To the Chairman of Examiners for Part III Mathematics.

Dear Sir,

I enclose the Part III essay of Temple He.

Signed ........................... (Director of Studies)
Part III Essay:
Varieties of Locality for Quantum Fields and Strings
(Essay Number 57)

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I declare that this essay is work done as part of the Part III Examination. I have read and understood the Statement on Plagiarism for Part III and Graduate Courses issued by the Faculty of Mathematics, and have abided by it. This essay is the result of my own work, and except where explicitly stated otherwise, only includes material undertaken since the publication of the list of essay titles, and includes nothing which was performed in collaboration. No part of this essay has been submitted, or is concurrently being submitted, for any degree, diploma or similar qualification at any university or similar institution.

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Part III Essay:

Varieties of Locality for Quantum Fields and Strings

(Essay Number 57)
Relinquishing Locality:
Entropy Bounds and the Holographic Principle

Abstract:
In recent years, a new paradigm in theoretical physics has emerged known as the holographic principle. This principle states that the number of degrees of freedom in a $d$-dimensional region of a $(d+1)$-dimensional spacetime is proportional to the area of a suitably defined surface associated with the region of spacetime. This principle was motivated by the fact that the entropy of a black hole is in fact proportional to the area, rather than the volume, of the event horizon. Over the years, this entropy bound on black holes has been extended to more general settings, culminating in the formulation of the covariant entropy bound. Nonetheless, there is still much controversy surrounding the holographic principle, as it violates locality and also contradicts the principles of traditional quantum field theories. This paper will both motivate the development of the holographic principle, beginning with the entropy of black holes and ending with the covariant entropy bound for arbitrary spacetime geometries, as well as explore the tensions between locality and the holographic principle.

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1 Introduction

Quantum field theory (QFT) is typically, and aptly, considered to be one of the spectacular achievements in theoretical physics in the twentieth century. It has been verified experimentally to an astonishing degree of accuracy and, together with classical general relativity (GR), forms the basis of our understanding in physics. Nonetheless, it is well known that unifying QFT and gravity is still a difficult open question, especially since it is currently unclear whether gravity is a theory with asymptotic safety or simply a nonrenormalizable theory [1]. Since neither gravity nor quantum effects is negligible in the vicinity of black holes, we may require a consistent theory of quantum gravity in order to correctly explain the physics occurring in this region. Thus, by examining how QFT and GR break down near black holes, we may be able to get insights into how to reconcile the tensions between these two theories.

Indeed, near a black hole, it becomes questionable whether the physics can be completely described by a local field theory. QFT, despite its successes, is still after all only an effective field theory, and hence we can apply perturbative techniques only in the low energy approximation [2, 3]. When energies become sufficiently high, nonrenormalizable terms in the Lagrangian are no longer negligible and thus must be considered, and the standard perturbation expansion employed in QFT breaks down. In this case, the QFT must be treated nonperturbatively, with the theory remaining well-defined only if the theory enjoys asymptotic freedom or safety (gravity as a local field theory is not asymptotically free, so we would require the theory to be asymptotically safe). Now, once we take into account black holes, however, even for low energy fields we cannot ignore the nonrenormalizable terms (and hence cannot apply perturbative methods) due to a phenomenon known as the UV/IR mixing.

Typically, in local field theories, the high energy modes are decoupled from the low energy modes, and this decoupling is a crucial aspect of Wilsonian renormalization. UV/IR mixing, however, is the “mixing” of high energy and low energy modes in a field so that low energy physics are now affected by physics at high energy scales, which means that different energy scales are no longer decoupled from each other. To see how UV/IR mixing arises for black holes, consider the collision of two particles with center-of-momentum (COM) energy $E$, taken to be above the Planck scale. If we do not consider gravity, then we expect this collision to produce high energy particles that reflect processes occurring at length scales of order $\frac{1}{E}$ (we are using natural units $\hbar = c = 1$). However, once we consider gravity, the collision results in the creation of a black hole with mass $E$ and Schwarzschild radius $R_s = 2GE$. The result is that the small length scale processes are all hidden behind the event horizon of the black hole, and the only particles produced are photons with energies of order $\frac{1}{R_s}$ due to Hawking radiation [4][Chap 10].

Thus, we see that due to black holes, increasing the COM energy of the initial collision increases the Schwarzschild radius of the resulting black hole, and hence the resulting particles
from Hawking radiation have lower energies. This illustrates that the low energy physics in the vicinity of black holes is not decoupled from high-energy physics. Therefore, perturbative techniques applied in analyzing field theories, which neglect the nonrenormalizable terms in the Lagrangian (the terms important for high-energy processes) under the assumption that the low-energy processes are isolated from high-energy processes, are no longer valid. Although it is possible that the field theory near black holes enjoys asymptotic safety and is thus still renormalizable under UV/IR mixing when we treat it nonperturbatively, we shall see later on that such a possibility is unviable if we want to preserve unitarity [1].

Due to the phenomenon of UV/IR mixing, it is clear now that to describe quantum gravity in the vicinity of black holes, we cannot naively use QFT in a curved background unless we are willing to assume that QFT enjoys asymptotic safety and thus can be treated nonperturbatively. However, even though black holes may be the source of the potential breakdown of QFTs, black holes also give us a glimpse into how we should alter our principles and surmount the issues we have uncovered. The first clues to how QFT may breakdown near black holes came from black hole thermodynamics, when in 1973, Bekenstein discovered that black holes may possess inherent entropy that is proportional to the area of the event horizon rather than the volume [5]. There has since been much interest in how to interpret this claim, as QFT predicts the entropy should always grow as the volume of the region. We will review in this paper the entropy bounds for black holes. We will then generalize these entropy bounds for black holes to entropy bounds for arbitrary physical systems, and this is known as “the holographic principle.” As we will see, the holographic principle relates the spacetime geometry of a system to the number of possible quantum states; in this respect, the holographic principle is a step towards a consistent theory of quantum gravity.

The structure of the paper is as follows. In Section 2, we will review in closer detail the shortcomings of attempting to describe physics near black holes using a local field theory. In Section 3, we will examine the spherical entropy bound for black holes. We will finally in Section 4 attempt to generalize the entropy bounds for black holes to entropy bounds in a general setting, known as “the covariant entropy bound.” To simplify notation, we will throughout this paper use natural units and take \( \hbar = c = G = 1 \).

2 Locality, Unitarity, and Other Issues

2.1 Degrees of Freedom

We begin by defining precisely what is meant by “degrees of freedom” in a quantum system. Let \( \mathcal{N} \) be the dimension of the Hilbert space of the quantum system, and define the number of degrees of freedom (DOF) \( N \) to be [6][Sec III.A]

\[
N = \log \mathcal{N}.
\]  

(2.1)
Hence, DOF in this context refers to the log of the number of possible quantum states in a system. For instance, consider a cubic lattice in a volume $V$ with lattice spacing $a$. At each lattice point we allow our quantum system to either be spin up or spin down, so there are a total of $V/a^3$ spins. Since there are two possibilities for each spin, there are then $2^{V/a^3}$ states in this quantum system, and hence the number of DOF is

$$N_s = \log 2^{V/a^3} \sim \frac{V}{a^3}. \quad (2.2)$$

Let us consider another example: a local quantum field [6][Sec III.C]. It is clear that by the definition of DOF given in eq. (2.1), the DOF for a quantum harmonic oscillator is $N_{qho} = \infty$ as the oscillator lives in an infinite dimensional Hilbert space. Thus, as a local quantum field is described by a harmonic oscillator at every point in space, it is clear that for a quantum field the DOF is given by $N_{qft} = \infty$, regardless of the volume $V$ under consideration. Note this is precisely the same divergence that crops up when calculating the vacuum energy of a QFT. To tame these infinities, we first assume that the resolution of spacetime is given by the Planck length $l_p = 1.6 \times 10^{-33}$ cm, so that the number of oscillators describing the field is $V/l_p^3$. Next, we also impose an ultraviolet cutoff, assuming that energy scales higher than the cutoff is beyond the scope of the QFT. We take this cutoff to be at the Planck energy $M_p = 1.3 \times 10^{19}$ GeV, so that number of states per oscillator is finite. For simplicity we will assume that every oscillator has $n$ possible states. (Note that for an actual quantum field, the oscillators are labeled by their momentum $\vec{p}$ and have energy spacing $\omega_{\vec{p}} = \sqrt{|\vec{p}|^2 + m^2}$, where $m$ is the mass parameter. This means the number of allowable states for each oscillator is actually different, but this technicality doesn’t affect our argument.) With these modifications, the number of DOF for the quantum field is

$$N_{qft} = \log n^{V/l_p^3} \sim \frac{V}{l_p^3}. \quad (2.3)$$

In both of the examples, we see that the number of DOF is directly proportional to the volume of the system. However, it is important to realize that in both cases above we have not taken into account gravity and the possibility of forming black holes. We will see in the next two subsections that upon including gravity, these conclusions will no longer necessarily hold.

## 2.2 Bekenstein Entropy of Black Holes and the Generalized Second Law

The motivation for postulating that black holes have inherent entropy stems from two theorems in classical GR: Hawking’s area theorem and the no-hair theorem [7, 8, 9]. Hawking’s area theorem states that the area of a black hole event horizon, denoted $A$, never decreases with time, that is,

$$dA \geq 0. \quad (2.4)$$
This is very reminiscent of the second law of thermodynamics, which states that entropy $S$ never decreases with time:

$$dS \geq 0. \quad (2.5)$$

This is the first clue that we may associate the area $A$ with the entropy $S$ for a black hole. The second clue, the no-hair theorem, states that any stationary black hole is characterized completely by its charge, mass, and angular momentum. This theorem immediately implies that the second law of thermodynamics is violated: A collapsing star is described by many more parameters than just its charge, mass, and angular momentum, but the no-hair theorem states that once the star collapses into a black hole, as long as the charge, mass, and angular momentum for the star are the same, regardless of the star’s other characteristics, its final state as a black hole is unique. We seem to have gone from an initial state of nonzero entropy (many possible states) to a final state of zero entropy (one possible state).

In order to avoid violating the second law, Bekenstein conjectured that black hole actually carries intrinsic entropy of its own and is proportional to its area, so that

$$S_{bh} = \eta A, \quad (2.6)$$

for some constant $\eta$. Bekenstein proposed that if we take this black hole entropy into account, then the second law is satisfied: an observer outside a black hole watching matter fall into the black hole will see the entropy of the infalling matter lost due to the no-hair theorem, but the area of the event horizon increases by the area theorem, with the overall result that, by eq. (2.6), the total entropy of the infalling matter and the black hole does not decrease. This result is known as the generalized second law (GSL).

Initially, the Bekenstein entropy of a black hole was considered to be a mere analogy of entropy since the area of the event horizon, like entropy, never decreases. However, after Hawking radiation was discovered [10], Bekenstein entropy was viewed as a physical quantity, as any object with a nonzero temperature has nonzero entropy. Furthermore, the discovery of Hawking radiation also allowed the proportionality constant $\eta$ in eq. (2.6) to be derived. We will now derive $\eta$ following [11]. First, we need to determine the Hawking temperature. An in-depth treatment on Hawking radiation is found in [11], but for our purposes it suffices to do a quick heuristic derivation of the Hawking temperature introduced in [12]. We begin by writing the Schwarzschild metric in Euclidean time $\tau$, where $\tau = it$:

$$ds_{Sch}^2 = \left(1 - \frac{2M}{r}\right) d\tau^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\Omega^2. \quad (2.7)$$

To explore the region near the event horizon at $r = 2M$, we move to Rindler coordinates via the coordinate transform $r = 2M + \frac{1}{2} \kappa x^2$, where $\kappa = \frac{1}{4M}$. Our metric then becomes

$$ds_{Sch}^2 = (\kappa x)^2 d\tau^2 + dx^2 + \frac{1}{4\kappa^2} d\Omega^2 + \ldots, \quad (2.8)$$
where “…” indicates higher order terms that can be neglected as \( x \to 0 \). Only the first two terms in the above metric are important since those two terms describes a Euclidean plane in polar coordinates. Assuming that the black hole is in thermoequilibrium, no singularities should occur at the event horizon \((x = 0)\), and so we must make the periodic identification as we do in ordinary polar coordinates:

\[
\tau \sim \tau + \frac{2\pi}{\kappa}.
\]

Thus, quantum fields that live in this curved background are periodic in imaginary time with period \( \frac{2\pi}{\kappa} \). However, the partition function for a QFT with period \( \beta \) in imaginary time is given by \( Z = \text{Tr} \left( e^{-\beta H} \right) \), where \( H \) is the Hamiltonian, so the QFT is in thermoequilibrium with temperature \( T = \frac{1}{\beta} \) (we set \( k_B = 1 \), where \( k_B \) is the Boltzmann constant). Thus, with \( \beta = \frac{2\pi}{\kappa} \) and \( \kappa = \frac{1}{4M} \), we see that the temperature of the black hole, i.e. the Hawking temperature, is given by

\[
T_h = \frac{1}{8\pi M}.
\]

The Schwarzschild radius of a black hole is \( R_s = 2M \), so the surface area of a Schwarzschild black hole is

\[
A = 4\pi R_s^2 = 16\pi M^2. \tag{2.11}
\]

Differentiating eq. (2.11) we get

\[
dM = \frac{1}{8\pi M} d \left( \frac{A}{4} \right). \tag{2.12}
\]

This is very similar to the first law of thermodynamics with fixed volume \( dE = TdS \), especially since \( M = E \) and we identified above in eq. (2.6) that \( S \) can be thought of as the area \( A \) of the black hole. With these observations, and substituting eq. (2.10) into eq. (2.12), we get

\[
dE = T_h d \left( \frac{A}{4} \right) \tag{2.13}
\]

which invites us by the first law of thermodynamics to identify

\[
S_{bh} = \frac{A}{4} \tag{2.14}
\]

i.e. \( \eta = \frac{1}{4} \) in eq. (2.6). We would like to emphasize that the discovery of Hawking radiation not only allowed us to determine the proportionality constant in the Bekenstein entropy, but more fundamentally showed that black holes are actually thermodynamic bodies that radiate, and hence indeed possess intrinsic entropy. In other words, the Bekenstein entropy is not just a mathematical analogy, but an actual physical property of the black hole.

Although the Bekenstein entropy is an intrinsic property of a black hole, in general entropy is not fundamental for a system but instead reflects the number of microstates
possible given that certain constraints, such as total energy, are satisfied. Typically, the entropy is then defined as the logarithm of the number of quantum states that satisfies these constraints in the thermodynamic system. Thus, depending on the number of constraints given for a particular system, the entropy of the system can also change. However, there is another concept known as “the maximal entropy,” which is the entropy of a system should there be no constraints at all. Maximal entropy is an intrinsic property of a physical system, and it is the logarithm of all possible quantum states the system can be in. This is what we are interested in since it is equivalent to the number of DOF (see eq. (2.1)), and we will throughout this paper refer to maximal entropy simply as “entropy.”

### 2.3 Breakdown of Local Field Theory

We will now show that the number of DOF in local field theory, given by eq. (2.3), is in fact a drastic overestimation once gravity is taken into account [6][Sec III.E]. We want the system to be gravitationally stable and prevent the formation of black holes. Obviously this means there cannot exist a spherical region of radius $R$ with mass greater than that of a black hole with radius $R$. Because the Schwarzschild radius is $R_s = 2M$, given any sphere of radius $R$ in our system, in order for no black holes to form, the mass found within the sphere must satisfy

$$M \leq \frac{R}{2} \sim R. \quad (2.15)$$

In our calculation of eq. (2.3), we attempted to restrict the number of high-energy modes in each Planck-sized cube by imposing a fixed cutoff $M_p$, i.e. $M \leq 1$ in natural units. For simplicity, we can approximate the Planck-sized cube as a sphere of radius one Planck length. This imposes condition eq. (2.15) for the smallest length scale $R = 1$, demanding that each sphere of radius one Planck length does not contain more than one Planck mass; otherwise a Planck-sized black hole will form.

However, this is a necessary but not sufficient bound; it does not prevent larger black holes from forming. For instance, consider a sphere of radius 1 cm. This is $10^{33}$ in Planck units, and hence has a volume of approximately $10^{99}$ in Planck units. Assuming we saturate the bound for each Planck volume, the mass in this 1 cm sphere is $10^{99} M_p$. However, by eq. (2.15) the maximum mass contained in this sphere before a black hole forms is $10^{33} M_p$. This demonstrates that we cannot simply impose an ultraviolet cutoff when calculating the DOF for a local field. Long before the high energy modes are excited, black holes have already formed, and the entropy, or the number of DOF, associated with a black hole is its Bekenstein entropy eq. (2.14), which is proportional to the black hole’s area and not volume.

This is another hint, in addition to the UV/IR mixing phenomenon, that local fields may not be suitable when attempting to formulate a theory unifying quantum mechanics and gravity. It is possible to claim that the extra DOFs that arise from the field theory calculation are actually correct, and that the over-counted DOFs are merely hidden behind the horizons.
of the miniature black holes that have formed. However, we will in the next subsection give
evidence for the unitary evolution of black holes, and then later using unitarity show that
neither asymptotic safety nor hiding DOFs behind event horizons are viable options, and
thus that local field theory indeed breaks down.

2.4 Unitary Evolution of Black Holes

In this subsection, we will briefly turn to the question of whether black holes evolve unitarily.
We will see that there is compelling evidence that the evolution is indeed unitary, and this
will in turn lend support to the holographic principle.

Originally, Hawking’s calculations in 1976 seemed to indicate that after a black hole
evaporates via Hawking radiation, all the states that went into the black hole are irretrievably
lost. This means that black holes do not evolve unitarily, and that information is lost when
black holes evaporate. However, unitarity is a property of physical processes that physicists
would like to preserve, since a violation of unitarity can also lead to violations in energy
conservation or causality [13]. Most notably, ’t Hooft, Susskind, and Preskill were all opposed
to Hawking’s claim that the information was destroyed, and numerous attempts were made
to ensure that the evolution of black holes is unitary. Perhaps the most natural solution
to this information-loss problem is for all the information inside a black hole to be carried
away by Hawking radiation before it evaporates, but this seems to introduce a paradox. If
Hawking radiation is indeed carrying away information, then consider the following thought
experiment [4][Chap 9.2]. A system $A$ containing some information falls into a black hole.
An observer $B$ then goes right to the edge of the event horizon and lingers there for a
while collecting Hawking radiation. If the Hawking radiation is carrying away information,
then observer $B$ will sooner or later be able to gather all the information about system
$A$. Observer $B$ can then dive into the black hole and obtain a second copy of information
regarding system $A$. On the other hand, according to the no-cloning theorem in quantum
mechanics, it is impossible to create identical copies of a general quantum state [14, 15].
Thus, it would appear that if Hawking radiation is indeed carrying information regarding
system $A$, then $B$ can create a copy of $A$ without destroying $A$, thus violating the no-cloning
theorem. There is still some controversies regarding how to resolve this paradox [16], but the
details of the resolution will not affect the important parts of what follows. For the purposes
of this paper, we will adopt the heuristic orthodox resolution to this paradox, which invokes
black hole complementarity [4][Chap 9.2].

One way to view black hole complementarity is that no observer can observe a phe-
nomenon that violates natural laws, though different observers may differ in their accounts
of the phenomenon [17, 18]. Indeed, this viewpoint is attractive because the general principle
of “complementarity” also appears in quantum mechanics when discussing, for example, the
wave-particle duality (the statement that light can be thought of as fundamentally both a
particle and a wave). There, it is said that it is logically inconsistent to claim that light has both particle-like and wave-like characteristics at the same time, but any experiment will only reveal one or the other description of light, never both. In this respect, it is consistent to think of light as both a particle and a wave, depending on the physical setting we are in.

We use a similar line of thinking for the case of black holes. For an observer at infinity, the no-cloning theorem is not violated because he will never have access to the copy of information about system \( A \) within the black hole. Therefore the only point of concern is that described in the previous paragraph, when the observer decides to follow system \( A \) into the black hole. To resolve the paradox, it is important to remember that system \( A \) can send information to observer \( B \) when both are inside the black hole only if system \( A \) has not yet hit the singularity. It was first shown by Page that the time it takes for observer \( B \) to gather one bit of information on system \( A \) via Hawking radiation is comparable to the lifetime of the black hole [19]. This then implies that any system \( A \) falling into the black hole would have hit the singularity before observer \( B \) jumps into the black hole (see fig. 1). Even if there is no true singularity within the black hole, as is predicted by, for example, string theory or loop quantum gravity, it is expected that new physics will be required to explain what occurs to system \( A \) when it is near the “approximate singularity.” Therefore, if there is a true singularity inside the black hole, the no-cloning theorem is not violated, and if there is only an “approximate singularity” as predicted by loop quantum gravity, new physics will most likely be needed to explain the evolution of system \( A \) near the “approximate singularity,” which falls beyond the scope of quantum mechanics and the no-cloning theorem. In either case, we see that the resolution of the paradox involves two complementary viewpoints: one for an outside observer and one for an infalling observer. While it does not make sense to consider both viewpoints simultaneously, each viewpoint by itself is logically sound.

Although we have shown above that unitary evolution of black holes does not result in a paradox, this does not guarantee that black holes actually have unitary evolution. Nevertheless, much evidence has come to support the view that black holes do evolve unitarily. In 1996, Strominger and Vafa were able to explicitly count the number of microstates for certain very special black holes in string theory following unitary evolution in string theory [20], and the AdS/CFT correspondence suggests that certain Anti-de Sitter (AdS) spacetimes and the black holes in them are described completely by a unitary supersymmetric conformal field theory (CFT) [21], and hence suggests that our own spacetime, which has an approximately de Sitter spacetime geometry, perhaps also obeys unitary evolution. Indeed, there is currently work by Strominger, among others, to extend the duality between CFT and AdS spacetimes by replacing AdS spacetimes with de Sitter (dS) spacetimes [22], and even Kerr black holes [23]. As Kerr black holes are “realistic” black holes, as opposed to black holes in AdS spacetime, this would lend evidence to the unitary evolution of “realistic” black holes. Thus, we will henceforth assume that black hole evolution is unitary, and demonstrate in Section 3.3 that this assumption will necessarily create tension with the principle of locality.
Figure 1: Possible resolution to the apparent violation of the no-cloning theorem. Quantum system $A$ hits the singularity before observer $B$ can gather enough information about $A$ via Hawking radiation to reconstruct $A$. This ensures $B$ cannot obtain two copies of $A$, even if he jumps into the black hole after reconstructing $A$ at the event horizon. Figure from [4][Chap 9.2].

3 The Spherical Entropy Bound

In Section 2.2 we showed using the first law of thermodynamics that the quantity $\frac{A^4}{\ell^4}$, where $A$ is the area of the event horizon, can be thought of as the entropy of the black hole (see eq. (2.13)). We then imposed the generalized second law (GSL) on the system, which states that the total entropy, including the Bekenstein entropy of the black hole, does not decrease over time. This result will give rise to an entropy bound in any spherical region in space via the Susskind process (see Section 3.1). We will then provide some examples to give intuition about the spherical entropy bound (see Section 3.2). Finally, using the Susskind process, we will be able to determine what is the correct interpretation of the number of quantum states in a region of spacetime; thus, we will arrive at the holographic principle (see Section 3.3).

3.1 The Susskind Process

We begin by assuming that the spacetime geometry we are in is spherically symmetric, and hence it makes sense to define spheres in it. Let $\mathcal{M}$ be a spacetime region containing an isolated matter system with entropy $S_m$, and let $A$ be the area of the circumscribing sphere $\mathcal{S}$, with radius $R$, so that for a fixed time in $\mathcal{M}$, the spacelike slice across $\mathcal{M}$ lies within $\mathcal{S}$. We assume that the matter system is not dominated by gravity and is stable enough so that that $A$ can be thought of as being time-independent.
The mass contained in $S$ must be less than that of a black hole of radius $R$, since if the matter system has mass greater than that of a black hole of radius $R$, then the matter system would be a black hole with radius greater than $R$ and hence would not be contained in $S$. Therefore, in order to convert the matter system into a black hole of radius $R$, we need to add mass into the system. Let $S_{m'}$ be the entropy associated with the added mass, and let $S_i$ be the initial entropy of the original matter system plus the added mass. Because entropy is additive, this means that $S_i$ is the sum of the entropy of the original matter ($S_m$) plus that of the added mass ($S_{m'}$):

$$S_i = S_m + S_{m'}.$$  \hfill (3.16)

But after adding the mass, the matter system has just the amount of mass required to be converted into a black hole of radius $R$. All the matter is required to convert the system into a black hole, so there is no residual ordinary matter left within $S$. Thus, in place of the ordinary matter system we now simply have a black hole, which means by eq. (2.14) that the entropy contained in $S$ after the black hole formed, denoted as $S_f$, is

$$S_f = S_{\text{black hole}} = \frac{A}{4}.$$  \hfill (3.17)

By the GSL, entropy never decreases, so it follows $S_i \leq S_f$, which implies in particular (since entropies are nonnegative quantities)

$$S_m \leq \frac{A}{4}.$$  \hfill (3.18)

This is the spherical entropy bound, and is a general statement for all quasistable isolated matter systems in spherically symmetric spacetimes [6][Sec II.C.1]. As we pointed out in Section 2 (see eq. (2.3)), this is directly at odds with the prediction given by local field theory. Nonetheless, we were in favor in Section 2.4 of giving up locality for unitarity, and this is the path we shall take for the rest of the essay.

Before continuing, it is worthwhile to emphasize the generality of eq. (3.18). Notice that the derivation above does not make any assumptions in the type of matter contained in $S$. $S$ could contain dark matter, neutron stars, or human beings, but regardless of what it contains, the number of DOF contained in $S$ does not change and is proportional to the area of $S$. Eq. (3.18) is a statement on the fundamental number of quantum mechanical states inherently present in any region contained in such a sphere $S$, and thus gives us a glimpse into what our universe is like at the most fundamental level. We now illustrate the spherical entropy bound with some explicit examples.

### 3.2 Explicit Examples of Spherical Entropy Bound

The examples we take in this section are those outlined in [6][Sec II.C.3]. First, we look at the trivial case where the sphere $S$ of radius $R$ and area $A$ contains just a black hole of
radius $R$. Then the entropy of the black hole is $\frac{A}{4}$ by eq. (2.14), so the spherical entropy bound is saturated.

Next, we look at the case when the sphere $S$ contains $n$ black holes. Let $M_1, \ldots, M_n$ be the masses of the black holes, so that the Schwarzschild radii of the black holes are $2M_1, \ldots, 2M_n$. The total entropy of the system is then given by eq. (2.14):

$$S_m = \sum_{i=1}^{n} \frac{4\pi R_i^2}{4} = \sum_{i=1}^{n} \frac{4\pi(2M_i)^2}{4} = 4\pi \sum_{i=1}^{n} M_i^2.$$  \quad (3.19)

Now, since the $n$ black holes contained in $S$ did not coalesce into a giant black hole of radius $R$ and mass $M = \frac{R}{2}$, this means that $\sum_{i=1}^{n} M_i < M$. By eq. (2.14) the entropy associated with the giant black hole is $\frac{4\pi R^2}{4} = 4\pi M^2$. As $A = 4\pi R^2$, it follows

$$\frac{A}{4} = 4\pi M^2 > 4\pi \left(\sum_{i=1}^{n} M_i\right)^2 \geq 4\pi \sum_{i=1}^{n} M_i^2 = S_m,$$  \quad (3.20)

where the last equality used eq. (3.19). This proves that again the spherical entropy bound is satisfied.

Finally, let us examine a gas of identical massless bosons with energy $E$ and temperature $T$ confined to the sphere $S$. We assume that $S$ is superPlanckian, i.e. that $S$ has a radius much larger than the Planck length, so we may still apply the results concerning Schwarzschild black holes derived from classical general relativity. The energy of this volume of gas is derivable using statistical mechanics (found in [24]) and is given to be

$$E \sim \frac{4}{3} \pi R^3 T^4 \sim R^3 T^4,$$  \quad (3.21)

where we have neglected constant prefactors of order unity. As the volume is constant, by the first law of thermodynamics $E = Q$, where $Q$ is the heat. Hence, the entropy of the gas is

$$S = \frac{Q}{T} \sim R^3 T^3.$$  \quad (3.22)

It follows that

$$S \sim R^{3/4} E^{3/4}.$$  \quad (3.23)

Because the gas has not formed a black hole, the energy of the gas must be less than the mass of a black hole with radius $R$, i.e. $E \leq M = \frac{R}{2}$. Substituting into eq. (3.23) we get

$$S \lesssim R^{3/2} \sim A^{3/4} \ll \frac{A}{4},$$  \quad (3.24)

where in the last inequality, we used the fact that in Planck units $A \gg 1$.

Notice that in the first example we gave above, the spherical entropy bound is in fact saturated when the region is a black hole, so it is the best possible bound given a spherical region in space.
3.3 Towards a Holographic Principle

We showed in Section 2.3 that describing a quantum gravity setting using local fields results in a drastic over-counting of DOF, since any attempt to excite one DOF per Planck volume (see eq. (2.3) with $l_p = 1$ in natural units) results in the formation of black holes, as eq. (2.15) is no longer satisfied. However, there are still two ways to interpret this breakdown of local field theory. The most conservative interpretation is that there is in fact no overcounting of DOF; the fact that black holes form in the system simply means that the extra DOF are merely hidden behind the event horizons of black holes. If this were true, then there are still $e^V$ quantum states, except we need to fall into a black hole to see those extra DOF.

This interpretation is shown to be unviable if we assume that black hole evolution is unitary [6][Sec III.F], which we are assuming following the suggestions of Section 2.4. This is because using the Susskind process, we can convert an isolated region of volume $V$ with mass $M$ into a black hole of mass $M + \Delta M$ by adding $\Delta M$ of mass into the system, and the Bekenstein entropy of the black hole is $\frac{A}{4}$ by eq. (2.14). Thus the number of quantum states the region initially had is $e^V$, and after the black hole forms, the number of states becomes $e^{\frac{A}{4}}$, so that a large number of quantum states is destroyed. This violates the unitarity condition on black hole evolution. Therefore, since the final number of states in the region is $e^{\frac{A}{4}}$, it follows that the region with mass $M$, plus the mass $\Delta M$ required to form the black hole, must start with only $e^{\frac{A}{4}}$ states. This in turn implies that the number of initial quantum states in the region with mass $M$ must be less than $e^{\frac{A}{4}}$ rather than equal to $e^V$.

This is an extremely counterintuitive result, and in particular suggests that the DOF of physical systems involving black holes is no longer local in space. We can no longer treat the DOF in a small volume inside the region in isolation from the DOFs in other small volumes, as this would imply that the total number of DOF in the region is proportional to the volume; rather, the excitation of a DOF in one small volume may very well influence the excitation of a DOF in another region, which is what we mean when we say the DOF of the physical system is no longer local in space.

This observation based on black holes suggests that in fact the DOF for every region of spacetime is proportional to an appropriately chosen boundary area. This is a rough statement of the holographic principle, and was initially conjectured by ’t Hooft and Susskind to be a fundamental law of nature [25, 26]. If this is true, then any fundamental theory of quantum gravity must respect this result [6][Sec III.F]. Hence, it appears from the above arguments that either we can preserve locality and claim that the DOF in a region of space is proportional to the volume and thus give up unitarity; or we can preserve unitarity and claim that the DOF in a region of spacetime is actually proportional to an appropriately chosen boundary area rather than the volume and thus give up locality. From arguments in Section 2.4, we choose to give up locality in favor of unitarity.

It is clear now from the spherical entropy bound that the number of DOF in a certain
spherical region of space satisfying certain assumptions should be proportional to the area rather than the volume. We would now like to generalize the spherical entropy bound to arbitrary regions in spacetime.

4 The Covariant Entropy Bound

The spherical entropy bound derived above assumes a number of constraints, including that the spacetime is spherically symmetric and that the matter system under consideration is isolated. We would however like to generalize this entropy bound to any matter system, isolated or not, in any arbitrary region in spacetime. After all, the maximum number of DOF in a region of spacetime should not depend fundamentally on whether the region is spherically symmetric or not. Because we are in a (3+1)-dimensional spacetime, given a 2-dimensional area $A$ in space, we can specify many 3-dimensional hypersurfaces that will have $A$ as their boundary areas. How do we know which particular hypersurface has its number of DOF constrained by the area of $A$? We will in this section first use the most straightforward generalization of the spherical entropy bound, which is to fix the time component and choose our 3-dimensional hypersurface to be the spacelike region of space contained inside $A$. This is known as “the spacelike entropy bound,” and will be incorrect in general (see Section 4.1). We will then give the correct entropy bound for an arbitrary region of spacetime, known as “the covariant entropy bound” (see Sections 4.2–4.4). The covariant entropy bound was formulated by Bousso in 1999 [27], and it is this bound that we call “the holographic principle.”

4.1 The Spacelike Entropy Bound?

The spherical entropy bound constrains the number of DOF in a spherical spacelike region of surface area $A$ to be less than $A^4$. Hence, the most natural generalization of the spherical entropy bound would be to claim that the number of DOF in any spatial region (of fixed time) with surface area $A$ is less than $A^4$. This is known as the spacelike entropy bound, and although it looks attractive, it is relatively easy to construct a counterexample to it [6][Sec IV.B.3].

Consider a spherical star collapsing into a black hole, and let our observer be on the surface of the star, so that he is following the trajectory of the star’s collapse. The star and the observer will fall through the horizon, and let $S_0$ be the entropy of the collapsing star as it passes its own horizon. Since the region inside the event horizon is isolated, the second law of thermodynamics applies, which means the entropy inside the event horizon does not decrease as the star collapses. Since the region inside the event horizon excluding the collapsing star is all vacuum, all the thermodynamic microstates must be within the collapsing star, so in particular this means the entropy of the star does not decrease below
as it collapses. On the other hand, the surface area of the star becomes arbitrarily small as it collapses into a black hole, so this means \( \frac{S}{A} \to \infty \) as the star collapses, where \( S \) is the entropy of the star and \( A \) the area of the star. This is a clear violation of the spacelike entropy bound, and results from the fact as the star collapses into a black hole, we are in a regime of dominant gravity. Thus, both the curvature and the time-dependence of the metric are nonnegligible, which violates the assumptions used in the Susskind process when formulating the spherical entropy bound.

### 4.2 Entropy in Asymptotic Minkowski Space

We showed above in our counterexample that the spacelike entropy bound fails. As we will show in the next few subsections, the way to remedy this is to consider an entropy bound not on spacelike regions, but on null hypersurfaces. This entropy bound is known as the covariant entropy bound. We will derive in this section the covariant entropy bound for arbitrary regions in spacetime geometries that are asymptotically Minkowski. This will provide the intuition for how to define the entropy bound in flat Friedman-Robertson-Walker geometries (see Section 4.3), and will lead eventually to Bousso’s generalization of the entropy bound to arbitrary spacetime geometries (see Section 4.4). We will follow the treatment of [4][Chap 11] for most of these subsections.

We begin by writing the metric for Minkowski space in lightcone coordinates (repeated indices imply summation):

\[
    ds^2 = -2dx^+dx^- + \delta_{ij}dx^i dx^j \quad \text{with} \quad i, j = 1, 2, \quad (4.25)
\]

where we will take \( x^+ \) to be the “time” coordinate. Note that the lightcone coordinates arise from the usual Minkowski metric

\[
    ds^2 = -dt^2 + \delta_{ij}dx^i dx^j \quad \text{with} \quad i, j = 1, 2, 3 \quad (4.26)
\]

with the coordinate change

\[
    x^+ = \frac{t + x^3}{\sqrt{2}}, \\
    x^- = \frac{t - x^3}{\sqrt{2}}. \quad (4.27)
\]

With our lightcone coordinates, we now define lightsheets to be hypersurfaces with \( x^+ = C \) for some constant \( C \). For each lightsheet \( x^+ = C \), we define the 2-dimensional “screen” to be the points at infinity on the lightsheet satisfying the additional constraint \( x^- = \infty \) (see fig. 2).

Given a point on a fixed lightsheet, it must lie on some null light ray originating from the screen with coordinates \((x^1, x^2) = (x^1_0, x^2_0)\). Thus, we can project any point on a fixed
Figure 2: A black hole passing through lightsheets. Each point on the event horizon can be projected via a null light ray onto the screen, so that the entire event horizon can be mapped to a region on the screen. The caustic is the region where the null light rays intersect. Figure from [4][Chap 11.2].

Let us now consider a Schwarzschild black hole moving through a fixed light sheet (see fig. 2). Note that since its event horizon is a null hypersurface, it is embedded in the lightsheet at any fixed \( x^+ = C \). Each point on the event horizon can then be projected onto the screen in the manner described in the previous paragraph. Let \( \Xi \) be a patch on the event horizon, and let \( P(\Xi) \) be the projection of \( \Xi \) onto the screen. Because the black hole’s entropy is described by the area of its event horizon, we can view the entropy as “living” on the event horizon, so that when we project the horizon of the black hole onto the screen, we are also mapping the entropy in each region of the horizon onto the corresponding region on the screen. In this fashion, we can “project” the entropy in \( \Xi \) onto the screen and define a corresponding entropy density in \( P(\Xi) \) (see fig. 3). Therefore, given that the entropy density of \( \Xi \) is \( \sigma(\Xi) \), the corresponding entropy density on the screen is \( \sigma(P(\Xi)) \). We now claim that the entropy density of the screen is bounded above by \( \frac{1}{4} \).

To prove this claim, we will use the focusing theorem from GR. Let \( \alpha \) be the cross-sectional area of a family of nonintersecting light rays, or a null congruence, beginning at
Figure 3: We can associate an entropy density to each region of the event horizon. By projecting each region of the horizon onto the screen, we are also mapping the entropy in each region onto the corresponding region on the screen, thus “projecting” the entropy of the black hole onto the screen. Figure from [4][Chap 11.2].

The horizon of the black hole and ending on the screen, and let $\lambda$ be the affine parameter characterizing this congruence. The focusing theorem then states

$$\frac{d^2 \alpha}{d\lambda^2} \leq 0.$$  \hfill (4.28)

This means that the graph of $\alpha$ as a function of $\lambda$ is concave down. As $\lambda$ increases, the null congruence approaches the screen, and the light rays become parallel to each other. This implies that $\frac{d\alpha}{d\lambda} \to 0$ as $\lambda \to \infty$. Because the graph is concave down, this means that $\alpha$ must be increasing as a function of $\lambda$, so the area of $P(\Xi)$, the projection of a patch of the horizon onto the screen, is larger than the patch $\Xi$. The entropy of the black hole is $A_4$, where $A$ is the area of the horizon. Thus, we can view the entropy of the black hole to be solely confined to the horizon, which allows us to define the entropy density of the horizon to be

$$\sigma_{bh} = \frac{1}{A} \cdot \frac{A}{4} = \frac{1}{4}.$$  \hfill (4.29)

Because the projection of a patch of black hole onto the screen has larger area than the original patch itself, the entropy density on the screen must also be less than that of the horizon. This proves our claim that the entropy density on the screen is bounded above by $\frac{1}{4}$.

We remark that this bound holds if we consider an arbitrary number of black holes. Because congruences of light rays do not intersect, different event horizons are projected onto different regions on the screen, so we may apply the identical argument from the previous paragraph to conclude that the entropy density on the screen is bounded above by $\frac{1}{4}$. Because black holes are considered to be the most entropic objects as they saturate the spherical
Figure 4: As the entropy of a matter system passes through the lightcone, we can associate each region of the lightcone with an entropy density. As in asymptotic Minkowski space, we can now “project” the entropy density in the lightcone onto the screen. This allows us to formulate a bound on the entropy density on the screen in FRW geometry. Figure from [4][Chap 11.3].

entropy bound (see Section 3.2), this leads us to the covariant entropy bound in Minkowski space: For any physical system projected onto the screen in the manner described above, the entropy density of the screen is bounded above by $\frac{1}{4}$.

4.3 Entropy in Flat Friedman-Robertson-Walker (FRW) Geometry

Next, we derive a similar bound for our own universe, which we assume to have a flat FRW geometry. The FRW metric is

$$ds^2 = dt^2 - a(t)^2d\Sigma^2,$$

(4.30)

where $d\Sigma$ is a metric over the 3-dimensional flat space independent of time (the time dependence is captured by $a(t)$). We will for simplicity assume that $a(t) = a_0 t^p$ for some $p$, and that the universe is homogeneous.

For a fixed time $t$, let $V$ be a spherical region in space of volume $V$, with surface $A$ of area $A$. The surface $A$ is now the screen considered in Section 4.2. We now trace a lightcone backwards in time with $A$ as the boundary of the lightcone (see fig. 4); this lightcone is the lightsheet from the previous section. As in Section 4.2, each spacetime point on the lightcone can be mapped onto a point on the screen $A$. Thus, when a black hole passes through the lightcone, the entropy density can be projected onto the screen $A$ in the same manner as that detailed in Section 4.2. By the focusing theorem it follows again that the entropy density on the screen cannot exceed $\frac{1}{4}$, i.e. the entropy projected onto the screen cannot exceed $\frac{4}{4}$. Because all the entropy passing through the lightcone is projected onto the
Figure 5: Entropy bound in flat FRW geometry when the screen $A$ is larger than the particle horizon. Notice that entropy in the sphere could have passed through the hole on the bottom of the lightcone and hence never pass through the lightcone itself. Figure from [4][Chap 11.3].

screen, the total entropy passing through the lightcone is bounded above by $\frac{A^4}{4}$. This is the covariant entropy bound in flat FRW geometry. It is clear that the same argument would work if $V$ is not spherical; the only change is that the light rays forming the lightcone will not converge in general to a point, but will instead intersect each other at different times. Such a region where the null light rays intersect is called a caustic.

Fig. 5 illustrates the fact that the covariant entropy bound is less restrictive than the spacelike entropy bound. If we take our surface $A$ to be larger than the particle horizon, then fig. 5 shows that the lightsheet we consider is a truncated lightcone, terminating at the big bang, rather than a full lightcone. This means there will be entropy contained in the spacelike sphere constrained by $A$ that will never pass through the lightcone, and so there will be entropy flowing through the spacelike sphere that will not pass through the lightsheet.

We illustrate the covariant entropy bound explicitly through an example by taking our screen $A$ to be the observed particle horizon. In this case, the backward lightcone is not truncated and begins at the big bang singularity (by the definition of the particle horizon). Therefore, there is no hole on the bottom of the lightcone like that shown in fig. 5, so the entropy passes through the lightcone if and only if the entropy passes through $V$, i.e. the observable universe. Let $\eta$ be the number of possible quantum states for each blackbody cosmic background photon, and let $n$ be the number of such photons. The number of quantum states is then $N = \eta^n$, so the entropy contained in the observable universe is given by

$$S = \log N \sim n.$$  \hspace{1cm} (4.31)

There are roughly $10^{80}$ cosmic background photons [4][Chap 11.3], so $S \sim 10^{80}$. The radius of the particle horizon is $10^{18}$ light-seconds; one Planck length is $10^{-43}$ light-seconds, so in Planck units the radius of the particle horizon is $R \sim \frac{10^{19}}{10^{-43}} = 10^{61}$. The area of the particle
horizon is
\[ A = 4\pi R^2 \sim 4\pi (10^{61})^2 \sim 10^{122}. \] (4.32)
Clearly
\[ S \sim 10^{90} \leq \frac{A}{4} \sim 10^{122}, \] (4.33)
so we see that the entropy bound is completely satisfied today.

### 4.4 Bousso’s Generalization to Arbitrary Geometries

We would now like to generalize the covariant entropy bound to any arbitrary geometry of dimension \( d+1 \). Let \( \Gamma \) be an arbitrary \( d \)-dimensional spacelike region, and \( \partial \Gamma \) be the \((d-1)\)-dimensional boundary of \( \Gamma \). We can then draw four null hypersurfaces originating from \( \partial \Gamma \), two of which will terminate at caustics and are hence bounded (see fig. 6). These two null hypersurfaces are our lightsheets. We will refer to \( F_1 \) and \( F_2 \) in fig. 6 to be future-directed and \( F_3 \) and \( F_4 \) to be past-directed. Also, \( F_1 \) and \( F_3 \) are ingoing while \( F_2 \) and \( F_4 \) are outgoing.

In flat Minkowski geometry, it is clear that the two null hypersurfaces that are bounded are the two ingoing lightcones. However, in more general geometries, other possibilities exist. For instance, if our universe underwent a period of inflation, then given a sufficiently large region, both past-directed lightcones will terminate at the big bang singularity while both future-directed lightcones will be unbounded since the space is expanding too quickly for the light rays in the future-directed ingoing lightcone to intersect at a point.

Bousso’s generalization is then the following statement: Given the \((d - 1)\)-dimensional boundary \( \partial \Gamma \) of a \( d \)-dimensional spacelike region \( \Gamma \), construct the two bounded lightsheets as described above. Then the entropy flowing through the lightsheets is bounded by \( \frac{A(\partial \Gamma)}{4} \), where \( A(\partial \Gamma) \) is the area of the boundary \( \partial \Gamma \) [27]. By Bousso’s generalization, we do not give any preference to any particular bounded lightcone emanating from the boundary hypersurface \( \partial \Gamma \) under consideration, and this is the holographic principle in arbitrary spacetime geometries.

Thus far, there has been no physically relevant counterexamples to Bousso’s generalization, often referred to as the covariant entropy bound. This is quite impressive, since it has been relatively easy to construct counterexamples to many of the other entropy bounds, i.e. the spacelike entropy bound. However, despite the lack of counterexamples, the covariant entropy bound would be much more established if it can be shown to arise out of a fundamental theory unifying quantum mechanics and gravity. One such theory could be string theory. While the covariant entropy bound does not explicitly arise out of perturbative string theory, because the perturbative expansion breaks down before the bound is violated [26], there are aspects of string theory that are reminiscent of the holographic principle. For instance, in perturbative string theory, the dynamical degrees of freedom for the wave function in a region of spacetime does not grow proportional to the volume of the region, since the wave
Figure 6: Given any $(d-1)$-dimensional spacelike region $B$, we can draw four null hypersurfaces, denoted $F_i$ in the figure, orthogonal to $B$. $F_1$ and $F_3$ are the relevant lightsheets when formulating the covariant entropy bound since they are bounded. Figure from [6][Sec V].

function is independent of the longitudinal coordinate in the lightcone frame [6]. One would hope that once string theory can be treated nonperturbatively, the holographic principle will emerge.

Although the statement of Bousso’s generalization is relatively straightforward, it is often quite tricky to implement, especially in cosmology. For instance, if we consider a large portion of a homogeneous universe, then we should be able to define the concept of entropy density, denoted $\sigma$, due to the homogeneity assumption. This means $S = \sigma V$, where $S$ is the total entropy of the region and $V$ is the volume of the region, so it appears that entropy is implicitly scaling as the volume and thus will grow faster than any boundary area of the region. However, Susskind, Fischler, and others showed in several different types of physically relevant cosmologies that as long as certain energy conditions are assumed, the holographic principle is still satisfied on the particle horizon [28, 29]. The crux of their arguments uses the fact that the universe is expanding, and it is possible using Einstein’s equations to show that on the particle horizon, the total entropy scales slower than the area of the particle horizon. However, the calculations are highly dependent on the type of cosmology and are done only on the particle horizon. Ultimately, one would like to show generally that given reasonable energy conditions, the holographic principle is satisfied for all surfaces in physically relevant cosmologies.
5 Conclusion

Although we have given compelling evidence motivating the covariant entropy bound in arbitrary geometries, i.e. the holographic principle, by starting with black holes in asymptotic Minkowski space, we must emphasize that the covariant entropy bound is not a bound derived from some fundamental theory. Rather, we believe in the covariant entropy bound because thus far there are no counterexamples to it and therefore we postulate it to be a fundamental law of nature. In this respect, the holographic principle is the starting point for a new fundamental theory in physics, just like the equivalence principle is the starting point for classical GR. This is a fundamental theory that bounds the amount of information content in a region of space by the area of the region rather than the volume, blatantly contradicting the results predicted by local field theory. However, by giving up locality, we are able to preserve unitarity and thus prevent the loss of information.

There are still many issues that need to be resolved. The covariant entropy bound, although precisely defined, can be quite difficult to calculate in certain situations since the regions we want to consider are not spacelike but null. Furthermore, it is still quite difficult to count precisely how the degrees of freedom are distributed in a region when they are proportional to the area rather than the volume. In 1996, Strominger and Vafa were able to successfully count the number of microstates in $\mathcal{N} = 4$ supersymmetric 5-dimensional extremal black holes [20], but the calculations are fairly involved and convoluted, although the final answer is as simple as $\frac{\text{Area}}{4}$ [30]. This contrast probably reflects the fact that we are missing something very deep regarding the holographic principle.

Perhaps the largest challenge the holographic principle currently faces is that thus far we have not yet been able to obtain the framework of QFT as a limiting case of the holographic principle. In QFT, we saw that the number of DOF in a region of space grows as the volume, and QFT has demonstrated itself to be correct in countless physical settings. Therefore, if the holographic principle is a fundamental principle of nature, it must be possible to give an explicit prescription of how a physical theory incorporating the holographic principle gives rise to QFT in certain situations and limits, and thus why the number of DOF appears to grow proportional to the volume rather than the area in those cases. In other words, locality is a property observed to a high degree of accuracy in many physical systems, and we need to understand how it arises seeing that the holographic principle is itself nonlocal. ¹

Despite these difficulties, the holographic principle is surely a step towards overcoming the vast differences between QFT and GR. By working to overcome these barriers, we will be able to get more glimpses into the rudiments of a new physical theory that will hopefully in the near future unite quantum physics with gravity.

¹Like string theory, the holographic principle does not imply that there is “action at a distance,” even though it is nonlocal.
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References


