_1^{\tilde{\mathfrak{A}},-} = \{3\}$, $c_1^{\tilde{\mathfrak{A}}} = 2$, and $\tilde{\Pi} = \varnothing$. Then $R_1 c$ and $\neg R_1 c$ are both quasi-true, while $\neg\exists x Rx$ is not.

In a partial structure, a relation symbol R_i has, in a sense, two separate interpretations. For one, there are its clear instances $R_i^{\tilde{\mathfrak{A}},+}$. They can be determined, for example, by their similarity to paradigmatic instances of R_i, or, more likely when it comes to scientific terms, by the fulfillment of some sufficient condition. Then there are also the clear non-instances $R_i^{\tilde{\mathfrak{A}},-}$. These are determined, for example, by their similarity to paradigmatic non-instances of R_i, or by the failure to fulfill some necessary condition. Determining whether some tuple is in $R_i^{\tilde{\mathfrak{A}},+}$ is thus more or less unrelated to determining whether some tuple is in $R_i^{\tilde{\mathfrak{A}},-}$. (That a tuple is in $R_i^{\tilde{\mathfrak{A}},\circ}$ will typically only be determined by its being in neither $R_i^{\tilde{\mathfrak{A}},+}$ nor $R_i^{\tilde{\mathfrak{A}},-}$.) Given the difference in determining the members of $R_i^{\tilde{\mathfrak{A}},+}$ and of $R_i^{\tilde{\mathfrak{A}},-}$, it is natural to assign separate symbols of a language to these two concepts, say, R_i^+ and R_i^-.

In a partial structure, the interpretation $F_j^{\tilde{\mathfrak{A}}}$ of an n_j-place function symbol F_j can be seen as the clear instances of an $n_j + 1$-ary relation. In analogy to the relation symbols in partial structures, it is natural to assign an $n_j + 1$-place relation symbol F_j^+ to the concept that determines these clear instances. $F_j^{\tilde{\mathfrak{A}}}$ does not have a value if its argument is not in $C_{\tilde{\mathfrak{A}},j}$, and thus for every $n_j + 1$-tuple not in the relation named by F_j^+, it is unknown whether it falls under the function or not. Thus there is no need for a relation symbol that names the clear non-instances of F_j.[5]

Since constant symbols are interpreted in the usual way, this leads to a new language $\mathscr{L}' = \{R_i^+, R_i^-, F_j^+, c_k\}_{i\in I, j\in J, k\in K}$, chosen so that $\{R_i^+, R_i^-, F_j^+\}_{i\in I, j\in J} \cap \mathscr{L} = \varnothing$. And any partial structure for $\mathscr{L}$ determines a structure for $\mathscr{L}'$:

Definition 3. $\mathscr{L}'$-structure $\mathfrak{A}$ *corresponds* to partial $\mathscr{L}$-structure $\tilde{\mathfrak{A}}$ if and only if $\mathrm{dom}(\mathfrak{A}) = \mathrm{dom}(\tilde{\mathfrak{A}})$, $R_i^{+\mathfrak{A}} = R_i^{\tilde{\mathfrak{A}},+}$ and $R_i^{-\mathfrak{A}} = R_i^{\tilde{\mathfrak{A}},-}$ for each $i \in I$, $F_j^{+\mathfrak{A}} = \{\bar{a}b \mid \bar{a} \in C_{\tilde{\mathfrak{A}},j} \text{ and } F_j^{\tilde{\mathfrak{A}}}(\bar{a}) = b\}$ for each $j \in J$, and $c_k^{\mathfrak{A}} = c_k^{\tilde{\mathfrak{A}}}$ for each $k \in K$.

$\bar{a}$ here stands for the tuple $(a_1, \ldots, a_{n_j}) \in A^{n_j}$, and $\bar{a}b$ for the tuple $(a_1, \ldots, a_{n_j}, b) \in A^{n_j+1}$. Note that for every partial structure $\tilde{\mathfrak{A}}$ there is exactly one structure $\mathfrak{A}$ that corresponds to $\tilde{\mathfrak{A}}$.

Despite having two separate interpretations, the relation symbols R_i^+ and R_i^- are of course connected, since they are known to refer to instances and, respectively, non-instances of the same relation symbol R_i from $\mathscr{L}$. This connection,

[5]Incidentally, this treatment of functions cannot capture situations in which the values of functions are only known up to a certain precision (cf. Lutz 2011, §3.2).

and the fact that over a restricted domain, F_j^+ is equivalent to a function F_j are thus background assumptions. They can therefore be described by primary statements in language $\mathscr{L}^* = \mathscr{L} \cup \mathscr{L}'$:

$$\begin{aligned}\Pi = \tilde{\Pi} &\cup \bigcup_{i \in I} \{\forall \bar{x}(R_i^+ \bar{x} \to R_i \bar{x}), \forall \bar{x}(R_i^- \bar{x} \to \neg R_i \bar{x})\} \\ &\cup \bigcup_{j \in J} \{\forall \bar{x} \forall y (F_j^+ \bar{x} y \to F_j \bar{x} = y)\}\end{aligned} \tag{3}$$

On the syntactic level, $\bar{x}$ stands for a *non-repeating* string of m_i or n_j variables, and $\bar{x}y$ for the non-repeating concatenation of $\bar{x}$ and y. In every structure $\mathfrak{A}$ that corresponds to a partial structure, relation $F_j^{+\mathfrak{A}}$ can provide a sufficient condition for function values because by definition 3, tuples in $F_j^{+\mathfrak{A}}$ differ in their last elements only if they also differ in one of their previous elements.

Since the structure $\mathfrak{A}$ corresponding to a partial $\mathscr{L}$-structure $\tilde{\mathfrak{A}}$ is itself an $\mathscr{L}'$-structure, Π cannot be true in $\mathfrak{A}$. However, Π may be true in an *expansion* of $\mathfrak{A}$ to $\mathscr{L}^*$, which differs from $\mathfrak{A}$ only in that it interprets the symbols in $\mathscr{L}^* - \mathscr{L}'$. With the help of corresponding structures, it is now possible to describe quasi-truth relative to $\tilde{\Pi}$:

Claim 1. *$\mathscr{L}$-sentence φ is quasi-true in partial $\mathscr{L}$-structure $\tilde{\mathfrak{A}}$ with respect to* $\tilde{\Pi}$ *if and only if the corresponding $\mathscr{L}'$-structure has an expansion $\mathfrak{C}$ such that*

$$\mathfrak{C} \vDash \{\varphi\} \cup \Pi\,. \tag{4}$$

Proof. '$\Leftarrow$': Let $\mathfrak{A}$ correspond to $\tilde{\mathfrak{A}}$ and $\mathfrak{C}$ be an expansion of $\mathfrak{A}$ such that $\mathfrak{C} \vDash \{\varphi\} \cup \Pi$. Then $\mathfrak{C}|_{\mathscr{L}} \vDash \{\varphi\} \cup \tilde{\Pi}$, $\mathrm{dom}(\mathfrak{C}|_{\mathscr{L}}) = \mathrm{dom}(\mathfrak{C}) = A$, and $R_i^{\tilde{\mathfrak{A}},+} = R_i^{+\mathfrak{A}} = R_i^{+\mathfrak{C}} \subseteq R_i^{\mathfrak{C}|_{\mathscr{L}}} \subseteq A^{m_i} - R_i^{-\mathfrak{C}} = A^{m_i} - R_i^{-\mathfrak{A}} = A^{m_i} - R_i^{\tilde{\mathfrak{A}},-}$ for each $i \in I$. Furthermore, for each $\bar{a} \in C_{\tilde{\mathfrak{A}},j}$, $F_j^{\mathfrak{C}|_{\mathscr{L}}}(\bar{a}) = b$ if $\bar{a}b \in F_j^{+\mathfrak{C}}$, and, since $F_j^{\mathfrak{C}|_{\mathscr{L}}}$ is a function, also *only* if $\bar{a}b \in F_j^{+\mathfrak{C}}$. Since further $F_j^{+\mathfrak{C}} = F_j^{+\mathfrak{A}}$, and $\bar{a}b \in F_j^{+\mathfrak{A}}$ if and only if $\bar{a} \in C_{\tilde{\mathfrak{A}},j}$ and $F_j^{\tilde{\mathfrak{A}}}(\bar{a}) = b$, it holds that $F_j^{\mathfrak{C}|_{\mathscr{L}}}|_{C_{\tilde{\mathfrak{A}},j}} = F_j^{\tilde{\mathfrak{A}}}$ for each $j \in J$. Finally, $c_k^{\mathfrak{C}|_{\mathscr{L}}} = c_k^{\mathfrak{A}} = c_k^{\tilde{\mathfrak{A}}}$. Thus $\mathfrak{C}|_{\mathscr{L}}$ is $\tilde{\mathfrak{A}}$-normal and hence φ is quasi-true in $\tilde{\mathfrak{A}}$.

'$\Rightarrow$': Let $\mathfrak{A}$ be the $\mathscr{L}'$-structure that corresponds to $\tilde{\mathfrak{A}}$ and let φ be quasi-true in $\tilde{\mathfrak{A}}$ with respect to $\tilde{\Pi}$. Then there is an $\mathscr{L}$-structure $\mathfrak{B}$ such that $\mathfrak{B} \vDash \tilde{\Pi} \cup \{\varphi\}$ and $R_i^{+\mathfrak{A}} = R_i^{\tilde{\mathfrak{A}},+} \subseteq R_i^{\mathfrak{B}} \subseteq A^{m_i} - R_i^{\tilde{\mathfrak{A}},-} = A^{m_i} - R_i^{-\mathfrak{A}}$ for each $i \in I$. Furthermore, $F_j^{\tilde{\mathfrak{A}}} = F_j^{\mathfrak{B}}|_{C_{\tilde{\mathfrak{A}},j}}$ and thus for each $\bar{a} \in A^{n_j}$ and $b \in A$, $\bar{a}b \in F_j^{+\mathfrak{A}}$ only if $F_j^{\mathfrak{B}}(\bar{a}) = b$ for each $j \in J$. Finally, $c_k^{\mathfrak{A}} = c_k^{\tilde{\mathfrak{A}}} = c_k^{\mathfrak{B}}$ for each $k \in K$. Define the $\mathscr{L}^*$-structure $\mathfrak{C}$ so that $\mathfrak{C}|_{\mathscr{L}'} = \mathfrak{A}$ and $\mathfrak{C}|_{\mathscr{L}} = \mathfrak{B}$. Then $\mathfrak{C} \vDash \{\varphi\} \cup \Pi$. □

Somewhat shorter, φ is quasi-true in $\tilde{\mathfrak{A}}$ with respect to $\tilde{\Pi}$ if and only if its corresponding structure has an expansion in which $\{\varphi\} \cup \Pi$ is true.[6]

In the new formalization of quasi-truth, the language $\mathscr{L}'$ is, in keeping with the basic motivation for partial structures, considered to be directly interpreted, while the interpretation of $\mathscr{L}^* - \mathscr{L}' = \{R_i, F_j\}_{i\in I, j\in J}$ is only given through the interpretation of $\mathscr{L}'$ and the primary statements Π. This notion of a basic vocabulary and an auxiliary vocabulary is the basis of many analyses in the received view (Carnap 1966, §23; Hempel 1958, §2).[7] In principle, all results from these analyses can therefore be used for partial structures. I want to present only one.

If $\mathscr{L}^*$ is finite, the Ramsey sentence $\mathrm{R}_{\mathscr{L}'}(\alpha)$ of an $\mathscr{L}^*$-sentence α is defined as $\exists_{i\in I} X_i \exists_{j\in J} Y_j \alpha^\dagger$. To arrive at $\alpha^\dagger$, one replaces in α the relation symbol R_i by the m_i-place relation variable X_i for every $i \in I$, and the function symbol F_j by the n_j-place function variable Y_j for every $j \in J$. This gives a new way to formulate quasi-truth:

Claim 2. *If $\tilde{\Pi}$ and $\mathscr{L}$ are finite, then $\mathscr{L}$-sentence φ is quasi-true in partial $\mathscr{L}$-structure $\tilde{\mathfrak{A}}$ with respect to $\tilde{\Pi}$ if and only if for the corresponding $\mathscr{L}'$-structure $\mathfrak{A}$ it holds that*

$$\mathfrak{A} \vDash \mathrm{R}_{\mathscr{L}'}(\varphi \wedge \bigwedge \Pi)\ . \tag{5}$$

Proof. Since $\tilde{\Pi}$ and $\mathscr{L}$ are finite, so are Π and $\mathscr{L}^*$. Therefore, by claim 1, φ is quasi-true in $\tilde{\mathfrak{A}}$ if and only if $\mathfrak{A}$ has an expansion $\mathfrak{C}$ such that $\mathfrak{C} \vDash \varphi \wedge \bigwedge \Pi$. Thus it has to be shown that there is such an expansion if and only if $\mathfrak{A} \vDash \mathrm{R}_{\mathscr{L}'}(\varphi \wedge \bigwedge \Pi)$.

'$\Leftarrow$': Since $\mathfrak{A} \vDash \mathrm{R}_{\mathscr{L}'}(\varphi \wedge \bigwedge \Pi)$, there is a relation $V_i \subseteq A^{m_i}$ for every $i \in I$ and a function $G_j : A^{n_j} \longrightarrow A$ for every $j \in J$ such that $\{V_i, G_j\}_{i\in I, j\in J}$ satisfies $(\varphi \wedge \bigwedge \Pi)^\dagger$ in $\mathfrak{A}$. Define $\mathfrak{C}$ so that $R_i^{\mathfrak{C}} = V_i$ for each $i \in I$, $F_j^{\mathfrak{C}} = G_j$ for each $j \in J$, and $\mathfrak{C}|_{\mathscr{L}'} = \mathfrak{A}$. Induction on the complexity of $\varphi \wedge \bigwedge \Pi$ shows that $\mathfrak{C} \vDash \varphi \wedge \bigwedge \Pi$.

'$\Rightarrow$': Induction shows that $\{R_i^{\mathfrak{C}}, F_j^{\mathfrak{C}}\}_{i\in I, j\in J}$ satisfies $(\varphi \wedge \bigwedge \Pi)^\dagger$ in $\mathfrak{A}$, so that $\mathfrak{A} \vDash \exists_{i\in I} X_i \exists_{j\in J} Y_j (\varphi \wedge \bigwedge \Pi)^\dagger$. □

Somewhat shorter, φ is quasi-true in $\tilde{\mathfrak{A}}$ with respect to $\tilde{\Pi}$ if and only if $\mathrm{R}_{\mathscr{L}'}(\varphi \wedge \bigwedge \Pi)$ is true in the structure corresponding to $\tilde{\mathfrak{A}}$.

The features of quasi-truth that follow from definition 2 can now also be recovered from claims 1 and 2. For example, that two incompatible sentences can

[6] Two further important concepts of the partial structures approach, partial homomorphism and partial isomorphism, can also be expressed with the help of corresponding structures (see appendix A).

[7] Incidentally, the sentences $\forall \bar{x}(R_i^+ \bar{x} \rightarrow \neg R_i^- \bar{x}), i \in I$ and $\forall \bar{x} \forall y \forall \bar{v} \forall w(F_j^+ \bar{x} y \wedge F_j^+ \bar{v} w \wedge \bigwedge_{1\leq r\leq n_j} x_r = v_r \rightarrow y = w), j \in J$, which follow from Π and contain only basic terms, express that in a partial structure $\tilde{\mathfrak{A}}$, $R_i^{\tilde{\mathfrak{A}},+} \cap R_i^{\tilde{\mathfrak{A}},-} = \varnothing$ for all $i \in I$ and $F_j^{\tilde{\mathfrak{A}}}$ is a partial function for all $j \in J$. Since they are therefore basic presumptions of the formalism, they are good candidates for analytic sentences in $\mathscr{L}'$ (cf. Carnap 1952).

both be quasi-true in the same partial structure follows from the fact that, given the primary statements Π, two incompatible sentences can have Ramsey sentences that are true in a structure that corresponds to a partial structure.

Van Fraassen (1980, p. 56) has famously and influentially argued that, like most results of logical empiricism, the Ramsey sentence is “off the mark”, a solution “to purely self-generated problems, and philosophically irrelevant.” If van Fraassen was right, the preceding results would establish a *reductio ad empirismum logicum* of the partial structures approach. But insofar as the partial structures approach has proven its merits, the inference has to go in the opposite direction: The tools developed within logical empiricism are more useful than its detractors have acknowledged.

A Partial homomorphisms and isomorphisms

Bueno et al. (2002, 503f) define partial homomorphisms between partial structures:

Definition 4. A *partial homomorphism* from partial structure $\tilde{\mathfrak{A}}$ to partial structure $\tilde{\mathfrak{B}}$ is a mapping $f : A \longrightarrow B$ for which the following holds: If $\bar{a} \in R_i^{\tilde{\mathfrak{A}},+}$ then $f(\bar{a}) \in R_i^{\tilde{\mathfrak{B}},+}$ for all $i \in I$, if $\bar{a} \in C_{\tilde{\mathfrak{A}},j}$ then $f(\bar{a}) \in C_{\tilde{\mathfrak{B}},j}$ and for all $\bar{a} \in C_{\tilde{\mathfrak{A}},j}$, $f\left(F_j^{\tilde{\mathfrak{A}}}(\bar{a})\right) = F_j^{\tilde{\mathfrak{B}}}\left(f(\bar{a})\right)$ for all $j \in J$, and $f\left(c_k^{\tilde{\mathfrak{A}}}\right) = c_k^{\tilde{\mathfrak{B}}}$ for all $k \in K$.[8]

Bueno (1997, 596) introduces the notion of a partial isomorphism between partial structures containing only relations, which can be generalized as follows:

Definition 5. A *partial isomorphism* from partial structure $\tilde{\mathfrak{A}}$ to partial structure $\tilde{\mathfrak{B}}$ is a bijection $f : A \longrightarrow B$ for which the following holds: $\bar{a} \in R_i^{\tilde{\mathfrak{A}},+}$ if and only if $f(\bar{a}) \in R_i^{\tilde{\mathfrak{B}},+}$ for all $i \in I$, $\bar{a} \in C_{\tilde{\mathfrak{A}},j}$ if and only if $f(\bar{a}) \in C_{\tilde{\mathfrak{B}},j}$ and for all $\bar{a} \in C_{\tilde{\mathfrak{A}},j}$, $f\left(F_j^{\tilde{\mathfrak{A}}}(\bar{a})\right) = F_j^{\tilde{\mathfrak{B}}}\left(f(\bar{a})\right)$ for all $j \in J$, and $f\left(c_k^{\tilde{\mathfrak{A}}}\right) = c_k^{\tilde{\mathfrak{B}}}$ for all $k \in K$.

The differences between the two definitions are analogous to the differences between the standard definitions of homomorphism and isomorphism between structures (Hodges 1993, 5), so that they can be easily discussed together:[9]

Claim 3. *Let $\mathfrak{A}$ correspond to $\tilde{\mathfrak{A}}$, and $\mathfrak{B}$ to $\tilde{\mathfrak{B}}$. Then f is a partial homomorphism/ partial isomorphism from $\tilde{\mathfrak{A}}$ to $\tilde{\mathfrak{B}}$ if and only if f is a homomorphism/isomorphism from $\mathfrak{A}$ to $\mathfrak{B}$.*

Proof. The proof for relations and constants is immediate. For functions, the following holds:

[8] For an n-tuple $\bar{a}$, $f(\bar{a}) = (f(a_1), \ldots, f(a_n))$.

[9] The left hand side and the right hand side of the slash denote separate conjuncts of claim 3 and its proof.

'$\Rightarrow$': For all $j \in J$, $\bar{a} \in A^{n_j}$, and $b \in A$, $\bar{a}b \in F_j^{+\mathfrak{A}}$ if and only if $\bar{a} \in C_{\tilde{\mathfrak{A}},j}$ and $F_j^{\tilde{\mathfrak{A}}}(\bar{a}) = b$. This holds only if/if and only if $f(\bar{a}) \in C_{\tilde{\mathfrak{B}},j}$ and $F_j^{\tilde{\mathfrak{B}}}(f(\bar{a})) = f(b)$, that is, $f(\bar{a})f(b) \in F_j^{+\mathfrak{B}}$.

'$\Leftarrow$': For all $j \in J$, $\bar{a} \in C_{\tilde{\mathfrak{A}},j}$ and $F_j^{\tilde{\mathfrak{A}}}(\bar{a}) = b$ if and only if $\bar{a}b \in F_j^{+\mathfrak{A}}$. This holds only if/if and only if $f(\bar{a})f(b) \in F_j^{+\mathfrak{B}}$, that is, $f(\bar{a}) \in C_{\tilde{\mathfrak{B}},j}$ and $F_j^{\tilde{\mathfrak{B}}}(f(\bar{a})) = f(b)$. □

Somewhat shorter, a mapping between two partial structures is a partial homomorphism/partial isomorphism if and only if it is a homomorphism/ isomorphism between their corresponding structures.

Claims 1 and 3 reduce the concepts of the partial structures approach to the model theory of first order logic, claims 2 and 3 reduce them to the model theory of second order logic. For example, since the truth-value of a sentence of second order logic is conserved under isomorphisms, it follows from claims 2 and 3 that the quasi-truth-value of a sentence is conserved under partial isomorphisms.

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