Sounds Like Light: Einstein's Special Theory of Relativity and Mach's Work in Acoustics and Aerodynamics

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1. **Introduction Mach** ***le bon mécanicien***

It is well known that Einstein did not regard his special theory of relativity as revolutionary, but, as he put it, as 'simply a systematic development' of electrodynamics (1919). Pais writes that he considered it 'the natural completion' of earlier physicists' work, and part of an 'orderly transition' (1982, p. 30). This was not a matter of modesty on Einstein's part; for, as Pais also points out, he did not shrink from describing another paper he published that year as 'very revolutionary'.[1] And, when he described the special theory as 'an astoundingly simple combination and generalization of the hypotheses […] on which electrodynamics was built' (1961, p. 41), Einstein was emphasizing its continuity with the theories of his precursors, especially Maxwell and Lorentz, but also going as far back as Newton.

It is also known that Einstein included Mach on the short list of his scientific precursors. Pais writes that, in the last interview Einstein gave, shortly before his death, 'he spoke of four people he admired: Newton, Lorentz, Planck, and Mach', and remarks: 'They, and Maxwell, and no others, are the only ones Einstein ever accepted as his true precursors'. Mach is also one of two philosophers Einstein said most affected him; he named David Hume and Ernst Mach as the two people who 'above all others' introduced the critical conception that achieved 'greater freedom in the formation of ideas or concepts' (Einstein, 1961, p. 142).

1 In Beck (1995, p. 20), Doc. 27. Einstein to Conrad Habicht dated 18 0r 25 May 1905.

However, Einstein also said that ‘to the extent that Mach was a good mechanician he was a deplorable philosopher’ (‘autant Mach fut un bon mécanicien, autant il fut un déplorable philosophe’).[2] Commentators sometimes treat this remark of Einstein’s as indicating that he had outgrown Mach’s philosophy, but it was a view Einstein held about scientists in general, including himself; he opens his 1936 essay ‘Physics and reality’ with an apologia for trying his hand at philosophy: ‘It has often been said, and certainly not without justification, that the man of science is a poor philosopher’ (1950, p. 30). And, elsewhere, when showing reservations about Mach’s philosophy, he is not dismissive of the scientific work. When ascribing to Mach a view of the aim of physics that he himself does not accept, Einstein adds the qualification that the ascription is to Mach the (deplorable) philosopher, rather than to Mach, the (superb) scientist: ‘From the point of view of theories of knowledge St. Mill and E. Mach took their stand approximately on this ground’ (1950, p. 30). And, in speaking of Mach and others he described as ‘skeptics’: The hostility of these scholars toward atomic theory can undoubtedly be traced back to their positivistic philosophical attitude[…] even scholars of audacious spirit and fine instinct can be hindered bin the interpretation of facts by philosophical prejudices’ (1992, p. 47). He even thought the motivation for such skepticism genuine in his obituary of Mach, he wrote: ‘Mach’s philosophical studies came entirely from the wish to develop a position which could help bring together the different directions to which his own scientific work had been devoted’ (1916, p. 158). These remarks suggest that it was a way of doing physics that Mach tried- and perhaps failed- to express as a theory of knowledge to which Einstein felt he owed so much.

Einstein’s ‘Physics and Reality’ is, like Mach’s *Science of Mechanics*, a critical-historical work, although his survey begins later, with Newton. And, it includes acoustics and hydrodynamics among the areas of physics discussed. Commentators on Einstein and special relativity tend not to look in the direction of work in acoustics for conceptual precursors to the special theory of relativity; expositions on special relativity that compare light and sound tend to associate the insight of special relativity with the contrast between light and sound, and the similarities between them with the (discredited) classical wave theory of light.[3] There is something to this, but if, as Einstein said, special relativity is a systematic development of

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2 Quoted in Nature, No. 2807, 112, 18 August, 1923, reprinted in Blackmore (1992, p. 162). He goes on to say: ‘[…] concerning the […] point, namely that concepts , I am in complete agreement with Mach’.

[3] Born (1962) is a notable exception.

classical electrodynamics, we should not be surprised to find that it is grounded in some similarities between light and sound as well; classical electrodynamics developed in concert with the development of the wave theory of light. Waves, including light waves, were generally conceived of as having a mechanical (or at least quasi-mechanical) basis,[4] and sound is the quintessential mechanical wave.

We shall see that Mach was exceptional here in that, in the very context of drawing an analogy between sound and light, he explicitly freed the notion of a wave from the necessity of having a mechanical basis. In this paper, I examine two of Mach's specific technical accomplishments: one in acoustics – his explanation of the Doppler effect and one in aerodynamics – his explanation of the shock waves accompanying supersonic flight. Specific points found in these two pieces of scientific work that are prescient of crucial moves Einstein made in developing the special theory are Mach's explanation of the Doppler effect as a kinematical effect arising from relative motion of bodies, and Mach's explanation of shock waves in terms of a principle of constancy of signal velocity as governing physical processes, including non-mechanical (i.e. thermodynamical) ones.

And, finally, there is a poignant twist to the story: Mach's work on the phenomenon of shock waves accompanying supersonic flight of projectiles has a close analogue in electrodynamics: Cerenkov radiation, the radiation that accompanies the motion of electrons travelling in a medium at superluminal velocities. But this electromagnetic analogue of Mach shock waves went unnoticed decades after Mach's widely known work was published; even though the radiation itself was observed, it was not recognized as the analogue of shock waves for electrodynamics. The twist is that, although Mach's work inspired Einstein – in fact, Einstein specifically mentioned the work on shock waves in his obituary of Mach[5] – the main reason that

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[4] A long survey article and a collection of papers on nineteenth-century author theories may be found in Schaffner (1972).

[5] In Einstein (1916, p. 158): 'In terms of his mental development, Mach was not a philosopher who chose the natural sciences as the object of his speculation, but a many-sided, interested, diligent scientist who also took visible pleasure in detailed questions outside the burning issues of general interest […]. The best-known among his physical investigations are those on shock waves which projectiles produce […]. He succeeded in photographing the thickness of the air front in the neighborhood of a speeding projectile, which threw light on a form of acoustical processes which we had been ignorant of before. His popular lecture on the phenomenon will please everyone who can take pleasure in physical matters'.

the analogous phenomenon for light was neglected was the fame of Einstein's special theory of relativity, with which many people associated the impossibility of superluminal velocities.

2. Mach in Einstein's Early Scientific Milieu

Before delving into Mach's work on these two specific topics, I want to fill in the personal historical picture to show that Mach was part of Einstein's general scientific milieu during his adolescent years. A neglect of Mach's scientific work may account for why Einstein scholars have not generally been as appreciative of Mach as Einstein appears to have been, and have, I think, often regarded his respect for Mach as due in part to a sentimental attachment to an adolescent hero, and not on a par with his regard for other scientists.[6] There is the recognition of Mach's rejection of absolute space, and the so-called Mach's principle ('in a truly reasonable theory inertia would have to depend upon the interaction of masses' (Einstein, 1992, p. 27)). But

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Incidentally, he mentions that lecture ['On Some Phenomena Attending the Flight of Projectiles'] at the end of the obituary again, to illustrate something else: 'the humane, friendly, and hopeful demeanor' characteristic of Mach, which, he says, 'protects him from the sickness of our time, the national fanaticism which few can avoid today' (1916, p. 158). For those interested in Einstein's political activity, some quotes from Mach's lecture (continued) foreshadow the difficult position the pacifist Einstein would later find himself in with regard to the use of his discoveries to develop atomic weapons: after discussing the scientific achievements exhibited in modern guns, Mach writes; 'We may surrender ourselves so completely to this impression as to forget the terrible purposes they serve […]. Think of our forefathers and of the times when club law ruled supreme […]. This state of affairs grew so oppressive that finally a thousand and one circumstances compelled people to put an end to it, and the cannon had most to say in accomplishing the work'. Mach continues: 'Questions of law will in a sense forever remain questions of might. Even in the United States where every one is as a matter of principle entitled to the same privileges, the ballot […] is but a milder substitute for the club'. He expresses hope that 'the intercourse of men will take on gentler forms', but adds: 'In the intercourse of the nations, however, the old club law still reigns supreme' (Mach, 1943, p. 3361).

[6] Besides the neglect of Mach's scientific work, another reason for this is undoubtedly the heavily advertised posthumously published preface in Mach's *Optics*, containing statements that he rescinds his views on the relativity theory. The authenticity of ascribing this to Ernst Mach is challenged in Wolters (1987), in which he presents his view that the preface is authored by Ernst Mach's son Ludwig. The publications that Wolters does ascribe to Mach indicate Mach's interest in the special theory of relativity: he saw it as progressive, but with some more development in order. This is exactly what he should have thoughts, for, as Einstein (1916, p. 158) said in his obituary of Mach, Mach's criticisms of Newton were a demand for relativity 'in the more general sense (relativity of acceleration)'. See footnote 34 for further discussion and references.

as significant as these are, they have not of late been seen as satisfying explanations for giving Mach the kind of credit Einstein did give him.[7] For Einstein made extreme remarks about Mach as a scientist. In his obituary of Mach, Einstein speculated on what Mach might have accomplished, making the remarkable claim: 'It is not improbable that when physicists were considering the significance of the constancy of the velocity of light that had Mach's mind been young and fresh at that time he would have come across the theory of relativity' (1916, p. 157). In his autobiography, he identifies Mach as being singular in having a critical attitude towards classical mechanics; he says that, except for Mach, 'all physicists of the previous century saw in classical mechanics a firm and delicate foundation for all physics, indeed for the whole of natural science, […] they never grew tired in their attempts to base Maxwell's theory of electromagnetism […] upon mechanics as well. Even Maxwell and H. Hertz […] in their conscious thinking consistently held fast to mechanics as the confirmed basics of physics' (1992, p. 19).

As Einstein ascribed importance to an understanding of the development of thermodynamics in the intellectual process that resulted in the special theory of relativity, one has to wonder why Mach's well-known *Principles of the Theory of Heat*, another critical-historical work that Einstein read and said impressed him deeply, is seldom mentioned in discussions of Mach's influence on Einstein. In his autobiography, Einstein (1992, p. 49) describes his earliest struggles in developing the special theory: 'Gradually I despaired of the possibility of discovering the true laws by means of constructive efforts based on known facts […] I came to the conviction that only the discovery of a universal formal principle could lead us to assured results. The example I saw before me was thermodynamics'. He identifies that formal principle of thermodynamics as the general principle of the impossibility of constructing a *perpetuum mobile*. In the next paragraph (1992, p. 51), he credits Hume and Mach with furthering the 'type of critical reasoning required for the discovery' of the central point of the special theory of relativity. Although he does not mentions Mach's *Principles of the Theory of*

[7] Earman (1989, p. 84) catalogues all the reasons and causes that might lead someone to give Mach some credit for the (general) theory of relativity, and argues that he deserves little. Blackmore (1972, p. 255-259) concludes that 'Einstein was neither Mach's philosophical nor scientific "successor"', and that (1972, p. 285) 'there is something grotesque in considering Ernst Mach, the leading opponent of theoretical physics, as the "forerunner" of Albert Einstein'.

Heat there, the following passage from it discusses the crucial role of transformations 'of a formal nature':

> The principles of the excluded *perpetuum mobile* can be seen in the clearest and easiest way in the domain of pure mechanics If, now, we conceive all physical processes as mechanical ones, we naturally conclude that there must be an analogous principle for the whole domain of physics. Even Helmholtz […] tried to give a firm basis for the principle of energy, from which it could be deduced, by assuming that all physical processes are conditioned by motions of atoms under the influence of central forces […]
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> It is only from experience that we can know whether and how thermal processes are connected with mechanical ones […]. The peculiarity in Carnot's idea consists in the fact that he was the first to exclude the *perpetuum mobile* in a wider domain than that of pure mechanics […].
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> A new transformation of a formal nature was necessary to enable the modern principle of energy to appear (Mach, 1986, p. 297-298).

Here Mach is explicit that the advance required for the emergence of what he calls the modern principle of energy lay not just in recording knowledge from experience, but in extending the domain to which a principle drawn from the experience was applied. The similarity of this formal principle in thermodynamics to Einstein's extension of the principle of relativity from classical (i.e. Newtonian) mechanics to electrodynamics is clear: it is a genuine extension of the principle to a larger domain.

Einstein himself speaks of having read Mach's *Principles of the Theory of Heat* by the time he would have been struggling to develop the special theory. Late in his life, he writes this to his friend Michele Besso about their adolescent years: 'I remember very well how, during my early years as a student, you directed my attention to [Mach's] treatise on mechanics and to his theory of heat, and how these two works made a deep impression on me'.[8] To be fair, in that letter written in old age, he says he cannot say just what the influence was, and he attributes a more direct influence to Hume, just as, as we saw, he mentioned Hume along with Mach in his autobiography when crediting Mach as a philosophical influence. Why would he bring in Hume when talking about Mach's treatises on mechanics and on the theory of heat? Well, Hume's

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[8] Einstein-Besso correspondence dated 6 January 1948, quoted in Miller (1981, p. 189).

approach to the study of human nature was to model it on the sciences. The notion of extending the range of a principle from one domain to a larger one, with the result that very different motions are then explained in terms of the same principle, is found in Hume as well. In a dialogue that treats the problem of the (cultural) relativism of moral judgments, the character expressing Hume's view prefaces the step whereby he will reconcile two apparently opposing judgments of character with an admonition to his conversant in trace matters 'a little higher' and to examine 'first principles', explaining: 'The Rhine flows north, the Rhone south, yet both spring from the same mountain, and are also actuated, in their opposite directions, by the *same* principle of gravity [Hume's emphasis]' (1983, p. 113). Hume, too, took inspiration for his philosophical views from Newton's mechanics, especially the theory of gravitation. The search for unifying principles inspired by the presence of Newtonian mechanics towering in the background is a common thread through Mach and Hume.

Einstein's interest in Mach is also exhibited in a letter he writes to Mileva Maric at the age of twenty: 'At present I'll be completely bookless for a week, since all libraries are now being inventoried, but within a week I can have the municipal library send me books by Helmholtz, Boltzmann, & Mach'.[9] A sentence later, he writes that he is thinking about the velocity of light; 'A good idea occurred to me in Aarau about a way of investigating how the bodies' relative motion with respect to the luminiferous ether affects the velocity of propagation of light in transparent bodies'. In a letter to her written three years later he mentions reading Mach again; 'I have almost finished reading Mach's book with tremendous interest, and will this evening'.[10] Besides *Science of Mechanics* and *Principles of the Theory of Heat*, another book by Mach that would have been available in the municipal library is his *Popular Scientific Lectures*. We know Einstein read some of Mach's popular lectures, as he mentions the popular lecture on supersonic flight ('On Some Phenomena Attending the Flight of Projectiles') in the obituary he wrote of Mach.[11] 'The Velocity of Light' appeared in the first edition of Mach's *popular Scientific Lectures*, the first German version of which was published in 1896; Einstein would

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[9] Einstein to Mileva Maric, 10 September 1899, Doc. 54 in Beck (1987, pp. 132-133).

[10] Einstein to Mileva Maric, February 1902, Doc. 136 in Beck 9(1987, pp. 192-193).

[11] The editorial notes given in Stachel (1987, p. 230 n. 8; p. 335, n. 9) suggest Mach's *Mechanik* (1897) and *Wärmelehre* (1896) as candidates for the reference to Mach's book found in these letters, but do not mention Mach's *Popular Scientific Lectures*, published in 1895 (English)/1896 (German).

have been sixteen turning seventeen that year. Einstein said that it was at the age of sixteen that he had the thought experiment in which the seed of the special theory was contained: the thought experiment about chasing a ray of light with velocity *c*. Mach's 'The Velocity of Light' is deceptively accessible; it is a layman's introduction to the history of techniques of measuring the velocity of light, but – and I think this highly significant – it includes Mach's illustrations of how the time intervals between the receipt of signals (light signals, sound signals, and information-bearing signals in general) in such a setup will lengthen or contract if the observer is moving with respect to the source of the signals. Another of Mach's works that would have been available at the time of that letter is a booklet consisting of three previously published paper on the Doppler effect (Mach, 1873). No one can say just which of Mach's papers in scientific journals Einstein actually read, but, as he singled out Mach as a scientific precursor, it is likely he read some such publications as well.

There are echoes of Mach's writings in Einstein's – in the questions to which interest is directed, in fundamental features of the approach to those questions (e.g. the ground rules set out for seeking resolutions to (apparent) paradoxes), and even in the concrete examples used to motivate and illustrate those questions (e.g. moving organ pipes, observers on train embankments, clocks side-by-side being moved apart). Perhaps Einstein should be given more credit for knowing what he was saying when he named Mach as both his scientific precursor and one of the most significant philosophical influences on his thought.

3. Some Themes in Einstein's 1905 Special Relativity Paper

First, a review of some points made in 'On the Electrodynamics of Moving Bodies', (hereafter called the 1905 special relativity paper) to have in thought, before going on to examine Mach's contributions to the explanation of the Doppler effect and supersonic head waves. What follows is not meant to be a summary of that paper – it is its own best summary – but a selection of points I will refer to later.

The paper opens with the remark: 'It is known that Maxwell's electrodynamics – as usually understood at the present time – when applied to moving bodies leads to asymmetries which do not appear to be inherent in the phenomena'. Einstein then says of moving magnets and conductors: 'The observable phenomenon […] depends only on the relative motion of the conductor and the magnet, whereas the customary view draws a sharp distinction between the two cases in which either the one or the other of these bodies is in motion'. The key things from this opening paragraph I want to highlight are: the focus on the application to moving bodies, the respect accorded 'observable phenomenon' in conflict with an established theory, and that a contrast is drawn between recognizing that something depends only on the relative motion of bodies, and 'customary practice'.

The next paragraph draws from this example, along with the failure to detect relative motion between the earth and a 'light medium' the conjecture that electrodynamics is *like* mechanics.[12] In particular, the conjecture is that 'the same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good'. This

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[12] In 'The Foundation of the General Theory of Relativity' (1952, p. 111), he is again explicit that the special principle of relativity is a principle retained from the classical mechanics: in prefatory remarks to presenting the more general version, he says, 'Thus the special theory of relativity does not depart from classical mechanics through the postulate of special relativity, but through the postulate of the constancy of the velocity of light *in vacuo*, from which, in combination with the special principle of relativity, there follow […] the relativity of simultaneity, the Lorentzian transformation, and the related laws for the behavior of moving bodies and clocks'.

It is sometimes said that the principle of relativity is not consistent with a (classical) wave theory of light. Though it is true that the principle in inconsistent with some of the many quasi-mechanical accounts of light waves, it is not accurate to say that the special principle of relativity is not consistent with classical (mechanical) wave mechanics itself.

Classical (Galilean) addition of velocities for particles differs from addition of velocities for mechanical waves in classical mechanics in that the velocity of a medium with respect to a particular inertial frame is a relevant factor for the latter but not the former. This means that the equations of motion for waves will be simpler in some frames than in others; the ones in which the medium is not in motion with respect to the frame. In that sense, one could say that there are 'preferred' reference frames for mechanical waves. But, as long as the motion of the wave medium is recognized as a relevant parameter, on a par with motions of (deformable as well as rigid) bodies, Newton's laws of mechanics do hold for classical waves in various inertial reference frames, just as they do for classical particles.

This the significance of the appeals to empty space being the same in every direction made in the 1905 special relativity paper (1952, p. 47) and in the Stafford Little Lectures (1956, p. 35), these pinpoint one respect in which light waves in empty space are *not* like sound waves considered without reference to a medium. For sound, if one describes a physical situation using descriptions that make no reference to the medium of transmission, one cannot assume that things will happen the same in all directions, as the medium of transmission may be in a state of motion with respect to one's frame of reference.

conjecture, identified as 'The principle of Relativity', is one of the two postulates he lays out as ground rules for applying Maxwell's equations for stationary bodies to moving bodies.

The principle as he states it in these opening paragraphs is noncommittal as to which frames of references these are; the principle just states that they are the ones for which the equations of mechanics hold good' (1905, p. 37). It is not presented as a challenge to mechanics, but as extending a feature of mechanics to electrodynamics.

In the same sentence in which he states that he will raise this conjecture to the status of a postulate, Einstein introduces another postulate; 'Light is always propagated in empty space with a definite velocity *c* which is independent of the state of motion of the emitting body' (1905, p. 38). Now, how does this second postulate compare with the physics of mechanical waves? Well, it is characteristic of mechanical waves that they travel in a medium with a definite velocity which is independent of the state of motion of the body creating the disturbance. So, the only difference between the Einstein postulate regarding the velocity of the propagation of light *in vacuo* and the corresponding statement for mechanical waves is the qualification 'in empty space'; mechanical waves cannot travel in empty spaces at all, but the statement would be true for sound if the phrase 'in a medium' were substituted for 'in empty space'.

This last point is worth keeping clear about, as many introductions to special relativity emphasize, not the significance of the constancy of velocity in empty space, but of the velocity of light signal's independence of the state of motion of the emitting body, as though this were special for light. Whereas it is a basic feature of wave mechanics that the velocity of a wave travelling in a medium is a characteristic of the medium, and is independent of the state of motion of the emitting body.[13]

[13] An example of such an exposition appears in the highly-regarded undergraduate physics text (Tipler, 1969, pp.2-5) that was my first introduction to special relativity. The text opens with a paraphrase of Einstein's postulates of special relativity, which are described later on as 'bold postulates': 'I. Absolute, uniform motion cannot be detected. II. The speed of light is independent of the motion of the source'. However, if 'speed of sound' were substituted for 'speed of light' in these paraphrased postulates, they would be entirely consistent with non-relativistic, classical wave mechanics. Although the motion of a wave with respect to the medium of transmission can be detected, it is certainly consistent with classical mechanics that absolute uniform motion cannot be detected. And, the speed of sound is also independent of the motion of the source; it is a characteristic of the medium. The important phrase in the Einstein postulates on which everything turns has been left out: the phrase 'in empty space' in the postulate

The rest of the 1905 special relativity paper is divided into two parts; 'I. Kinematical Part' containing Sections 1 through 5, and 'II. Electrodynamical Part', containing Sections 6 through 10. After giving a definition of 'time' and 'simultaneous' in Section 1, he opens Section 2 by defining the principles of relativity and of the constancy of the velocity of light more precisely:

1. The laws by which the states of physical systems undergo change are not affected, whether there changes of state be referred to the one or the other of two systems of coordinates in uniform translator motion.
2. Any ray of light moves in the 'stationary' system of co-ordinates with the determined velocity *c*, whether the ray be emitted by a stationary or by a moving body (1905, p. 41).

As in the previous formulation, the first postulate is a principle of mechanics.[14] But, the second has subtly changed: the more precise statement states that a light ray moves in the 'stationary' system of coordinates with velocity *c*. The 'stationary' system of coordinates (Einstein's terminology) is an arbitrarily chosen (but particular) system of coordinates in which the equations of Newtonian mechanics hold good. For sound, the analogous statement would now be obtained by replacing the phrase: the '"stationary" system of co-ordinates' by 'a system of coordinates stationary with respect to the medium of transmission'. In fact, if light is travelling in a transparent medium, the statement for light needs to be clarified, too. Einstein certainly recognized this: here is a statement from his popular account of relativity theory, made in the context of considering the addition of velocities for light travelling in a stream of water flowing with respect to an observer: 'in accordance with the principle of relativity we shall

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about the constancy of the velocity of light. Captions to figures 1, 2, and 3 on pp. 3 and 4 misstate the role of the motion of the sound source. The figures do not appear in later versions of the text.

(continued) I mention Tipler here only because his happened to be the textbook used in my undergraduate physics course. I believe the error is quite common. One recent example is Pais' (1994, p. 60) 'mini-briefing on relativity, for the layman'.

[14] Michael Friedman (1983, pp. 149-159) argues that the most popular way of formulating the special principle of relativity ('the laws of nature are the same 9or take the same form) in all inertial reference frames') can be interpreted in such a way that it is (trivially) satisfied by classical electrodynamics (expressed in general covariant form) and proposes instead a formulation in terms of a criterion of experimental equivalence of two reference frames.

certainly have to take for granted that the propagation of light always takes place with the same velocity with respect to the liquid, whether the latter is in motion with respect to the bodies or not'. So, if we ask of these more specific statements of the two postulates given in Section 2: 'How do the Einstein postulates compare with the physics of mechanical waves?', we would put our finger on the phrase: '"stationary" system of coordinates in the statement 'any ray of light moves in the "stationary" system of coordinates with the determined velocity *c*'.

What is not so obvious from the statement in which "stationary" system of coordinates' replaces 'in a medium' in the classical analogue is the significance the revision gives to the presence or absence of a medium whose motion is relevant to the physics of the situation. For although, in Einstein's presentation, any system of co-ordinates in uniform translatory motion could have been chosen as the 'stationary' system, in general, whether two physical situations are equivalent or not may depend on whether the motions whose equations are sough include motions with respect to a medium. We just saw that there was a difference that had to be taken account of between the case of light travelling in empty space and light travelling in glass or water. Even though light does not require a medium of transmission, if it is travelling through a transparent medium, the motion of the medium becomes a factor in determine the measured velocity of light, because the motion of the light with respect to the medium, although constant, ins different from the universal constant *c* of light travelling in empty space. Einstein gives this subject a chapter of its own ('Theorem of Addition of Velocities: The Experiment of Fizeau') in his popular account (1961, pp. 38-41).

The importance of the postulate of the constancy of the velocity of light *in vacuo* is that it eliminates some relative motions that have to be considered, and, as will be emphasized later, this makes some situations equivalent for light travelling *in vacuo* that are distinguished for sound (since, unlike light, sound requires a medium). That is, it requires the second postulate (in conjunction with the principle of relativity) to conclude that, as far as measuring the speed of light *in vacuo*, which system of coordinates was chosen as 'stationary' is irrelevant – even among ones that are moving with respect to each other. This is not true for sound – or, even, for light travelling in a medium – for this reason: there are relative motions that arise in the situation in which a wave travels with a constant velocity with respect to a medium, but that do not arise for light travelling at a constant velocity *in vacuo*. The key notion here is the elimination of relative motions from consideration, not the non-existence of the ether *per se*: Einstein would later write that ‘the hypothesis of ether in itself is not in conflict with the special theory of relativity. Only we must be on our guard against ascribing a state of motion to the ether’ (1920, p. 15).

There are a few other sections from the kinematical part of the 1905 special relativity paper that I want to set out here for later reference. In Section 3, entitled ‘Theory of the Transformation of Co-ordinates and Times from a Stationary System to another System in Uniform Motion of Translation Relativity to the Former’, the task Einstein carries out is to show that the two postulates he has proposed are compatible. To illustrate that they are consistent, he considers the case of a spherical wave propagated in empty space with velocity *c*, and concludes that ‘the wave under consideration is therefore no less a spherical wave with velocity of propagation *c* when viewed in the moving system (1905, p. 46). This, he has shown the consistency of the postulate concerning the constancy of the velocity of light travelling in empty space and the principle of relativity. What I want to point out about this derivation is that (i) the velocity of the source of the spherical wave does not enter the discussion, and (ii) the illustration of the compatibility of the two proposed principles does rely upon the light wave travelling in empty space, and that this is because there is no velocity with respect to a medium that must be considered.

Some readers may wonder at my statements that the novelty of Einstein’s postulate is all in the phrase ‘in empty space’. But the claim is mundane: if a light wave is not travelling in empty space, the principle that the velocity of light is equal to the universal constant *c* and is the same for all observers need not hold, nor did Einstein claim it would.[15] In fact, one of the

[15] Except that the popular book Einstein nominally co-authored with Leopold Infeld could in places give rise to this misconception, because it is not clear about the relevance of the motion of the light source and the motion of the medium to the addition of velocities for sound or light (1938, p. 168-169): ‘[…] if ether is carried with the room moving with its light sources and if the mechanical laws are valid, then the velocity of light must depend on the velocity of the light source. Light reaching our eyes from a moving light source would have greater velocity if the motion is toward us and small if it is away from us’. This is very misleading; it is the motion of the ether, not the motion of the light source, that affects the measured velocity of the light. The statement should read instead: ‘[…] if the mechanical laws are valid, then the velocity of light must depend upon the velocity of the medium in which it is travelling […]’. Because the example is set up such that the source is not moving with respect to the medium, it is not literally false. But that passage as a whole (as well as others that follow) is entirely misleading; it deliberately gives the impression that a crucial difference between light and sound is that the velocity of light is independent of

consequences of special relativity is the equation for Fresnel's dragging coefficient, which is an equation for how the velocity of light measured by an observer is affected by the motion of the fluid in which the light is travelling. The bold move Einstein made was to consider the constancy of the velocity of light travelling in empty space as constancy in an arbitrarily chosen inertial system that may be regarded as stationary. Whether or not a light medium exists, this means that motion with respect to it cannot be a relevant factor in the physics of the situation. That changed everything, for, since the motions with respect to a medium for light waves were out of the picture, the conflict between the principle of the constancy of the velocity of light and the principle of relativity disappeared with them. This reinforces the point that the significant phrase is 'with respect to a "stationary" system of coordinates', used in place of 'with respect to the medium of transmission'. It is this phrase of the postulate (and not the independence of the velocity of light from the motion of the light source) that allows the proof of its compatibility with the principle of relativity.

In the next section of the 1905 special relativity paper, entitled 'Physical Meaning of the Equations Obtained in respect to Moving Rigid Bodies and Moving Clocks', Einstein compares the shape of a rigid body measured at rest with its shape when measured in a state of motion: 'A rigid body which, measured in a state of rest, has the form of a sphere, therefore has in a state of motion – viewed from the stationary system – the form of an ellipsoid'. He explains that the 'shortening' will increase as the velocity of the body increased, and, for a velocity equal to the velocity of light *in vacuo*, 'all moving objects – viewed from the "stationary" system – shrivel up into plain [*sic*] figures' (1905, p. 48). Similarly, the rate of a moving clock, as measured in the stationary system with respect to which it is considered moving, is retarded.

Notice, first, his acceptance of these results; he does not portray the slowing of a clock's ticking or the shriveling up of moving bodies as a sign that the equations cannot be correct; rather, he goes on to say that '[…] the velocity of light plays the part, physically, of an infinitely great velocity' 91905, p. 48). Second, he does not look for an explanation of this contraction in terms of causes; that the time period between two events in a moving system depends on its velocity is treated as a kinematical fact. And, thirdly, he remarks upon the symmetry of the equations, i.e. that the same results are obtained for bodies at rest in the 'stationary' system when

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'whether or not the emitting source moves, or how it moves'. Einstein's own popular account (1961) is clear on the subject.

viewed from the system of uniform motion. We shall see that the last two of these attitudes were championed by Mach in his explanation of the Doppler effect. The first, accepting of singularities, is exhibited in Mach's investigations into shock waves. Although, we shall see that Einstein did not go as far as Mach did: Mach not only came to accept the velocity of sound as a physical limit of a sort, but he discovered the physical phenomena that result when that limit is exceeded.

In Section 5, 'The Composition of Velocities', the focus on symmetry and kinematics is again in evidence: Einstein draws attention to the fact that the two velocities 'enter the expression for the resultant velocity in a similar manner'. He then concludes the kinematical part of the paper thus: 'We have now deduced the requisite laws of the theory of kinematics corresponding to our two principles, and we proceed to show their application to thermodynamics' (1905, p. 51). In Section 6, which begins the electrodynamical part of the paper, we see once more the significance with which symmetry is regarded: 'it is clear that the asymmetry mentioned in the introduction as arising when we consider the currents produced by the relative motion of a magnet and a conductor, now disappears', as well as the significance of the elimination of causes to be accounted for: 'Moreover, questions as to the "seat" of electrodynamic electromotive forces (unipolar machines) now have no point' (1905, p. 55).

The very first special topic to which this newly-born electrodynamics is applied is the derivation of Doppler's equation for the frequency (color) shift of light perceived by an observer moving with respect to an infinitely distant sources of light. It is the first result obtained in Section 7, 'Theory of Doppler's Principle and of Aberration'. Again, without attempting a summary, I want to point out some features of the exposition. Here the velocity of the observer with respect to the source of waves does arise as a matter of importance – in the context of explaining frequency shift – as it did *not* arise earlier, in the context of the topics of the addition of velocities or compatibility of two postulates. In fact, it is precisely the relative velocity of the observer and the source of light (electromagnetic waves) that is responsible for the Doppler effect (i.e. the change in frequency, or color shift). Also, the explanation given is kinematical, in that it explains the shift as a consequence of motion, rather than in terms of any forces due to, for instance, fluid pressure or changes at the molecular level. These two points echo sticking points in Mach's explanation of the Doppler effect.

And, once more, we see Einstein accepting that the equations he has derived contain singularities: he notes that the expression derived for the frequency shift observed due to motion with respect to the light source has the consequence that, for an observer moving towards the source at velocity *c*, the observed frequency becomes infinite. And, if the observer is approaching a source of light with the velocity *c*, 'this source of light must appear of infinite intensity' (1905, p. 57).

There is much more in the 1905 special relativity paper; here I only remark on how it closes: after showing that the expression for energy of motion of an electron becomes infinite when its velocity equals *c*, he writes: 'Velocities greater than that of light have – as in our previous results – no possibility of existence' (1905, p. 64). Taken out of context, this is ambiguous; does he mean velocities greater than the velocity that light has in empty space, or does he mean that, no matter what the medium, nothing travelling in that particular medium can ever exceed the velocity that light attains in that particular medium? Of course the answer is that the velocity of light referred to in Einstein's postulates is the universal constant *c*, the velocity of light in empty space. Whether anything travelling in a substance could exceed the velocity with which light travels in that substance is a different question.

4. Motion with Respect to a Medium for Signal Transmission

Some readers may feel that the significance that relativity of simultaneity holds in special relativity has been underplayed in the preceding discussion, and that it is this feature, rather than the equivalence of uniformly moving reference frames for observable phenomena due to propagation of waves in empty space, that is the logically prior insight. Certainly it is the relativity of simultaneity that allowed Einstein to show that the two principles he raised to the level of postulates could be consistent. However, the two features are intimately connected in his exposition, in which, under the postulate of the principle of relativity, absolute simultaneity is still tenable if time measurement is based on wave propagation in a medium, but becomes untenable for wave propagation (i.e. propagation characterized by a finite, constant velocity), in empty space. This can be seen by considering simultaneity of events as measured by sound signals. It is a commonplace that whether two sound signals reach an observer simultaneously or not depends upon the standpoint of that observer. In fact, this is the basis for measuring the velocity of sound. Mach describes a method of measuring the velocity of sound in his popular scientific lecture, 'The Velocity of Light':

> The apparatus is very simple. It consists of two electric clock-works which strike simultaneously, with perfect precision, tenth of seconds. If we place the two clock-works side by side, we hear their strokes simultaneously, wherever we stand. But if we take our stand by the side of one of the works and place the other at some distance from us, in general a coincidence of strokes will now not be heard. The comparison strokes of the remote clock-work arrive as sound, later […]. But by increasing the distance we may produce again a coincidence of the strokes […] plainly that distance is travelled over by sound in a tenth of a second.[16]

The velocity of the sound signal is thus derived from measurements of the distance and the time interval between signals; all three measurements are then independent of the standpoint of the observer, in a system of coordinates that is fixed with respect to the medium of transmission of the sound wave, i.e. the air.

In addition, corrections have to be made for relative motion between the observer and the medium; e.g. in windy weather, the sound signal would travel (with respect to an observer standing on the ground) faster in the direction of the wind. And the time interval between signals is affected by the relative motion of a moving observer and a source of periodic sound signals; Mach mentions this in the context o0f explaining the relation of the velocity of light to apparent periodic changes in the time of revolution of Jupiter's satellite. His apparatus for experimentally verifying the Doppler effect used a moving sound-emitting source the apparatus he designed for

[16] Mach (1895, p. 58). Compare Einstein's treatment of 'time' in terms of simultaneous receipt of signals, in Section 1 of the 1905 special relativity paper. The connections between time, simultaneity, and signal velocity were fundamental to his resolution of the two postulates. Pais (1982, p. 139) quotes from Einstein's Kyoto lecture: 'My solution was really for the very concept of time, that is, that time is not absolutely defined but there is an inseparable connection between time and the signal velocity'. This is in the context of how it was in conversation with Besso that he 'could suddenly comprehend the matter'. Einstein met Besso at age sixteen, and Besso recommended some of Mach's works to Einstein.

In his biography of Einstein, Banesh Hoffman (1973, p. 78) remarks that 'Einstein's treatment of simultaneity in terms of specific synchronizing procedures clearly shows Mach's influence', but it is not clear to me whether he means that it was the treatment in terms of specific procedures, or the specific procedures themselves, that show Mach's influence.

it was a rotating rod with a whistle at one end. In this popular lecture there is a discussion of a rotating cross as well; it is prefaced with two examples that draw attention to the change in time intervals measured in a system moving with respect to the source. In the first, the information is 'news' from a political capitol that travels 'by the post'; in the second, music travelling by sound waves: 'At rest, we hear a piece of music played in the same tempo at all distances. But the tempo will be seemingly accelerated if we are carried rapidly towards the band, or retarded if we are carried rapidly away from it'.[17] The, he suggests thinking of the cross as a windmill, the information in this case being conveyed by light: 'Clearly, the rotation of the cross will appear to you more slowly executed if you are carried very rapidly away from it' (1895, p. 53).

For transformations between two reference frames both of which may be in uniform motion with respect to a medium, one can actually develop an analogue of the Lorentz transformations for time and distance between reference frames using the velocity of sound rather than the velocity of light. In fact, Max Born has worked this out: '[…] if we use sound signals to regulate the clocks, Einstein's kinematics can be applied in its entirety to ships that move through motionless air. The symbol *c* would then denote the velocity of sound in all formulae. Every moving ship would have its own units of length and time according to its velocity, and the Lorentz transformations would hold between the systems of measurement of the various ships. We should have before us a consistent Einsteinian world on a small scale' (1962, p. 251). This 'world on a small scale' differs from the real one of which it is supposed to be a model in that, if rigid meter rods were constructed to represent units of length, a meter rod on one ship need not work as a meter rod on another. Nor would a clock on one ship necessarily serve as a clock on another, thus violating one of Einstein's requirements for clocks.

[17] Those who are familiar with the significance Einstein attributed to a 'thought experiment' he had at the age of sixteen about pursuing a beam of light with the speed of light, may be interested in the date of these lectures. The anthology *Popular Scientific Lectures* was published in 1895 in the U.S. and in Europe in 1896, the year Einstein was sixteen/seventeen, although this particular lecture was first published elsewhere in 1867. Here Mach is describing a special case of the Doppler effect, which was probably not news to even a sixteen year-old Einstein. However, that Mach points out that the time interval/tempo whose measurement is based on sound signals/waves will appear to lengthen or be retarded to a moving observer, in a lecture entitled 'The Velocity of Light' is worth remarking, especially since the point is that the measurement of the velocity of light is based on techniques for measuring the velocity of sound. I discuss this in more detail at the end of Section 5.

Still, using the units specific to each ship's reference frame, what results is a consistent set of equations for transforming times and distances in one reference frame into another that satisfies a principle of relativity (for transmission of sound signals) and the constancy of the (round-trip) velocity of sound in each reference frame. Of course the equations for sound transmission between ships are not really Lorentz-invariant, but we do have an analogue of Lorentz-invariance. The point I want to make[18] has to do with the contrivance involved in building a model of the analogue of Lorentz invariance obtained from the Lorentz transformations by using the speed of sound in place of the speed of light *in vacuo*. What is contrived about the model is that the measuring devices on each ship depend upon the sip's motion with respect to the medium of sound transmission (i.e. the air). Thus, the medium's state of motion is an essential part of the physics of the situation. In Lorentz' theory, time was specific to a particular reference frame in motion with respect to the ether. As in Lorentz' theory, there can be judgments of absolute simultaneity in this analogue of Lorentz-invariance. For differences in judgments about whether events are simultaneous or not can be regarded as only apparent, attributable to the ships' relative motions with respects to the medium. Differences could be explained as dynamical effects for which correlations could be made, and it could be determined which ship was at rest with respect to the medium (on it, sound transmission would be isotropic). Hence my claim that, in Einstein's exposition, even with the first of his postulates (the principle of relativity), and the kinematics given in Sections 1 and 2 of the 1905 special relativity paper, absolute simultaneity would still be tenable, were there a medium for light transmission with respect to which motions were relevant to the physics of the situation.

Thus the importance of 'empty space' in the result obtained of the equivalence of inertial reference frames for observing the velocity of wave propagation in empty space (where empty space means devoid of a medium to which states of motion can be ascribed) for Einstein's conclusion that 'we cannot attach any absolute signification to the concept of simultaneity, […] two events which, viewed from a system of coordinates, are simultaneous, can no longer be looked upon as simultaneous events when envisaged from a system which is in motion relatively to that system' (1905, p. 42).

The acceptance of the constancy of the velocity of light in empty space precedes the claim of relativity of simultaneity in the 1905 paper. It is tucked into Section 1; just after the

[18] See Born (1962, pp. 225-255) for a more fully-worked out explanation of the analogue.

definition of ‘simultaneous’ and ‘time’, Einstein writes this single-sentence paragraph: ‘In agreement with experience we further assume the quantity $(2AB/(t'_A - t_A)) = c$ to be a universal constant – the velocity of light in empty space’ (1905, p. 40). Likewise, in the abbreviated account of the development of special relativity in his autobiography, as soon as he mentions the relativity of simultaneity, he remarks on the ‘presuppositions’ involved, and they include the propagation of light *in vacuo*: ‘a light signal that is reflected back and forth between the ends of a rigid rod constitutes an ideal clock, provided that the postulate of the constancy of the light velocity in a vacuum does not lead to contradictions’ (1992, p. 53). And, in the Stafford Little Lectures (1974, p.29), he explains why the propagation of light *in vacuo* is given such logical priority: ‘In order to give physical significance to the concept of time, processes of some kind are required which enable relations to be established between different places. It is immaterial what kind of processes one chooses for such a definition of time. It is advantageous, however, for the theory, to choose only those processes concerning which we know something certain. This holds for the propagation of light *in vacuo* in a higher degree than for any other process which could be considered.’ This move of Einstein’s seems a natural response to Mach’s remark: ‘There is no physical standard of time corresponding to the standards of length and mass’, which appears in the book that Einstein said exerted such a strong influence on him: Mach’s *Science of Mechanics*. Finding a physical standard of time fills the lacuna in classical mechanics that Mach identified. And, looking towards the behavior of light to fill it is also in keeping with Mach’s remark that ‘[a]ll our knowledge of the universe comes from light’ (1862, p. 553).

What I want to do next is show that there is a prescience in Mach’s *scientific* work of these crucial point in Einstein’s 1905 special relativity paper, shown in Mach’s attentiveness to the same points in his investigations, in (i) looking to relative motion, and not considering a medium of transmission an essential feature of waves, in explaining the Doppler effect, and (ii) treating the constancy of signal propagation as a principle, in explaining shock waves. We shall see that Mach’s explanation of the Doppler effect permits constructing an explanation for the phenomenon that explains it in the same way for sound as for light, because it renders irrelevant motion with respect to a medium of transmission. The analogy used in extending the Doppler effect relies upon treating constancy of velocity of propagation as a feature common to both sound and light waves; in his explanation of supersonic shock waves he again treats the constancy of signal velocity as an over-arching principle.

5. Mach Against 'The Law of Conservation of the Period of Oscillation' – The Doppler Effect for Sound[19]

Whereas Einstein's two principles are about things that remain invariant (e.g. the velocity of light, the laws by which the states of physical systems undergo change), the insight he had about how to show their compatibility was instead about something that *did* vary between inertial reference frames: simultaneity of events, or, more generally, the time interval between two events not in the neighborhood of the observer. In his popular account of special relativity, in which, he said, he 'has spared himself no pains in his endeavor to present the main ideas […] on the whole, in the sequence and connection in which they actually originated' (19612, p. v), he mentions the basic apparatus Mach designed to validate the Doppler effect for sound. It is easy to zip right by the remark comparing an organ-pipe placed with its axis parallel to the direction of travel with one in which the axis of the pipe is perpendicular to the direction of travel. But, as we shall see, the appearance of such an apparatus in this exposition is telling: understanding the similarities between Mach's approach to explaining the Doppler effect and Einstein's approach to explaining the Lorentz contractions explains why Einstein could have made the remarkable speculation that Mach might have 'come across' the theory of relativity had his mind been young and fresh 'when physicists were considering the significance of the constancy of the velocity of light' (1916, p. 157).

The historical story of the validation of the Doppler effect goes from light to sound and back: Mach worked on the Doppler effect for sound only because he could not make the analogous measurements for light in a laboratory. The relevant factor is the ratio of the velocity of the relative motion between observer and source to the velocity *c*, where *c* is the velocity of sound for a change in pitch, or the velocity of light for color shift. The velocity of motion required to create an observable change in pitch is much less than that required to

[19] I have been fortunate to have found W. W. G. J. Swoboda's dissertation (1973), which contains an extensive discussion of Mach's work on the Doppler effect in Chapter III (pp. 43-74). This section owes much to his discussion. Quotes from Mach's writing given in English in this section are Swoboda's translations, unless otherwise indicated. Other accounts of Mach's work on the Doppler effect are given in Blackmore (1972, pp. 17-19), Ratliff (1970, pp. 24-27), and Hiebert (1970, pp. 80-81).

create an observable change in color. For Mach, as for Doppler and Einstein, the phenomenon that aroused interest in explanations of frequency shift were astronomical/cosmological observations of color shifts of stars; if Doppler were correct that the color shift was due to changes in relative velocities of stars, Mach pointed out, then observations of color shifts could be used to calculate motions of heavenly bodies.

The story begins in 1842, with Doppler's publication of an explanation for the observed changes in colors of the double stars, by showing he could predict the change in the observed frequency in cases where exists relative motion between the observer and a source of vibrations. Doppler addressed the color shift of stars; Ballot's experiments soon confirmed analogous predictions for sound.

In 1850, when Doppler reviewed these experimental results before the Academy of Sciences, Petzval, who had been Mach's mathematics professor, presented objections to Doppler's theory. These objections included a mathematical analysis intended to show that the motion of the source of light or sound could not alter the period of oscillation, and hence could not account for the change in pitch of sound or color of light. Petzval's mathematical analysis actually applied to the frequency of oscillations with respect to the medium of transmission, rather than with respect to an observer, but he did not seem to realize this. He confidently formulated the results in a principle, which he dubbed, rather grandiosely, the 'Law of the Conservation of the Period of Oscillation': '[F]or every oscillatory motion, the period of oscillation is a constant magnitude, neither dependent on the medium or on any motions which may be present in it'. Petzval also thought that Doppler's predictions of a singularity in the limiting case of the source of sound approaching an observer at the speed of sound showed his reasoning could not be correct; Doppler predicted that 'all the separate wave-pulses arrive at the same moment at the observer, or, what is the same, in infinitely short time intervals, which circumstance would cause a tone of indefinitely high pitch'. Petzval considered an infinitely high pitch 'impossible' and dismissed Doppler's explanation of the observed frequency shift in terms of relative motion of the observer and wave-source. Although he did not offer an alternate explanation, he proposed that the answer lay in an analysis of the behavior of the elastic medium in which the waves travelled (Swoboda, 1973, pp. 28-30, 41-42).

In 1860, Mach published a critical paper on the Doppler effect in which he reported on experiments he had designed and performed as part of his *Habilitationsschrift*, concluding that 'the fluctuation in the pitch is dependent on no other circumstance than the direction and speed [of the source] with respect to the observer' (1860, p. 551). The experimental apparatus consisted of a rotating rod with a whistle attached to its end; the whistle could be made to produce a sound by a bellows.[20] An observer situated along the axis of rotation hears a constant pitch, but if he is situated perpendicular to it, the pitch fluctuates as the rod rotates, increasing in pitch as the tip of the rod approaches the observer, and decreasing in pitch as it recedes from the observer. Actually, for sound, there will be some effect on the frequency due to motions with respect to the medium, but Mach regarded them as no more than secondary effects. This is correct, in that the Doppler effect exists only if relative motion exists between the source and the observer, for the secondary effects due to motion with respect to the medium exactly cancel otherwise. He regarded the relevant factor to be the relative motion of the sound source with respect to the observer, casting the explanation of the apparent lengthening and contraction of time intervals (the period of oscillation) as a *kinematical* matter.[21] That is, from the characteristic wave motion alone, without regard to the forces causing it, he derives the Doppler equation. This is in sharp contrast to Petzval, who suggested looking to the behavior of the elastic medium (which Mach did not consider an essential characteristic of wave motion) for causes of the apparent frequency shifts, rather than to the relative motion of the observer and the source of oscillation.

One can see a similar break with his respected predecessors in Einstein's approach to the Lorentz contraction: whereas Lorentz, among others, looked for a causal explanation (e.g. molecular forces) of the contraction of times and lengths accompanying moving bodies in electrodynamics, Einstein concluded that the contractions were a matter of kinematics.[22] The

[20] Swoboda regards this experimental setup as a variation on a pendulum as the emitting body, and this as directed towards Petzval's claim that the motion of a pendulum whose bob contained a sound-producing body would produce no variations in the pitch of the sound (Swoboda, 1973, p. 50).

[21] Miller's discussions of the Doppler effect in his (1981, pp. 36-39 and pp. 301-307) do not mention Mach. Pais (1982, pp. 121-122) does not mention Mach in connection with the Doppler effect, either, but does mention Voigt's 1887 paper on the Doppler principle, in which transformations equivalent to Lorentz transformations are given.

[22] Harvey Brown (1993, p. 253) cautions against taking kinematic/dynamic distinction too seriously. Thanks to Rob Clifton for telling me of this interesting paper.

difference is that he had to come up with a definition of 'time', whereas Mach did not need to rewrite kinematics to show that the Doppler effect (the contraction or lengthening of the time period of oscillation) was kinematical in nature. But even this lacuna of a physical standard for 'time' in classical kinematics was noted by Mach, as pointed out above.

Now for the significance of the appearance of an apparatus resembling Mach's in Einstein's popular account of relativity. In a prefatory chapter on the principle of relativity, he writes: 'We should expect, for instance, that the mote emitted by a pipe-organ placed with its axis parallel to the direction of travel would be different from that emitted if the axis of the pipe were placed perpendicular to this direction' (1961, p. 14). This is an example of a description of natural phenomena for which reference frames in relative uniform motion are *not* equivalent; constancy of pitch (tone) provides an example of something to which the principle of relativity does not apply. That is, constancy of pitch is not a law of nature, as constancy of the light velocity in empty space is. Keeping in mind that Mach used this apparatus against claims of a conservation 'law' for the time period of an oscillation, we can see a connection between this prefatory remark of Einstein's and what he plans to do next in that exposition: derive a concept of 'time' for which the observed behavior (time interval between beats) of a clock will be different depending on its motion relative to the observer. According to the concept of time he is about to derive, whether two events are simultaneous or not will depend on the direction and velocity of the observer's motion, just as pitch does. As to why he would use an example from sound to make the point: 'One see that a priori it is not at all necessary that the "times" thus defined in different inertial systems agree with one another. One would have noticed this long ago if, for the practical experience of everyday life, light did not present (because of its large value of (c) the means for fixing an absolute simultaneity' (1992, p. 53). Laws of nature are absolute, and they will be independent of choice of inertial reference frame, but simultaneity (and 'time' in general) is not absolute, and so need not be independent of choice of an inertial reference frame. The move is parallel to Mach's move in showing that constancy of the time period of oscillation was not a 'law', i.e. it was not invariant between reference frames in uniform motion with respect to each other.[23]

[23] If the reader stops to absorb the remark about the organ-pipe, it catches him by surprise, for two reasons. First, since the section is on the invariance of phenomena with respect to the direction of the earth's motion, I think the

Picking up the story of Mach's verification of the Doppler effect: having settled the issue for sound, Mach went on to draw consequences for the astronomical-cosmological phenomenon that Doppler had wanted to explain: the color shift of stars. It spoke in favor of the Doppler account, he said, that it did not require the introduction of *ad hoc* assumptions, Although extending the consequences beyond is experiments for sound meant he was on less firm ground, Mach's remarks on why such consequences are of importance involves his interest in the behavior of light: 'Through light alone we arrive at our knowledge of the universe, thought light we have learned all that is known of the physical nature and the movement of the heavenly bodies' (1860, p. 553).

Petzval was not impressed by Mach's experimental results; he promptly responded by reasserting his 'Law of the Conservation of the Period of Oscillation'. His attitude towards Mach bordered on the sarcastic: '[…] now a young admirer of science is actually found, who wishes to demonstrate the Dopplerian theory by its correspondence with a certain experiment. Obviously the experiment is here being used for a purpose for which it is not suited. No experiment can remove clearly demonstrated logical or mathematical contradictions and deformed facts' (Petzval, 1860, quoted in Swoboda, 1973, p. 55).

Mach responded by stating firmly that known appearances were to be accepted, and structured his conclusion as a resolution of two apparently incompatible propositions:

> If one wishes to summarize the case in generally comprehensible terms, one would say:
>
> 1. Petzval has shown through his deduction that windy weather has no influence on the pitch of a tone.
> 2. Doppler investigates how the pitch of a tone is affected by the relative motion of source and observer.
>
> The results of these two investigations cannot contradict each other (Mach, 1861, p. 123).

(continued) reader is at this point expecting an example for which the phenomenon is *not*, in fact, dependent on the direction of motion, and instead is faced with contemplating one in which the direction of the motion makes a difference. Second, it is striking that Einstein has chosen a phenomenon that arises for sound as well as for light, rather than one that would illustrate that light is not like sound.

Thus, rather than being drawn into taking side between 'experiment' (Doppler) and 'mathematics and logic' (Petzval), Mach sets out to show their compatibility. There are some similarities here between Mach's presentation of the issue and how Einstein would later structure his presentation of special relativity emphasizing that known appearances were to be included as a starting point in theoretical investigations, and laying out two apparently incompatible propositions that will be shown to be only apparently so. And the essence of the resolution is to point out a mistaken reliance on, and eliminate as a relevant factor, motions with respect to a medium of transmission. Mach showed that the two statements were compatible, and pointed out that the fault in the reasoning that regarded them as incompatible lay in the application of Petzval's mathematical analysis. His criticism of Petzval's reasoning appeals to symmetry: 'If one considers a single point in a medium, it is obviously immaterial whether one regards the point as moving and the medium at rest, or the reverse. On the other hand, it will never do to replace the relative motion of two points against each other with a motion of the medium' (Swoboda, 1973, p. 60). Mach's position was that the crucial factor in causing the frequency shift was the relative motion of source and observer. By the time Einstein was a young man, this explanation of the Doppler effect had become the standard one, and Mach's inventive apparatus for illustrating it commonly known.[24]

6. Mach for the Constancy of the Velocity of Light – The Doppler Effect for Light

Years passed, and in 1873, Mach published his three papers on the Doppler effect in a booklet. In 1878, he published on the Doppler effect again, specifically discussing the extension of the acoustical Doppler effect to optics.[25] In that paper, he says experiment is unnecessary to

[24] The explanation of the Doppler principle in Lord Rayleigh's landmark *The Theory of Sound*, also published in 1896 (the year Einstein was sixteen/seventeen) is clearly the one Mach gave; it even includes his specific corrections of Petzval's objections. Mach is credited there only for his inventive laboratory apparatus: 'The principle of the alteration of pitch by relative motion was first enunciated by Doppler […] Strangely enough its legitimacy was disputed by Petzval, whose objection was the result of a confusion between two perfectly distinct cases, that in which there is a relative motion of the course and recipient, and that in which the medium is in motion while the source and the recipient are at rest […]. A laboratory instrument for proving the change of pitch due to motion has been invented by Mach' (Rayleigh, 1945, p. 155).

[25] The paraphrases that follow are my own. The text on which they are based reads: 'Es ist für die meisten und so auch für unsere Frage gar nicht von Belang, ob das Licht eine mechanische Wellebewegung ist. Man kann sich das

verify the Doppler effect for light, as the effect follows from characteristics of light that are common to both sound and light waves. It is of absolutely no significance for the question of whether Doppler's principle applies to light, he says, whether or not light is a mechanical wave motion. One could just as well think of light as chemical oscillation, for many of the appearances, such as anomalous dispersion and fluorescence, can be better understood, in many respects, under such a notion of light. He is no longer hesitant about extending the Doppler principle into the realm of optics; he confidently states that the Doppler principle can be applied to light in the same way as four sound, on the basis that light and sound are propagated in time, have spatial and temporal periodicity, and can be algebraically summoned. This is in keeping with his earlier remarks on the Doppler effect for sound, in which he kept clear of appealing to any causes arising from the mechanical nature of sound waves, and stuck to kinematical consideration (Mach, 1878, p. 308).

Mach offers a thought-experiment to prove the Doppler effect applies to light. It involves two light sources, one moving away from the observer. He reasons from the kinematics of the situation that a shift in a lengthening of the period of oscillation will result, and that *necessarily* the shift is due to the motion of that source. He also provides an argument that makes an explicit connection between the constancy of the velocity of light ('ungeänderte Lichtgeschwindigkeit') and the lengthening of the period of oscillation. He appeals to the constancy of the velocity of light to show that the period *must* lengthen if the second source recedes from the observer. And he emphasizes the role of time in this consequence, by explicitly pointing out the dependence on the shortening or lengthening of the light wave on the velocity of the propagation of light: 'So long as one concedes the temporal (finite, or non-instantaneous) propagation of light, so must one also concede that at the beginning of the motion of B [the light source that recedes in his thought experiment], […] the waves will expand (or contract)' (1878, pp. 309-310).[26] So, clearly, Mach is treating the Doppler effect as applicable to both sound and light on the basis that the

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(continued) Licht ebenso gut als chemische Schwingung denken und manche Erscheinungen, wie die anomale Dispersion, die Fluorescenz u.s.w., warden dadurach viel verständlicher.

Das Doppler'sche Princip ist, alle Hypothesen bei Seite gelassen, auf das licht in gleicher Weise anwendbar wie auf den Schall, da Licht und Schall sich zeitlich fortpflanzen, eine räumliche und zeitliche Periodicität haben und algebraisch summirbar sind' (Mach, 1878, p. 308).

[26] 'So lange man die zeitliche Fortpflanzung des Lichtes zugibt, muss man auch zugeben, dass beim Beginn der Bewegung von B […] sich die Wellen dehnen (oder verküzen)'.

effect is a consequence of the kinematics of wave motion, and so is independent of the nature of the oscillation. Perhaps less obvious in his exposition, but certainly significant to Einstein, is the elimination of relative motions with respect to the medium of transmission in his explanation.

For sound, as for light, the frequency shift known as the Doppler effect is due to the relative motion of observer and source, in the sense that if there is no relative motion between them, there is no frequency shift. However, in calculating the magnitude of the shift, the existence of a medium of transmission actually does make a difference for sound. For mechanical waves such as sound,. The relative motions of the source and observer with respect to the medium are relevant; the frequency shift for the motion of the observer moving with respect to the medium, towards a source that is at rest with respect to the medium, is a factor $(1 - v_{obs}/c)$ for the one-dimensional case, or $(1 - v_{obs}/c)\cos\Theta_{obs})$ for the two dimensional case. Here v_{obs} is the relative motion of the observer in uniform motion with respect to the medium, c is the characteristic velocity of sound in the medium, and Θ_{obs} is the angle between the line of the observer's motion and the direction of travel of the sound wave. But the frequency shift for the motion of a source with respect to the medium is the inverse of this, $(1 - v_{source}/c)^{-1}$ for the one-dimensional case, and $(1 - (v_{source}/c)\cos\Theta_{source})^{-1}$ for the two-dimensional case (Elmore and Heald, 1985, pp. 164-166). In the laboratory experiments resulting in a change of pitch performed for his *Habilitationsschrift*, the only one of these two velocities that was ever non-zero was the velocity of the source with respect to the medium. The other acoustical experiments referred to were of observers seated on a bank listening to an approaching (and then receding) train. Was Mach's experimental work flawed, then, in that he only considered one of the two cases? Not really, for we can see that, if v is much less than c, the two factors are approximately equal, so, for low speeds, the experimental results would not significantly differ between the two cases, which somewhat vindicates Mach's attitude towards effects due to motions with respect to the medium as secondary at most.

However, given that it was Mach who clarified the confusions in the controversies surrounding the Doppler effect, this aggressive neglect of discussing the effects from motions with respect to the medium in cases where they are relevant is noteworthy; one would expect him to explain the secondary effects due to motions with respect to the medium as well, if only to cast his precursors' mistaken view in the right perspective. I think his focus is best explained by his interest in emphasizing the similarities between sound and light that would validate extending his experiments with sound to the realm of light, as his initial interest in the Doppler effect was in the color shift of the stars. And, in keeping with his principles of avoiding reification of anything he did not have experimental evidence for, he would have preferred not to involve the ether in explanations of the Doppler effect for light, and so was uninterested in secondary effects for sound due to motions with respect to the air. This is borne out in his final paper, in which he identifies the essential characteristics of wave motion in such a way that wave motion need not mean mechanical wave motion.

It turns out that Mach's eye for non-medium based similarities between sound and light led him on a precocious course in this case: for the Doppler effect for light in empty space, i.e. the relativistic Doppler effect, the only relevant velocity *is* the relative velocity of source and observer, and, as a result, the equation is the same whether we consider the source to be moving or the observer to be moving. This can be seen from Einstein's expression for the one-dimensional case: the factor by which the frequency increases or decreases is $(1-\square/\square)/(1+\square/\square)$.[27] And Einstein's derivation of the expression is purely kinematical, as was Mach's argument for the applicability of the Doppler effect to light.

For light, as for sound, there is a singularity. Einstein points out that when the observer moves along the line connecting the source and the observer, the equation expressing Doppler's principle 'assumes the perspicuous form' of $(1-\square/\square)/(1+\square/\square)$, and remarks: 'We see that, in contrast with the customary view, when $v = -c$, $v' = \infty$' (1905, p. 56). This, following Mach, he does not think an infinitely high pitch 'impossible' (as Petzval charged), either. Exploring how observations of oscillatory motion vary with the observer's motion is reminiscent of the thought experiment he had at age sixteen. There, he explored the limit at the other end: the observer is

[27] Feynman gives several different derivations of the expression for the relativistic Doppler effect, and provides a nice insight: '[…] the necessary equality of the two expressions [for the frequency that we would observe if we move toward a source and for the frequency that we would see if the source moved toward us] is one of the ways by which some people like to demonstrate that relativity requires a time dilation, because if we did not put those square-root factors in, they would no longer be equal' (1963, pp. 34-38). Both Feynmann and Einstein's derivations are meant to apply only to the case of light propagated *in vacuo*; Born (1962, p. 299) provides a more precise derivation that distinguishes between the '*c*' that occurs in the term added for the relativistic Doppler effect that refers to the universal constant *c*, the velocity of light *in vacuo*, and the '*c*' that refers to the velocity at which the wave is being propagated. This, the symmetry arguments used by Einstein and Feynmann would not carry through for sound, or for light in glass or water.

travelling away from the source (chasing a ray of light) with the velocity *c*; Einstein concluded that he would observe an electromagnetic field standing still ('ruhend') though 'spatially oscillating' ['räumlich oszillatorisch'] (1992, p. 49). That thought experiment echoes Mach's discussion of how a windmill would appear to rotate more slowly as one was carried away form it in the 'Velocity of Light' lecture, published in an anthology around this time (1895, pp. 48-65). Carried to the limit, where the velocity with which one was carried away from the windmill were the velocity of light, the windmill would appear to be standing still even though it is actually oscillating in space. I suppose the paradox is that, for travel at light velocity, there would be no difference between observations of a rotating windmill and one that is not rotating. Notice that this is different from the non-paradoxical situation for objects in motion in one coordinate system that appear to be at rest to an observer moving with the same velocity. Thus, the paradox (from a coordinate system in uniform motion with respect to an object, that object is observed as stationary even though it is actually oscillating) arises in classical mechanics as for electrodynamics, and for sound as well as for light. For sound, the observed frequency for an observer moving away from a sound source at the velocity of sound would be zero. This paradox arises only for things that oscillate (or are in some sort of non-uniform motion).[28]

[28] This is my own reading of the paradox. Einstein's comments on his thought-experiment only made sense to me after reading Mach's lecture 'The Velocity of Light' and thinking of how the same paradox for the windmill relies crucially on oscillation in space, and so on the wave characteristic of spatial periodicity.

Notice that, on this reading, the paradox is not a problem about the addition of velocities, and so differs form interpretations suck as Miller's (1991, p. 190) on which the issue is about undetectable ether drift. It seems to me that, even if ether drift were detectable, there would be two indistinguishable but non-identical cases: observer moving with respect to the ether away from a windmill that is oscillating, and from a windmill that is not oscillating.

Although Einstein's remark at the end of the paragraph in which he introduces the paradox may seem to bring in the issue of ether drift ('For how should [that] observer know, or be able to determine, that he is in a state of fast uniform motion?'), a close reading of the passage shows that the remark is an explication of the insight '[… it appeared to me intuitively clear that, judged from the standpoint of such an observer, everything would have to happen according to the same laws as for an observer who, relative to the earth, was at rest' (1992, p. 51). So, I read that remark as an explication of a starting point from which the resolution will come, rather than an explication of how the paradox arises or what it consists in. For light, the impossibility of travel at the speed of light shown by special relativity resolves the paradox of travel *in vacuo*. But what resolves the paradox for sound? This is the interesting part: there really is a singularity at sonic velocities; discontinuities in the medium arise in this case. The discontinuities are called shock waves, and they were discovered by Mach.

The image of riding the crest of a wave arises in another phenomenon that Mach had discovered and described by the time Einstein was sixteen: the work Einstein described as 'the best known' among Mach's physical investigations, the shock waves produced by projectiles. Mach's first laboratory investigations involving supersonic waves arose out of his attempts to build a time-measuring apparatus based on the constancy of velocity of sound. This failed, as the waves he was employing (audible waves accompanying electric sparks) were not normal acoustical waves, as evidenced by the fact that they travelled faster than the speed of sound.[29] So he turned to exploring how the velocity of sound is involved in the phenomena arising from supersonic flight. His explanation provides an answer, in a concrete way, to what happens when one travels at the velocity of sound; in fact, Mach provided photographs of the phenomenon.

7. Mach for the Constancy of the Velocity of Sound – Supersonic Shock Waves

The story of Mach shock waves is similar to that of the Doppler effect in that it is inspired by a comparison between analogous phenomena for light and sound, is performed in the laboratory for sound, and is later shown to apply to light. However, Mach's own work covers only the first two parts of the story for shock waves. It would provide a happy ending to this story were it true that, as for the Doppler effect, for which Einstein's derivation of the relativistic expression for the Doppler effect completed the last part of the jo0urney back to light, Einstein also completed the story of shock waves with a recognition of an analogous phenomenon for

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(continued) The astute reader will have noticed that one paradox remains untreated: the case for light that is not *in vacuo*, and an observer moving in a medium in which the velocity of light is less than *c*. What if the observer is carried away from the oscillator at the speed with which light travels in that medium? Would not the same paradox arise? Yes, it would. The discontinuity that arises there is real, too: Cerenkov radiation. These latter two phenomena are treated in Section 7.

What is common in these three cases is that one can construct the case of a system in uniform motion and information (either of an oscillation or some other non-uniform motion) that cannot travel to the system that has achieved the characteristic velocity of wave propagation in that medium. I think it a recommendation for this reading of the paradox that it is general for various kinds of waves (e.g. light waves, surface waves, and sound waves).

After developing this reading, I read Grünbaum's (1961) analysis of the passage; his conclusion that Einstein's reason for ruling out stationary light waves could only have been his intuitive grounds for the principle of relativity (1961, p. 48) seems right to me and I believe the reading I have given coheres with his analysis.

[29] Blackmore (1972, p. 106). Also in Merzkirch (1970, p. 47).

light. But, the story ends even more sadly than in mere neglect; for the phenomenon of shock waves, misinterpretations of Einstein's special theory of relativity actually interfered with the recognition of the analogous phenomenon for light: Cerenkov radiation, an electromagnetic analogue of Mach shock waves.

In his popular scientific lecture, 'On Some Phenomena Attending the Flight of Projectiles', Mach rather modestly presented the results of his investigations into the head waves accompanying the flight of bullets travelling at supersonic velocities as a technical advance in laboratory methods, rather than as a new scientific discovery. Einstein reflected the assessment in recounting this work in Mach's obituary, stating that 'the basic thoughts which he applied were not new in principle', although 'they illustrated an exceptional experimental gift. He successes in photographing the thickness of the air front in the neighborhood of a speeding projectile, which threw light on a form of acoustical processes which we had been ignorant of before' (1916, p. 158). Probably both are referring, not to an earlier discovery of shock waves, but, rather, to the fact that, as Mach put it in his lecture, upon exhibiting a photograph of a shot from an Austrian Mannlicher rifle and remaking that it resembled a bird's eye view of a boat moving rapidly on a lake: 'the dark hyperboloid arc which streams from the tip of the projectile really is a compressed wave of air exactly analogous to the bow-wave produced by a ship moving through the water, with the exception that the wave of air is not a surface-wave'. The description he gives is in terms of an appeal to an extension of a principle from another domain, which tends to downplay the significance of his discovery: 'The explanation of the bow-wave of a ship and that of the head-wave of a body travelling in atmospheric space both repose upon the same principle, long ago employed by Huygens' (1943, pp. 324-325).

There is also a reference to Huygens in the technical paper in which the ratio *v/c* (Mach number) first occurs in his discussion of this topic. The equation Mach derives and later uses to determine the bullet's velocity is based on an equation developed for refraction of light waves: 'If we consider Huygens' principle applicable to the lines of the out-going sound wave, the angle determines the velocity of propagation of the explosion' (1885, p. 630). In that technical paper, Mach applies Huygens' treatment for light at a refractive boundary to the discontinuity in the medium that occurs at the shock wave, not just as a qualitative analogy, but for quantitative results as well. This was not for lack of a more direct approach: in a footnote, he remarks that numerical results achieved by others thought very complicated methods are of the same order.

Thus Mach's determination of supersonic velocities began with an analogy between light and sound. One point of the experimental work was to render the imperceptible perceptible: 'It is always peculiarly fascinating […] to render palpable to the senses, something which we have only theoretically excogitated or surmised' (1943, p. 310). As it turns out, he shows that the conjecture he is testing – Melsen's suggestion that the projectile carries along with it masses of air which it forces into the bodies [of the people] struck – is not correct; the photographs show that the condensation is a sound wave, and '[a] forward-moving sound wave is not a forward-moving mass of matter, but a forward-moving form of motion, just as a water wave or the waves of a field of wheat are only forward-moving forms of motion and not movements of masses of water or masses of wheat' (1943, p. 327).

More importantly, Mach found that the head wave travelled ahead of the projectile, and although having the character of a sound wave, it travelled faster than the characteristic velocity of sound in the surrounding air. Mach does not present this as a paradox, but explains it; his explanation involves the local velocity of sound, which, being dependent on the thermodynamic properties of air, can be different from the velocity of sound in the rest of the surrounding air:

> If the projectile moves faster than sound, the air ahead of it cannot recede from it quickly enough. The air is condensed[30] and warmed, and thereupon, as all know, the velocity of sound is augmented until the head-wave travels forward as rapidly as the projectile itself, so that there is no need whatever of any additional augmentation of the velocity of propagation (1943, p. 330).

The significance of the change in the local sonic velocity is that the sonic velocity is a physical limit; no pressure pulse (mechanical signal) can travel faster than sonic velocity. Thus, for the head wave to travel ahead of the projectile, the 'information' that the projectile communicates to the head-wave has to be transmitted at the same speed as sound travels. The explanation Mach gave was this: the only way for the projectile (bullet) to travel faster than the speed of sound in the surrounding air is for the local air temperature at the tip of the bullet to increase enough so that the sonic velocity in the air between the tip of the projectile and the head wave becomes at least as high as the velocity of the speeding bullet. Thus, the general rule that pressure pulses

[30] The dominant factor for the velocity of sound in gas is the temperature, so Mach's explanation is that condensing the air causes the air to heat up, and the local sonic velocity is higher than in the surrounding air.

cannot travel faster than the speed of sound still holds for the projectile's disturbance. And the head wave's breaking of the sound barrier is qualified:

> If such a wave were left entirely to itself, it would increase in length and soon pass into an ordinary sound wave, travelling with less velocity. But the projectile is always behind it and so maintains it at its proper density and velocity (1943, p. 330).

Here, unlike in his explanation of the Doppler effect, the medium is key to explaining the phenomenon. But, the explanation, although it turns on changes in signal velocity, actually puts the emphasis on a principle that will carry over to special relativity: Mach puts the emphasis on the *constancy* of the speed of sound (albeit in a particular medium in a particular state), gives an explanation that regards it as a governing principle, and notes that the characteristic speed of sound represents a *limit* for moving objects. In fact, the principle of the constancy of the signal velocity is presented as a principle governing physical processes, including thermodynamical ones, rather than as arising from them. This is a remarkable thing to pull out of the phenomenon of supersonic flight. Treating the constancy of the speed of light as a principle (rather than as derived from laws) is, of course, the distinctive feature of Einstein's move in formulating the light postulate in developing special relativity.[31] It is also what he uses to synchronize clocks.

But there is part of Mach's explanation that Einstein does *not* carry over when discussing the role of limiting velocities in electrodynamics: the other side of the velocity limit. Mach's explanation does not shy away from recognizing the discontinuity in behavior that occurs at the point where the bullet first reaches the velocity of sound in the surrounding air. He says we should '[not be] puzzled at learning that in actuality the law of resistance changes as soon as the speed of the projectile exceeds the velocity of sound, for this is the precise point at which one important element of resistance, […] the formation of waves, first comes into play'. This is presented as being at odds with theory:

[31] Einstein (1954, p. 228) identifies special relativity as a 'principle-theory', which he characterizes as: 'The elements which form their basis and starting-point are not hypothetically constructed but empirically discovered ones,, general characteristics of natural processes, principles that give rise to mathematically formulated criteria which the separate processes or the theoretical representations of them have to satisfy'. He explicitly identifies the principle of the constancy of the velocity of light *in vacuo* as such as principle in special relativity.

> […] the resistance, it is said, increases with the square of the velocity. This is all very pretty and simple and obvious. But practice and theory are at daggers' point here. Practice tells us that when we increase the velocity, the law of the resistance changes (1943, p. 330).

To fill out the description of supersonic flight of projectiles: the difference between the subsonic and supersonic cases can be described in terms of signal travel; from the standpoint of the reference frame of the moving bullet (or boat) the disturbances made will be felt throughout the region if the flow is subsonic. In contrast, for supersonic flow, the region at the tip of the moving bullet divides into areas in which the disturbance will be felt (e.g. in the subsonic are between the bullet tip and the shock front) and areas to which it will not propagated (outside the hyperbolic cone). The shock wave is a discontinuity in pressure (and so also of temperature), and the governing equations are developed by considering a system at rest with respect to the shock (i.e. moving along with the shock wave), using conservation laws (of thermodynamical as well as mechanical) energy, continuity, and momentum. The flow is always subsonic on the upstream side of the shock and supersonic on the downstream side. And the general behavior of many fluid properties change; many properties that would decrease with increasing velocity for subsonic flow, will instead increase for supersonic flow.

We do not see any attempt in the 1905 special relativity paper to imagine cases where a velocity limit is exceeded. What we do see, several times, is discussion of what happens when a limit is reach (e.g. frequency '=∞' (he boldly writes the symbol for infinity on one side of the identity), the source of light appears 'of infinite intensity', and '[the energy of motion] *W* becomes infinite').

8. Optical Analogues of Supersonic Shock Waves

The conceptual return journey from sound (sonic booms) to light (Cerenkov radiation) was not taken for many years after Mach published his work on shock waves. But in the Nobel Lecture Igor' Tamm gave in accepting his prize for work in developing the theoretical interpretation of the radiation of electrons moving through matter faster than light, he opens with a statement explaining, as Mach did for sound waves, that the principle is not new:

> 'The mechanism of radiation of light by a system moving with a super-light velocity is a very simple one and common to the radiation at corresponding conditions of all kinds of waves – electromagnetic as well as sound waves, waves on the surface of the water, etc.' (Tamm, 1958, p. 470)

The analogy to shock waves is explicit: 'We perceive the Mach waves radiated by a projectile as its familiar hissing or roaring, that is why, having understood the quite similar mechanism of [Cerenkov] radiation of light by fast electrons, we have nicknamed it "the singing electrons"'. The mechanism itself is explained for the general case of waves: if the velocity of a system capable of emitting radiation (e.g. an electron of light radiation, an airplane for sound, etc.) is less than the velocity of propagation of waves (e.g. light, sound, etc.) in the surrounding medium [if there is one], radiation only occurs if the velocity of some or all of the system is non-uniform (rotation of a propeller, oscillation of an electron in an atom). However, if the velocity of this system exceeds that of the velocity of wave propagation, 'quite a new mechanism is introduced, by means of which even systems possessing a constant velocity radiate'. After remarking that all the general properties involved in this type of radiation had been known much earlier in aerodynamics, he explains: 'The air waves emitted at supersonic velocities are called Mach waves. The emission of these waves sets in when the velocity of a projectile […] begins to exceed the velocity of sound in the air. Emitting waves means losing energy' (Tamm, 1958, p. 472).

As with the shock waves, when the velocity of wave propagation is exceeded, some properties that would tend to decrease tend to increase; an especially significant one id internal energy: 'But at super-light velocities […] the radiation of energy by the system may be accompanied by a positive increase […] of its internal energy *U*. For example, an atom, being originally in the stable state, radiates light and at the same time becomes excited!' (Tamm, 1958, p. 474).

Then he asks: 'The phenomenon could have easily been predicted on the basis of classical electrodynamics many decades before its actual discovery. Why then was this discovery so much delayed?' The answer should make one pause before dismissing Mach's reservations about the theory of relativity, if indeed he had them, as evidence of lack of insight:

> For many decades all young physicists were taught that light (and electromagnetic waves in general) can be produced only by non-uniform motions of electric charges. When

> proving this theorem one has – whether explicitly or implicitly – to make use of the fact that super-light velocities are forbidden by the theory of relativity (according to this theory no material body can ever attain the velocity of light).
>
> […] only on the very next day after our first talk on our theory […] we perceived the simple truth: the limiting velocity for material bodies is the velocity of light *in vacuo* (denoted by *c*) whereas a charge, moving in a medium with a constant velocity *v*, will radiate under [the condition where it s velocity exceeds the velocity of light in that medium] (Tamm, 1958, p. 474).

Although there is some controversy as to whether Mach actually wrote the preface to the *Optics* in which he retracted his earlier support of relativity theory,[32] Tamm's remarks show there would not be any need to make excuses for Mach if he had in fact said something along the lines that 'present-day relativity theory' was 'becoming more dogmatical'. Given that he had done research on supersonic flight, and that he explicitly looked for analogues of light and sound, it makes sense that Mach would object to a superficially characterized rejection of the possibility of superluminal velocities – something Einstein's relativity theory did not actually exclude, but which 'present-day relativity theory' as described by many of its proponents, did set out as a hallmark of the theory.[33] What is harder to explain is that Einstein did not predict the electromagnetic analogue of Mach shock waves.

[32] Gereon Wolters (1987) holds that Mach did not write this preface. My point does not turn on whether Mach did or did not write it. But I have in common with Wolters an (independently developed) appreciation of Mach, and sense of injustice at the underappreciation of Mach's work, especially by Einstein's scientific biographers. Since the topic of my paper is Einstein's appreciation of Mach, rather than Mach's appreciation of Einstein, I leave the latter aside in my discussion. For further reading on that subject: both Holton (1968) and Wolters (1987) cite numerous remarks by Mach showing a positive attitude towards Einstein's theory of relativity, for example: 'Space and time are not here conceived as independent entities, but as forms of the dependence of the phenomena on one another. I subscribe, then, to the principle of relativity, which is also firmly upheld in my Mechanics and Wärmlehre' (Mach, 1911, p. 95). In Beck (1995), the four letters from Einstein to Mach, which refer to positive comments received from Mach, are Documents 174, 175 (p. 130), 448 (p. 340), and 495 (pp. 370-371). Robert DiSalle's (1990) critical notice of Wolters' books provides a summary of Wolters' argument. DiSalle also makes the excellent point that Mach explicitly approved of a four-dimensional continuum combining space with time.

[33] The criticism in the *Optics* preface was of dogmatism in 'present-day relativity theory'; Mach's remarks about Einstein himself were all positive (Wolters, 1987; DiSalle, 1990). However, when taken out of context, Einstein's remarks in the special relativity paper do lend themselves to the regrettable misreading Tamm blames for the

9. Philosophers and Mechanicians

The philosopher in me has been taught to shrink from having any association with the skepticism of Mach's empiricism. But the mechanician in me has to grant the excellence of thought arising from the type of skepticism Mach employed in his scientific investigations. He saw value in rendering things palpable to the senses, and doing so often required summoning deep analogies in science. In turn, the search for analogies required an informed skepticism in jettisoning the right irrelevancies and being suspicious of the right presumptions: this led him to find formal characterizations and principles that did not occur to others who approached the same problems theoretically. On an approach to physics on which one searches for equivalent situations, skepticism towards the right things becomes a potent resource for a theoretician.

Mach is remembered for writing a philosophical critique of classical mechanics; it is probably just as relevant to appreciating the significance he ascribed to his critical outlook that it could be used in solving puzzling problems – determining what causes the Doppler effect and then using thought experiments to prove that his laboratory results for sound extended to light, and investigating the cause of bizarre bullet wounds by photographing the supersonic shock waves accompanying the bullet's flight, then using analogies of principles for light waves to

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(continued) misconceptions promulgated in training of physicists, for Einstein's paper (1905, p. 63) reads: 'As the electron is to be slowly accelerated, and consequently may not give off any energy in the form of radiation, the energy withdrawn from the electrostatic field must be put down as equal to the energy of motion *W* of the electron'. Str5ictly speaking, this is correct, since he is starting with an electron at rest, in empty space. However, the wording of his conclusion there that 'velocities greater than that of light have – as in our previous results – no possibility of existence' does not contain a qualification emphasizing that he means the velocity of light *in vacuo.*

Einstein later discussed the issue of whether Maxwell's theory rules out superluminal velocities in correspondence with Wien in 1907 (Beck, 1995, Docs 49, 50, 51, 52, 53, and 55, pp. 33-41), and with Sommerfeld in 1908 (Beck, 1995, Docs 72 and 73, pp. 49-50). However, although he admits there that Maxwell's equations do not rule out superluminal signal velocities, his interest is in indentifying the qualifying conditions that do rule them out; he seems uninterested in exploring the other side of the limit, even in the case where it is the velocity of light in a medium that is exceeded. That even velocities greater than the speed of light *in vacuo* were not disallowed by the special theory of relativity was later more widely recognized, although it is true that the barrier cannot be passed. (For the basis of these claims, see Feinberg (1967, 1970) and Bilaniuk et al. (1962); Kreisler (1973) provides a survey of the search for superluminal travel; some related philosophical discussions appear in Fox (1970), who also discusses superluminal velocities of sound waves in ultradense matter, Earman (1972) and Redhead (1993), among others.

explain them. These were problems other shad tried the hand at unsuccessfully using more mechanistic means. I have tried to show that Mach's approach in these practical investigations involved just what Einstein said he saw was needed to resolve the paradox he came upon at the age of sixteen: a universal formal principle. And, that Einstein followed Mach in looking to experience for such a principle. Einstein's regard for Mach as 'un mécanicien' who affected him philosophically begins to make more sense when we take Mach's scientific writings into account; I have also suggested how this scientific work could explain both Mach's (undisputed) appreciation of Einstein's theory of relativity and Mach's (disputed) charge of dogmatism in his contemporaries' understanding of it.

Many scientists of the nineteenth century thought, as Mach did, that light looked something like sound, too, but with the consequence that they looked for mechanical causes for electromagnetic phenomena. Mach was novel in that he looked instead for the most formal, least mechanistic characterization of waves that would explain the observed phenomena. For the Doppler effect, his explanation was purely a matter of kinematics; for shock waves, kinematics supplemented with thermodynamical principles (and in a way that recognizes the principle of the constancy of the velocity of signal propagation as governing, rather than violate din, supersonic flight).

I think Einstein noticed the formal move to kinematics Mach made, understood its significance, and never, ever, forgot who made it.[34] That is the personal story I have just told here. The larger story reflected in it is the story of wave mechanics outgrowing its mechanistic upbringing.

[34] In saying that Einstein never forgot who made the move, I am referring to quotes and anecdotes mentioned earlier in this paper: Pais' (1982, p. 283) account of Einstein's remarks made near the end of his life, Einstein's (1916) remarks in his obituary of Mach, and, especially, the remarks in his autobiography (1992, p. 19) in which he singles out Mach as *the* only one of that century who did not regard classical mechanics as a firm and definitive foundation for all physics, and as *the* person who 'upset this dogmatic faith'.

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