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Quantum decoherence in a pragmatist view: Part I

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Abstract The quantum theory of decoherence plays an important role in a pragmatist interpretation of quantum theory. It governs the descriptive content of claims about values of physical magnitudes and offers advice on when to use quantum probabilities as a guide to their truth. The content of a claim is to be understood in terms of its role in inferences. This promises a better treatment of meaning than that offered by Bohr. Quantum theory models physical systems with no mention of measurement: it is decoherence, not measurement, that licenses application of Born's probability rule. So quantum theory also offers advice on its own application. I show how this works in a simple model of decoherence, and then in applications to both laboratory experiments and natural systems. Applications to quantum field theory and the measurement problem will be discussed elsewhere.

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1 Introduction

One can treat the delocalization of phase of a quantum system through interaction with its environment simply by applying the formalism of quantum theory without regard to its interpretation. But the interest of many researchers in the conceptual foundations of quantum theory has been piqued by the potential contributions of environmental decoherence to an interpretation of quantum theory capable of solving long-standing puzzles about the role of measurement.

The quantum theory of decoherence is important for the pragmatist interpretation of quantum theory outlined in [1]. It governs the descriptive

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content of claims about values of physical magnitudes and advises an agent on when to apply the Born Rule as a guide to their truth. But, on this interpretation, it does so without representing the dynamic behavior of physical systems: for while it is quantum states that are subject to environmental decoherence, the quantum state does not serve to represent or describe physical systems. Because the familiar view that quantum models of environmental decoherence offer representations of a physical process conflicts with a non-representational view of the quantum state, I first explain how the quantum state functions, according to this pragmatist interpretation.

Section 3 then shows in a simple model how decoherence governs the content of descriptive claims about a qubit. The content of a claim is to be understood in terms of what inferences an agent may draw from that claim and what would entitle an agent to make it. This inferentialist pragmatism about conceptual content promises a better treatment of meaning than that offered by Bohr and his followers.[1]

Quantum theory assigns probabilities to claims about the values of magnitudes through the Born Rule. But it is now well established that these magnitudes cannot consistently all be taken simultaneously to possess precise values on each system, distributed over a collection of similar systems in such a way that the fractions of systems with particular values for each magnitude match the corresponding probabilities flowing from the Born Rule.[2] Consistency then restricts each application of the Born Rule to a proper subset of all magnitudes. Conventionally, one specifies probabilities only for *measurement outcomes*, and postulates that only those magnitudes represented by pairwise commuting self-adjoint operators are simultaneously measurable. But Bell([14], [11]) raised powerful objections against incorporating 'measurement' into the basic principles of quantum theory, either as a primitive term or when cashed out in equally unsatisfactory terms such as "irreversible amplification", "classical system", or "conscious observation". The pragmatist interpretation of [1] relies on quantum models of environmental decoherence that involve no reference to measurement to secure consistent application of the Born Rule. Section 3 shows how this works.

The general account of section 3 is illustrated in section 4 by applying it to some more realistic examples, both in a laboratory setting and in the universe at large. These are intended to show how the details of environmental decoherence can affect the significance of descriptive claims licensed by a quantum state, and to exhibit both the practical use of the Born Rule and its limitations.

The paper ends by summarizing its conclusions and pointing to questions that still need to be answered to achieve a fully adequate account of environmental decoherence within the pragmatist interpretation of quantum theory outlined in [1].

[1] Here I am indebted to the writings of Brandom([2], [3]) and Price([4], [5]). Bohr expressed his views in a number of essays collected in Bohr([6], [7], [8]).

[2] See, for example, [9], [10], [11], [12], [13].

2 The function of the quantum state

The delocalization of phase in a system's quantum state due to interaction with systems constituting its environment is generally regarded as a physical process. Quantum models compare and contrast this process with other, more familiar, physical processes such as dissipation due to energy loss into the environment. Quantum decoherence need not be accompanied by dissipation, though when it is, it is typically a much faster process. Some say that quantum decoherence occurs as a result of a system acting on its environment, whereas dissipation occurs because the system is acted on by its environment. Such language in which discussions of environmental decoherence are couched is thoroughly physical. Master equations and other mathematical treatments of quantum decoherence are taken to represent how the physical condition of a system changes in response to its interaction with its environment when that interaction is represented by an interaction Hamiltonian. But the immediate content of such treatments concerns the evolution of a system's quantum state. To read a representation of the evolution of a quantum state as a description of the changing condition of the system to which it pertains is to adopt a particular interpretative stance toward quantum states. It is to assume that a quantum state provides a description of the physical condition of a system to which it is assigned.

But this is only one, disputed, view of the function of the quantum state. On the present pragmatist understanding, the function of the quantum state is not to describe but to prescribe: A quantum state does not provide even an *incomplete* description of a physical system to which is assigned. Instead, by assigning a quantum state, a user G of quantum theory takes the first step in a procedure that licenses G to express claims about physical systems in descriptive language and then warrants G in adopting appropriate epistemic attitudes toward some such claims. The language in which these claims are expressed is not the language of quantum states or operators, and the claims are not about probabilities or measurement results: they are about the values of magnitudes. That is why I refer to such claims as NQMCs—Non-Quantum Magnitude Claims. Here are some typical examples of NQMCs:

A helium atom with energy -24.6 electron volts has zero angular momentum.

Silver atoms emerging from a Stern-Gerlach device each have angular-momentum component either $+\hbar/2$ or $-\hbar/2$ in the z-direction.

The fourth photon will strike the left-hand side of the screen.

When a constant voltage V is applied across a Josephson junction, an alternating current I with frequency $2(e/h)V$ flows across the junction.

(Notice that two of these *non-quantum* claims are stated in terms of Planck's constant.)

Any user of quantum theory is a physically situated cognitive agent. That includes physicists and other humans in a position to benefit from quantum advice in a wide variety of circumstances. But nothing rules out the possibility of non-human, or even non-conscious agents. A user of quantum theory must be physically situated because a quantum state and consequent Born probabilities can be assigned to a system only relative to the physical situation of an (actual or hypothetical) agent for whom these assignments would yield good epistemic advice. What one agent should believe may be quite different from what another agent in a different physical, and therefore epistemic, situation should find credible. This relational character of quantum states and Born probabilities does not make these subjective, and it may be neglected whenever users of quantum theory find themselves in relevantly similar physical situations. NQMCs are also objective, but, unlike claims pertaining to quantum states and Born probabilities, they are not relational in this way: Their truth-values do not depend on the physical situation of any actual or hypothetical agent.

A quantum state is objective because it provides authoritative guidance to an agent on two important matters. It provides sound advice both on the content of NQMCs concerning physical systems and on the credibility of some of these claims. Environmental decoherence figures in both these roles of the quantum state. Section 3.2 shows how decoherence enables the quantum state to play the first advisory role: section 3.3 is concerned with its contribution to the second. Note that this pragmatist interpretation does not deny that environmental decoherence *involves* a physical process: but it does deny that a system's quantum state plays any role in describing or representing such a process. An agent requires guidance in assessing the content of NQMCs about systems of interest. It is often said that assignment of a value to an observable on a system is meaningful only in the presence of some apparatus capable of measuring the value of that observable. But some account of meaning must be offered in support of this assertion, and the extreme operationist account that is most naturally associated with it would be unacceptably vague even if it were otherwise defensible. Exactly what counts as the presence of an apparatus capable of measuring the value of an observable?

Contemporary pragmatist accounts of meaning have the resources to provide a better account of the meaning of a NQMC about a system, as entertained by an agent, in a context in which that system features. A pragmatist like Brandom([2], [3]) takes the content of any claim to be articulated by the material inferences (practical as well as theoretical) in which it may figure as premise or conclusion. These inferences may vary with the context in which a claim arises, so the content of the claim depends on that context. The quantum state of a system modulates the content of NQMCs about that system by specifying the context in which they arise. This depends on the nature and degree of environmental decoherence suffered by this quantum state. A NQMC about a system whose quantum state has extensively decohered in a basis of eigenstates of the operator corresponding to that magnitude has a correspondingly well-defined meaning: a rich content accrues to it via the large variety of material inferences that may legitimately be drawn to and from the NQMC in that context.

Only when the content of a canonical NQMC (of the form $\mathbf{Q} \in \mathbf{\Delta}$, where Q is a magnitude and Δ is a Borel set of real numbers) is sufficiently well articulated in this way is it appropriate to apply the Born Rule to assess the credibility of that claim. With a sufficiently extended license, an agent may apply the Born Rule to evaluate the probability of each licensed NQMC of the form $\mathbf{Q} \in \mathbf{\Delta}$ using the appropriate quantum state.

3 The content and credibility of NQMCs

3.1 A simple model of decoherence

Consider a simple model of decoherence introduced by Zurek[15] and further discussed in Cucchetti, Paz and Zurek[16]. This features a single quantum system A interacting with a second "environment" system E as in [16].[3] A is a single qubit, and its environment E is modeled by a collection of N qubits. One can think of each qubit as realized by a spin $\frac{1}{2}$ system, so that $|\Uparrow\rangle$ ($|\Downarrow\rangle$) represent z-spin up (down) eigenstates of the Pauli spin operator $\hat{\sigma}_z$ of A, while $|\uparrow\rangle_k$ ($|\downarrow\rangle_k$) represent z-spin up (down) eigenstates of $\hat{\sigma}_z^k$ for the the kth environment spin subsystem.

The individual Hamiltonians $\hat{H}^A$, $\hat{H}^E$ of A and E are assumed to be zero, while the interaction Hamiltonian $\hat{H}^{AE}$ has the form

$$\hat{H}^{AE} = \frac{1}{2}\hat{\sigma}_z \otimes \sum_{k=1}^{N} g_k \hat{\sigma}_z^k. \tag{1}$$

If A, E are assumed to be initially assigned pure, uncorrelated states

$$\psi_A = (a\,|\Uparrow\rangle + b\,|\Downarrow\rangle)\,, \tag{2}$$

$$\psi_E = \prod_{k=1}^{N} (\alpha_k\,|\uparrow\rangle_k + \beta_k\,|\downarrow\rangle_k) \tag{3}$$

then the initial state

$$\Psi(0) = \psi_A \otimes \psi_E \tag{4}$$

evolves according to the Schrödinger equation, becoming

$$\Psi(t) = (a\,|\Uparrow\rangle\,|\mathcal{E}_\Uparrow(t)\rangle + b\,|\Downarrow\rangle\,|\mathcal{E}_\Downarrow(t)\rangle) \tag{5}$$

at time t where

$$|\mathcal{E}_\Uparrow(t)\rangle = \prod_{k=1}^{N} \left(\alpha_k e^{ig_k t}\,|\uparrow\rangle_k + \beta_k e^{-ig_k t}\,|\downarrow\rangle_k\right) = |\mathcal{E}_\Downarrow(-t)\rangle\,. \tag{6}$$

The state of A, calculated by tracing over the Hilbert space of E, is therefore

$$\hat{\rho}_A(t) = |a|^2\,|\Uparrow\rangle\langle\Uparrow| + ab^* r(t)\,|\Uparrow\rangle\langle\Downarrow| + a^* b r^*(t)\,|\Downarrow\rangle\langle\Uparrow| + |b|^2\,|\Downarrow\rangle\langle\Downarrow|\,. \tag{7}$$

The coefficient $r(t) = \langle\mathcal{E}_\Uparrow(t)|\mathcal{E}_\Downarrow(t)\rangle$ appearing in the off-diagonal terms of $\hat{\rho}_A$ here is

$$r(t) = \prod_{k=1}^{N} \left[\cos 2g_k t + i\left(|\alpha_k|^2 - |\beta_k|^2\right)\sin 2g_k t\right]. \tag{8}$$

[3] Zurek's original model also included a third system S: his choice of notation then was intended to help his reader bear in mind an application of the model to a system S interacting with a quantum apparatus A.

Cucchetti, Paz and Zurek[16] show that $|r(t)|$ tends to decrease rapidly with increasing N and very quickly approaches zero with increasing t. More precisely, while $|r(t)|^2$ fluctuates, its average magnitude at any time is proportional to 2^{-N}, and, for fairly generic values of the g_k, it decreases with time according to the Gaussian rule $|r(t)|^2 \propto e^{-\Gamma^2 t^2}$, where Γ depends on the distribution of the g_k as well as the initial state of E. This result is relatively insensitive to the initial state of E, which need not be assumed to have the product form (3), though if the environment is initially in an eigenstate of (1) $|r(t)| = 1$ so the state of A will suffer no decoherence. Since $r(t)$ is an almost periodic function of t for finite N, it will continue to return arbitrarily closely to 1 at various times: but for N corresponding to a macroscopic environment Zurek[15] estimated that the corresponding "recurrence" time exceeds the age of the universe.

3.2 NQMCs in this simple model

Suppose an agent G is considering what claims to entertain about the system A in this simple model. G is not explicitly represented in the model itself. But since any assignment of quantum states is from the perspective of some actual or hypothetical physically instantiated agent, we must assume that G has implicitly adopted such a perspective by assigning the states that figure in the model. Magnitudes pertaining to A correspond to self-adjoint operators on the Hilbert space $\mathcal{H}_A$. In this simple model, any such operator $\hat{Q}$ may be expressed as a real linear sum of Pauli spin operators and the identity operator on $\mathcal{H}_A$ as follows

$$\hat{Q} = x\hat{\sigma}_x + y\hat{\sigma}_y + z\hat{\sigma}_z + c\hat{I}. \tag{9}$$

So the canonical NQMCs under consideration by G are claims about A of the form K: $\mathbf{Q} \in \mathbf{\Delta}$, where Δ is a Borel set of real numbers, and Q corresponds uniquely to the operator $\hat{Q}$. After setting $S_i \equiv (\hbar/2)\,\sigma_i$ $(i = x, y, z)$ these include

$$I = 1 \tag{A}$$
$$S_x \in \{+\hbar/2, -\hbar/2\} \tag{B}$$
$$S_x = +\hbar/2 \tag{C}$$
$$S_z \in \{+\hbar/2, -\hbar/2\} \tag{D}$$
$$S_z = +\hbar/2 \tag{E}$$

G's primary interest is in his entitlement to believe a claim K about A, including each of the claims (A)-(E). But his first concern is what *content* is expressed by such a claim, and so he should consult the quantum state of A.

Claim (A) is vacuous. It never warrants further claims about A. What is the content of each of (B)-(E) given *only* the initial state (2) G assigns to A? That depends on the inferential role of each claim. G may be tempted to infer claim (D) about A from the fact that (2) expresses this state as a superposition of eigenstates of $\hat{\sigma}_z$. But the initial state of A may be expressed equally well as a superposition of eigenstates $|\Longleftarrow\rangle$, $|\Longrightarrow\rangle$ of $\hat{\sigma}_x$

$$\psi_A = (c\,|\Longleftarrow\rangle + d\,|\Longrightarrow\rangle)\,, \text{ where } c = \frac{1}{\sqrt{2}}\,(a+b)\,,\ d = \frac{1}{\sqrt{2}}\,(a-b)\,. \tag{10}$$

(or indeed of any operator of the form $\hat{\mathbf{Q}}$.) So if the content of a claim of the form K then depended only on the state (2) then G should be equally tempted to make claim (B) (as well as every other similar claim assigning some eigenvalue of $\hat{Q}$ to every magnitude Q in that state.) Feynman[17, vol.III, 1.9] warned against an analogous temptation in the famous 2-slit experiment in these words:

> if one has a piece of apparatus which is capable of determining whether the electrons go through hole 1 or hole 2, then one can say it goes through either hole 1 or hole 2. [otherwise] one may not say that an electron goes through either hole 1 or hole 2. If one does say that, and starts to make any deductions from the statement, he will make errors in the analysis. This is the logical tightrope on which we must walk if we wish to describe nature successfully.

The simple model represents no analogous piece of apparatus capable of determining whether (C) or (C′): $S_x = -\hbar/2$ is true, so Feynman would warn G against saying (B) *unless G declines to make any inferences from (B).* (B) is an exclusive disjunction, and the problematic inferences to be barred would proceed by disjunction elimination—by deriving a (false) conclusion from each disjunct separately and hence drawing that conclusion on the basis of the disjunction alone. In the two-slit experiment, the assumption that each electron goes through one slit or the other leads to the false conclusion that the interference pattern on the screen is the sum of a pattern formed by electrons going through "hole" 1 and a pattern formed by electrons going through "hole" 2. But to derive that conclusion, one needs further to assume that the behavior of an electron going through "hole" 1(2) is the same whether or not "hole" 2(1) is open—an assumption rejected by Bohmians, among others.

This illustrates an important point. The inferences that contribute to the content of an NQMC are not restricted to mathematically and logically valid inferences, but include what Sellars[18] called material inferences. Indeed, according to a pragmatist inferentialist account of content it is precisely such material inferences that contribute essentially to empirical content. But, as in this case, what material inferences a claim licenses will depend on what other assumptions are made.

The content of (C) and (C′) must be restricted so as to exclude their use even in hypothetical material inferences when A is in state (2). Without such a restriction, G could infer that S_x,S_z (and indeed all other spin components) have precise real values together in state (2). While this is not a contradiction, it does conflict with generally accepted background assumptions.[4]

[4] A similar conclusion in the 3-dimensional Hilbert space of a spin 1 system *would* be inconsistent with Gleason's theorem [9].

State (2) is an eigenstate of the operator

$$\hat{\sigma}_{\theta\varphi} \equiv (\sin\theta\cos\varphi)\,\hat{\sigma}_x + (\sin\theta\sin\varphi)\,\hat{\sigma}_y + \cos\theta\hat{\sigma}_z, \tag{11}$$

where $a = \cos\theta/2\exp(-i\varphi/2)$, $b = \sin\theta/2\exp(+i\varphi/2)$. This state may be represented on the Bloch sphere by a unit vector $\mathbf{n}$ with angular coordinates (θ, φ). The operator $\hat{S}_{\mathbf{n}} = (\hbar/2)\,\hat{\sigma}_{\theta\varphi}$ corresponds to a component $S_{\mathbf{n}}$ of angular momentum in a spatial direction $\mathbf{n}$ with spherical coordinates (θ, φ) defined with respect to the (x, y, z) Cartesian coordinate system. Consider the claim

$$S_{\mathbf{n}} = +\hbar/2 \tag{F}$$

Since (2) is an eigenstate of $\hat{S}_{\mathbf{n}}$ with eigenvalue $+\hbar/2$, G may be tempted to make claim (F) solely on the basis of that initial state assignment to A. But before doing so, G should assess (F)'s content.

Since the content of (F) is a function of its inferential role, G must consider what could entitle him to infer (F) and what he could infer from (F). G could immediately infer (F) from (2) in accordance with this interpretative principle (EVI):

> If a system's quantum state $\hat{\rho}$ satisfies $\hat{P}_i\hat{\rho} = \hat{\rho}$, where $\hat{P}_i$ projects onto the eigenspace with eigenvalue q_i of an operator $\hat{Q}$ corresponding to magnitude Q, then Q has value q_i.

But G should reject (EVI) as incompatible with the pragmatist denial that a system's quantum state provides any kind of description of that system. Alternatively, G might think to infer (F) using the EPR [19, p.777] sufficient condition of reality.

> If, without in any way disturbing a system, we can predict with certainty (i.e. with probability equal to unity) the value of a physical quantity, then there exists an element of reality corresponding to this physical quantity.

His thought might be that application of the Born Rule to (2) would assign probability unity to the claim $S_{\mathbf{n}} = +\hbar/2$, and then (F) follows from (EPR)'s reality condition. But this thought is mistaken whatever the status of that criterion. G is entitled to apply the Born Rule to state (2) to assign a probability (unity) to (F) only if (F) has sufficient content to permit that application. But the empirical content of (F) is exactly what is in question. Clearly, it would be circular for G to *assume* that (F) has sufficient content to entitle him to apply the Born Rule to (F) in state (2) in order to argue that (F) has any significant empirical content! In fact the Born Rule is not applicable to this claim in the simple model, in which the only interaction to which A is subject is modeled by (1): For G to be entitled to apply the Born Rule to assign a probability to (F) at $t = 0$, A would have to be subject to an interaction that decohered eigenstates of $\hat{S}_{\mathbf{n}}$ at $t = 0$.

In state (2) G may infer from (F) to any claims validly deducible by logic and mathematics alone, such as these:

$$S_{\mathbf{n}} \in \{+\hbar/2, -\hbar/2\}$$
$$(S_{\mathbf{n}} = +\hbar/2) \text{ or } (S_{\mathbf{n}} = +\hbar)$$
$$S_{\mathbf{n}}^2 = \hbar^2/4$$

So it is not strictly correct to say that (F) is vacuous in this case. But in order to have any *physical* content, (F) would have to permit material inferences that are neither logically nor mathematically valid.

In classical physics, NQMCs typically permit material inferences of two kinds: dynamic inferences and measurement inferences. An assumption of continuity guarantees that ascription to a magnitude of a value in set Δ at time t licenses a material inference to its value in set Δ_ε at $t+\varepsilon$, where Δ_ε is "close" to Δ for sufficiently small ε: and ascription to a magnitude of a value in set Δ at time t licenses a material inference that the result of a sufficiently carefully conducted measurement at $t+\varepsilon$ would find a value in Δ_ε. Since G can make neither kind of material inference from (F), (F) lacks physical content here—it is empirically vacuous, as are (C) and (C′). The initial state (2) licenses G to make no physically significant claims about A.

But G's resources are not confined to the assignment of an initial state to A. Using this simple model of decoherence, G also specifies how the initial quantum state of A evolves under the influence of interaction with its environment. It is the role of decoherence here that endows certain NQMCs about A with empirical significance, according to the interpretation outlined in [1].

The quantum state initially assigned by G to A evolves, so that after a remarkably short time T it will come to approach the diagonal form

$$\hat{\rho}_A = |a|^2 \left|\Uparrow\right\rangle \left\langle\Uparrow\right| + |b|^2 \left|\Downarrow\right\rangle \left\langle\Downarrow\right| . \tag{12}$$

If $\theta = 0$ or π (i.e. $|a|^2$ or $|b|^2 = 1$) in state (2), then claim (F) reduces respectively to (E) or to (E′): $S_z = -\hbar/2$. Each of these *is* now physically significant—not because of the initial state (2), but as a result of how the simple model treats A's interaction with E. It is only because of this environmental decoherence that G can entertain any physically significant NQMCs about A: these include (D), (E) and (E′), but *not* (A)-(C).

While (A)-(C) remain empirically vacuous in state (12), (D), (E) and (E′) have empirical content because of the material inferences each supports. G may use (D) in inferences that assume that just one of its disjuncts is true in state (12), even if he has no empirical basis for claiming (E) as against (E′) (or *vice versa*). The material inference to (D) at time T is not justified by the state of A alone, but because the state of A continues to remain very close to (12) for an extended interval including T, and is in that sense stable against the environmental interaction G models by (1).

While (EVI) is false, a pragmatist interpretation of the quantum state does endorse this particular consequence of (EVI) because at T each of (E), (E′) permits the dynamic and measurement inferences discussed four paragraphs back. This illustrates the importance of environmental decoherence in endowing an NQMC with empirical content. Deployment of the simple model of decoherence cannot by itself justify G in making a material inference either to (E) or to (E$'$). Rather, as section 3.3 explains, assignment of state (12) to A in the context of this model justifies G in making the *practical* inference involved in adopting degree of belief (credence) $|a|^2$ in (E) and credence $|b|^2$ in (E$'$). On this pragmatist interpretation it is a basic assumption of any application of a quantum model that an agent such as G can subsequently come to be warranted in believing (E) as against (E$'$) (or *vice versa*) *by experience*. It follows that the process of observation or experimentation which would give rise to such an experience of an agent G applying a quantum model can nowhere be represented within the model G is applying. It does not follow that this process cannot *itself* be modeled by another agent G', although G''s model could not be extended to encompass processes that give rise to any experiences of G' it may lead G' to expect.

Environmental decoherence is not perfect, even in this simple model. As it evolves, the state of A arrived at by tracing over E in state (5) will be exactly diagonal in some orthogonal basis of eigenstates of $\hat{\sigma}_{\psi\varkappa}$ for ψ almost always extremely close, but not equal, to zero. (Here ψ varies over the angle of inclination to the z axis, and χ over the azimuthal angle from the x axis). Consider a claim about A of the form L for some pair (ψ, χ):

$$S_{\psi\chi} \in \{+\hbar/2, -\hbar/2\} \qquad L$$

If ψ is close enough to zero, then (1) will very rapidly, and quite stably, bring the state of A almost as close to diagonal in a basis of eigenstates of $\hat{S}_{\psi\chi}$ as of $\hat{S}_z$. A material inference to a claim of the form L in the context of the model will be almost as good as the inference to (D), and the inferential power of a claim of the form L will be almost as great as that of (D). More generally, the empirical content of a claim of the form L here is a function of ψ, varying continuously from its maximum value for $\psi = 0, \pi$ to zero for $\psi = \pi/2$. It corresponds to the reliability of the inference from the claim that $S_{\psi\chi}$ has one of its eigenvalues at time t to the conclusion that $S_{\psi\chi}$ has that same eigenvalue at $t + \varepsilon$ and that this would also be the result of a well-conducted measurement of $S_{\psi\chi}$ at time $t + \varepsilon$.

What is the relation between the interaction modeled by (1), the possibility of measuring the value of a magnitude Q in the simple model, and what it takes to have a piece of apparatus which is capable of determining the value of a magnitude Q? The model itself makes no mention of any apparatus or measurement. But in *applying* the model, an agent G is effectively committed to counting the interaction modeled by (1) as itself a potential measurement of S_z that excludes measurements of other magnitudes $S_{\psi\chi}$ with ψ far from $0, \pi$ while simultaneously serving as a somewhat less reliable measurement of magnitudes $S_{\psi\chi}$ with ψ very close to $0, \pi$. G makes this commitment by taking it for granted that the process being modeled had or will have a determinate outcome that G could come to recognize as indicating the value of S_z by examining either A itself or some part of E. G is thereby committed to regarding the whole system being modeled as effectively including an apparatus capable of determining the value of S_z, but excluding any apparatus capable of determining the value of any magnitude $S_{\psi\chi}$ with ψ far from $0, \pi$.

3.3 The Born Rule in the simple model

The references of footnote 2 show there is no consistent simultaneous assignment of a Born probability to every NQMC ascribing a precise value to a magnitude on a system. According to [1], the function of the Born Rule is to advise an agent on what credence to attach to certain NQMCs. Since an agent can attach some credence only to an empirically significant claim, this immediately restricts applications of the Born Rule to empirically significant canonical NQMCs.

As section 3.2 made clear, not every NQMC concerning a system is equally empirically significant even when that system is subject to environmental decoherence: Empirical significance comes in degrees here. As ψ varies, the empirical significance of a claim of the form L varies accordingly, as does that of any claim about A of the form M:

$$S_{\psi\chi} \in \Delta \qquad M$$

Empirical significance has no natural cut-off here, or in any application of quantum theory. So this restriction on application of the Born Rule does not yield a precise selection criterion. This is a classic case of vagueness. The Born Rule is clearly applicable to claims (D), (E), (E′) in the simple model, and clearly inapplicable to claims (A), (B), (C), (C′). But the limits of applicability of the Born Rule to a claim of the form L or M may be set anywhere within a wide (but equally indeterminate) range of values of ψ in the neighborhoods of $0, \pi$. Does this vagueness matter?

Some take quantum theory to be fundamental because it provides our most accurate descriptions of nature—call this fundamental$_a$. But Bell [11, pp.125-6] criticized contemporary formulations of quantum theory on the grounds that these are fundamentally approximate and intrinsically inexact. "Surely", he asked in [14], "after 62 years we should have an exact formulation of some serious part of quantum mechanics?" Bell denied that quantum theory is fundamental$_a$, because contemporary formulations are in terms of observables rather than what he called 'beables':

> It is not easy to identify precisely which physical processes are to be given the status of "observations" and which are to be relegated to the limbo between one observation and another. So it could be hoped that some increase in precision might be possible by concentration on the beables, which can be described "in classical terms", because they are there. [11, p.52]

Bell would surely have rejected the pragmatist interpretation of [1]. He would have taken the vagueness inherent in the conditions of applicability of the Born Rule to introduce an unacceptable imprecision into the theory, and regarded quantum theory under this interpretation as not a serious theory—a serious candidate for the job of truly describing nature. But, in this pragmatist view, quantum theory achieves its unprecedented success *without* describing nature, either vaguely or precisely.

Interpreted along the lines of [1], an agent does not use a quantum-theoretic model to represent physical systems: quantum theory is not itself in the business of describing physical reality. Since quantum theory does not yield descriptions of nature, it is clearly not a fundamental$_a$ theory. But it is fundamental in another sense: it gives us our best and only way of predicting and explaining a host of otherwise puzzling phenomena. We do this using quantum models—not to describe reality but to advise us on what to believe about it. Any such use depends on application of the Born Rule. So the predictive and explanatory successes we achieve using quantum theory depend on judicious application of that rule.

Now one can see why any vagueness associated with application of the Born Rule does not matter. Application of a theory or rule always requires judgment, and this is no exception. In applying any physical theory one must first decide how to model the part or aspect of the physical world on which the application is targeted. The model of section 3.1 was called simple because it has few if any real world targets—a wise agent would rarely if ever decide to apply it. When applying a quantum-theoretic model, an agent must make a further decision about which NQMCs are apt for application of the Born Rule. Here, too, good judgment is called for.

Models of quantum theory are not inherently imprecise. Their specification need contain none of Bell's[11, p.215] "proscribed words" 'measurement', 'apparatus', 'environment', 'microscopic', 'macroscopic', 'reversible', 'irreversible', 'observable', 'information' or 'measurement', though one may use any of these words harmlessly in commenting on the model with a view to its intended applications, as several of these words were used in section 3.1.[5] Any element of imprecision or inexactness can enter only when a quantum model is *applied* to a specific physical situation.

The Born Rule itself is in no way imprecise or inexact: specifically, a statement of the Rule should not contain 'measurement' or any other similarly problematic terms The Born Rule simply assigns a mathematical probability measure to all canonical NQMCs about A of the form K for every single magnitude Q in the simple model. In a model with a higher-dimensional Hilbert space for A, the Born Rule also assigns a joint probability measure to sets of NMQCs $\{\mathrm{K}_i\}$, where the corresponding $\hat{Q}_i$ pairwise commute. In applying the model, an agent needs to judge which of these mathematical measures should be taken to govern credence and which lack cognitive significance in this application. Previous experience, as filtered through vague categories such as those criticized by Bell may improve this judgment. But, just as in classical physics, quantum theory can help structure the agent's deliberation by making available enlarged models that take account of the interaction of the target system with its environment, whether this is thought of as an experimental arrangement or just the natural physical situation in which the target system finds itself. In application, quantum theory is no more inexact than classical physics.

[5] It does seem necessary to use a word like 'system' to say what is ascribed a quantum state in a quantum model. But with no mention of any apparatus, the model cannot enshrine the "shifty split" between system and apparatus of which Bell complained.

4 Examples of NQMCs and use of the Born Rule

In this section I extend the discussion to more realistic examples. Laboratory experiments in a controlled environment provide the clearest examples. But the universe at large supplies an environment for natural processes that furnish some of the most interesting applications of quantum theory.

4.1 Molecular interference lithography of C_{60}

Juffman *et al.* [20] prepared a beam of C_{60} molecules with well-defined velocity v, passed them through two gratings of a Talbot-Lau interferometer in a high vacuum, and collected them on a carefully prepared silicon surface placed at the Talbot distance. They then moved the silicon about a meter into a second high vacuum chamber and scanned the surface with a scanning tunneling electron microscope (STM) capable of imaging individual atoms on the surface of the silicon. After running the microscope over a square area of approximately $2\mu\text{m}^2$ they were able to produce an image of some one to two thousand C_{60} molecules forming an interference pattern. They reported that the surface binding of the fullerenes was so strong that they could not observe any clustering, even over two weeks. Clearly they felt no compunction in attributing very well defined, stable, positions to the molecules on the silicon surface, and even recommended developing this experiment into a technique for controlled deposition for nano-technological applications.

Assuming the process is stationary, one can assign fullerenes in the beam a quantum state at each position z along the interferometer axis by replacing t in the Schrödinger equation by z/v. To account for the interference pattern in this experiment, one can then use the Schrödinger equation to calculate the quantum state at the silicon surface and apply the Born Rule to calculate a fullerene probability density at a particular location at distance x from the interferometer axis in a direction perpendicular to the slit orientation. Application of the Born Rule would lead one confidently to expect formation of the observed interference pattern for a large enough number of fullerenes, while acknowledging that this confidence falls short of certainty.

There was no mention of decoherence in this account, which simply assumed one can apply the Born Rule to NQMCs of the form $\mathbf{x}\epsilon\mathbf{\Delta}$ for fullerenes at the silicon surface. But the account involved no such application to fullerenes passing through the interferometer from the oven to the surface. The title of Juffman *et al.*[20] points to a common way of talking about single-particle interference experiments like this. One says that a C_{60} molecule acts like a particle at the surface (so it is meaningful to ascribe it a determinate position there) but a wave in the interferometer (so it is meaningless to say that it passed through just one slit of the central grating). The pragmatist interpretation outlined in [1] endorses this way of talking, but only as a gloss on the more nuanced account made possible by the application of an inferentialist view of empirical content within a quantum model of decoherence.

It is impracticable to formulate and solve the Schrödinger equation for the entire many-body quantum interaction that begins with the binding of a C_{60} molecule to the silicon surface. It is clear that this will rapidly and strongly couple the C_{60} molecule to an environment of an exponentially increasing number of degrees of freedom, eventually involving the entire laboratory and beyond. But it is not unreasonable to apply a canonical model of quantum Brownian motion in which the center of mass x-position of a C_{60} molecule is linearly coupled to a bath of harmonic oscillators corresponding to the modes of the entire silicon crystal to which it is bound by an assumed simple harmonic potential. In this model, the relevant degree of freedom of the molecule picks out a system A interacting with an environment system E modeling the silicon surface. This model has been studied by Paz, Habib and Zurek[21] and others.[6] An agent may appeal to this model in assessing the empirical significance of NQMCs concerning a C_{60} molecule at the silicon surface, and to justify application of the Born Rule to certain of these, even while acknowledging that a more complex model might offer wiser counsel.

In this case, the model of quantum Brownian motion shows that after a remarkably short time the quantum state of A will be expressible as a mixture of narrow Gaussians, each approximating a point (x, p_x) in classical phase space. Moreover, the weights of these states will be, and will for a long time remain, equal to the corresponding phase space probability densities as calculated from the Wigner functions of such Gaussians. An agent may therefore associate a high degree of empirical content with a claim locating a C_{60} molecule at a particular place on the silicon surface, and is therefore entitled to apply the Born Rule to NQMCs of the form $\mathbf{x}\epsilon\mathbf{\Gamma}$, $\mathbf{p}_x\epsilon\mathbf{\Delta}$ using the quantum state deduced by applying the Schrödinger equation to calculate how the initial quantum state of a C_{60} molecule evolves before it reaches the silicon surface. The high empirical content of such claims follows from the justifiability of material inferences to claims about values of (x, p_x) at different times, as attestable by repeated measurements by the STM.

In the experiment of Juffman *et al.*[20], the center of mass fullerene wave-function suffered negligible decoherence in the interferometer. So here an agent should assign NQMCs of the form $\mathbf{x}\epsilon\mathbf{\Gamma}$ about C_{60} molecules minuscule empirical content before they reach the silicon surface and so decline to apply the Born Rule to then. The next experiment is interesting precisely because it incorporates just such decoherence within the interferometer.

4.2 Influence of molecular temperature on C_{70} coherence

Hackermüller *et al.*[23] investigated the effects of increased temperature in matter wave interferometer experiments in which fullerenes lose their quantum behavior by thermal emission of radiation. They prepared a beam of C_{70} molecules of well-defined velocity, passed them through two gratings of a Talbot-Lau interferometer in a high vacuum, and detected those that passed through a third movable grating set at the appropriate Talbot distance and used as a scanning mask, by ionizing them and collecting the ions at a detector. Each molecule is sufficiently large and complex to be assigned a temperature as it stores a considerable amount of energy in its internal

[6] Schlosshauer[22] gives a more recent review.

degrees of freedom.[7] Interaction with the electromagnetic vacuum may result in emission of photons with an intensity and frequency that increases as the internal temperature is raised. Entanglement between such photon states and the state of the emitting molecule tends to induce environmental decoherence.

Hackermüller *et al.*[23] present a theoretical model of this decoherence that fits their observations quite well, as the observed interference dies away when the molecules' temperature is raised from 1000°K to 3000°K. Hornberger, Sipe and Arndt[24] give a more detailed exposition. An agent can use this model to assess the empirical significance of NQMCs concerning a C_{70} molecule as it traverses the interferometer. The model treats photon emission by a fullerene as a sequence of independent, separate events, analogous to collisions with gas particles (which occur relatively rarely, given the extremely low pressure inside the interferometer). Since the process is stationary, the reduced quantum state for the C_{70} molecules' center of mass degree of freedom may be expressed as a function of position $\mathbf{R}$ in the interferometer as

$$\rho' (\mathbf{R}_1, \mathbf{R}_2) = \rho (\mathbf{R}_1, \mathbf{R}_2) \, \eta (\mathbf{R}_1 - \mathbf{R}_2) \tag{13}$$

where the decoherence function $\eta (\mathbf{R}_1 - \mathbf{R}_2)$ represents the effect of decoherence on the off-diagonal elements of the un-decohered reduced state $\rho (\mathbf{R}_1, \mathbf{R}_2)$. (It has no effect on the diagonal elements, since $\lim_{\mathbf{R}_1 \to \mathbf{R}_2} \eta (\mathbf{R}_1 - \mathbf{R}_2) = 1$.)

Assuming photon emission is both isotropic and independent of $\mathbf{R}$, η is a function only of the spectral photon emission rate. This is not the same as for a macroscopic black body since the emitting particle is small and not in thermal equilibrium with a heat bath, and the emission is assumed to take place at a fixed internal energy rather than temperature. Given the assumption that the emitting molecule has a definite energy E, it can be associated with a microcanonical temperature T^* given in terms of the entropy $S(E)$ by

$$T^*(E) = \left[\frac{\partial S(E)}{\partial E} \right]^{-1} \tag{14}$$

So to assign an internal temperature to a beam of C_{70} molecules at each point in the interferometer one must assume that each is always in a state of definite energy except during photon emission, which is associated with a transition between energy states. But what justifies this assumption?

While it is decoherence of their center of mass quantum state that is the focus of the model, one must also consider decoherence of the *internal* state of the C_{70} molecules. Here a different model of decoherence is more appropriate. While the fullerenes interact strongly with the electromagnetic field of the laser beams that heat them before entering the interferometer, their electromagnetic interactions inside it are very weak at room temperature. Paz and Zurek[25] showed that in this "quantum limit", the reduced internal quantum state of a system rapidly becomes approximately diagonal in a basis of energy eigenstates as result of its interaction with the environment. This is what justifies an agent in claiming that each fullerene has a

[7] In fact, assignment of a temperature to each molecule is a step that requires justification, as we shall see.

definite internal energy within the interferometer, which may change if it emits a photon. So decoherence plays a double role here. The model of (center of mass) decoherence relevant to assigning empirical content to NQMCs concerning the position of a fullerene in the interferometer shows how that decoherence depends on the fullerene's temperature. But the justification for assigning empirical content to a claim asserting such dependence rests on an independent model of the fullerene's (internal energy) decoherence.[8]

Hackermüller *et al.*[23] detected their fullerenes by ionizing them after the scanning mask and measuring the intensity of detected ions. The detection process involves focusing any ions produced on a conversion electrode, then detecting the emitted electrons. How far does this indirect method of observing the interference pattern affect the application of the Born Rule and the assignment of content to NQMCs of the form $\mathbf{x}\epsilon\mathbf{\Gamma}$ about fullerenes at the scanning mask? As [24] shows, the geometry of the interferometer in the experiment of Hackermüller *et al.*[23] is such that the probability density for fullerene x-position at the scanning mask predicted by unreflective application of the Born Rule to the un-decohered reduced state $\rho(\mathbf{R}_1, \mathbf{R}_2)$ corresponds to a smoothed image of the first grating—a pattern with the same period d as the first grating, with maximum intensity at the center of the image of a slit window in the first grating and minimum intensity at the center of the image of a wall in the first grating.

The detector system employed by Hackermüller *et al.*[23] operates by measuring the ionization intensity for *all* C_{70} molecules passing through the scanning mask at a particular x-setting. So it is insensitive to through which slit in that third grating any particular fullerene may (or may not) have passed. Since no apparatus is capable of detecting through which slit of the second *or third* grating a fullerene passes, Feynman would forbid one to say the fullerene passed through slit1 or... or slit i or...or slit N_j of either the 2nd or 3rd (scanning mask) grating ($j = 2, 3$).

On the present pragmatist interpretation, because no significant decoherence occurred at either grating, a NQMC of the form $x\epsilon\Gamma_i$ about a fullerene at the second or third grating has little or no empirical content, where Γ_i specifies the opening interval of the ith slit of either grating ($i = 1, 2, ...N_j$). The exclusive disjunction

$$(x\epsilon\Gamma_1) \veebar ... (x\epsilon\Gamma_i) \veebar ... (x\epsilon\Gamma_{N_3}) \tag{15}$$

regarding the position of a fullerene at the scanning mask therefore also lacks the empirical content required to license its use as a premise in any inference. ((15) asserts that exactly one $(x\epsilon\Gamma_i)$ $(i = 1, ..., N_3)$ holds.)

Since the object of the experiment of Hackermüller *et al.*[23] is to investigate the effect of thermal decoherence on the fringes produced by quantum

[8] There is a further subtlety here, since no claim about the entropy S or the microcanonical temperature T^* defined in terms of it is an NQMC. The entropy is a function of the quantum state ρ given by the von Neumann expression $S = -\mathrm{Tr}(\rho \log \rho)$, and so the interpretation of [1] denies that a claim assigning a value either to S or to T^* is an NQMC. But both S and T^* are still just as objective as the quantum state assignment ρ, and an NQMC ascribing a value to E has objective and well-defined empirical content.

interference of single fullerenes, it is obviously important to be able to justify application of the Born Rule to both decohered and un-decohered reduced states of fullerenes to compare their predicted detection intensities. However, neither an NQMC of the form $\mathbf{x}\epsilon\boldsymbol{\Gamma}_i$ nor an exclusive disjunction (15) of such NQMCs has enough empirical content to license application of the Born Rule to such a claim about a fullerene at the third grating.

Yet there is a related NQMC to which the Born Rule may perhaps be justifiably applied, namely the *inclusive* disjunction

$$(x\epsilon\Gamma_1) \vee \ldots (x\epsilon\Gamma_i) \vee \ldots (x\epsilon\Gamma_{N_3})\,. \tag{16}$$

One could use a claim of this form to say that a fullerene passed through the third grating, without thereby implying that it passed through some particular slit in that grating. Such a claim has some inferential power in this situation. It supports the material inference to the true conclusion that fullerenes will be detected by the ionization detector if, but only if, the slit windows in the third grating are not blocked: One is entitled to make the claim if, but only if, a beam of fullerenes emerges from the oven (as confirmed by ionization detectors placed in front of the first grating to measure the initial beam temperature). But one may question whether these inferences alone endow (16) with enough empirical content to permit one to apply the Born Rule here, since interaction with the third grating produces little decoherence in the fullerenes' quantum state.

The blue laser beam used to ionize fullerenes after the third grating does interact strongly with them and substantially decoheres the beam, effectively localizing ionized fullerenes. At this point, a NQMC of the form $\mathbf{x}\epsilon\boldsymbol{\Gamma}_i$ about an ionized fullerene has acquired a high degree of empirical content, and so one is clearly entitled to apply the Born Rule to assign a probability to it, as well as to (15) and (16). But the rule is here applied to the quantum state *after* the third grating—a state that played no part in the analysis of the experiment! Ionization-induced decoherence is irrelevant to an application of the Born Rule to the quantum state of the fullerenes at the third grating.

If one remains dissatisfied by the justification for applying the Born Rule offered two paragraphs back, another approach is available. This is to focus instead on application of the Born Rule to a claim of the form $\mathbf{x}\epsilon\boldsymbol{\Gamma}_{TOT}$, where Γ_{TOT} is an interval covering the whole range of x-positions where the blue laser is capable of ionizing fullerenes so that the resulting ion can impact the detector electrode and elicit a detection signal. Localization of ionization events gives a high degree of empirical significance to such a NQMC about a fullerene in an ionization event. The experiment can be taken to show directly how the frequency of these events varies with the x-setting of the scanning mask when the fullerenes are heated to a specific temperature before entering the interferometer. One can redescribe such a frequency as a measure of the probability of *detecting* a fullerene at an x-position in a slit window rather than a wall in the scanning mask while denying the significance of the claim that the fullerene *had* any x-position as it encountered the scanning mask. This guarded way of speaking is common when quantum physicists discuss applications of the Born Rule. It was subjected to withering criticism by Bell[11]. But Bell's objections do not apply when such talk is cashed out in terms of empirically significant NQMCs to which the Born Rule may be justifiably applied, such as $\mathbf{x}\epsilon\mathbf{\Gamma}_{TOT}$ in this example.

4.3 Quantum theory in the universe

Quantum theory has been applied successfully to a wide range of terrestrial phenomena outside the laboratory. Such applications have already provided the basis for a thriving technology. We have overwhelming reasons to believe that some natural phenomena to which quantum theory has been successfully applied occurred long before there were laboratories, physicists or any other agents capable of observing or applying quantum theory to them: we use the quantum theory of radioactive decay to date them!

Quantum theory has been successfully applied to yield an understanding of such extraterrestrial phenomena as the structure of the sun and many other kinds of stars in our galaxy and far beyond, as well as their modes of energy production, nucleosynthesis, birth and death. Quantum theories of the Standard Model have been successfully applied to give us a detailed, quantitative understanding of the evolution of matter in the early universe. Quantum theories have been applied (albeit as yet more speculatively) to predict the existence and nature of radiation from black holes and to the development of large-scale structure in the extremely early universe. How is quantum decoherence relevant to such applications of quantum theory "in the wild"?

Much of what we know about the solar system, and almost everything we know about what lies outside it, is based on evidence provided by analyzing electromagnetic radiation, especially that emitted or absorbed by excited atoms and molecules. Since quantum theory provides us with our best understanding of the processes involved in the emission and absorption of radiation by atoms and molecules, it is only by applying quantum theory to phenomena that occurred far away (and in many cases long ago) that we can justify knowledge claims based on this evidence. No single, simple model of decoherence can be expected to encompass all such phenomena. But in many cases the atoms and molecules involved will be in an environment that decoheres their internal states in an energy basis (cf. the discussion of emission by the C_{70} molecules in section 4.2). It is such decoherence that justifies one in assuming that emission or absorption occurs between states of well-definide internal energy, and so applying the Born Rule to calculate absorption or emission probabilities.

Environmental decoherence in an energy basis provides a similar justification for application of the Born Rule to calculate rates of nuclear reaction and decay in stellar nucleosynthesis. Here it is the nuclear energy levels that decohere in consequence of environmental interactions.

Applications of the Standard Model to primordial nucleosynthesis in the first three minutes of the Big Bang have successfully accounted for observed cosmic abundance of helium, deuterium and lithium-7. These applications depend on calculations of rates for weak interaction processes including conversion of a proton and electron into a neutron and neutrino. Such a calculation proceeds by applying the Born Rule in the context of perturbation theory *via*

Fermi's golden rule (or some elaboration to take account of higher order terms in a perturbation expansion). This application may once again be justified by the decoherence due to interaction with the early universe environment provided by the highly excited state of the quantized electromagnetic field. Part II will discuss quantum field-theoretic models of decoherence.

5 Summary and outlook

If a quantum state does not describe or represent physical properties of a system to which it is assigned, it must have some other function. In the pragmatist interpretation outlined in [1] it has one role within a model of quantum theory and a second role in guiding the application of a model. Within a model, the quantum state functions as input to the Born Rule for calculating probabilities of canonical NQMCs of the form $\mathbf{Q}\epsilon\mathbf{\Delta}$. There is no mention of *measurement*, either in these NQMCs or in the Born Rule itself. Within a model, Bell's([14],[11]) requirement is met—that the theory should be fully formulated in mathematical terms, with nothing left to the discretion of the theoretical physicist. However, in order to *apply* a quantum model it is necessary to assess the significance of NQMCs concerning the system(s) which are the intended target of the application.

Physicists have acquired practical expertise in this task, guided by informal maxims like Wheeler's "No phenomenon is a real phenomenon until it is an observed phenomenon" and by Bohr's view that the entire experimental setup provides us with the defining conditions for the application of classical concepts in the domain of quantum physics. But while the application of models in physics always requires skill and judgment, Bell([14],[11]) was surely right to complain about reliance here on such vague and anthropocentric terms as 'measurement', 'observation' and 'experimental setup'.

There is nothing anthropocentric about environmental delocalization of coherence. In the pragmatist interpretation outlined in [1] quantum models of decoherence govern the significance of NQMCs and their suitability for application of the Born Rule. The extension of a quantum model of a target system to include its environment yields a principled and non-anthropocentric way of using quantum theory to guide its own application.

This does not eliminate the need for judgment and discretion. Existing quantum models of environmental decoherence incorporate many idealizations and/or approximations known to hold only in highly controlled laboratory experiments. Choosing one model rather than another as a basis for assessing the significance of NQMCs and their suitability for application of the Born Rule requires skill and judgment, especially when the intended target system is in an uncontrolled and epistemically inaccessible environment. By using an inappropriate model of decoherence, an agent might come to apply the Born Rule to an unsuitable NQMC. But any such mistake would be subject to correction by standard scientific methods.

The foundational significance of environmental delocalization of coherence in the pragmatist interpretation outlined in [1] makes it important to improve our understanding by constructing, analyzing and (if possible) testing new and more realistic models. To my knowledge, there has not been much interest in modeling the environmental decoherence suffered by astrophysical or cosmological systems including those mentioned in section 4.3. This attitude may seem justifiable if quantum theory is simply a tool for calculating probabilities for outcomes of laboratory experiments. But it is indefensible from the pragmatist view of quantum theory outlined in [1].

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