Why the De Broglie-Bohm Theory Goes Astray

Shan Gao*

December 18, 2011

Abstract

We show that the de Broglie-Bohm theory is inconsistent with the established parts of quantum mechanics concerning its physical content. According to the de Broglie-Bohm theory, the mass and charge of an electron are localized in a position where its Bohmian particle is. However, protective measurement implies that they are not localized in one position but distributed throughout space, and the mass and charge density of the electron in each position is proportional to the modulus square of its wave function there.

The de Broglie-Bohm theory is an alternative to standard quantum mechanics initially proposed by de Broglie (1928) and later developed by Bohm (1952). According to the theory, a complete realistic description of a quantum system is provided by the configuration defined by the positions of its particles together with its wave function. The wave function follows the linear Schrödinger equation and never collapses. The particles, called Bohmian particles, are guided by the wave function via the guiding equation. Although the de Broglie-Bohm theory is mathematically equivalent to standard quantum mechanics, there is no clear consensus with regard to its physical interpretation. In this paper, we will show that the de Broglie-Bohm theory is inconsistent with the established parts of quantum mechanics concerning the mass and charge distributions of a single quantum system such as an electron.

Let's first see how the mass and charge of an electron distribute according to the de Broglie-Bohm theory. In the minimum formulation of the theory, which is usually called Bohmian mechanics (Goldstein 2009), the guiding equation for the Bohmian particle of a one-particle system with mass m and charge e in the presence of an external electromagnetic field is[1]

$$m\frac{d\boldsymbol{x}}{dt}=\hbar\Im[\frac{\nabla\psi_t}{\psi_t}]-e\boldsymbol{A}(\boldsymbol{x},t), \tag{1}$$

where $\boldsymbol{x}$ is the position of the Bohmian particle, ψ_t is the wave function of the system that obeys the Schrödinger equation, $\boldsymbol{A}(\boldsymbol{x},t)$ is the magnetic vector potential in position $\boldsymbol{x}$. According to this equation, the motion of a Bohmian

*Unit for HPS and Centre for Time, University of Sydney, NSW 2006, Australia. E-mail: sgao7319@uni.sydney.edu.au.

[1]Note that this guiding equation applies only for spin 0 particles, and for spin 1/2 particles there is also a spin-dependent term (Holland and Philippidis 2003).

particle is not only guided by the wave function, but also influenced by the external vector potential $\boldsymbol{A}(\boldsymbol{x},t)$. The existence of the term $e\boldsymbol{A}(\boldsymbol{x},t)$ in the guiding equation indicates that the Bohmian particle has charge e localized in its position. Therefore, according to the theory, the charge of a single quantum system such as an electron is an attribute of its Bohmian particle, and it is localized in the position of the Bohmian particle. For example, in the ground state of a hydrogen atom, the charge of the electron in the atom is localized in a random position near the nucleus where the Bohmian particle of the electron is at rest.

That the Bohmian particle of a one-particle system has the mass and charge of the system can be seen more clearly from the quantum potential formulation of the de Broglie-Bohm theory. By differentiating both sides of Eq. (1) relative to time and including an external gravitational potential V_G, we obtain

$$m\frac{d\dot{\boldsymbol{x}}}{dt} = -\nabla Q - m\nabla V_G - e[\nabla A_0 + \frac{\partial \boldsymbol{A}}{\partial t} - \dot{\boldsymbol{x}} \times (\nabla \times \boldsymbol{A})], \tag{2}$$

where $\frac{d}{dt} = \frac{\partial}{\partial t} + \dot{\boldsymbol{x}} \cdot \nabla$, A_0 is the electric scalar potential, and $Q = -\frac{\hbar^2}{2m}\frac{\nabla^2|\psi_t|}{|\psi_t|}$ is the so-called quantum potential. The gravitational interaction term $-m\nabla V_G$ indicates that the Bohmian particle has mass m, and the electromagnetic interaction term $-e[\nabla A_0 + \frac{\partial \boldsymbol{A}}{\partial t} - \dot{\boldsymbol{x}} \times (\nabla \times \boldsymbol{A})]$ indicates that the Bohmian particle also has charge e. Moreover, the mass and charge of the Bohmian particle are localized in its position.

The question is whether the mass and charge of a single quantum system such as an electron really distribute only in one position as the de Broglie-Bohm theory claims. In the following, we will show that protective measurement (Aharonov and Vaidman 1993; Aharonov, Anandan and Vaidman 1993; Aharonov, Anandan and Vaidman 1996; Vaidman 2009), whose principle is based on the established parts of quantum mechanics, gives a negative answer to this question.

Like the conventional impulse measurement, protective measurement also uses the standard measuring procedure, but with a weak, adiabatic coupling and an appropriate protection. Its general method is to let the measured system be in a nondegenerate eigenstate of the whole Hamiltonian using a suitable protective interaction (in some situations the protection is provided by the measured system itself), and then make the measurement adiabatically so that the state of the system neither changes nor becomes entangled with the measuring device appreciably. In this way, such protective measurements can measure the expectation values of observables on a single quantum system. Since the principle of protective measurement is irrelevant to the controversial process of wavefunction collapse and only based on the linear Schrödinger evolution and the Born rule, its results can be used to examine the alternatives to standard quantum mechanics such as the de Broglie-Bohm theory.

According to protective measurement, the mass and charge density of a quantum system, as well as its wave function, can be measured as expectation values of certain observables. For example, a protective measurement of the flux of the electric field of a charged quantum system out of a certain region will yield the expectation value of its charge inside this region, namely the integral of its charge density over this region. Similarly, the mass density of a quantum system can also be measured by a protective measurement of the flux of its

gravitational field in principle. Here is a simple example. Consider a quantum system in a discrete nondegenerate energy eigenstate $\psi(x)$. Let the measured observable A_n be (normalized) projection operators on small spatial regions V_n having volume v_n:

$$A_n = \begin{cases} \frac{1}{v_n}, & \text{if } x \in V_n, \\ 0, & \text{if } x \notin V_n. \end{cases} \tag{3}$$

The protective measurement of A_n yields

$$\langle A_n \rangle = \frac{1}{v_n} \int_{V_n} |\psi(x)|^2 dv = |\psi_n|^2, \tag{4}$$

where $|\psi_n|^2$ is the average of the density $\rho(x) = |\psi(x)|^2$ over the small region V_n. Then when $v_n \to 0$ and after performing measurements in sufficiently many regions V_n we can measure $\rho(x)$ everywhere in space. When the observable A_n and the corresponding interaction Hamiltonian are physically realized by the electromagnetic or gravitational interaction between the measured system and the measuring device, what the above protective measurement measures is just the charge or mass density of the quantum system, and its result indicates that the mass and charge density of the system in each position x is proportional to the modulus square of its wave function there, namely the density $\rho(x)$. In the Appendix, we give a concrete example to illustrate this important result (see also Gao 2011a, 2011b).

If an electron indeed has only one basic charge as usually thought, then the above result of protective measurement will have already refuted the de Broglie-Bohm theory. The guiding equation in the theory requires that the mass and charge of an electron are localized in a position where its Bohmian particle is. But protective measurement shows that they are not localized in one position but distributed throughout space, and the mass and charge density in each position is proportional to the modulus square of the wave function of the electron there.

In fact, there is already evidence indicating that the physical explanation of the guiding equation imposed by the de Broglie-Bohm theory is improper. To begin with, the guiding equation is only a mathematical transformation of the relation between the density ρ and the flux density $\mathbf{j}$ for the wave function; the relation is $\mathbf{j} = \rho\mathbf{v}$, while the guiding equation is $\mathbf{v} = \mathbf{j}/\rho$. Since the wave function is not merely a probability amplitude for the predictions of measurement results, but also a realistic description of the physical state of a quantum system as the results of protective measurement imply[2], the guiding equation already has a physical explanation relating only to the realistic wave function. Inasmuch as a mathematical equation in a physical theory has a unique physical explanation, the additional explanation of the guiding equation relating to the Bohmian particles will be improper. This further implies that the Bohmian particles, whose existence is assumed by the de Broglie-Bohm theory, do not really exist.

Next, explaining the guiding equation as a description of the motion of the Bohmian particles will lead to various inconsistencies. For example, in Bohmian mechanics, a charged Bohmian particle responds not to the electric scalar potential, but only to the magnetic vector potential, and it has no gravitational mass

[2]This is also admitted by most explanations of the de Broglie-Bohm theory.

but only inertial mass. Although these inconsistencies disappear in the quantum potential formulation, there are still other problems. For instance, although the charged Bohmian particles respond to external electromagnetic potential, they don't have electromagnetic interaction with each other. Moreover, the charged Bohmian particles of a quantum system do not have any electromagnetic influence on other charged quantum systems. Note that these apparent inconsistencies do not exist for the explanation of the guiding equation relating only to the wave function; the evolution of the wave function of a charged quantum system is influenced by both electric scalar potential and magnetic vector potential, and the wave functions of two charged quantum systems also have electromagnetic interaction with each other.

If one would like to bite the bullet, one may even assume that an electron has two basic charges: one is for its wave function, and the other is for its Bohmian particle. However, this bizarre picture will introduce more problems. First of all, there is a dilemma concerning the electromagnetic interaction between the wave function and the Bohmian particle of an electron. If they do have usual electromagnetic interaction, then the theory will be inconsistent with quantum mechanics and experiments. If they have no electromagnetic interaction, then this will add more inconsistencies for the theory. Next, there is a further problem about the superposition of the two charges in space. For example, for an electron in the state $\delta(x-x_0)$ at a given instant, there are two basic charges being in the same position x_0. Then how can each charge "know" whether it belongs to the wave function or the Bohmian particle? If it belongs to the wave function, it will respond to both electric scalar potential and magnetic vector potential, and it will also influence another charged quantum system; while if it belongs to the Bohmian particle, it will respond only to magnetic vector potential (according to Bohmian mechanics), and it will not influence another charged quantum system. Moreover, the superposition of two charges in one position seems also inconsistent with the impenetrability of particles.

Lastly, we note that the improperness of the de Broglie-Bohm theory can be seen more clearly from the meaning of the wave function[3]. It has been argued that the meaning of the wave function can be derived based on the established parts of quantum mechanics, independent of the alternatives to standard quantum mechanics such as the de Broglie-Bohm theory (Gao 2011a, 2011b). It turns out that the wave function describes the state of random discontinuous motion of particles, and at a deeper level, it represents the intrinsic property of the particles that determines their random discontinuous motion. In particular, the modulus square of the wave function determines the probability density of the particles appearing in certain positions in space. Therefore, the wave function guides the motion of particles in a probabilistic way, rather than in a deterministic way as assumed by the de Broglie-Bohm theory.

In conclusion, we have shown that the de Broglie-Bohm theory is inconsistent with the established parts of quantum mechanics concerning its physical content, though they may give the same predictions of measurement results.

[3]Note that the interpretation of the wave function in the de Broglie-Bohm theory has been debated by its proponents. For example, the wave function has been regarded as a field similar to electromagnetic field (Bohm 1952), an active information field (Bohm and Hiley 1993), a field carrying energy and momentum (Holland 1993), a causal agent more abstract than ordinary fields (Valentini 1997), a component of physical law (Dürr, Goldstein and Zanghì 1997), and a dispositional property of Bohmian particles (Belot 2011) etc.

Appendix: Protective measurement of the charge distribution of a quantum system

Consider the spatial wave function of a single quantum system with negative charge Q (e.g. $Q = -e$):

$$\psi(x,t) = a\psi_1(x,t) + b\psi_2(x,t), \tag{5}$$

where $\psi_1(x,t)$ and $\psi_2(x,t)$ are two normalized wave functions respectively localized in their ground states in two small identical boxes 1 and 2, and $|a|^2+|b|^2 = 1$. An electron, which initial state is a small localized wave packet, is shot along a straight line near box 1 and perpendicular to the line of separation between the boxes. The electron is detected on a screen after passing by box 1. Suppose the separation between the boxes is large enough so that a charge Q in box 2 has no observable influence on the electron. Then if the system were in box 2, namely $|a|^2 = 0$, the trajectory of the electron wave packet would be a straight line as indicated by position "0" in Fig.1. By contrast, if the system were in box 1, namely $|a|^2 = 1$, the trajectory of the electron wave packet would be deviated by the electric field of the system by a maximum amount as indicated by position "Q" in Fig.1.

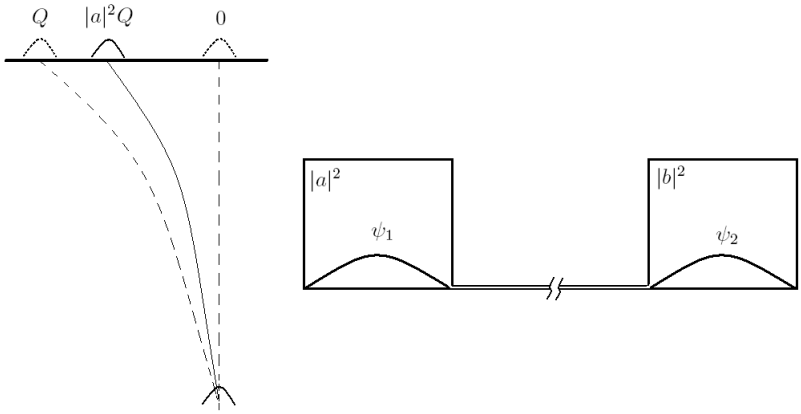

Fig.1 Scheme of a protective measurement of the charge density of a single quantum system

We first suppose that $\psi(x,t)$ is unprotected, then the wave function of the combined system after interaction will be

$$\psi(x,x',t) = a\varphi_1(x',t)\psi_1(x,t) + b\varphi_2(x',t)\psi_2(x,t), \tag{6}$$

where $\varphi_1(x',t)$ and $\varphi_2(x',t)$ are the wave functions of the electron influenced by the electric fields of the system in box 1 and box 2, respectively, the trajectory of $\varphi_1(x',t)$ is deviated by a maximum amount, and the trajectory of $\varphi_2(x',t)$ is not deviated and still a straight line. When the electron is detected on the screen, the above wave function will collapse to $\varphi_1(x',t)\psi_1(x,t)$ or $\varphi_2(x',t)\psi_2(x,t)$. As a result, the detected position of the electron will be either "Q" or "0" on the screen, indicating that the system is in box 1 or 2 *after* the detection. This is a conventional impulse measurement of the projection operator on the spatial

region of box 1, denoted by A_1. A_1 has two eigenstates corresponding to the system being in box 1 and 2, respectively, and the corresponding eigenvalues are 1 and 0, respectively. Since the measurement is accomplished through the electrostatic interaction between two charges, the measured observable A_1, when multiplied by the charge Q, is actually the observable for the charge of the system in box 1, and its eigenvalues are Q and 0, corresponding to the charge Q being in box 1 and 2, respectively. Such a measurement cannot tell us the charge distribution of the system in each box *before* the measurement.

Now let's make a protective measurement of A_1. Since $\psi(x,t)$ is degenerate with its orthogonal state $\psi^{'}(x,t) = b^*\psi_1(x,t) - a^*\psi_2(x,t)$, we need an artificial protection procedure to remove the degeneracy, e.g. joining the two boxes with a long tube whose diameter is small compared to the size of the box[4]. By this protection $\psi(x,t)$ will be a nondegenerate energy eigenstate. The adiabaticity condition and the weakly interacting condition, which are required for a protective measurement, can be further satisfied when assuming that (1) the measuring time of the electron is long compared to $\hbar/\Delta E$, where ΔE is the smallest of the energy differences between $\psi(x,t)$ and the other energy eigenstates, and (2) at all times the potential energy of interaction between the electron and the system is small compared to ΔE. Then the measurement of A_1 by means of the electron trajectory is a protective measurement, and the trajectory of the electron is determined by the expectation value of the charge of the system in box 1. In particular, when the size of box 1 can be ignored compared with the separation between it and the electron wave packet, the wave function of the electron will obey the following Schrödinger equation:

$$i\hbar\frac{\partial\psi(\vec{r},t)}{\partial t} = -\frac{\hbar^2}{2m_e}\nabla^2\psi(\vec{r},t) - k\frac{e\cdot|a|^2Q}{|\vec{r}-\vec{r_1}|}\psi(\vec{r},t), \tag{7}$$

where m_e is the mass of electron, k is the Coulomb constant, $\vec{r_1}$ is the position of the center of box 1, and $|a|^2Q$ is the expectation value of the charge Q in box 1. Correspondingly, the trajectory of the center of the electron wave packet, $\vec{r_c}(t)$, will satisfy the following equation by Ehrenfest's theorem:

$$m_e\frac{d^2\vec{r_c}}{dt^2} = -k\frac{e\cdot|a|^2Q}{|\vec{r_c}-\vec{r_1}|(\vec{r_c}-\vec{r_1})}. \tag{8}$$

Then the electron wave packet will reach the position "$|a|^2Q$" between "0" and "Q" on the screen as denoted in Fig.1. This shows that the result of the protective measurement is the expectation value of the charge Q in the state $\psi_1(x,t)$ in box 1, namely the integral of the charge density $Q|\psi(x)|^2$ in the region of box 1.

The result of the above protective measurement can tell us the charge distribution of the system in each box *before* the measurement. Suppose we can continuously change the measured state from $|a|^2 = 0$ to $|a|^2 = 1$ (and adjust the protective interaction correspondingly). When $|a|^2 = 0$, the single electron will reach the position "0" of the screen one by one, and it is incontrovertible that no charge is in box 1. When $|a|^2 = 1$, the single electron will reach the position "Q" of the screen one by one, and it is also incontrovertible that there is a charge Q in box 1. Then when $|a|^2$ assumes a numerical value between

[4]It is worth noting that the added protection procedure depends on the measured state, and different states need different protection procedures in general.

0 and 1 and the single electron reaches the position "$|a|^2Q$" between "0" and "Q" on the screen one by one, the result will similarly indicate that there is a charge $|a|^2Q$ in the box by continuity. The point is that the definite deviation of the trajectory of the electron will reflect that there exists a definite amount of charge in box 1.[5] Moreover, the above equation that determines the result of the protective measurement, namely Eq. (8), gives a more direct support for the existence of a charge $|a|^2Q$ in box 1. The r.h.s of Eq. (8) is the formula of the electric force between two charges located in different spatial regions. It is incontrovertible that e is the charge of the electron, and it exists in position $\vec{r}$. Then $|a|^2Q$ should be the other charge that exists in position $\vec{r_1}$. In other words, there exists a charge $|a|^2Q$ in box 1.

To sum up, protective measurement shows that the charge of a charged quantum system is distributed throughout space, and the charge density in each position is proportional to the modulus square of its wave function there. This conclusion is based on two established parts of quantum mechanics, namely the linear Schrödinger evolution and the Born rule. In the above example, the linear Schrödinger evolution determines the deviation of the electron wave packet, and the Born rule is needed to obtain the information about the center of the electron wave packet detected on the screen.

Acknowledgments

I am grateful to Antony Valentini and Hans Westman for helpful discussions. This work was supported by the Postgraduate Scholarship in Quantum Foundations provided by the Unit for HPS and Centre for Time (SOPHI) of the University of Sydney.

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