SSB: QSM vs. QFT

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Abstract: Philosophical analysis of spontaneous symmetry breaking (SSB) in particle physics has been hindered by the unavailability of rigorous formulations of models in quantum field theory (QFT). A strategy for addressing this problem is to use the rigorous models that have been constructed for SSB in quantum statistical mechanics (QSM) systems as a basis for drawing analogous conclusions about SSB in QFT. Based on an analysis of this strategy as an instance of the application of the same mathematical formalism to different domains and as an instance of drawing analogies between domains, I conclude that certain structural explanations can be exported from QSM to QFT, but that causal explanations cannot.

Key words: spontaneous symmetry breaking, quantum field theory, quantum statistical mechanics, applicability of mathematics, analogical reasoning

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1 Introduction

Spontaneous symmetry breaking (SSB) in particle physics is difficult to subject to philosophical analysis because of the many complications involved in formulating the theory. In all cases in particle physics, formulation of a theory of SSB requires quantization of a classical field theory to obtain a quantum field theory (QFT)[1]. In the particularly interesting case of the Higgs mechanism for the electroweak theory, the fact that the classical field theory is a non-Abelian gauge theory is both an enabling condition for obtaining a renormalizable QFT and a source of interpretive puzzles. Ideally, both philosophers and theoretical physicists would like a rigorous theory of SSB in the electroweak theory. Within the algebraic approach to QFT, SSB can be given a clear and simple characterization: "[a] symmetry α [of an algebra $\mathcal{U}$] is broken in the state ω on $\mathcal{U}$ just in case α is not unitarily implementable on ω's GNS representation" (Ruetsche 2011, 300). That is, the characteristic features of SSB are the inability to represent the algebraic symmetry α as a unitary operator on the Hilbert space representation associated with the state ω and the existence of unitarily inequivalent Hilbert space representations associated with algebraic states related by the algebraic symmetry α.[2]

The standard textbook presentations of the Higgs mechanism in the electroweak theory fall short of the ideal of a rigorous theory satisfying a set of algebraic axioms in a number of respects. First, Strocchi draws attention to the fact "[m]ost of the wisdom on [SSB], especially for elementary particle theory, relies on approximations and/or a perturbative expansion" (115). This is natural because, in order to establish the existence of a symmetry-breaking order parameter, renormalization–i.e., control of the vacuum expectation values–is required, and renormalization schemes often invoke approximations and perturbative expansions (Strocchi 2008, 127). Strocchi argues that a non-perturbative treatment is desirable because the standard perturbative treatment which employs semi-classical approximations "leaves some basic questions open": "it is known that mean field approximations are often not reliable and the results on the triviality of the φ^4 theory in four space time dimensions seem to indicate that the perturbative expansion, which might be, at best, an asymptotic expansion, may have little to do with the non-perturbative solution" (128). Second, as Earman stresses, standard presentations of the Higgs mechanism are not gauge-invariant. This poses an obstacle for interpreting the theory because it makes it difficult to disentangle the physical content from features of the mode of mathematical presentation which have no physical significance (Earman 2004). Third, Ruetsche points out that important underlying questions are which quantization strategies can be rigorously applied to gauge theories and, if more than one is applicable, which quantization strategy is correct (2011, 335). In particular, it is not clear that the Dirac quantization procedure is both applicable and correct. As she observes, this is not merely a technical or calculational problem, but a conceptual problem the resolution of which will require development of the physics, and not merely computational power or new mathematical techniques for solving equations (335). See Stoeltzner's paper in this volume for further exploration of these issues.

Thus, at present there are a number of outstanding questions about how to rigorously formu-

[1] Terminological note: in this paper "quantum field theory" refers to the theory by that name developed for application to particle physics. (This excludes, for example, usage of the term to refer to theories of condensed matter physics.)

[2] For background in the algebraic approach to QFT that is presupposed by this short paper, see Ruetsche 2011 or Liu and Emch 2005.

late SSB for QFTs for realistic systems in general and for the electroweak theory in particular. One strategy for trying to make progress on philosophical questions without being hampered by these unresolved issues is to shift attention to simpler cases in which at least some of these problems do not arise. For example, Struyve 2011 focuses on the Abelian Higgs model in classical field theory in order to evaluate the gauge-invariant content of the Higgs mechanism in this case. In this paper I will examine whether it is feasible to adopt the same strategy, but to shift attention to cases of SSB quantum statistical mechanics (QSM) rather than a classical Abelian field theory. The chief advantage of treatments of SSB in QSM over QFT is that there exist rigorous algebraic models of SSB for a number of realistic QSM systems. Of course, this strategy is only useful for addressing philosophical questions about SSB in QFT if we can be reasonably confident that elements of the analysis of SSB in QSM carry over to SSB in QFT. One piece of evidence that apparently supports confidence about this is the well-known fact that analogies between systems exhibiting SSB[3] in QSM–particularly the BCS theory of superconductivity–and systems in particle physics were the source of inspiration for the introduction of SSB in QFT in the first place. The primary goal of this paper is to assess the nature of the analogies drawn between SSB in QSM and SSB in QFT in order to determine what types of conclusions about SSB in QSM are likely to transfer over to SSB in QFT.

As a framework in which to conduct this assessment, it is helpful to consider how analysis of this case study falls under two broader topics in philosophy of science. One topic is the applicability of mathematics. At an informal level, theoretical physicists have established that the same piece of mathematical formalism can be used to describe SSB in QSM and SSB in QFT. More rigorously, the assumption adopted here for the purpose of making progress on philosophical issues raised by SSB in QFT is that the rigorous algebraic models for SSB in QSM systems could also be constructed for QFT systems. Roughly, regarding the situation as one in which the same mathematical formalism is applicable in two different physical domains produces a picture in which there is a shared mathematical formalism which is common to the two cases and which is given a different physical interpretation in QSM than it is in QFT. The second broader topic in philosophy of science under which this analysis falls is analogical reasoning. Historically, cases of SSB in QSM systems were regarded as helpful analogies for formulating QFTs in particle physics. Construing the case study as an instance of analogical reasoning produces a different picture of how the case works: mappings are made from elements of a QSM source model (a physically interpreted mathematical model) to elements of a QFT target model (also a physically interpreted mathematical model) which reflect similarities between the two cases. These two ways of conceiving of the case study are compatible and both are correct. Both pictures will prove useful for my project of analyzing the types of conclusions that can be exported from models of SSB in QSM to models of SSB in QFT. The applicability of mathematics picture is useful because it separates the similarities in mathematical structure from the differences in physical interpretation. The analogical reasoning picture is useful because it draws out the similarities between the physically interpreted models. The nature of these similarities is the crucial question for determining what sorts of conclusions can be safely transferred from the context of QSM to the context of QFT. (There is a further project of determining which elaborated accounts of the applicability of mathematics and analogical reasoning apply to SSB; however, these brief sketches

[3] I note that this use of the term "SSB" is anachronistic: the term "spontaneous symmetry breaking" was not introduced until after the connection to particle physics had been established.

suffice for the purposes of the analysis of the case study undertaken here.)

My thesis is that the applicability of the same mathematical formalism to treat SSB in QSM and QFT licenses the transfer of some conclusions about the formal structure of SSB from QSM to QFT, but that physical disanalogies undercut the transfer of conclusions about the causal structure of SSB from QSM to QFT. As a starting point for making the case for this thesis, I will draw on the very helpful analysis in Liu and Emch 2005. Liu and Emch defend the strong position that for an understanding of SSB in quantum theories "...it is neither necessary nor advisable to tackle the recondite cases [of SSB] in QFT... Theories and models of SSB in QSM provide just the right material for the purpose" (157). I will argue that this is the right view to take about what they label structural explanations, but not correct when it comes to causal explanations.

2 Structural explanation and the applicability of mathematics

The primary thesis defended in Liu and Emch 2005 is that the decompositional account of SSB furnishes a better understanding of SSB in quantum theories than the representational account of SSB. Briefly, both the representational and decompositional accounts of SSB in a quantum theory are formulated within the algebraic framework; the difference lies in the features that are taken to be the defining characteristics of SSB. The representational account (abbreviated [RA]) is limited to the features of SSB highlighted in Sec. 1 above: either the algebraic symmetry α is not unitarily implementable or fundamental states[4] related by α generate unitarily inequivalent representations of the algebra (139). The decompositional account [DA] for a QSM system gives a more sophisticated characterization of the fundamental states (using the KMS condition), which allows for the decomposition of the state and the introduction of a *witness*, which is a macroscopic observable that takes different values in the component states. (See Liu and Emch 2005, Secs 2 and 3 for details). For our purposes, the example of the [DA] for the Weiss-Heisenberg model for the ferromagnet will be sufficient (Liu and Emch 2005, 147-49). The ferromagnet is modeled as an infinite one-dimensional chain of quantum spins with long-range, Ising-type interactions. The symmetry α is the 'flip-flop' symmetry (180° rotation about the y-axis), a witness is the net magnetization in the z-direction, and the component states are a state of the infinite spin chain with all spins oriented in the $+z$ direction and a state with all spins oriented in the $-z$ direction. Liu and Emch show that, if a system satisfies the definition of SSB in [DA], then it also satisfies the definition of SSB in [RA] (145-46). At the present time, [DA] models have been rigorously constructed for some QSM systems; some QFT systems have [RA] models, but no [DA] model has been rigorously constructed for any QFT system (Liu and Emch 2005, 142). Furthermore, "there are simple and experimentally implementable models in QSM for [DA], while models in QFT for [RA] are either simple but unrealistic (i.e. not experimentally realizable) or realistic but complex (i.e. too complicated to be used to exhibit in controllable or simple terms what the account means)" (143). Given this situation, Liu and Emch advise that [DA] models for QSM provide a better guide for understanding SSB in QFT than do the extant representational models for QFT.

[4] i.e., equilibrium states in QSM and vacuum states in QFT (138).

Liu and Emch argue that there are two respects in which [DA] provides a superior explanation of SSB to [RA]. First, [DA] provides a clearer structural explanation of SSB than [RA] (154-55). A *structural* explanation of SSB is an explanation that "tells us under what conditions an SSB is possible"; in contrast, a *causal* explanation "tells us the mechanisms for actual breakings: how systems harboring SSB end up in those symmetry-breaking fundamental states" (152). Structural explanations are closely associated with the kinematics, while causal explanations are closely associated with the dynamics. The second respect in which [DA] is superior is that, while [DA] does not identify a specific causal mechanism for symmetry breaking (because this is a system-specific matter), [DA] is "more conducive" to supporting a causal explanation for symmetry breaking than [RA] (152-54).

The argument in support of the first claim is that [DA] provides a clearer structural explanation of SSB because, unlike the [RA], the account does not merely assert that characteristic features of SSB are that the automorphism is not unitarily implementable and fundamental states belong to unitarily inequivalent representations; [DA] explains why these features are associated with the physical phenomenon of spontaneously broken symmetry. For example, the [DA] model of the Weiss-Heisenberg ferromagnet supplies an explanation of under what (kinematical) conditions SSB is possible: when there is a non-trivial centre. Moreover, the [DA] model of the Weiss-Heisenberg ferromagnet contains the resources to explain why the broken symmetry (180° rotation about the y-axis) is not unitarily implementable and relates states belonging to unitarily inequivalent representations: the infinite spin-up state is physically inequivalent to the infinite spin-down state–as indicated, for example, by the fact that the witness global magnetization takes different values in the two states; "[i]f unitarily equivalent representations can be regarded as different mathematical descriptions of the same physical situation,"[5] then the representations generated by the infinite spin-up and infinite spin-down states should be unitarily inequivalent (154). Liu and Emch also stress that [DA] "restitutes the correct conceptual order of explanation (or understanding)," which gets reversed in [RA] (145). That is, the conditions that the automorphism is not unitarily implementable and that the fundamental states belong to unitarily inequivalent representations should not be taken to "define the concept" of SSB in quantum theories, which is their role in [RA], but should follow as consequences of the physical phenomenon of the spontaneous breaking of the symmetry (145). [DA] also supplies a better structural explanation than [RA] to address the following question: "[w]hy do quantum SSBs occur only in infinite systems?" (138). (See 140-41 for details).

These structural explanations rely on the mathematical structure of the [DA] model. That is, the fact that, for example, the [DA] model distinguishes a witness which marks a physical difference between two fundamental states which break the symmetry is relevant; further details about the physical interpretation of the witness in a given system are not relevant. (Contrast this situation to the causal explanations considered in the following section.) Consequently, if the same mathematical formalism–a [DA] model–is applicable in QSM and QFT, then these structural explanations can be offered in both contexts. However, the physical interpretations of the models can affect whether a particular structural explanation is called for or not. For example, the question about why SSBs only occur in infinite systems seems to demand an answer in the context of QSM, where the idealization of the thermodynamic limit is introduced into the model to obtain an infinite number of degrees of freedom, but is less pressing in QFT,

[5] See Chapter 3 of Ruetsche 2011 for a discussion of this assumption.

where continuous spacetime is a natural assumption. Also, physical interpretation can generate additional demands for structural explanations (e.g., in gauge theories).

If [DA] models can be rigorously constructed for both QSM and QFT, then structural explanations can be transferred from QSM to QFT. The satisfaction of the antecedent of this conditional is, of course, non-trivial, as Liu and Emch themselves acknowledge (142). However, while it is certainly an important question whether QFT systems of physical interest admit [DA] models and efforts should continue to be made to rigorously construct such models, the fact that this question is open does not undermine the usefulness of [DA] models as supplying candidate (i.e., provisional and defeasible) structural explanations for SSB in QFT. Recall, the obstacle to understanding SSB in QFT is that rigorous models for realistic QFT systems have not yet been obtained. Our strategy for dealing with this obstacle is to, when it is reasonable to do so, transfer conclusions drawn from the rigorous models of QSM, which do exist, to QFT. If rigorous [DA] models of QFT existed then we would not need to employ this strategy![6]

The second respect in which, according to Liu and Emch, [DA] provides a superior explanation of SSB to [RA] is that it is better suited to supporting a causal explanation of the phenomenon. Liu and Emch's argument for this claim rests on an explanation of how [DA] provides the materials for a generic causal explanation of SSB in the Weiss-Heisenberg ferromagnet and on the observation that, when causal explanations of SSB in QFT are attempted, they rely on an analogy to QSM systems (143, 153). Roughly, the causal explanation of SSB in the Weiss-Heisenberg ferromagnet which is accommodated by [DA] explains that when the temperature of the system is decreased below the critical value, spontaneous magnetization can occur because the tendency for the interactions to align the spins in the same direction is stronger than the tendency of thermal agitation to knock the spins out of alignment (153). [DA] includes the net magnetization–the witness–while [RA] does not.

Turning to QFT, the pertinent question is whether the fact that the [DA] model supports a causal explanation for QSM systems carries over to QFT systems: does the suitability of the [DA] model for causal explanations of QSM systems entail that a [DA] model is similarly well-suited to supporting causal explanations of QFT systems? As the argument given by Liu and Emch indicates, answering this question requires going beyond the consideration that the same mathematical formalism is applicable in both domains; what is needed is a careful examination of the physical similarities between QSM and QFT which motivate the analogy drawn between causal explanations in the two domains.

3 Causal explanation, analogies, and disanalogies

To evaluate the nature of the similarities between SSB in QSM and SSB in QFT, we will focus on the ferromagnet and a QFT with a scalar ϕ^4 self-interaction. Liu and Emch give the following sketch of the causal explanation of the spontaneous breaking of the symmetry afforded by the [DA] model for the system:

> Two opposing tendencies exist in a Weiss–Heisenberg system, namely, the one caused

[6] Admittedly, the question of whether there exists a [DA] model for QFT does get entangled with the physical interpretation. For example, one of the outstanding questions is whether there is an analogue of the KMS condition which is both mathematically and physically suitable (Liu and Emch 2005, 146; Ruetsche 2011, 303-04).

> by the interactions to align the spins in the same direction and the other caused by thermal agitation to randomize the directions of individual spins. The strength of the latter obviously depends on the system's temperature and the strength of the former depends on how many spins are already aligned in parts of the system. When the temperature is above the critical value, the balance of the strengths is in favor of thermal agitation. Any large fluctuation of aligned segments of spins will quickly disappear rather than growing larger. But when the temperature drops below the critical value, the balance tilts in favor of interactional alignment of spins that will tend to grow larger and eventually result in spontaneous magnetization, i.e. having an average net magnetization even when no external field is present. (153)

The quantities which play instrumental roles in this causal explanation are the temperature, the change in temperature (decreasing to critical value), fluctuations in the aligned segment of spins, and the average net magnetization. To indicate how the analogy is drawn to QFT, I will sketch how elements of the source domain (the model of the ferromagnet) are mapped to the target domain (the model of a scalar ϕ^4 QFT). Note that the analysis that follows is based on a standard textbook account, so mathematically informal models will take the place of the rigorous algebraic models. This is in keeping with Liu and Emch's appeal to this literature in their argument (143, 153). As with any analogy, the mappings pick out respects in which the systems are similar, and there are other respects in which the systems are dissimilar. I will highlight two respects in which the systems are physically dissimilar that are relevant to providing a causal explanation. These dissimilarities suggest that the causal explanation of the ferromagnet cannot be straightforwardly transferred to particle physics. The following sketch is based on Peskin and Schroeder 2005, Sec. 11.3.

In the early 1960's, the analogy with SSB in QSM was exploited in order to determine the vacuum states for QFTs with spontaneous symmetry breaking. As Jona-Lasinio, one of the physicists who first made the analogy to particle physics, explains, one of the keys was recognizing that the vacuum states for QFT could be determined by applying analogues of the variational principles used to determine the stable states in QSM (Jona-Lasinio 2002, 146-47). The starting point for the mapping is the following equation:

$$\Gamma_{n,m} = \varsigma^2 D_m(-in\tau) \text{ where } \varsigma \text{ is a scale factor} \tag{1}$$

(Wilson and Kogut 1974, 150). On the lefthandside, $\Gamma_{n,m}$ is a spin-spin correlation function for the ferromagnet on a spatial lattice. On the righthandside, $D_m(-in\tau)$ is a propagator for the quantum field on a spatial lattice. (Crudely interpreted, the propagator represents the amplitude for a free particle to propagate between spatial locations $x = 0$ and $x = ma$, where a is the lattice spacing (Peskin and Schroeder 1995, 92).) Note that, on the QSM side, the thermodynamic limit has already been taken. (That is, the number of particles and the volume have both been taken to infinity.) On the QFT side, the time variable has been Wick rotated: $t \to -i\tau$.[7]

The variational analysis proceeds by considering the generating functionals for the expressions on each side of the equation. (Following Peskin and Schroeder, we will now assume that the spin

[7] As an aside, this reflects the fact that the QSM system is in Euclidean space and the QFT system is in Minkowski space. This physical difference between the systems does not play any role in the analysis that follows (cf. Liu and Emch 2005, 143).

field $s(x)$ and quantum field $\phi(x)$ are defined on continuous space(-time). This will become important below.) On the QSM side, the generating functional of the correlation function is the partition function, $Z(H)$:

$$Z(H) = \int \mathcal{D}s \exp\left[-\beta \int d^3x \left(\mathcal{H}[s] - Hs(x)\right)\right] \tag{2}$$

where $\beta = \frac{1}{kT}$ is the inverse temperature, $\mathcal{H}[s]$ is the spin energy density, and H is an external magnetic field. For comparison, in QFT the generating functional of the propagator is

$$Z[J] = \int \mathcal{D}\phi \exp\left[i \int d^4x \left(\mathcal{L}[\phi] + J\phi\right)\right] \tag{3}$$

where $\mathcal{L}[\phi]$ is the Lagrangian and $J(x)$ is an external source. Comparing these generating functionals, $\mathcal{H}[s]$ and $\mathcal{L}[\phi]$ play analogous roles as do H and $J(x)$.

In QSM, the thermodynamically most stable states are found by rewriting $Z(H)$ as an exponential of the Helmholtz free energy, performing a sequence of differentiations, and finally setting $H = 0$ in the resulting extremum equation and solving it. The vacuum states in the QFT are found by working through the analogues of each of these steps. For our purposes of tracing the aspects of the analogy relevant to the causal explanation of spontaneously broken symmetry, what is noteworthy about these analogue derivations is that the net magnetization–$M \equiv \int d^3x \langle s(x) \rangle$–is defined in the course of taking the sequence of differentiations; there is an analogue quantity–$\Phi(x) \equiv -\langle \Omega | \phi(x) | \Omega \rangle_J$–in the QFT derivation. Also, in both cases an effective potential function can be derived. (In the QSM case, this potential function is the Gibbs energy.) The minimization of the potential function picks out the fundamental state in both cases, including all fluctuations. In QSM, the fundamental state is the equilibrium state and the fluctuations are thermal fluctuations; in QFT, the fundamental state is the vacuum state and the fluctuations are quantum fluctuations.

There are two physical dissimilarities apparent in this set of mappings between the QSM system and the QFT system which are relevant to causal explanation. First, equation (1) maps a QSM system on d-dimensional space to a QFT system on d-dimensional space*time*. For causation, time is a privileged parameter. The fact that time is playing disanalogous roles in the two derivations provides a strong indication that the account of causal explanation that we get for the QSM system cannot be straightforwardly carried over to the QFT system using this analogy. The mapping of space to spacetime has consequences that cascade through the whole analogue derivation. The analogue of the spin-energy density $\mathcal{H}[s]$ is not the energy density of the quantum field, but the Lagrangian $\mathcal{L}[\phi]$. The disanalogy between space and spacetime does not affect the variational analysis used to determine the vacuum states–which is the goal of the analogue derivation–but it does weigh against the assumption that the causal structure of the QSM system is mirrored by the QFT system.

The second physical dissimilarity pertains to the role of temperature. In the analogue derivation for QFT sketched above, there is no quantity that gets mapped to the inverse temperature β. (Compare equations (2) and (3)) Already, this seems problematic for the prospects of carrying the causal explanation for the QSM system over to the QFT system because the decrease in the temperature of the ferromagnet to the critical value plays a crucial role in the causal explanation of symmetry breaking for this system. The analogue of temperature for the QFT system is revealed by considering equation (1) from a different point of view. In practice, in QFT it is difficult calculate the propagator $D(x,t)$ on continuous spacetime. Renormalization group methods furnish a strategy for gaining control of the divergences: start with the propagator on a spatial lattice (or else impose a high momentum cutoff) then exploit the connection with the QSM system set out in (1) to remove the cutoff. When this strategy gets carried out for the scalar ϕ^4 QFT system, taking $T \to T_c$ (where T_c is the critical value) corresponds to taking $\mu_0 \to \mu_{0c}$ (where μ_0 is the bare mass and μ_{0c} is the "critical" value of bare mass which permits the high momentum cutoff to be taken to the infinite limit while the renormalized mass remains finite). (For the details, see a contemporary QFT textbook, such as Peskin and Schroeder 1995, Chapter 12, or Wilson and Kogut 1974. For a rigorous treatment, see Glimm and Jaffe 1987.) There are a number of relevant physical differences between $T \to T_c$ and $\mu_0 \to \mu_{0c}$. For $T \to T_c$, there is a single physical system–the ferromagnet–that undergoes a physical process described by a sequence of states with different temperatures. The temperature–and other properties of the system–are experimentally accessible. In contrast, $\mu_0 \to \mu_{0c}$ does not represent a physical process in which a single system is in a sequence of distinct physical states. The bare mass μ_0 is not an experimentally accessible quantity; the renormalized mass corresponds to the measured quantity. If different values of μ_0 represent anything physical, they represent the masses of different physical systems–or perhaps the values of mass that a single system would have in different possible worlds (cf. Strocchi 2008, Liu and Emch 2005, 154-55). However, given that the purpose of the construction is to control the divergences that would otherwise arise, another plausible interpretation of $\mu_0 \to \mu_{0c}$ is as not representing anything physical at all, but as a mathematical device for achieving the calculational aim of deriving (finite) values for the vacuum expectation values.

Regardless of which of these interpretations of $\mu_0 \to \mu_{0c}$ one favours, there are significant dissimilarities between $T \to T_c$ and $\mu_0 \to \mu_{0c}$ which undermine the attempt to base a causal explanation for spontaneously broken symmetry on μ_0 in analogy to the explanation for the QSM system based on T. A decrease in T towards the critical value has the effect of weakening the thermal agitation; spontaneous magnetization can occur at the critical value, when the thermal agitation is no longer strong enough to disrupt the tendency of the interactions to align the spins. At a minimum, for it to be possible to offer this causal explanation, there must be a single physical system which can occupy states at different temperatures. The analogue of this minimal condition for μ_0 does not hold in the QFT model. Furthermore, the causal explanation in QSM is compelling because it can be supported by experiments. T is a control parameter that experimenters can vary and there is plenty of experimental data about how the system behaves at different temperatures. Again, this is not the case for μ_0 in the QFT model. (More generally, there is even *more* experimental data available to support QSM models of critical phenomena because a wide variety of systems behave in similar ways–e.g., possess nearly the same values of their critical exponents–as they approach their critical points.)

4 Conclusion

Given the difficulty of formulating a rigorous theory of SSB in QFT, exploiting the analogy between QSM and QFT by transferring conclusions gleaned from rigorous QSM models of SSB to QFT seems like a promising strategy for making progress on philosophical issues surrounding

SSB in QFT. I have argued that we can be reasonably confident that the [DA] model of SSB in QSM provides a guide for understanding some of the structural aspects of SSB in QFT, but that we have reason to doubt that even the rudiments of causal explanations of SSB in QSM can be exported to QFT. The presumed applicability of the same mathematical formalism–the [DA] model of SSB–to both QSM and QFT systems licenses the exportation of structural explanations from QSM to QFT. I have also argued that physical dissimilarities between the models of SSB in QSM and QFT constitute grounds for skepticism about whether causal explanations of the sort sustained by QSM models can also be sustained by QFT models. Both the mapping of space in QSM to spacetime in QFT and the mapping of T to μ_0 are indications that causal relationships are not preserved by the analogy between SSB in QSM and SSB in QFT.

The upshot of these arguments is that philosophers should be careful to use rigorous models of SSB in QSM as guides to SSB in QFT only for the appropriate philosophical ends. Quite possibly, achieving a better understanding of SSB in gauge theories counts as an appropriate use of QSM models according to my argument; QSM models could be used–provisionally, defeasibly–for the purpose of addressing mathematical or formal questions.

One might worry that in denying that causal explanations carry over from QSM to QFT, my analysis fails to account for the uses to which physicists put the analogy between SSB in QSM and SSB in QFT. As Liu and Emch remark, "attributing the causal mechanism of SSB in a gauge field to a correlation or coherence of phases within a vacuum state is helpful when one recalls what actually happens in a phase transition in a thermo-system, but not so helpful without such an analogy" (153). However, the fact that particle physicists can successfully work with the more intuitive picture of SSB that goes along with QSM systems, even though it does not describe the causal structure of QFT systems, can be explained by the applicability of the same mathematical formalism in the two theories. In the same way that Maxwell used his mechanical ether models as intuitive devices to help him formulate his theory of electromagnetism even though electromagnetic phenomena are not so produced, contemporary particle physicists can use the picture of how symmetry is spontaneously broken which is given by QSM models to help them formulate a theory of SSB in QFT even though it seems that spontaneously broken symmetries in QFT are not so produced.

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