Probabilistic Justification and the Regress Problem

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Abstract
We discuss two objections that foundationalists have raised against infinite chains of probabilistic justification. We demonstrate that neither of the objections can be maintained.

Keywords
Probabilistic justification, regress problem, foundationalism, infinitism.

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Introduction

The definition of 'knowledge' as 'justified true belief' is part and parcel of many a textbook in epistemology, even though Gettier-like counterexamples date back to Plato's *Theaetetus*. The existence of counterexamples is however not the only problem that the definition faces. It also runs into the Regress Problem: if my belief in a true proposition S_0 is justified by my belief in another true proposition, S_1, which in turn is justified by my belief in still another true proposition S_2, and so on, *ad infinitum*, then I have no epistemic justification whatsoever for a belief in *any* of the propositions $S_0, S_1, S_2, \ldots$ etc.

One might try to take the sting out of the Regress Problem by toning down the definition to a more modest version, one in which we content ourselves with probabilistic support rather than full-blown justification and where we are satisfied with degree of belief rather than belief *tout court*. But this move clearly does not help us, for a probabilistic variant of the Regress Problem is readily made: if I partially believe S_0 because S_0 is probabilistically supported by S_1, and partially believe S_1 because it is probabilistically supported by S_2, and so on, then it seems that we are not even partially justified in believing *any* of the propositions $S_0, S_1, S_2, \ldots$ etc.

Although the Regress Problem is a threat to both foundationalists and coherentists, we will concentrate on its significance for the former. As is well known, foundationalists hold that every process of inferential or conditional justification must come to an end in a basic proposition that is itself non-inferentially or immediately justified. Foundationalists of the old, non-probabilistic school claim that this basic proposition must be absolutely certain, and some of them maintain that it is even a priori. Contemporary foundationalists however mostly wear probabilistic colours, and they argue that the basic proposition need not amount to absolute certainty. It is enough if its unconditional probability can be established, and if it is such that it probabilistically supports another proposition p, which means that the conditional probability of p given that the basic proposition is true, is greater than the conditional probability of p given that the basic proposition is false. But no matter what their different views may be on basic propositions and on the nature of justification, both factions share the same *horror infinitatis*: the concept of an endless chain of inferential justifications not only bewilders the old-style foundationalists, but remains anathema for their probabilistic offspring as well.[1]

In this paper we focus on the probabilistic variant: we will deal with foundationalism and with the Regress Problem only insofar as they are clothed in probabilistic raiment. In particular, we will discuss two objections that probabilistic foundationalists have raised against infinite chains of probabilistic justification. We shall first recall the reply of Peter Klein to these objections, but then we will give a better reply, one which shows that neither of the objections hold water.

1. Two objections

When one looks at the plurality of objections that probabilistic foundationalists have raised against infinite chains of probabilistic justification, two in particular stand out. Each of them can in principle also be deployed by 'ordinary' foundationalists in arguing against 'ordinary' infinite chains, but here we are especially interested in the way they are used within a probabilistic setting. Both objections are explicitly discussed in the polemic between Carl Ginet and Peter Klein [2, p. 131–155], but they can be found at many other places in the literature.

[1] The distinction between probabilistic and non-probabilistic foundationalism corresponds to Bonjour's distinction between moderate and strong foundationalism [1, p. 26–30]

The first objection refers to our limited cognitive capacity and we will call it the *finite mind objection*. The idea is that we, with our finite minds, are unable to complete an infinite probabilistic chain:

> "Even if I could have an infinite number of beliefs, how would I ever know anything if knowledge required an infinite epistemic chain?" [3, p. 183].

> "... finite minds cannot complete an infinitely long chain of reasoning and so, if all justification were inferential, no-one would be justified in believing anything at all to any extent whatsoever" [4, p. 40]; [5, p. 2]; [6, p. 150].

The second objection implies that (partial or probabilistic) justification can never be created by inferences alone. We dub it the *transfer objection*, after its formulation by Carl Ginet:

> "Inference cannot *originate* justification, it can only *transfer* it from premises to conclusion. And so it cannot be that, if there actually occurs justification, it is all inferential. .... [T]here can be no justification to be transferred unless ultimately something else, something other than the inferential relation, does create justification" [7, p. 148] (italics by the author).

Here the thought is that propositions in an endless chain of probabilistic justifications are not *really* justified. The only way to justify them would be to end the chain with a proposition that is itself noninferentially justified. Since the chain in question is supposed to be probabilistic, this last proposition must be an unconditional probability statement of the form $P(S_{n+1})$, where it is understood that S_{n+1} probabilistically supports S_n, so that $P(S_n|S_{n+1}) > P(S_n|\neg S_{n+1})$. In an attempt to lend force to this objection, Ginet cites Jonathan Dancy:

> "Justification by inference is conditional justification only; [when we justify A by inferring it from B and C] A's justification is conditional upon the justification of B and C. But if all justification is conditional in this sense, then nothing can be shown to be actually, non-conditionally justified." [8, p. 55].

So in Dancy's view, a proposition is actually or really justified if and only if its justification is in the end non-conditional. In the context of probabilistic justification, this view implies that a proposition S_0 is only really justified if in the end we can calculate its non-conditional probability, $P(S_0)$. But if $P(S_0)$ were to depend on an infinite chain of conditional probabilities of the form $P(S_n|S_{n+1})$, then, according to Dancy, there is no non-conditional probability to be justified.

As a long standing supporter of epistemic infinitism, Peter Klein has repeatedly argued against the foundationalists' dislike of infinite chains [9], [10], [11], [12], [13]. In his view there is nothing troublesome with infinite regresses, the reason being that an infinite chain of reasoning need not be completed. What is more, the requirement that an infinite chain must be completed "would be tantamount to rejecting infinitism" [9, p. 920]. The only thing that an infinitist requires, Klein argues, is that for every proposition S_n in the probabilistic chain, there is a proposition S_{n+1} such that the conditional probability $P(S_n|S_{n+1})$ is known to be greater than $P(S_n|\neg S_{n+1})$, for *that* is the condition under which we can properly say that S_{n+1} probabilistically supports S_n.

As Klein sees it, the Regress Problem is only a problem because it has been given a faulty formulation. Once we phrase it correctly, the problem loses its force. Consider again our infinite chain of propositions $S_0, S_1, S_2, \ldots$ etc., where each S_n is probabilistically justified by S_{n+1}. If we now were to ask how a belief in *any* of those propositions can be justified, we would be asking too much. What we should ask is how a belief in *each* of those propositions can be justified, and the answer to this question is simple: the justification comes from the proposition one step up in the chain [11, p. 729]. Claiming that a particular proposition S_n is (partially or probabilistically) justified means no more than being able to point to another proposition S_{n+1} that bestows the support in question, and that in turn receives support from still another proposition. This, and nothing more, is what infinitism claims.[2]

Given these views, it is clear what Klein's replies to the two objections are. His reply to the finite mind objection is that we need not, and should not, attempt to complete an infinite chain: any completion of such a chain would fly in the face of infinitism. As to the transfer objection, Klein's reply is plainly to deny that real justification implies unconditional justification, or

[2]Actually there is rather more. At several places Klein intimates that *all* justifications have this structure, and that might be questionable.

that an infinite chain of conditional probabilities can culminate in a particular value for an unconditional probability. In Klein's view,

> "Infinitism ... depicts justification as emerging when the set of propositions that are appropriately adduced as reasons expands. Of course, were the foundationalist to insist on thinking of warrant as originating in some propositions and then being transferred by inference to other propositions, he or she would be begging the question at hand. For it is this very concept of warrant that infinitism is challenging." [13, p. 152].

Klein's replies to the two objections are interesting, but in our view he concedes too much. He grants the foundationalists that finite minds cannot complete infinite chains and he also grants them that, were 'real warrant' the same as 'unconditional warrant' or as unconditional probability, justification on the basis of an infinite epistemic chain would be impossible. Neither of these concessions is necessary, as will be explained in the next section.

2. The objections refuted

Consider once more our chain $S_0, S_1, S_2, \ldots$ etc., where each S_{n+1} probabilistically justifies S_n. Are we able to justify S_0, in the sense that we can compute $P(S_0)$? As we have seen, Audi and Fumerton deny that we can by referring to the finite mind objection; Ginet and Dancy deny the same on the basis of the transfer objection. Below we propose to deal with both objections, starting with the latter.

Dancy's text makes it clear that there is a striking similarity between the transfer objection and C.I. Lewis's reasons for claiming that an endless epistemic chain does not make sense [8, p. 57]. It is true that Lewis is an old style foundationalist, but his ideas are easily adapted to foundationalism of probabilistic stripe.[3] What Dancy does not say, however, is that Lewis's critique of infinite epistemic chains is ambiguous or even incoherent, and that

[3]For example, consider: "The supposition that the probability of anything whatever always depends on something which is only probable itself, is flatly incompatible with the assignment of any probability at all." [14, p. 173]. This is easily probabilized to: "The supposition that the *unconditional* probability of anything whatever always depends on something which is only *conditionally* probable itself, is flatly incompatible with the assignment of any *unconditional* probability at all."

the incoherence may be present in one and the same article (for example in [14, p. 173]). On the one hand Lewis suggests that, where any regress of probability values is involved, the value of $P(S_0)$ cannot be computed. On the other hand he claims that, under these circumstances, the value of $P(S_0)$ will always be zero.

It seems to us that the transfer objection suffers from a similar ambiguity. Consider Dancy's phrase "if all justification is conditional, then nothing can be shown to be actually, non-conditionally justified". What does this phrase mean for the value of $P(S_0)$, if this value were to be determined by an infinite chain of conditional probabilities? Does it mean that it is impossible to say what exactly this value is? Or does it mean that the value becomes zero? Under the first interpretation, the transfer objection seems to boil down to the finite mind objection; for the latter assumes that $P(S_0)$ *has* a definite value, but that our tiny brains lack the equipment to compute it. Under the second interpretation, on the other hand, the transfer objection once was supported by no one less than Bertrand Russell. Russell argued that, in the case at hand, the value of $P(S_0)$ is given by the product

$$P(S_0) = P(S_0|S_1) \times P(S_1|S_2) \times P(S_2|S_3) \times \ldots \text{and so on, } \textit{ad infinitum}.$$

Since all the factors in this product are less than one, Russell concludes that $P(S_0)$ "may be expected to be zero" [15, p. 434]. However, as Hans Reichenbach first pointed out in a letter to Russell on March 28, 1949, this argument is flawed. For it is simply not true that the value of $P(S_0)$ is given by the product above. Rather $P(S_0)$ is given by the rule of total probability

$$P(S_0) = P(S_0|S_1)P(S_1) + P(S_0|\neg S_1)P(\neg S_1), \tag{1}$$

where $P(S_1)$ is

$$P(S_1) = P(S_1|S_2)P(S_2) + P(S_1|\neg S_2)P(\neg S_2), \tag{2}$$

and $P(\neg S_1)$ is

$$P(\neg S_1) = P(\neg S_1|S_2)P(S_2) + P(\neg S_1|\neg S_2)P(\neg S_2), \tag{3}$$

and so on, *ad infinitum*. Since (1)–(3) are sums rather than products, the fact that all the terms in these equations are less than one does *not* imply that the outcome generally tends to zero. Two weeks after having received Reichenbach's letter Russell replied, admitting his error.[4]

One may accept that $P(S_0)$ need not be zero, but still think that it cannot be computed. For how should we compute an unconditional probability if the only thing we know is an infinite number of conditional probabilities? This question takes us back to the finite mind objection. In the rest of this paper, we nullify this objection by actually completing a particular infinite probabilistic chain.

In our chain of propositions S_n, with $n = 0, 1, 2, \dots$ etc., it is supposed that the conditional probabilities α_n and β_n

$$\begin{aligned} \alpha_n &= P(S_n|S_{n+1}) \\ \beta_n &= P(S_n|\neg S_{n+1}) \end{aligned}$$

are known for all n. We may write

$$P(S_n) = \alpha_n P(S_{n+1}) + \beta_n P(\neg S_{n+1})\,, \tag{4}$$

where the unconditional probabilities $P(S_n)$ and $P(\neg S_n)$ are unknown. If we now write $P(\neg S_{n+1})$ as $1 - P(S_{n+1})$ and introduce the abbreviation

$$\gamma_n = \alpha_n - \beta_n = P(S_n|S_{n+1}) - P(S_n|\neg S_{n+1})\,,$$

then we can rewrite Eq.(4) as

$$P(S_n) = \alpha_n P(S_{n+1}) + \beta_n - \beta_n P(S_{n+1}) = \beta_n + \gamma_n P(S_{n+1})\,. \tag{5}$$

In order to calculate $P(S_0)$, we can now simply iterate (5), obtaining

$$P(S_0) = \beta_0 + \gamma_0\beta_1 + \gamma_0\gamma_1\beta_2 + \dots + \gamma_0\gamma_1 \dots \gamma_{n-1}\beta_n + \gamma_0\gamma_1 \dots \gamma_n P(S_{n+1})\,. \tag{6}$$

If $P(S_{n+1})$ were known, (6) could be used to calculate $P(S_0)$. But the difficulty is of course that $P(S_{n+1})$ is not known. In contradistinction to the first $n+1$ terms of (6), which are all conditional probabilities and therefore known, the last term in (6), like the last term in (5), contains the unknown

[4] "... you are right as to the mathematical error that I committed ..." (Russell to Reichenbach in a letter of April 22, 1949). We thank Carl Spadoni of the Mills Memorial Library, McMaster University (Hamilton, Canada), for sending us a copy of the letter. C.I. Lewis remained unconvinced, however. See [16] for a further analysis of the debate between Lewis, Reichenbach and Russell.

unconditional probability $P(S_{n+1})$. So it would seem that we have come to a dead end, and that old and new foundationalists have a point after all. The old ones claim that the iteration can only be terminated by a supposed certainty, $P(S_{n+1}) = 1$; the new ones say that a smaller value will serve, e.g. $P(S_{n+1}) = \frac{1}{4}$, but both insist that a definite value of $P(S_{n+1})$ is necessary to enable one to calculate $P(S_0)$.

However, here is a counterexample that demonstrates the falsity of these foundationalist claims. Elsewhere one of us has given a counterexample based on a geometrical series, and one based on an exponential series [17]. The following example has the advantage that is much simpler than either of these, in that the series reduces in the end to the sum of two terms only. Take

$$\alpha_n = 1 - \frac{1}{n+3} \qquad\qquad \beta_n = \frac{1}{n+3}\,, \tag{7}$$

so that

$$\begin{aligned} \gamma_n \;=\; \alpha_n - \beta_n \;&=\; \left(1 - \tfrac{1}{n+3}\right) - \left(\tfrac{1}{n+3}\right) \\ &=\; \tfrac{n+3}{n+3} - \tfrac{2}{n+3} = \tfrac{n+1}{n+3}\,. \end{aligned} \tag{8}$$

In order to calculate (6), let us begin by evaluating parts of it, notably the last term and the penultimate one. The last term is $\gamma_0\gamma_1 \ldots \gamma_n P(S_{n+1})$ and the coefficient in this term can be computed as

$$\begin{aligned} \gamma_0\gamma_1 \ldots \gamma_{n-1}\gamma_n \;&=\; \tfrac{1}{3} \times \tfrac{2}{4} \times \tfrac{3}{5} \times \tfrac{4}{6} \times \ldots \times \tfrac{n-1}{n+1} \times \tfrac{n}{n+2} \times \tfrac{n+1}{n+3} \\ &=\; \tfrac{2}{(n+2)(n+3)}\,. \end{aligned} \tag{9}$$

This is a very compact result, thanks to the cancellations between numerators and denominators of the factors $3, 4, 5, \ldots, n-1, n, n+1$. The penultimate term in (6) can be written as

$$\begin{aligned} \gamma_0\gamma_1 \ldots \gamma_{n-1}\beta_n \;&=\; \tfrac{1}{3} \times \tfrac{2}{4} \times \ldots \times \tfrac{n-1}{n+1} \times \tfrac{n}{n+2} \times \tfrac{1}{n+3} \\ &=\; \tfrac{2}{(n+1)(n+2)(n+3)} \\ &=\; \tfrac{1}{n+1} - \tfrac{2}{n+2} + \tfrac{1}{n+3}\,. \end{aligned} \tag{10}$$

With the definition

$$\delta_n = \tfrac{1}{n+1} - \tfrac{1}{n+2}\,, \tag{11}$$

one can check that

$$\delta_n - \delta_{n+1} = \tfrac{1}{n+1} - \tfrac{2}{n+2} + \tfrac{1}{n+3}\,. \tag{12}$$

So from (10) and (12) it is clear that

$$\gamma_0\gamma_1\ldots\gamma_{n-1}\beta_n = \delta_n - \delta_{n+1}\,. \tag{13}$$

Plugging (9) and (13) into (6), we find

$$P(S_0) = \beta_0 + (\delta_1 - \delta_2) + (\delta_2 - \delta_3) + \ldots + (\delta_n - \delta_{n+1}) + \tfrac{2}{(n+2)(n+3)} P(S_{n+1}). \tag{14}$$

Since $\delta_2, \delta_3, \ldots \delta_n$ cancel out, (14) can be simplified to

$$P(S_0) = \beta_0 + \delta_1 - \delta_{n+1} + \frac{2P(S_{n+1})}{(n+2)(n+3)}. \tag{15}$$

In the limit that n tends to infinity, $(n+2)(n+3)$ in the denominator grows without bound. Consequently the last term in (15) vanishes, since $P(S_{n+1})$, whatever it is, cannot be greater than unity. Moreover, δ_{n+1} disappears in the limit, as can be seen from (11). On taking the limit in (15) we are left with

$$P(S_0) = \beta_0 + \delta_1 = \tfrac{1}{3} + \tfrac{1}{6} = \tfrac{1}{2}\,. \tag{16}$$

This example undermines all claims that an infinite regress of probabilities cannot make sense, or that it must always leads to zero, or that it can never be completed.

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