On the Metaphysics of Quantum Mechanics

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1. Introduction

What is quantum mechanics about? The most natural way to interpret quantum mechanics realistically as a theory about the world might seem to be what is called *wave function ontology*: the view according to which the wave function mathematically represents in a complete way fundamentally all there is in the world [Monton2002]. Erwin Schrödinger was one of the first proponents of such a view, but he dismissed it after he realized it led to macroscopic superpositions (if the wave function evolves in time according to the equations that has his name). The Many-Worlds interpretation[2] accepts the existence of such macroscopic superpositions but takes it that they can never be observed. Superposed objects and superposed observers split together in different worlds of the type of the one we appear to live in. For these who, like Schrödinger, think that macroscopic superpositions are a problem, the common wisdom is that there are two alternative views: “Either the wave function, as given by the Schrödinger equation, is not everything, or is not right” [Bell 1987]. The deBroglie-Bohm theory, now commonly known as Bohmian Mechanics[3], takes the first option: the description provided by a Schrödinger-evolving wave function is supplemented by the information provided by the configuration of the particles. The second possibility consists in assuming that, while the wave function provides the complete description of the system, its temporal evolution is not given by the Schrödinger equation. Rather,

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[2] This theory has been first developed by Hugh Everett [Everett 1957] and later developed by many, among which [Barer 2008], [Vaidman 2002], and [Wallace 2002].

[3] This theory has been presented for the first time by Louis deBroglie [deBroglie 1928] and later developed by David Bohm [Bohm 1952].

the usual Schrödinger evolution is interrupted by random and sudden "collapses." The most promising theory of this kind is the GRW theory, named after the scientists that developed it: Gian Carlo Ghirardi, Alberto Rimini and Tullio Weber [Ghirardi Rimini and Weber 1986].

It seems tempting to think that in GRW we can take the wave function ontologically seriously and avoid the problem of macroscopic superpositions just allowing for quantum jumps. In this paper we will argue that such "bare" wave function ontology is not possible, neither for GRW nor for any other quantum theory: quantum mechanics cannot be about the wave function *simpliciter*. That is, we need more structure than the one provided by the wave function. As a response, quantum theories about the wave function can be supplemented with structure, without taking it as an additional ontology. We argue in reply that such "dressed-up" versions of wave function ontology are not sensible, since they compromise the acceptability of the theory as a satisfactory fundamental physical theory. Therefore we maintain that:

- Strictly speaking, it is not possible to interpret quantum theories as theories about the wave function;
- Even if the wave function is supplemented by additional non-ontological structures, there are reasons not to take the resulting theory seriously.

Moreover, we will argue that any of the traditional responses to the measurement problem of quantum mechanics (Bohmian mechanics, GRW and Many-Worlds), contrarily to what commonly believed, share a common structure. That is, we maintain that[4]:

- All quantum theories should be regarded as theories in which physical objects are constituted by a *primitive ontology*. The primitive ontology is mathematically represented in the theory by a mathematical entity in three-dimensional space, or space-time.

Even if the discussion will start out with the GRW theory, the same conclusions will apply also to Bohmian mechanics and to other theories in which the wave function is taken to represent physical objects, such as Many--Worlds as traditionally intended.

2. Bare Wave Function Ontology

In the following section we present a bare wave function ontology for the GRW formalism, discuss its problems and the proposed solutions.

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[4] Here we follow the work of [Bassi and Ghirradi 2003], [Benatti *et al.*1995], [Godlstein 1998], [Dürr *et al.* 1997], [Allori *et al.* 2008b], [Allori *et al.* 2011].

2.1 The Claim

One of the main proponents of such a view is David Albert. He maintains that the wave function represents a real, physical field, “just like electromagnetic fields in classical electrodynamics” [Albert 1996]. One difference, though, is that the wave function lives in a much bigger space than three-dimensional space: it lives in a space that combines all the positions of all the particles in the universe. So, if there are N particles in the universe, this space - called configuration space - has dimension $M=3N$; this is what physical space really is. “And whatever impression we have to the contrary (whatever impression we have, say, of living in a three-dimensional space or in a four-dimensional space-time) is somehow flatly illusory” [Albert 1996]. A similar approach has been endorsed by Peter Lewis [Lewis 2005].

Clearly, not only it seems possible but also very natural to interpret GRW as a theory about the wave function: isn't it the case that in the theory there is just one fundamental equation that involves the wave function? And isn't it the case that when similar situations have happened in previous fundamental physical theories (like classical mechanics) we interpreted those entities as representing physical objects?

2.2 The Problems

But even if the view is to some extent attractive, there are some problems. First, the fundamental space is not the usual three-dimensional space anymore: rather, it is configuration space. So we need to explain why it appears *as if* we live in a three-dimensional space. Under the current assumption, we do not have enough resources to get three-dimensional space without making use of the very definition of configuration. In fact, if the theory concerns the behavior of *stuff* in this space of dimension M, then the whole world is just mathematically represented by a function in that space: the wave function is $\psi(q)$, where q belongs to $\mathbb{R}^M$. We might be tempted to regard the coordinates of q as grouped into triples, representing the three spatial coordinates of the N particles. But the only way we could accomplish the suggested partition into triplets is to *already* know that the configuration can be divided as such, and that amounts to assigning to the word “configuration'” what we think it means: collection of positions of particles. And this amounts to saying that there *are* particles in three-dimensional space, implicitly adding them to the furniture of the universe, something that we have explicitly denied from the start. In short, if one wants to insist that the world is “made of”[5] wave functions, she needs to specify some rule or map from the M-dimensional space to three-dimensional space.

Connected to this problem, we should also explain why the world is *as if* there are (macroscopic) three-dimensional objects that move around in three-dimensional space. We think that macroscopic objects have properties, among which to be located in some position in three-dimensional space, or to have a given temperature, and so on. A fundamental physical theory should be able to describe, at least in principle, the behavior and the motion of these objects together with their properties, in one way or another. In orthodox quantum mechanics there is a rule, called the *eigenstate-eigenvalue rule* (EER), that is used to connect properties with the wave function: “an observable (i.e. any genuine physical property) has a well-defined value for a system S when and only when S's quantum state is an eigenstate of that observable.” In GRW the evolution for the wave function is constructed by modifying the Schrödinger equation to get rid of macroscopic superpositions. As a consequence, the wave function of a macroscopic object “collapses” very rapidly into one of the terms of the superposition but, because of the properties of the stochastic equation, it has tails that are never exactly zero. Since such a wave function is not an eigenstate of any operator that is supposed to represent properties, we cannot use the usual EER in GRW to determine the properties of macroscopic objects. So again, bare wave function ontology fails, leaving macroscopic objects with indefinite properties.

3. Dressed-up Wave Function Ontology

Albert realizes these failures, and consequently proposes to solve them as follows. He first suggests [Albert 1996] that the Hamiltonian provides the required map to get three-dimensional space from configuration space. Suppose that physical space is $\mathbb{R}^M$, where *it happens* to be the case that $M=3N$. The total Hamiltonian of the world is something like the following: $H=\nabla^2_q+V(q)$, where q belongs to $\mathbb{R}^M$. Without any further restrictions, this Hamiltonian could apply to a space of any dimension M. But, Albert claims, it is an *empirical* fact of the world that the potentials V should be written as $V(q)=\Sigma_{i<j}V(|q_i-q_j|)$, where $q=(q_1, \dots, q_N)$, q_i in $\mathbb{R}^3$, for any $i=1, \dots, N$. And this is what *ensures* us of the appearances of the world as three-dimensional. The structure of the actual Hamiltonian, Albert says, is what explains why we *think we live* in a three-dimensional space, and there is no further explanation of why the Hamiltonian is the way it is, or the dimensionality M of physical space is what it is (for example, there is no

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[5] For simplicity, here and in the following we might use the locution “is made of” as short for: “is mathematically represented by.”

explanation of why *M* cannot be any number but must be a multiple of 3; in particular, there is no further explanation of why *M* cannot be a prime number[6]).

To solve the problem of indefinite properties, Albert and Loewer [Albert and Loewer 1996] propose to replace EER with a different rule. For the property of localization of a macroscopic object, they put forward the following proposal: "particle *x* is in region *R* if and only if the proportion of the total square amplitude of *x*'s wave function which is associated with points in *R* is greater than or equal to $1-p$," where the parameter p is a conventional matter. It is a supervenience rule, since it is a rule that explains how our talk about macroscopic objects and properties (the macroscopic talk) supervene on the talk in terms of wave function (the microscopic talk). In this way, they say, it is possible to recover what we usually mean when we talk about localizable objects on the macroscopic level and the appearances of those objects to be localized while they are not. Note that here neither of the rules implies any additional ontology in any way. Rather, they are just practical rules that should be added for our epistemic purposes.

4. Quantum Theories with a Primitive Ontology

In this section we discuss a completely different approach to quantum theories, in which some mathematical object *other than the wave function* is representing physical objects.

4.1 Bohmian Mechanics

Bohmian Mechanics (BM), together with GRW and Many-Worlds, is one of the quantum theories that solves the measurement problem of ordinary quantum mechanics. It is a theory in which the wave function does not provide the complete description of a physical system, and in the usual yet unfortunate terminology, the actual positions of the particles are the *hidden variables* of the theory: the variables which, together with the wave function, provide a complete description of the system.

Commonly, BM is understood as a theory about the behavior of particles, whose evolution is defined in terms of a velocity field. Such velocity field involves an equation that contains the wave function, which in turns evolves according to the usual Schrödinger equation[7]. In BM as traditionally intended by those working on it the wave function does not really describe matter, the particles do.

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[6] This has been suggested by Tim Maudlin [Maudlin, p.c.].

[7] See [Dürr *et al.* 1997] for details.

4.2 Primitive Ontology

Some scholars[8] think that the way in which we understand BM should extend to any quantum theory: the wave function, living in configuration space and not in three--dimensional space, is not the right kind of mathematical entity to describe physical objects. Rather, given that matter appears to live in three--dimensional space and evolves in time, the obvious choice to represent matter seems to be a mathematical object in three--dimensional space, or in four--dimensional space-time!

This attitude is motivated by a more general approach according to which all fundamental physical theories have a common structure grounded on the notion of primitive ontology. Here we will present the basic idea, for more details about this notion and about its connection with the structure of fundamental physical theories, see [Allori forth.]. Any fundamental physical theory must always contain a metaphysical hypothesis about what are the fundamental constituents of physical objects. We will call this the *primitive ontology* of the theory: entities living in three-dimensional space or in space-time, which are the fundamental building blocks of everything else, and whose histories through time provide a picture of the world according to the theory. In the formalism of the theory, the primitive ontology is represented by some variables that we could name, for obvious reasons, primitive variables. Since any theory aims to describe not only what physical bodies exist but also how they evolve in time, in addition to the variables describing the primitive ontology the theory contains some other non-primitive variables. They are necessary to implement the equations whose solutions will describe how the primitive ontology moves through space in time - the so-called *laws of motion*. For this reason such variables are sometimes called "nomological" variables. Once these ingredients are given, all the properties of macroscopic objects of our everyday life follow from a clear explanatory scheme developed on the lines of the one used in classical mechanics.

A note of clarification about the "primitiveness" of the primitive ontology: as discussed in [Allori *et al.* 2008b], even if the primitive ontology does not exhaust all the ontology, it is the one that makes direct contact between the manifest and the scientific image. Since the primitive ontology describes matter *in the theory*, we can directly compare its macroscopic behavior to the behavior of matter *in the world of our everyday experience*. Not so for the other non-primitive variables, that can only be compared indirectly in terms of the way in which they affect the behavior of the primitive ontology.

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[8] See [Goldstein 1998], [Dürr *et al.* 1997], [Goldstein *et al.* unpublished], [Allori *et al.* 2008b], [Allori *et al.* 2011], [Tumulka 2006], [Allori *et al.* 2005], [Ghirardi 2007] and to some extent [Maudlin 2007a].

One of the motivations for this position is that all fundamental physical theories, up to quantum mechanics, are straightforwardly theories about the temporal evolution of a primitive ontology. For example, in Newtonian mechanics the primitive ontology is given by point-particles, whose time evolution is determined by the Newton's law of motion and laws of the force. A similar case can be made for classical electrodynamics, which can be seen as a theory about the behavior of charged particles. Since the theory of general relativity is a theory of space-time structure, the metric is part of the primitive ontology of the theory. Also, as we just saw, BM as traditionally intended is a theory with a primitive ontology: the particles. With this in mind, let us come back to the other quantum theories.

4.3 GRW

By definition, given that the wave function does not live on three-dimensional space, it cannot be a possible primitive variable. Hence, Albert's take on GRW, being about the wave function, does not have any primitive ontology and therefore could be dubbed "GRW0." Let us see now two distinct approaches to GRW based on the notion of primitive ontology.

First of all, we have John Stuart Bell's proposal [Bell 1987]. Consider the space-time points (x_i,t_i) in which the wave function collapses. One could call these events "flashes [Tumulka 2006]. Bell's proposal is to take these events as the primitive ontology of the theory: forget about the fact that they are collapse points, the flashes are what the world is made of. For this reason, it seems appropriate to call this reformulation of GRW "GRWf."This is, admittedly, an unusual primitive ontology, but it is a possible one nonetheless. In GRWf matter is made neither of particles (such as in classical or Bohmian mechanics), nor of a continuous distribution of matter (like in GRWm or Sm, as we will see next). Rather, matter is made of a collection of discrete events in space-time. The flashes happen randomly with a given temporal frequency and their probability is determined (quantitatively) by the wave function. In particular, once a particular history of the wave function has been chosen in a given time interval, the set of these events in space-time in such interval is determined[9].

The second GRW theory with a primitive ontology was proposed by Gian Carlo Ghirardi - the 'G' in GRW: the primitive ontology is a scalar field, called the mass density field. The mass density field is on three--dimensional space, and it is determined by the wave function as specified in [Bassi and Ghirardi 2003]. In this theory matter is continuous, rather than corpuscular, as in classical or Bohmian

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[9] See [Bell 1987], [Tumulka 2006], and [Allori *et al.* 2008b] for technical details about GRWf.

mechanics. Since the primitive ontology of this theory is the mass density field, this theory is called "GRWm"[10].

GRWm and GRWf therefore are theories that describe the temporal evolution of a primitive ontology: in GRWm the primitive ontology is the mass density field, in GRWf it is the flashes. In both cases the temporal evolution of the primitive ontology is determined by the wave function, which in turn evolves stochastically according to the modified Schrödinger equation.

Given GRWf and GRWm, we can now draw a first parallel between BM and GRW theory. Even if, traditionally, BM and GRW are always presented almost as dichotomical solutions to the measurement problem of quantum mechanics[11], the suggestion here is instead that BM and GRW (at least in its GRWf and GRWm versions) have much more in common than one would expect at first sight: both are theories about a primitive ontology, described in the formalism by the primitive variables, while the non-primitive wave function serves as a tool for generating the law of evolution for the primitive ontology.

Note that, contrary to the understanding of BM in terms of primitive ontology but in line with his view about GRW, Albert [Albert 1996] proposes a different take on BM. The motivation for Albert's position is simple and entirely parallel to the one provided in the context of GRW: in BM there are two fundamental equations - one for the particles, the other for the wave function - and if we are to interpret the theory realistically, the most natural way to do so is to assume they both represent physical bodies. As a consequence, Albert holds that in BM physical space is represented by configuration space, and it is a theory about a single particle *and* the field represented by the wave function. Alternatively, we could think of the theory even more dualistically, both in the ontologies and in the spaces: there are two different physical substances, the particles in three-dimensional space, and the field represented by the wave function in configuration space. However, as we will see in section 7, the main problem here is again to take configuration space as (part of) physical space.

4.4 Many Worlds

To exhaust the solutions to the measurement problem, we should also consider Many-Worlds (MW): if the Bohmian route is to deny that the wave function provides the complete description of a system, and the GRW one is to change the wave function temporal evolution, the strategy of MW is to claim that the superpositions states are

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[10] The names "GRWf" and "GRWm" were introduced in [Colin *et al.* 2006]. Other names has been proposed for GRWf, like for example "flash-GRW," "flashy-GRW" or "Bell-GRW."

[11] See for example the recent paper by [Putnam 2005].

never observed because they are not happening in the same world. The idea in MW is that what is represented by a Schrödinger evolving wave function is all there is, and that it "branches" appropriately into different worlds.

The most common way to interpret the MW formalism, in which the wave function represents matter, can be called "MW0" to denote the absence of a primitive ontology. In contrast to the common understanding of MW but in line with the understanding of fundamental physical theories expressed in this paper, we can consider different Many-Worlds theories with a primitive ontology.

The first example is MWm, in which the primitive ontology is given by the mass density field in three-dimensional space defined in terms of a Schrödinger evolving wave function. The primitive ontologies of GRWm and MWm are the same but, while in GRWm the wave function collapses, in MWm it does not.

There is also a Many-Worlds theory with flashes, MWf, in which the distribution of the flashes is determined by a Schrödinger evolving wave function. The flashes are generated by the wave function exactly as in GRWf: the algorithm, whose output provides the set of flashes, is the same as the one for GRWf with the exception that no collapse takes place in the evolution of the wave function. The random set of flashes can be generated both using a collapsing wave function (as in GRWf) and using a non-collapsing one (and this is what happens in MWf)[12].

MWf and MWm can be regarded as Many-Worlds theories: if one considers the flashes (or the mass density) that correspond to macroscopic superpositions, one sees that they form independent families of correlated flashes (or mass density) associated with the terms of the superposition, with no interaction between the families. The families can indeed be regarded as comprising many worlds, superimposed on a single space-time. Metaphorically speaking, the universe according to MWf or MWm resembles the situation of a TV set that is not correctly tuned, so that one always sees a mixture of two channels. In principle, one might watch two movies at the same time in this way, with each movie conveying its own story composed of temporally and spatially correlated events.

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[12] One might have thought that the use of a flash ontology has to be tightly connected with the nonlinearity of the evolution of the wave function in GRWf, but this is not the case. In fact the flashes in GRWf *just happen* to be the points in which the wave function collapses, while in MWf they are not: both in GRWf and in MWf the flashes are some random events in space-time that happen at a given rate determined by the wave function, which evolves stochastically in GRWf and according to Schrödinger's equation in MWf. These theories have been discussed in [Allori *et al.* 2008b] and in [Allori *et al.* 2011]. They have been called "Sm" and "Sf" respectively, 'S' denoting the Schrödinger evolution of the wave function, so that Bohmian Mechanics as commonly described as a theory of particles should be called "Sp."

Note that MWm and MWf are different from MW0 because in the latter physical space is configuration space and matter is described by the wave function, while in both MWm and MWf physical space is three-dimensional space, and matter is described by the mass density in three-dimensional space, and by the flashes in space-time in MWm and MWf respectively. For more details on MWm and MWf, see Allori *et al.* 2008b] and in [Allori *et al.* 2011].

4.5 Common Structure

To conclude, it seems that quantum theories can be interpreted as sharing the same *common structure* as the other fundamental physical theories: there are primitive variables in three-dimensional space or in space-time which represent the fundamental constituents of macroscopic physical objects, and then there is the wave function whose role in the theory is to implement the dynamics for the primitive ontology. The specification of the primitive and non-primitive variables completely determines the theory[13]. In other words, each of these theories is about matter in space-time, what might be called a decoration of space-time. Each theory involves a dual structure (X,ψ): the primitive ontology X providing the decoration, and the wave function ψ governing the primitive ontology. The wave function in each of these theories, given that it has the role of generating the dynamics for the primitive ontology, has therefore a nomological character utterly absent in the primitive ontology.

It is interesting to note that even the orthodox quantum theory (OQT, the theory originally proposed by Bohr in which there are two separate words: a classical and a quantum one) involves such a dual structure: what might be regarded as its primitive ontology is the classical description of macroscopic objects, including in particular pointer orientations, while the wave function serves to determine the probability relations between the successive states of these objects. In this way, also in the case of OQT, the wave function governs the behavior of the primitive ontology. An important difference, however, between OQT on the one hand and the other theories on the other is that in the latter the primitive ontology is microscopic while in the former it is macroscopic. This makes OQT rather vague, even noncommittal, since the notion of `macroscopic' is intrinsically vague: of how many atoms need an object consist in order to be macroscopic? And, what exactly constitutes a 'classical description' of a macroscopic object?[14]

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[13] This common structure has been first proposed in [Dürr *et al.* 1997], and discussed at length in [Allori *et al.* 2008b] and [Allori forth.].

[14] Also the notion of “microscopic” could be considered vague, but it is not a problem for the theories with a primitive ontology since it is not central to these theories as “macroscopic” is for OQT.

It is interesting to observe that within this approach we could develop a variety of quantum theories with a primitive ontology as follows[15]:

- Choose the primitive ontology (particles, mass density field, flashes, strings, and so on): it could be defined independently of the wave function (as for example in Bohmian mechanics) or as a function of it (as for example in GRWm);
- Choose the law of evolution for the primitive ontology: this would involve the wave function, and could be deterministic (as for example in MWm), or stochastic (as for example in GRWm);
- Choose the law for the temporal evolution of the wave function: again, it could be deterministic (as for instance in MWf), or stochastic (as for instance in GRWf).

When these ingredients are appropriately chosen, we will obtain theories that are roughly[16] empirically equivalent to OQT, and that are indistinguishable on the basis of experiments. Therefore, the choice among which one we should take ontologically seriously should be based on other factors.

4.6 Symmetries

As a last remark, let us note that the notion of primitive ontology allows for a better understanding of the symmetry properties of a theory. To say that a theory has a given symmetry is to say that: the possible histories of the primitive ontology, when transformed according to the symmetry, will again be possible histories for the theory, and the possible probability distributions on the histories, when transformed according to the symmetry, will again be possible probability distributions for the theory. That is, a quantum theory with a primitive ontology is symmetric (invariant) with respect to a given transformation when both the histories of the primitive ontology and their transformed counterparts describe possible physical states of the world. This is so because the histories of the primitive ontology provide the image of the world. And if a theory has a given symmetry, this image should not change under the transformation associated to that symmetry. Because of their role in the theory, the non-primitive variables will transform in such a way to ensure that the histories of the primitive ontology are invariant. In other words, the wave function could also be transformed when transforming the trajectory of the primitive ontology. However, while there is a natural transformation of the trajectory of the primitive ontology (determined by the

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[15] See [Allori *et al.* 2008b] for a discussion of some of them.

[16] While BM and MW are strictly speaking empirical equivalent to OQT, GRW is not. For more on the origin of this, see [Allori *et al.* 2008b].

kind of mathematical object that represents it), there is not necessarily a corresponding natural change of the wave function. The latter is allowed to change in any way, solely determined by its relationship to the primitive ontology.For example, consider the Galilean symmetry of BM understood as a theory of particles. In the case of a single particle system evolving freely, the theory will be Galilei invariant only if the transformed configuration is also solution of the equation of motion. And to accomplish that we need a very particular transformation of the wave function under Galilei boosts.

Particularly important for quantum mechanics is the question of relativistic invariance: it is usual to assume that a theory is relativistic invariant if the law of evolution of the wave function is of a particular sort. But that is a mistake, since what is important is the evolution of the primitive ontology. It is worthwhile mentioning that the recognition of the importance of the primitive ontology has led to the construction of a relativistic invariant extension of GRWf [Tumulka 2006], while GRWm still has no relativistically invariant formulation. This example underlines the dependence of the symmetry on the choice of the primitive ontology. In fact there is no reason to believe that when changing the primitive ontology of a theory the symmetry properties of the theory will remain unchanged. Thus, one should not ask whether, say, GRW as such is Lorentz invariant, since the answer to this question depends on the choice of primitive ontology for GRW. Relativistically invariant single particle extensions of Bohmian mechanics based on the notion of primitive ontology have been proposed: in [Bohm and Hiley 1993] and [Dürr *et al.* 1999] using a wave function evolving according to Dirac's equation, while in [Berndl *et al.* 1995] and [Nikolic 2005] the wave function used to implement the dynamic of the primitive ontology evolves according to the Klein-Gordon equation. Also, [Allori *et al.* 2011] have developed relativistically invariant extensions of MWm and MWf.

5. Objections to Quantum Theories with a Primitive Ontology

Let us now discuss possible concerns for quantum theories with a primitive ontology and possible ways to respond to them.

5.1 The Mysterious Wave Function

The first charge is that such theories have the problem of explaining how the wave function should be understood: if physical objects are described by the primitive variables, the wave function has to be something else. But what?

As we saw in the previous section, the wave function in all theories we analyzed has a common role: while the primitive variables specify what physical bodies are, the wave function specifies how these objects move. For this reason, Dürr, Goldstein and Zanghì [Dürr *et al.* 1997] have proposed that the wave function should be intended as a law of nature.

Objections have been raised to this view, especially by Harvey Brown and David Wallace [Brown and Wallace 2005]. First of all, laws are time independent, while the wave function evolves in time. Dürr, Goldstein and Zanghì [Dürr *et al.* 1997] and more recently Shelly Goldstein and Stefan Teufel [Goldstein and Teufel 2001] have anticipated and replied to this objection claiming that even if it might be difficult to accept the wave function as a law in the current theories, it will become straightforward once we reach a theory of quantum cosmology in which the wave function is static.

Another objection to the view of the wave function as a law focuses on the fact that there seem to be multiple degrees of reality: there are material entities, represented by the primitive ontology, and there are nomological entities, which the wave function is intended to capture. One could avoid the problem endorsing a nominalist point of view for laws. As an alternative, one could maintain that laws exist as abstract entities. One could insist in fact that, even if the view has problems, they are not strong enough to make one abandon the view altogether. This, of course, needs to be argued, and indeed it has[17]. Another possible option is to try to eliminate the wave function completely from the theory, as it has been attempted by Fay Dowker and collaborators [Dowker and Henson 2004], [Dowken and Herbauts 2004], [Dowker and Herbauts 2005], that have developed some toy models of quantum mechanics without using the wave function at all.

5.2 The Artificial Primitive Ontology

Another charge against such theories (but not BM) is that they are artificial: there is just one equation in GRW and MW, and it is about the wave function. What else could describe physical objects?

We think this objection is question begging: the whole point is to establish whether in such theories there is only one evolution equation or not, and if not, which one is fundamental. GRW0 and MW0 assume there is just one equation, while the corresponding theories with a primitive ontology explicitly *deny* that. And one cannot assume as a premise something that is supposed to be established.

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[17] See [Maudlin 2007b] for a recent realist proposal about laws of nature.

Another objection to GRW and MW with primitive ontology, similar to the previous one, could be that these theories add the primitive ontology even though it is of no use: everything can be done without it. But again, the very issue is to determine whether it is possible to derive everything with the wave function only, and we are about to argue that it is not.

5.3 The Complexity of the Theory

On a different note, even if we assume that it is not true that GRW0 and MW0 are the most natural interpretations of the formalism of GRW and MW respectively, one could claim that they are the simplest in the sense that they postulate the existence of just one thing, the wave function. In contrast, the corresponding interpretation of the GRW and MW formalism in terms of primitive ontology postulate also the existence of the primitive ontology. So, appealing to Occam's razor, GRW0 and MW0 should be preferred.

To respond, first of all let us say that this formulation of the objection is misleading: if one takes laws to be abstract entities, or if one takes a nominalist position towards laws, then theories with a primitive ontology postulate the existence of only one thing: the primitive ontology. If, on the other hand, one takes a realist view on laws, as Maudlin does, then one could argue that simplicity is only one of many criteria for theory choice. Explanatory power is another one, and one could argue that theories with a primitive ontology allow for a more straightforward explanation than theories without one. This is what we will discuss in section 6, in which we will focus on how theories with a primitive ontology are able to explain macroscopic phenomena. In section 7 instead we will see the amount of work theories without a primitive ontology will have to do to account for the world of our everyday experience.

6. The Explanatory Scheme of Theories with a Primitive Ontology

GRW and MW with a primitive ontology, as well as BM as a theory of particles, are in line with the traditional realistic interpretation of classical mechanics: there is matter in space-time. How do these theory account for the macroscopic world?

First of all, they do not have to explain the *appearance* of three-dimensionality, since the world *is* three--dimensional.

There seem to be no fundamental problems also in the case of macroscopic properties. In fact in principle we are in the same situation as classical mechanics, and arguably in classical mechanics one can identify macroscopic properties more or less naturally given by how the fundamental objects (the primitive ontology) clump together to form more complex bodies interacting in a variety of ways.

In this sense the primitive ontology is the most fundamental ingredient of the theory. Also, the primitive ontology grounds the "architecture" of the theory: first we describe matter through the primitive variables, then we describe its dynamics, implemented by some non-primitive variables. Once these ingredients are given, all the macroscopic properties are recoverable. In other words, any property of the macroscopic physical world can be appropriately "read off" from the histories of the primitive ontology. For example, we can explain why a table is solid on the basis of the fact that it is composed of particles that interact electromagnetically such that it is impossible for another object, like for instance my hand, to penetrate them. Next, suppose we wish to account for the fact that a comet is an object with a given localization at a given time. One can accomplish this in terms of the microscopic components of the comet and their interaction with each other: there are particles that interact with each other to form a solid object whose motion (and therefore its localization at different temporal instants) can be just as effectively described by the motion of its center of mass. Also, the transparency of an object such a pair of glasses can be explained in terms of the electromagnetic forces acting between the particles composing the glasses, that are such that incoming light-rays will completely pass through them. Similarly for fluids: a property like the liquidity of water can be explained in terms of the very weak interaction between the microscopic constituents of water that allow it to change shape with the container. In addition, the behavior of gases is accounted for considering them as composed of non-interacting particles colliding with one another. This is what happens when we derive thermodynamics from statistical mechanics: what in thermodynamics we call pressure, volume, temperature of a gas are derived from the fact that gases are composed of moving particles. Given that air is a gas, and given that a gas is just a collection of non-interacting particles, we can also explain why it is compressible: it is possible to reduce the distance between the particles in it almost as much as we want.

These examples show how in the classical framework we have a clear and straightforward scheme of explanation: given the primitive ontology at the microscopic level, one can employ standard methods to determine the properties of familiar macroscopic objects. Since in classical theories this is possible because the theories have a primitive ontology, for any other fundamental physical theory with a primitive ontology, like the quantum theories we just discussed, we could employ an explanatory scheme derived along the lines of the classical one. This is true for BM[18]. Also in the other quantum theories with a primitive ontology we start from a primitive ontology in space or space-time. Therefore, also in these theories we should be able to recover (at least in principle) all macroscopic properties of physical objects with a transparent and

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[18] See [Dürr *et al.* 2004] and [Allori *et al.* 2008a].

well tested mathematical method, even if in the frameworks of GRW and MW with a primitive ontology more work needs to be done[19].

An antireductionist could object all this, but the point here is that in quantum theories with a primitive ontology we are not in any way worse off than in classical mechanics. That is, whatever can be said against reductionism in classical mechanics, in principle can be said for quantum theories with a primitive ontology. But there seem to be no *additional* problem for reductionism in these theories due just to the fact that they are quantum theories with a primitive ontology. In contrast, if we think the wave function describes physical objects, since it cannot be a primitive variable, we have to radically revise the explanatory scheme of classical mechanics. We will see what the problems are with that in the next section.

7. Objections to Dressed-up Wave Function Ontology

The common problem to theories in which the wave function is taken as describing matter is that they are incredibly radical in an unnecessary way: if less far-fetched alternatives work, why go radical? Even if we grant that there are reasons to go that way, still the theory does not seem to provide (at least at the moment) something to be compared to the explanatory we just described which is available to theories with a primitive ontology. And even if we set these issues aside, there are worries related to the potential involvement of mental states into the formulation of the theory which suggest that the wave function ontology research program might not even be able to get off the ground.

7.1 The Radicality of the Metaphysics

The pictures of the world provided by the quantum theories with a primitive ontology are, more or less, not too revisionary. In fact there is space-time and there are histories of the primitive ontology in it. Given these two ingredients, we have seen in the previous section that we can implement an explanatory scheme similar to the classical one[20]. By contrast, the picture of the world given by GRW0 or MW0 is at best

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[19] In any case, see [Bassi and Ghirardi 2003] and [Goldstein *et al.* unpublished] for some related comments on the matter for GRW and [Allori *et al.* 2011] for MW.

[20] One could question that quantum theories with a primitive ontology are not too revisionary: for example, in the case of BM its nonlocality seems to make the theory sufficiently distant from our common understanding. That is certainly true: the idea that the world is local is surely deeply ingrained in our intuition. Nonetheless, this intuition is mistaken. In fact Bell, with his famous inequality, has shown [Bell 1964] that *any* quantum theory (and therefore, not only BM)

extremely bizarre: physical space is a highly dimensional space, and all there is is a material field in that space. All the complexity, all the variety, all the individuality, all the multiplicity of things is in that object: planets, stars, tables, chairs, apples, trees, cat, reptiles, electrons, quarks, humans, aliens, me, you, Mother Theresa, George Bush, are not made of particles, are not made of fields in three--dimensional space, rather they are ``all there together" somewhere meshed in the wave function. As also pointed out in [Monton 2002], GRW0 and MW0 are even more radical than the brain-in-a-vat scenario: at least in that case brains are in space-time, while in this view there are basically no brains at all.

These theories seem far too revisionary than needed: it is *possible* that the world is as described by these theories but there seem to be no reason to believe it to be like that. In fact, it seems we can do perfectly fine without assuming that the wave function represents physical objects: as we discussed, nothing is deeply wrong with the alternative view that the world *is* actually three-dimensional with three-dimensional objects moving around. As a matter of methodology, we think that we should not opt for some radical view if there are no strong reasons to reject less revisionary perspectives.

7.2 The Inadequacy of the Hamiltonian Rule

The arguments presented in the previous subsection do not prevent in principle to find the rules that are needed to the wave function ontology approach to recover the macroscopic appearances. Indeed, Albert has proposed in the context of GRW theory

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has to be nonlocal. For more on this, see [Dürr *et al.* 2004] or read directly [Bell 1964]. In the case of GRWf, also, the metaphysics is very peculiar: matter is made of flashes, random events in space-time that happen with rate such that at almost every time space is empty, containing no flashes and thus no matter. This peculiarity, though, is no more than that: something contrary to our intuition. While the atomic theory of matter entails that space is not everywhere continuously filled with matter but rather is largely void, GRWf entails that at most times space is entirely void. If the number of the degrees of freedom in the wave function is large enough, as in the case of a macroscopic object, the number of flashes is also large. So large numbers of flashes can form macroscopic shapes, such as tables and chairs. Therefore, adopting this theory we will not have to entirely quit understanding things as we did so far, something that instead we will have to do for theories without a primitive ontology. The bottom line is this: even if the metaphysical picture given us by a quantum theory with a primitive ontology is distant from our common intuition or from the classical picture, the method we use to go from the level of the (microscopic) constituents of matter to its (macroscopic) everyday properties is very much alike.

his own rules, as we have seen, and most likely he would expand the use of these rules also for other theories without a primitive ontology like MW0.

As we saw, quantum theories without a primitive ontology first of all need a rule to recover the three-dimensionality of space from configuration space. Albert's proposal is that the Hamiltonian provides the correct correspondence rule between the two spaces.

Arguments against this are given in [Monton 2002]. We do not wish to focus on those arguments in this paper: even if Albert's argument is sound and the Hamiltonian could in principle be enough to define a suitable rule, we still find the argument unconvincing. In fact, let us grant Albert that the Hamiltonian is enough to explain why it seems that we are living in a three-dimensional world even if we are not. What are the reasons for which the Hamiltonian is the way we write it? It seems straightforward that the reason we *use* in physics books a certain Hamiltonian, and not some other, is that we *already assume* that we are in three-dimensional space, and not the other way round. That is, we do not deduce the three-dimensionality of space from the fact that in the physics book we find a particular kind of Hamiltonian. Therefore, it seems that the explanatory structure in Albert's view is upside down: is it the structure of the Hamiltonian that explains the appearance of the three-dimensional world, or the existence of such a world that explains the structure of the Hamiltonian?

Additional problems arise if we consider MW0 and BM as theories about a configuration space field represented by the wave function: in MW0, since there are the different elements of the superposition and they interfere, studies needs to be done to ensure that these interferences are suppressed[21]. In the case of BM, if physical space is composed of both three-dimensional space (in which there are the particles) and configuration space (on which the wave function lives), in addition to the usual concerns, all sorts of problems arise about the interaction between the two spaces, some of them reminding of the problems of dualistic theories of the mind: how is it possible for the material wave function to interact with the material particles?

7.3 The Plurality of the Supervenience Rules

Second, the proponents of the wave function ontology need to account for macroscopic properties, and Albert and Loewer have proposed a supervenience rule to account for the property of localization.

This rule obviously work: it allows us to define a clear correspondence between the microscopic language (in terms of wave functions) and the macroscopic everyday language (in terms of macroscopic properties). But notice that not only do we need to

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[21] See for instance [Wallace forth.] and references therein.

explain the appearances of three-dimensionality and localizability, but also of *all the other* macroscopic properties of all other macroscopic objects. That is, not only do we need to supplement the theory with the Hamiltonian rule for three-dimensionality and the supervenience rule for localization: we also need other rules to account for *every* single property we think of every single macroscopic object (including "us") can have! For example, not only is it the case that there is a cup of coffee localized on the table, but also the cup of coffee *is* a cup of coffee - it has a particular shape. Also, as a matter of fact, the cup of coffee is white, and it is fragile. In addition, not only is Fred localized in his office in front of his computer and close to the cup of coffee, but also he has the property of thinking certain thoughts. Hence, these theories should be able to account for all these properties, and many many more. Albert's idea is to account for them introducing a supervenience rule for each property. For example: "An object is a cup of coffee if and only if the wave function is localized in a cup-shaped region of three-dimensional space." To account for the color of an object, the situation is more complicated since it requires us to talk about the light deflected by the object. But both the object and the light are part of the wave function, so one needs to find a way to accommodate this. The same situation (or maybe a more complex one) arises for fragility, since it involves what would happen to the object under certain circumstances.

Even if the worries just described could be solved, as a matter of fact they are still with no answer. Thus, it seems that wave function ontology approach is, at best, more like a research program rather than a fully developed account. There is of course nothing particularly wrong with that, but remember that it was maintained that wave function ontology theories are simpler than the alternatives since they involve just the wave function. Given what we have seen so far, is it really so?

7.4 The Non-justification of the Supervenience Rules

Here is another concern for the wave function ontologist: there is no deep justification for the additional rules the theories about the wave function need. In fact, the answer to the question "Why these rules?" is nothing but "Because they work." The problem is therefore that the wave function ontology account of macroscopic properties does not seem to provide a genuine account of these properties at all.

Let us contrast this with the explanatory schema of theories with a primitive ontology, in which we can explain macroscopic properties starting off from three-dimensionality and compositionality (there is microscopic *stuff* in space that evolves in time, and the macroscopic objects are composed of this microscopic *stuff*). We do not have to invert any supervenience rules, and therefore we do not have to justify them.

Every macroscopic property just “arises” from the motion of the primitive ontology. In contrast, in theories without a primitive ontology one has to derive macroscopic properties without three--dimensionality and without compositionality, just using plenty of *ad hoc* rules.

7.5 The Lack of Important Symmetries

Another worry for quantum theories about the wave function is the following: it is difficult to account for symmetry properties. This could be taken as a strong objection, at least from the point of view of the physicist. In fact, it is common in scientific practice to give a lot of importance to symmetry properties of physical theories candidate to be fundamental. For one thing, for example, conservation laws of nature originate in symmetries. Moreover, gauge symmetries in the Standard Model are used to describe the fundamental interactions (excluding gravity), and are based on a particular symmetry group. Not to mention that there is the problem of constructing a relativistic quantum theory, namely a theory that has the property of being symmetric (invariant) under the relativistic transformations.

In the wave function ontology framework, symmetries are presumably defined as properties of the evolution of the most fundamental object of the theory, the wave function: a theory is invariant under a given symmetry if the original wave function and the one obtained after the transformation both represent possible physical states of the world. With this definition it turns out that theories of the wave function lack important symmetries [Albert, p.c.]. In fact symmetry transformations are linked to the kind of mathematical object we consider the theory to be about. In the case of BM as a theory about particles *and* the wave function, since the wave function is a scalar field it will naturally transform under a pure Galilei transformation of magnitude v in a way that will not preserve the Galilei invariance of the theory[22]. As a consequence, Albert concludes that particle- *and* -wave function BM is *not* Galilei invariant. A similar reasoning can be made for other theories of the wave function, given that physical objects are ``made of'' the wave function, and the wave function is a scalar field.

Albert does not find the fact that a theory is not Galilei-invariant as problematic. We beg to differ: a theory better have some symmetries if one wishes to build a relativistic quantum theory. Of course, one could object that the fact that current theories do not have any symmetries, does not imply that the future ones will be like them. This might be true, but this is the present situation if we accept Albert's approach. Rather, if we reject it and we think in terms of theories with a primitive

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[22] The transformation is: $\psi'(q,t)= \psi (q+vt, t)$. If $\psi '(q,t)$ is a solution to the Schrödinger equation, then $\psi (q+vt, t)$ is not.

ontology, theories indeed possess the relevant symmetries. As we already saw, when talking about symmetries in the primitive ontology framework, we should not focus on the wave function. This has led toward some progress in finding relativistic quantum theories: as a matter of fact, the flashy ontology was introduced by Bell *explicitly* as a step toward a relativistic GRW theory [Bell 1987]. As we already mentioned, if symmetries concern the histories of the primitive ontology (and not the wave function) a different primitive ontology may lead to different symmetries, as we saw in the case of GRWf and GRWm.

7.6 The Mind-Body Problem

To conclude, let us even grant Albert that his rules to recover the macroscopic world of our experience are plausible and successful. We think that there is still a deeper and bigger problem. In theories with a primitive ontology, there seem to be no mystery about the fact that there are three-dimensional macroscopic objects, like tables and chairs: they are particular clusters of the fundamental constituents of the world, the primitive ontology. The only mystery there is (if any, depending of what position one takes with respect to the mind-body problem) is that (some of?) these objects have conscious experiences. Of course, if there is a problem here, there is also a problem for every physical theory, starting from classical mechanics.

It is worth noting, though, that theories with a primitive ontology do not need to solve the mind--body problem in order to account for what (physically) happens in the world around us: they simply leave it out from the beginning, claiming that all physics does is to account for the behavior of *material* objects. In the case of theories without a primitive ontology instead, since physics ought to explain the *appearances that we have*, the mind-body problem is right in from the beginning and cannot be left out from the discussion. In fact, suppose we want to account for Fred's seeing of the motion of a projectile in a gravitational field. In classical mechanics this is equivalent to the request of accounting for the motion of a projectile in a gravitational field. There is no need to invoke Fred's visual perception of the projectile. The evidence (i.e. the motion of the projectile) is stated in the language of physical facts, and not in the language of experience. The same can be said in the case of quantum theories with a primitive ontology. In theories without it, instead, one has to stick with the original request and has to account for *Fred's perception* of the motion of the projectile *to start with*. If we don't have a primitive ontology, the connection between physics and the behavior of ordinary objects has to be made at the level of experiences. Hence, one cannot avoid to discuss about how conscious experiences come about. In other words, whether one considers the gap between the physical and the mental in principle unsolvable by physics or not, this issue has no implication for theories with a primitive ontology, while this is not the case for theories without it.

Therefore, while theories with a primitive ontology can develop independently to the solution of the mind-body problem, the success of theories without a primitive ontology crucially depends on it: if the mind-body problem cannot be resolved within physics, then the wave function ontology research program *cannot even start*. This is another good reason for which it seems more sensible to choose right from the beginning an ontology in three-dimensional space - that does not require any mentioning of the mind-body problem - instead of having an ontology in some other space and then find ourselves dealing with the mind-body problem right from the beginning.

To put it differently and to sum up, there are two problems we have to face when a theory describes the world around us: an “easy” one and a “difficult” one. The easy one is to explain the behavior of macroscopic objects in three-dimensional space in terms of the motion of microscopic objects in three-dimensional space. This is arguably solved by some explanatory schema similar to the classical one. The difficult problem is to explain perceptions, as for example the perception we have that physical space is $\mathbb{R}^3$, while it is actually $\mathbb{R}^M$. In the approach in terms of primitive ontology the difficult problem is left to a future theory (of consciousness or a more complex physics), while physics is concerned only about the easy one: once we have left perceptions out, we can dedicate physics to the explanation of the motion of macroscopic bodies in three-dimensional space. On Albert's approach, in contrast, we are *required* to explain perceptions in order to begin explaining everything else. Physics and the theory of perception are completely merged, and thus everything becomes more difficult, if not impossible, at any level. Notice that we do not claim that we should not aim at such a complete theory. But since it happens that we can do quantum mechanics without having to have a theory of consciousness, why not continue to do so?

8. String Theory

One might think that, since a problem with configuration space ontologies is that they live in a space with an elevated number of dimensions, then also string theory will be in trouble, given that in this theory space has ten dimensions.

We think that there are some relevant differences between string theory and the wave function ontology approach that make the two cases not analogous in this respect. On the one hand, in fact, it is true that in string theory the number of dimensions of physical space is greater than three. Indeed, without entering into any detail, if we assume the existence of extra dimensions, string theory can explain features of our world (such as the nature of the vacuum state) which were left unexplained by the previous theories. On the other hand, though, in string theory all dimensions except three are "compactified." That is, they are wrapped up on themselves very tightly like little rolls, such that they are not visible to us. To understand this, let us consider a pipe. When the pipe is observed from far away (so that its diameter is small compared to the distance of observation) the observer just sees a one-dimensional object. But as soon as the observer gets closer, the pipe is revealed as a two dimensional object. Similar is the case in string theory: space seems three-dimensional to a macroscopic observer, but if she could probe it more closely (to a distance smaller than 10^{-35} m), she could appreciate the existence of the other compactified dimensions.

This should make clear that the approach of string theory is very different from the one based on configuration space: in the case of wave function ontology, extra dimensions are simply added to the usual three; in string theory they are added but then promptly compactified, in order to keep the world macroscopically always like $\mathbb{R}^3$. In this way, the space of string theory is, for all practical purposes, three--dimensional, and an explanatory scheme similar to the classical one could be employed to account for the macroscopic world.

9. Conclusion

We have argued that any quantum theory should not be interpreted as a theory about a material field in configuration space represented by the wave function. Quantum theories, instead, should be regarded as theories about a primitive ontology in three-dimensional space or space-time, that constitute the building blocks of everything else. We have argued in this paper that this approach to quantum theories is more desirable than the alternative based on the wave function. In fact quantum theories with a primitive ontology, contrarily to what is commonly believed, are in line with classical theories: they share with them a common structure that grounds their explanatory power. In this way, the quantum metaphysics is straightforward: no weirdness and no paradoxes are involved, we just have stuff in space-time.

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