Comment on "How to protect the interpretation of the wave function against protective measurements" by Jos Uffink

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Abstract

It is shown that Uffink's attempt to protect the interpretation of the wave function against protective measurements fails due to several errors in his arguments.

This comment is motivated by a recent review of my manuscript "Protective Measurement and the Meaning of the Wave Function" (Gao 2011a). The reviewer said, "the manuscript fails to deal with the most important of such objections, i.e. J. Uffink in Phys. Rev. A 60: 3474-3481 (1999), a paper that argues against AAV that the concept of protective measurements has no implication for the interpretation of the wave function." A critical analysis of Uffink's objection may be indeed necessary, as no answers have been given by the proponents of protective measurements[1]. Vaidman (2009) regarded this objection as a misunderstanding, but he gave no concrete rebuttal. Maybe it is just this objection that made most (?) people blind to the important implications of protective measurements for the interpretation of the wave function. In this comment, I will analyze Uffink's objection to protective measurements in detail.

The main claim of Uffink's paper is that only observables that commute with the system's Hamiltonian can be protectively measured (Uffink 1999). Based on this claim, he argued that protective measurements have no implications for the interpretation of the wave function, and the coherence of alternative interpretations of quantum mechanics can be saved. Uffink gave a strict proof of this claim in Section IV of his paper. However, there is a deadly error in the proof concerning the two equations between Eq. (23) and Eq. (24) (in the eprint between Eq. (24) and Eq. (25)):

$$\langle\phi_m|e^{i(\tau H_S+pO)}H_S e^{-i(\tau H_S+pO)}|\phi_n\rangle \to E_n\delta_{mn}, \tag{1}$$

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[1]The earlier objections to the validity and meaning of protective measurements have been answered (Aharonov, Anandan and Vaidman 1996; Dass and Qureshi 1999).

$$e^{i\tau(E_m-E_n)}\langle\phi_m|e^{ipO}H_S e^{-ipO}|\phi_n\rangle \to E_n\delta_{mn}. \tag{2}$$

In the derivation from the first equation to the second, it is implicitly assumed that the two operators O (the observable) and H_S (the system Hamiltonian) are commutative. The exponential function satisfies the equality $e^{X+Y} = e^X e^Y$ only if the two operators X and Y commute[2]. But the aim of the proof is to prove the commutativity of O and H_S. Thus Uffink's proof fails.

In fact, the validity of the first order perturbation theory, which has been widely used and confirmed in quantum mechanics, already implies that Uffink's proof is doomed to failure. For according to the theory, Eq. (1) can be satisfied when the two operators O and H_S are noncommutative. Since the derivation of the result of a protective measurement is only based on the first order perturbation theory, it should have no problem.

Another claim of Uffink's paper is that protective measurement does not measure an arbitrary observable O that may not commute with the system Hamiltonian, but rather a related observable $\tilde{O}$ that commutes with the system Hamiltonian:

$$\tilde{O} = \sum_n P_n O P_n, \tag{3}$$

where $P_n = |\phi_n\rangle\langle\phi_n|$. This claim is partly based on the above failed proof. Note that the measurement of this related observable $\tilde{O}$ on a system in an eigenstate $|\phi_n\rangle$ of H_S yields the expectation value $\langle O\rangle_n$.

However, this claim is also problematic. First of all, the measurement of the observable $\tilde{O}$, which commutes with the system Hamiltonian, results in neither entanglement between the measured system and the measuring device nor collapse of the measured state. By contrast, for a protective measurement of O, the entanglement and collapse can never be completely avoided (as the measuring time cannot be made infinitely long), though their effects can be made arbitrarily small. Next, the measurement of $\tilde{O}$ requires a full *a priori* knowledge of the system Hamiltonian, while a protective measurement of O can be made without this knowledge. For instance, it may only require to know the measured state is a non-degenerate eigenstate of an unknown Hamiltonian[3]. Last but not least, the observable $\tilde{O}$ already contains the information about the measurement result $\langle O\rangle_n$:

$$\tilde{O} = \sum_n \langle O\rangle_n P_n. \tag{4}$$

Thus it is not a measurement of the expectation value $\langle O\rangle_n$ at all. By contrast, the protective measurement of an arbitrary observable O does measure its expectation value, which is unknown before the measurement.

In his paper, Uffink also used an example, which has been discussed by Aharonov, Anandan and Vaidman (1993), to illustrate his conclusions. Unfortunately, his analysis of the example is also problematic. In the example, a

[2] In the noncommutative case, we can use the Baker-Campbell-Hausdorff formula to calculate e^{X+Y}. But the expansion is very complex in general, and thus the perturbation theory is widely used.

[3] This important point has been repeatedly stressed by the proponents of protective measurement and other authors (Aharonov, Anandan and Vaidman 1996; Dass and Qureshi 1999).

charged particle is in a superposition of two states localized in distant boxes L and R:

$$|\phi_+\rangle = \frac{1}{\sqrt{2}}(|\phi_L\rangle + |\phi_R\rangle), \tag{5}$$

where $|\phi_L\rangle$ and $|\phi_R\rangle$ are the ground states of the box potentials. The question is whether a protective measurement can demonstrate that the particle is in this delocalized state. Since this state degenerates with

$$|\phi_-\rangle = \frac{1}{\sqrt{2}}(|\phi_L\rangle - |\phi_R\rangle), \tag{6}$$

a protective procedure is needed to lift the degeneracy. For example, by arranging that in the region between the two boxes the potential has a large but finite constant value V as Uffink suggested, one can achieve that these two states are no longer degenerate. Then a protective measurement of the observable:

$$O = -|\phi_L\rangle\langle\phi_L| + |\phi_R\rangle\langle\phi_R| \tag{7}$$

on this state will yield its expectation value $\langle O\rangle_+ = 0$. This measurement can be done by sending a charged test particle straight through the middle between the boxes, perpendicular to the line joining the two boxes. Since the trajectory of the test particle is only sensitive to $\langle O\rangle_+$ under the condition of protective measurement, it will continue through the boxes without deviation.

In order to demonstrate a negative answer to the above question, Uffink further considered the case where the measurement is carried out on a charged particle prepared in a localized state $|\phi_L\rangle$. Since this state is not protected under the above experimental setup, one obtains the evolution:

$$|\phi_L\rangle|\chi\rangle = \frac{1}{\sqrt{2}}(|\phi_+\rangle + |\phi_-\rangle)|\chi\rangle \to \frac{1}{\sqrt{2}}(|\phi_+\rangle|\chi_+\rangle + |\phi_-\rangle|\chi_-\rangle), \tag{8}$$

where $|\chi\rangle$ is the initial state of the test particle, $|\chi_+\rangle$ and $|\chi_-\rangle$ are its final states in the cases when the measured particle was initially in the states $|\phi_+\rangle$ and $|\phi_-\rangle$. Since $\langle O\rangle_+ = \langle O\rangle_- = 0$, the test particle travels a straight trajectory in the state $|\chi_+\rangle$ as well as in $|\chi_-\rangle$. Then the test particle will travel on a straight path for the measured state $|\phi_L\rangle$. This means that for both delocalized state $|\phi_+\rangle$ and localized state $|\phi_L\rangle$, the test particle always travels on a straight path. Based on this result, Uffink concluded that a protective measurement provides no evidence for the spatial delocalization of the measured particle.

At first sight this argument seems reasonable. However, it is not difficult to find its problem by a careful analysis. The key is to realize that in order to measure the state of a measured system, e.g. whether the system is in a delocalized state or not, the state must be protected before the measurement. This is a basic requirement of protective measurement. It is obvious that in the above example the measured state $|\phi_L\rangle$ is not protected during the measurement, which was also admitted by Uffink. Accordingly, the result of this measurement can neither tell us the state $|\phi_L\rangle$ is localized nor tell us the state $|\phi_L\rangle$ is delocalized, and thus it cannot be used to support Uffink's conclusion. In other words, only when the result of the protective measurement of $|\phi_L\rangle$ is the same as the result of the protective measurement of $|\phi_+\rangle$, can Uffink's argument hold true. But these two results are obviously different; for the former the trajectory of the test particle deviates, while for the latter the trajectory is a straight path. Here it is also worth noting that under the same experimental setup (e.g. in the above example) no measurement consistent with quantum mechanics can measure whether the measured particle is in the state $|\phi_+\rangle$ or $|\phi_L\rangle$, as this requires the distinguishability of two non-orthogonal states, which is prohibited by quantum mechanics.

A possible reason leading Uffink to the wrong conclusion is that the above example is very special; that the result of the non-protective (adiabatic) measurement of $|\phi_L\rangle$ is always the same as the result of the protective measurement of $|\phi_+\rangle$ is only a coincidence. Although a non-protective measurement results in wavefunction collapse, whose effect cannot be made arbitrarily small, the effect of the wavefunction collapse is very tiny for the non-protective measurement of $|\phi_L\rangle$ in the above special example. In order to see the effect more clearly, we had better consider the protective measurement of a general state:

$$|\phi_+\rangle = a\,|\phi_L\rangle + b\,|\phi_R\rangle\,, \tag{9}$$

where $|a| \neq |b|$, and $|a|^2 + |b|^2 = 1$. Since this state degenerates with

$$|\phi_-\rangle = b^*\,|\phi_L\rangle - a^*\,|\phi_R\rangle\,, \tag{10}$$

a similar protective procedure is also needed to lift the degeneracy. For this general case, the results of the protective measurements of $|\phi_+\rangle$ and $|\phi_-\rangle$ will be obviously different: $\langle O\rangle_+ = |b|^2 - |a|^2$, $\langle O\rangle_- = |a|^2 - |b|^2$. Therefore, the non-protective measurement of $|\phi_L\rangle = a^*\,|\phi_+\rangle + b\,|\phi_-\rangle$ will lead to remarkable wavefunction collapse; its result will be either the result of the protective measurement of $|\phi_+\rangle$ with probability $|a|^2$ or the result of the protective measurement of $|\phi_-\rangle$ with probability $|b|^2$, and correspondingly the measured state $|\phi_L\rangle$ will collapse to one of these two states with the same probabilities. This analysis also confirms that the result of a non-protective measurement cannot reflect the measured state and indicate whether the measured particle is in a localized state or not due to the resulting wavefunction collapse.

To sum up, I have shown that Uffink's attempt to protect the interpretation of the wave function against protective measurements fails due to several errors in his arguments. However, I am still puzzled by his attitude towards the possible implications of protective measurements for the interpretation of the wave function, which may be shared by many people. It seems that the errors in Uffink's arguments were made at least partly due to his biased philosophical opinions. Why protect the interpretation of the wave function against protective measurements? Why make the different views on the meaning of the wave function peacefully coexist? Is it not very exciting and satisfying if we can decide the issue of the interpretation of the wave function someday? Is it not one of the ultimate objectives of our explorations in quantum foundations? In my opinion, a recent important work by Pusey, Barrett and Rudolph (2011) further strengthens our confidence in fathoming the meaning of the wave function, and protective measurement is probably the golden key to open the door to the real quantum world (Gao 2011a, 2011b, 2011c).

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