

On The Role Of Simplicity In Science

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Abstract

Simple assumptions represent a decisive reason to prefer one theory to another in everyday scientific praxis. But this praxis has little philosophical justification, since there exist many notions of simplicity, and those that can be defined precisely strongly depend on the language in which the theory is formulated. The language dependence is a natural feature—to some extent—but it is also believed to be a fatal problem, because, according to a common general argument, the simplicity of a theory is always trivial in a suitably chosen language. But, this *trivialization argument* is typically either applied to toy-models of scientific theories or applied with little regard for the empirical content of the theory. This paper shows that *the trivialization argument fails, when one considers realistic theories and requires their empirical content to be preserved*. In fact, the concepts that enable a very simple formulation, are not necessarily measurable, in general. Moreover, the inspection of a theory describing a *chaotic billiard* shows that precisely those concepts that naturally make the theory extremely simple are provably not measurable. This suggests that—whenever a theory possesses sufficiently complex consequences—the constraint of measurability prevents too simple formulations in any language. This explains why the scientists often regard their assessments of simplicity as largely unambiguous. In order to reveal a cultural bias in the scientists' assessment, one should explicitly identify different characterizations of simplicity of the assumptions that lead to different theory selections. General arguments are not sufficient.

1 Introduction

In order to appreciate the important role of the idea of simplicity, it is worth reviewing one of the most challenging open questions, concerning our understanding of science.

Most scientists believe that the main goal of their work, namely that of finding *better theories* than those representing the state of the art, is well defined and the criteria for success are reliable and do not depend on the particular culture dominating the scientific community to which they belong. Although the scientists are not immune to disputes, even bitter, the latter occur on rather minor issues, compared to the common grounds that unite the scientific community. In particular, it is certainly not true that *for any* two competing theories, all scientists agree on which one is better, but *there do exist many and significant* pairs of theories where all scientists agree that one is unambiguously better than the other. Moreover, many issues, that divided the scientists in the past, are now fully settled.

This high level of convergence begs for an explanation. A challenge for philosophy of science is to understand whether the standards that scientists perceive as reliable are actually well-grounded on unambiguous cognitive values—and, if so, identify such values—or, alternatively, identify the cultural bias in the scientists' assessments, and show how different—but in principle equally admissible—cultural

prejudices would lead to different assessments, *even in questions where the scientists unanimously agree*. In order to justify the first conclusion, one should identify *general* and *durable* criteria for comparing two scientific theories, which are based on *unambiguous cognitive values*. Moreover, the criteria should be *usable in practice* to select among real scientific theories.

Incommensurability (Kuhn, 1996; Bird, 2004) is sometimes believed to be a stumbling block undermining any *general* criterion for the comparison of scientific theories. The alternative is to acknowledge the necessity of irreducibly different criteria of theory appraisal, for different scientific domains. This is a favored view among many philosophers, which is also not strongly opposed by scientists, who have limited authority to judge beyond their own disciplines (and might even be seduced by the shortsighted illusion that granting the full responsibility of the judgment to *experts* is good for them). But, it should be clear that the lack of a *general* criterion is ultimately equivalent to no reliable criterion at all, with the consequence that *anything goes* (Feyerabend, 1975). In fact, it is not uncommon that a dispute over the scientific value of a method, or of a theory, results in the foundation of a new discipline, with its own alleged scientific standards and experts. If we deny any general standard in science, we have to accept such practices as perfectly justified ways of doing science.

A general criterion for the comparison of different scientific theories—which has also an obvious cognitive value—is empirical adequacy¹, but it cannot be the only one. In fact, empirical adequacy can be easily improved by introducing ad-hoc assumptions and building more and more complex theories that adapt themselves to the data, without producing any cognitive advantage. It has been argued (Sokal and Bricmont, 2001) that there is often just one theory—at best—that is compatible with the data and it is not *crazy* (such as theories that might be motivated by solipsism, radical skepticism and other implausible speculations). This suggests that empirical adequacy should be sufficient for theory appraisal, provided that one excludes crazy theories. But unfortunately, there is no sharp distinction between crazy and non-crazy theories. How many ad-hoc assumptions are we willing to accept before declaring a theory crazy? For example, a full class of gravitational theories within the parametrized post Newtonian (ppN) formalism (Will, 2006) are in agreement with the experimental data as precisely as general relativity (GR). But GR is still unanimously regarded as unambiguously better than most of those theories². These are not crazy theories at all, but we should nevertheless be able to tell precisely why GR is a better theory than the other empirically equivalent ppN ones, otherwise we might have no strong argument against publishing also, say, post Ptolemaic terms in scientific journals... It is therefore necessary to define some other epistemologically relevant measure, besides agreement with the data. But, which one?

The ability of a theory to *predict* nontrivial, yet unobserved, phenomena is rightly considered a strong evidence of success (see Douglas, 2009, which contains a recent review). Predictions are certainly invaluable tools of theory selection, in everyday practice of science. But, defining precisely what *predictions* are, turns out to be subtler than one might expect. For instance, it is not too hard to hit a prediction by producing many possible extensions of an already successful theory. Are such shots in the dark also 'predictions'? Predictions are valuable only if their alternatives cannot be equally well justified, which, essentially, leads again to the necessity of characterizing ad-hoc assumptions, in the first place.

Scientific theories are often evaluated for the opportunities of *technological applications* that they promise to open. But, either these advantages can be reformulated simply in terms of better empirical adequacy, or, if not, it is interesting to know *why* some theories seem to offer more opportunities than others *in spite of being empirically equivalent*. Hence, applications do not answer our question (they are rather one of the motivations for our question).

One of the most popular tools for theory selection is *falsifiability* (Popper, 1959). But, because of the Quine-Duhem thesis (Quine, 1950), almost no theory can be falsified, as long as any ad-hoc assumption may be freely added to it. Therefore, discriminating between the introduction of ad-hoc assumptions and

¹In this paper, the precise definition of *empirical adequacy* does not play any important role. Only the concept of *empirical equivalence* matters, and it is defined later.

²This does *not* refer to those ppN theories that are in *better* agreement with some experimental data than GR, like those used to model Dark Matter. These do represent interesting alternatives, and are the reason why the ppN formalism is studied.

truly new theories is necessary also to ensure the effectiveness of the criterion of falsifiability.

The idea of *reduction* of a theory to a more fundamental one (Nagel, 1961)—even if only partially (Kemeny and Oppenheim, 1956) or in some limit (Nowakowa and Nowak, 2000)—together with the related idea of *unification*, singles out essential aspects of true scientific progress. However, from a logical point of view, nothing prevents the reducing (or unifying) theory from being an artificial superposition of old theories, made of many and complex assumptions. Reductions and unifications represent true progress only if, at the end of the process, some old assumptions can be dropped.

All this strongly suggests that defining some measure of the amount and/or complexity of the *assumptions* does not only represent a cognitive value in itself, but also a prerequisite for a precise characterization of many other classic goals of science as well. The idea is not new. Many philosophers and scientists (e.g., Mach, 1882; Poincaré, 1902, to mention only two of the most influential and modern authors) have stressed the importance of *simplicity*, *economy of thought* and related concepts³. But, a precise and general definition is problematic (see e.g., Sober, 2002). The main obstacle lies in the fact that any conceivable characterization of simplicity inevitably depends either on the language in which the theory is formulated, or on some other choice which is equally hard to justify.

A few prominent examples can better clarify this point. A theory is usually defined as more *parsimonious* (Baker, 2004)⁴ if it postulates less entities. But there is no natural and general way to count the number of entities, and any prescription in this sense inevitably introduces an arbitrary subdivision of the world into elementary kinds, without convincing justification. Alternatively, parsimony can be made precise by identifying the ontological commitment of a theory with the domain of its logical quantifiers (Baker, 2004). But this property is not invariant under reformulation of the theory (Quine, 1951). Another famous definition of simplicity counts the *number of free parameters* that appear in the formulation of the theory (Popper, 1959). This is well defined within a fixed class of theories with a fixed common parameterization, but it becomes arbitrary beyond that. A further well known example is the proposal of Goodman (1977), that stimulated much interest and further developments, especially in the 50s and the 60s. In this case, the complexity of the theory depends on the choice of the set of *primitive predicates*, which is effectively analogous to the choice of the language (Schwartz, 2011). Finally, the concept of simplicity derived from *Kolmogorov complexity* (KC) (Solomonoff, 1964; Kolmogorov, 1965; Chaitin, 1969) has been used by many authors, in recent years, to determine the so-called universal prior probabilities in a Bayesian context (see Li and Vitanyi, 1997; Grünwald and Vitanyi, 2008 for reviews). It is well known that KC is defined only up to a constant, that depends on the language. KC is well suited to study asymptotic properties of theories describing an increasing amount of empirical data, while keeping the language fixed. But, KC cannot be used to compare the simplicity of different theories (each expressed in its own preferred language) with fixed empirical data. In fact, for any scientific theory, it is always possible to find a suitable language in which the theory assumes a trivially simple form (Kelly, 2009).

It should be stressed that the language dependence that characterizes any precise definition of simplicity is not a problem in itself: an awkward language should obviously produce a complex formulation of the theory. But, if *any* theory can be made trivially simple by a suitable choice of the language, then the concept of simplicity loses any interest. The idea of simplicity is only meaningful if the *simplest* formulation of realistic theories is *not trivial*. Unfortunately, a common, general argument (hereafter called *trivialization* argument) shows that all previous examples suffer this problem, unless the admissible languages are somehow limited. But, how should we justify such limitations?

It is sometimes argued (see, e.g., Psillos, 1999, chap. 11) that the special language that can reduce a theory to a trivial form is artificial and not based on *natural kinds*. This shifts the problem to the one of

³The previous discussion makes clear that what matters, in order to assess the cognitive value of a theory, is always the complexity of its *assumptions*. By contrast, the complexity of its *consequences* and *results* may very well be high, which is desirable in a theory that aims at describing the world and its manifest complexity.

⁴The review of Baker (2004) distinguishes *syntactic* from *ontological* definitions of simplicity. However, any general definition of simplicity, once it is made precise, it becomes syntactic, in some sense. This is the case also for parsimony. In this paper, simplicity is always to be understood as syntactic simplicity

characterizing what natural kinds are, which has no convincing solution either (Bird and Tobin, 2008). But there is also a deeper reason to be skeptical about this approach: one of the main tasks of science is precisely to discover new kinds (and new languages), which may look weird now, but eventually enable a deeper understanding of the laws of nature. The revision of the concept of *time* introduced by Einstein and the formulation of particle physics in terms of *quarks* are obvious examples.

In this paper it is stressed that *measurability*, rather than *naturalness* is the key. In fact, scientific theories typically contain concepts that are *in principle not measurable*. Such unmeasurable concepts should obviously not be used to ground the empirical content of a scientific theory. Unmeasurable concepts can certainly be used to formulate the principles of a theory, but then, in order to compute the complexity of the theory, also the cost of defining the measurable concepts from those used in the principles should be taken into account.

This idea can be applied to any of the characterizations of simplicity mentioned above. It should be stressed that this paper does *not* propose a new notion of complexity, but rather shows how the proper consideration of the empirical content of a scientific theory prevents a trivialization of essentially any notion of simplicity. The obstacles preventing trivialization are illustrated in detail with reference to the definition of simplicity given in Section 3.1 (*conciseness*). But the same ideas can be applied to essentially any acceptable characterization of the simplicity of the assumptions, as discussed in Section 3.5.

The requirement that the formulation of a theory should provide a connection to its measurable concepts may seem too weak and easy to fulfill. In fact, as shown in Section 3.2, this requirement does not rule out such theories as “*all emeralds are grue*” (Goodman, 1955), and it also does not offer a solution to the curve fitting problem (see e.g., Sober, 2002). But, such *toy-models* of scientific theories are only significant if they capture the relevant features of *realistic* theories. The arguments in Sections 3.3 and 3.4 show that those models are indeed inadequate. It is only when the theory becomes sufficiently rich of consequences that qualitatively new features appear: the connection with measurable concepts becomes difficult to achieve for those languages that are designed to make most *realistic* scientific theories trivially concise. In particular, it can be proved that the simple (but not too simple) theory analyzed in Section 3.3 contains unmeasurable concepts. Moreover, such concepts appear naturally, when one tries to reformulate the theory in a very concise form. This provides evidence that the general trivialization argument reviewed in Section 3.2 is not conclusive, and it also suggests that the obstacles to trivialization are unlikely to be evaded.

Lacking evidence to the contrary, the fact that some theories can be formulated more concisely than others cannot be regarded as purely conventional. Achieving a concise formulation of a realistic scientific theory is far from easy and highly valuable.

The discussion above makes clear that the notions of simplicity which are significant for science cannot be properties of the logical or syntactic structure of the theory alone. Instead, they must depend also on the connection between the theory and the experience. For this reason, before examining any concept of simplicity, it is necessary to define precisely what the *empirical content* of a theory is, and what its *empirical (i.e. measurable) concepts* are. The traditional approach to these issues is represented by the syntactic *received* view of scientific theories, originally formulated by the logical empiricists (Carnap, 1966, 1958). The main problem with that view is its reliance on a theory-independent observational language, in order to verify the empirical adequacy of a theory and compare different theories among each others. But no such language exists, as it has been convincingly shown by a vast literature (e.g., Kuhn, 1996; Quine, 1950; Putnam, 1962; Suppe, 1972; van Fraassen, 1980). Perception itself is theory-laden (Quine, 1993) and a self-sufficient phenomenal language is an illusion. The causal theory of reference for physical magnitude terms (Putnam, 1975a) is often regarded as a way to achieve stability of reference—and hence enable the comparison of theories with the experience and among each others—in spite of the theory ladenness of observations. In this paper, the causal theory of reference is not regarded as a tool to *ensure* the stability of the reference, but rather as a framework to examine *under which assumptions* the

reference is sufficiently reliable for the present purposes. These observations lead to the identification of those syntactic elements that are necessary to describe the interplay between the empirical content of a theory and its simplicity, without running into the pitfalls of the received view and while being consistent with the now widely accepted semantic view (van Fraassen, 2008) of theories. The main message of this paper is that *a clear identification of the empirical content of a theory lies at the heart of the problem of simplicity*.

The paper is organized as follows. Section 2 introduces those elements of scientific theories which are needed to provide a relevant characterization of simplicity. These are further analyzed in Appendix A, in order to show their consistency. Section 3 introduces and examines the notion of conciseness. In particular, Sections 3.3 and 3.4 show that most realistic theories cannot be made arbitrarily concise by any known procedure. Section 3.5 extends the previous result to other definitions of simplicity. Finally, Section 4 examines the possibility that different definitions of simplicity may converge to produce a consistent characterization of the goals of science.

2 Scientific Theories And Empirical Concepts

As stressed in the Introduction, in order to provide a characterization of simplicity which is significant for science, we need to identify precisely a few elements that are part of any scientific theory. In particular, we need to specify the role of the *principles* and that of the *empirical concepts* of a theory. Similar concepts occupied the central stage in the traditional syntactic view of scientific theories (Carnap, 1966, 1958; Feigl, 1970), but the latter included unacceptable assumptions that have been the object of detailed criticisms in the past 50 years, that are briefly reviewed later. On the other hand, modern semantic views (Suppes, 1967; van Fraassen, 1980), concentrate on other aspects of scientific theories (e.g., models), which are not directly usable for our purposes. However, Putnam (1975a) has shown that the empirical concepts (*physical magnitude terms*) of a scientific theory can be characterized without running into the inconsistencies of the traditional view. In this section, we introduce those elements in a way that mimics the received view, where the latter is unproblematic, but also introduces the crucial corrections dictated by the causal theory of reference for physical magnitude terms (Putnam, 1975a). Many comments are postponed to Appendix A. In particular, it is shown in Section A.3 that this approach is not inconsistent with a semantic view.

To our purposes, a **scientific theory**, may be viewed as the union of the following elements: a set of abstract *principles*, a set of *results*, a set of *empirical concepts* and the *language* that is used to express all the previous elements. The **principles** are abstract, in the sense that they make use of concepts which are only defined implicitly through the principles themselves. They merely describe a network of symbols (Feigl, 1970), and can be seen as a set of mathematical axioms⁵. Each theory is regarded as a multidisciplinary collection of principles that include *all* assumptions (from the logical rules of deduction to the modeling of the experimental devices and of the process of human perception) which are needed to derive the results of the theory and compare them with the experiments, including a complete estimate of the uncertainties. All such principles have the same epistemological status: even logic rules are to be considered working assumptions and there may be theories that adopt different ones.

The **results** comprise all theorems, formulae, rules, solutions of equations, models etc. that have been derived from the principles of the theory. The set of results is introduced as a distinct element of the theory, because its derivation from the principles is not automatic, but requires original intuitions. Moreover, when a new theorem is proved, the theory may acquire new empirical consequences and become richer.

The principles and the results are necessarily formulated in some **language**⁶. Its terms may be

⁵In this paper, the words *principles*, *postulates*, *laws*, *axioms*, *assumptions* and *hypotheses* are regarded as equivalent. No restriction to first order logic is assumed.

⁶Because some languages may complicate the comparison with the experiments, as shown in Section 3.3, it is convenient,

conventionally divided (Feigl, 1970), into **derived concepts**, if they have an *explicit* definition in terms of other concepts of the theory, or **primitive concepts**, if they are only implicitly defined through the principles.

The **empirical (or measurable) concepts** (ECs) have a double characterization: they are concepts of the theory (either primitive or derived), and they are also endowed with a set of *prototypes* (Rosch, 1978). The **prototypes** of a concept are the subjective examples that a person bears in mind as typical instances of that concept. When we need to decide whether a particular phenomenon is an occurrence of a concept or not, we can compare what we observe to our personal set of prototypes and decide by analogy. In other words, a prototype for an EC is a typical member of the *extension* of that EC. Obviously, this does not yet explain how such prototypes could provide a solid base to science. This is where the causal theory of reference (Putnam, 1975a) plays a role, but the discussion is postponed to Appendix A.

The ECs are further distinguished into **basic empirical concepts** (BECs), that are empirically characterized (interpreted) *only* through a set of prototypes, and **operationally defined empirical concepts** (ODEC), for whom the theory allows the deduction of a precise operational definition in terms of the BECs⁷. All concepts for which we have neither prototypes nor rules to build them are **not empirical** (NEC). The fact that we do not have prototypes or rules associated to a certain concept does not mean, in general, that it is impossible to build one. In fact, some NECs may turn out to be ECs after the discovery of some new experimental technique. There are, however, also NECs that could not possibly become ECs. This crucial observation is discussed in Section 2.2.

Note, that there is no relation, in general, between primitive concepts and BECs. The former are related to the logical structure of the theory, while the latter to the availability of prototypes. In other words, there is no obstacle to the existence of primitive-NECs or derived-BECs, as shown in the example in Section 2.1.

The division into ECs and NECs evokes the traditional distinction between observational and theoretical terms (Carnap, 1958). However—contrary to Carnap’s observational terms—the ECs are theory *dependent*. In the received view, the observational terms were supposed to represent theory independent sense-data and provided the basis for radical reductionism and verification theory and also the basis for the comparison of different theories. This reconstruction cannot be defended anymore after Quine (1950)⁸: no universal concept can be assumed to be translatable into a purely sense-data language and hence must be assumed to have a meaning only within some theory. For this reason, the ECs are here introduced as an additional label for some theoretical concepts⁹. It is of course not obvious how the BECs can enable the comparison of the empirical statements of different theories. This is discussed in Appendix A.2.

A different objection (Putnam, 1962; Suppe, 1972) against the observational-theoretical division deserves special attention, because it is independent of the theory-ladenness of the observational terms. Putnam (1962) has observed that there are no terms in the English dictionary that may be regarded as univocally either observational or theoretical. For example, the property of being *red* can be empirically verified in ordinary objects of macroscopic size, but its observability is questionable, or certainly impossible, for sufficiently small objects. Suppe (1972) has further recognized that the observational-theoretical division could be more complex than a simple bipartition of dictionary *terms*, and could involve the context in which the terms are used. But he has also argued that such division, if it exists, would be extremely complex, in a way that it is hopeless to characterize. These observations are correct: the ECs

in general, to regard different formulations as different theories. Nevertheless, we may, for brevity, still refer to two different formulations of the same theory, if one is simply the translation of the other in a different language.

⁷We are not interested in defining the concept of *directly* measurable: if—under the assumptions of the theory—measuring *A* implies a definite value of *B*, both *A* and *B* are ECs. It is up to the theory to decide which one, if any, is also a BEC.

⁸Note that Quine (1993) himself defends the usefulness of observation sentences, once their theory-ladenness is made clear.

⁹Note that the prototypes themselves, like any experiment, do not depend on any theory: they are historical events. But this does not allow to produce theory independent BECs, because both the selection and the description of those prototypes that should be relevant to characterize a BEC can only be theory dependent.

are not simple dictionary terms. They include the full specification of the experimental conditions that the theory considers relevant (and this reinforces their theoretical dependence). Moreover, understanding which setup may allow which measurement is the hard and ingenious work of experimental scientists. Drawing the complete distinction between the ECs and the NECs would require the classification of all realizable experimental arrangements where any quantity could be measured. This is clearly not feasible. Moreover, the boundary between ECs and NECs is populated by concepts associated to quantities that can be measured only with such poor precision that it is questionable whether they are ECs at all. However, from a philosophical point of view, a precise and comprehensive compilation of all the ECs is unnecessary: it is sufficient to recognize that for each scientific theory at least some ECs exist and they can all be constructed on the basis of both the theory and a small set of BECs. Only the full list of BECs must be made explicit, as discussed in Section 3. Also the BECs may not be just dictionary terms: they are rather selected because of their assumed unambiguity. For example, most modern scientific theories tend to reduce all the BECs to the reading of the digital displays of some experimental devices, for which suitable models are assumed. The classes introduced in this section are summarized in the table below.

Concepts of the theory		
ECs		NECs
BECs	ODECs	

2.1 An Example

Consider, for example, a theory that, besides standard mathematical and logical axioms, also assumes the Gay-Lussac’s law of gases at fixed volume: $P = cT$, where P represents the pressure, T the temperature and c is a constant. Here, P , T and c are primitive concepts. Let us also assume a suitable model for the thermometer and the barometer, which can be used, however, only in limited ranges. As a result P and T are ECs within those ranges and NECs outside them. These allow the definition of other ECs such as $c = P/T$, which is hence a ODEC. A typical prototype for the EC of T at a reference temperature $T_{\text{ref}} \pm \Delta T$ consists in a sample of real gas equipped with a thermometer that displays the value T_{ref} with a precision of at least ΔT . The ECs corresponding to measurements of different temperatures can be characterized by similar prototypes, but they can also be *operationally* defined using the theory (in particular a model for the thermometer) and a single BEC at the reference temperature $T = T_{\text{ref}} \pm \Delta T$. The choice of the temperature T_{ref} which is selected as BEC is arbitrary. But it is important that the *necessary* prototypes can be reduced to those at a single temperature $T = T_{\text{ref}} \pm \Delta T$, while all other (measurable) T correspond to ODECs.

2.2 A Crucial Property Of The ECs

With no loss of generality, it can be always assumed that the ECs represent properties whose value is either yes or no. In fact, any measurement of a real-valued quantity is equivalent to assess whether its value lies or not within some intervals $[x \pm \Delta x]$, for some x and Δx . (Given the limited precision of all measurements, this is also closer to the experimental praxis.) In this case *a valid prototype should be associated to a single connected interval*. This requirement is necessary to comply with the intuitive idea of prototype: a single prototype must correspond to a single outcome of a measurement—as inaccurate as it might be—and not to many precise outcomes at the same time. If this is not the case for one prototype (e.g. because the outcome was poorly recorded), a clearer prototype should be provided. If this is also not possible, one can only conclude that the corresponding concept is not empirical.

In the example of the previous section, a prototype was represented by an experimental setup where the temperature of a given sample of gas was measured. Typically, the thermometer would let us read a number somewhere between 30.1 °C and 30.2 °C. We can accept some uncertainty, which is, in this case ~ 0.1 °C. Now, imagine that we find a report of the previous day stating that the temperature was measured once and the result was “either 29.31 ± 0.01 °C or 32.05 ± 0.01 °C”. We would conclude that

there was a mistake in taking or recording that measurement and we would repeat it. Experimental results cannot be in macroscopic quantum mechanical superposition states!

This remark plays a central role in this work. Section 3.3 shows that the requirement stated here—which is indispensable¹⁰, in order that the ECs have any chance of actually being empirical—cannot be fulfilled by those very concepts that would naturally make a theory trivially concise.

2.3 Empirically Equivalent Theories

Consistently with the motivations given in the Introduction, we are only interested in considering the relative simplicity of *empirically equivalent* theories. Empirical equivalence is defined here.

Each scientific theory is motivated by some questions. A **question** for the theory T consists in the specification of the values of some concepts of the theory (e.g., the initial conditions or other choices within the alternatives offered by the principles) and a list of concepts that the theory is expected to determine. For example, in astronomy a valid question is: determine the motions of the planets in the sky, knowing the positions and velocities at some initial time. It is convenient to distinguish two kinds of questions: **empirical questions**, that contain *only* ECs, and **technical questions**, that also contain non-empirical concepts of the theory. Examples of the latter are questions concerning what cannot be measured in principle, such as the quantum mechanical wave function, or in practice, because of technical limitations that may be overcome eventually.

Two theories T and T' are said **empirically comparable**, relatively to the sets of ECs \mathcal{E} of T and \mathcal{E}' of T' , if there is a one-to-one correspondence \mathcal{I} between \mathcal{E} and \mathcal{E}' and—under this correspondence—the experimental outcomes are interpreted in the same way by the two theories, i.e. those concepts that are identified via \mathcal{I} possess the same prototypes. Note that, if T and T' are comparable for some ECs, then all the empirical questions—limited to those ECs—of one theory are also empirical questions for the other. Finally, two theories T and T' are said **empirically equivalent**, relatively to \mathcal{E} and \mathcal{E}' , if they are comparable and all their results concerning the ECs in \mathcal{E} and \mathcal{E}' are equal (within errors) under the correspondence \mathcal{I} .

3 Simple But Not Trivial

This central section shows that there is no reason to expect that realistic theories can be expressed in an arbitrarily simple form by a suitable choice of the language, while also preserving their empirical content.

First, for the sake of definiteness, a particular definition of simplicity (*conciseness*) is introduced in Section 3.1. The *trivialization* argument, according to which a trivial formulation of any theory *always* exists, is reviewed in Section 3.2. But, a gap in the argument is also pointed out, inasmuch the measurability of the concepts used in the trivial formulation is not granted. This is not a remote possibility: in Section 3.3 an elementary theory, that involves chaotic phenomena, is analyzed in detail. It is actually easy to identify a very concise formulation for it, but precisely those concepts that naturally enable such trivial formulation can be *proved* to be non measurable. This simple (but not too simple) theory underlines a serious difficulty in closing the gap of the trivialization argument.

In Section 3.4 it is stressed that the obstacles identified in Section 3.3 are not due to some very peculiar features of that theory, but they are rather general. In fact, they are expected to emerge whenever a theory possesses sufficiently complex consequences. In view of this, it seems very unlikely that the gap in the trivialization argument might be closed, for any relevant set of realistic scientific theories.

Finally, Section 3.5 considers other possible characterizations of the simplicity of the assumptions, besides conciseness. It is shown that any acceptable (as defined below) characterization of the complexity of the assumptions poses the same obstacles to its trivialization, as conciseness does.

¹⁰Note that this is certainly not a *sufficient* condition in order that a concept is an EC.

The fact that different characterizations of simplicity are nontrivial does not imply that they are equivalent, when used for theory selection. This interesting issue is addressed in Section 4.

3.1 Definition Of Conciseness

Let $\sigma(T^{(L)})$ denote the **string encoding all the principles** of a theory $T^{(L)}$, where it is emphasized that the theory is formulated in the language L . As already stressed, it is crucial that the string $\sigma(T^{(L)})$ include also the definitions of all the BECs in terms of the primitive concepts of the theory. In this way, anybody able to recognize the BECs of $T^{(L)}$ would find in $\sigma(T^{(L)})$ all the ingredients that are needed to check¹¹ which results are correctly deduced from $T^{(L)}$, which questions they answer, and compare them with the experiments. The **complexity** $\mathcal{C}(T^{(L)})$ is defined as the length of $\sigma(T^{(L)})$ ^{12 13}. The length of the string is measured in the alphabet associated to the language L . Note that one cannot tell, in general, whether a given $\sigma(T^{(L)})$ represents the shortest possible formulation of the principles of T in the language L . The string $\sigma(T^{(L)})$ is simply the shortest *known* formulation¹⁴ in the language L . The discovery of a shorter encoding represents the discovery of a new result of the theory, enabling a higher conciseness. Finally, the **conciseness** of $T^{(L)}$ is defined as the inverse of the complexity $\mathcal{C}(T^{(L)})$.

3.2 Arguments For The Triviality Of Conciseness

The philosophical literature contains many examples of theories that can be expressed in a very simple form by a suitable choice of the language. The classic example is the theory asserting: *all emeralds are green if they were first observed before January 1st 2020 and blue if first observed after that date* (Goodman, 1955). This statement can be shortened to *all emeralds are grue*, by a suitable definition of *grue*. Another example is provided by the curve fitting problem (Sober, 2002). Higher degree polynomials may appear more complex than lower degree ones, but the complexity disappears under a suitable change of variables.

The concept of conciseness does *not* help in deciding which formulation is simpler in these cases. In fact, both concepts of green and grue are perfectly measurable and hence acceptable as BECs. Similarly, high degree polynomials may look unappealing, but they can be defined and computed precisely in terms of the original (measurable) variables. The problem with these toy-models is that they miss some essential features of realistic scientific theories, insofar as they have very few consequences. As soon as the theory becomes sufficiently rich of consequences, qualitatively new obstacles appear, and the path toward a concise *and* measurable formulation is lost, as shown in the example of the next section.

There is also a common *general* argument holding that the formulation of *any* theory can be made arbitrarily simple. In the case of conciseness, such **trivialization argument** goes as follows¹⁵. Imagine that, in the language L , the long string $\sigma(T^{(L)})$ cannot be compressed further with any known method.

¹¹In order to *derive* the results of $T^{(L)}$, the string $\sigma(T^{(L)})$ is not sufficient, without further original ideas. However, $\sigma(T^{(L)})$ is sufficient to *check* the validity of any given derivation.

¹²It is interesting to compare this definition with Kolmogorov complexity. The Kolmogorov complexity of a string x is defined as the length of the shortest program written in a *fixed* Turing-complete language, that outputs x . We could have defined also our complexity as the length of the shortest program that outputs the string $\sigma(T^{(L)})$. However, in the present context, the language depends on the theory. It is therefore equivalent and simpler to define the complexity directly as the length of $\sigma(T^{(L)})$, because if we find a shorter program, we can choose that program as $\sigma(T^{(L)})$. Note that $\sigma(T^{(L)})$ is *not* expected to produce theorems or formulae automatically (see footnote 11). Finally, Kolmogorov theory does not distinguish ECs from NECs, although it would not be difficult to introduce an equivalent distinction between realizable and unrealizable Turing machines.

¹³Note that $\sigma(T^{(L)})$ includes all the principles, but not the questions, which are potentially unlimited. However, a theory T cannot cheat by hiding the principles inside the questions, because the empirical questions translated from another theory T' through the correspondence \mathcal{I} (see Section 2.3) would miss this information and would have no answer in T .

¹⁴This is analogous to the fact that the Kolmogorov complexity function is not computable in general (Li and Vitanyi, 1997), and most applications of Kolmogorov theory refer to the available compression methods.

¹⁵In the context of Kolmogorov complexity, the corresponding argument has been presented in Kelly (2009); Delahaye and Zenil (2008).

Then one can always define a new language L' , which is identical to L except that it represents the long string $\sigma(T^{(L)})$ with the single character Σ . Obviously, it is impossible to deduce any nontrivial result from a theory whose principles are just ' Σ '. However, this might not be necessary, if all the results of T could still be implicit in the *interpretation* of Σ . In general, one should expect the concept Σ to be difficult to interpret in terms of the empirical data. But the fact that Σ may be *difficult* to measure is not sufficient to exclude the formulation of the theory in the language L' : difficult measurements can be learned and are routinely conducted by experimental scientists.

The key point is that there exist concepts that are *provably not measurable* (examples are given in the next section). In order to be conclusive, the trivialization argument should demonstrate that Σ can always be chosen among the measurable concepts of the theory. This task has never been undertaken in the literature¹⁶. The proof that Σ can be chosen—in general—to be measurable is not only missing, it also looks quite unrealistic. In fact, the following section illustrates an example of a theory where the natural choices of Σ can be *proved* to be unmeasurable. Alternative choices of Σ cannot be excluded. But, on the basis of this example, assuming the general existence of measurable Σ is definitely not plausible.

Even if the primitive concepts of the theory are not measurable, it is still possible to define other measurable concepts and select them as BECs. In fact, any sentence in the new language L' can still be translated into the original language L and vice versa. However, the definition of conciseness requires to take into account also the length of the string that defines all the BECs in terms of the primitive concepts of the theory. In the following example, also this approach is considered, but it happens to lead to lengthier expressions.

3.3 A Not-too-simple Theory

The goal of this section is to show that there exist concepts that are *provably not measurable*, and that such concepts appear naturally when trying to reduce a theory to a trivial form. This demonstrates a serious gap in the trivialization argument, which does not ensure that unmeasurable concepts can be avoided.

To this end, we consider the theory (called \mathcal{B}) which is defined by the laws of classical mechanics applied to a single small (approximately point-like) ball on a billiard table with a mushroom shape (see, e.g., Porter and Lansel, 2006 and Figure 1). This is defined by a curved boundary on the top side (the cap) joint to a rectangular boundary with sharp corners on the bottom side (the stem). Such billiards possess chaotic behaviors, when the initial conditions are chosen within certain values, which are assumed in the following. The nice feature of such billiards is that the trajectory of the ball can be computed exactly at any time—in spite of its chaotic nature. This enables precise statements about the (non-)measurability of the quantities relevant to this discussion.

The theory \mathcal{B} can be naturally expressed in the language L that makes use of the coordinates z , where $z := (\vec{q}, \vec{p})$ denotes together the position $\vec{q} := (q_x, q_y)$ and momentum $\vec{p} := (p_x, p_y)$ of the ball. The only BEC that needs to be assumed corresponds to assessing, within some fixed precision, whether the ball at time t_0 lies at a reference point \vec{q}_{ref} in the table, and whether it has a reference momentum \vec{p}_{ref} . Any other measurements of position or momentum (at any time) can be operationally defined from this single BEC and the principles of the theory. In fact, the measurement procedures are exactly the same at any time, since the theory is manifestly time invariant, when expressed in the coordinates z (this does not hold in the coordinates ξ , introduced below).

Measurements of position and time have necessarily limited precision, which is assumed, for definiteness, at the level of a millimeter and a tenth of a second, respectively. It is assumed also, for simplicity, that the walls are perfectly elastic, that the ball does not spin and the friction is negligible for a time sufficient for the ball to perform a large number of bounces.

Assuming the standard Hamiltonian formalism, the dynamics of this system is completely defined

¹⁶Remarkably, simplicity and measurability—both classic topics in philosophy of science—have been rarely combined.

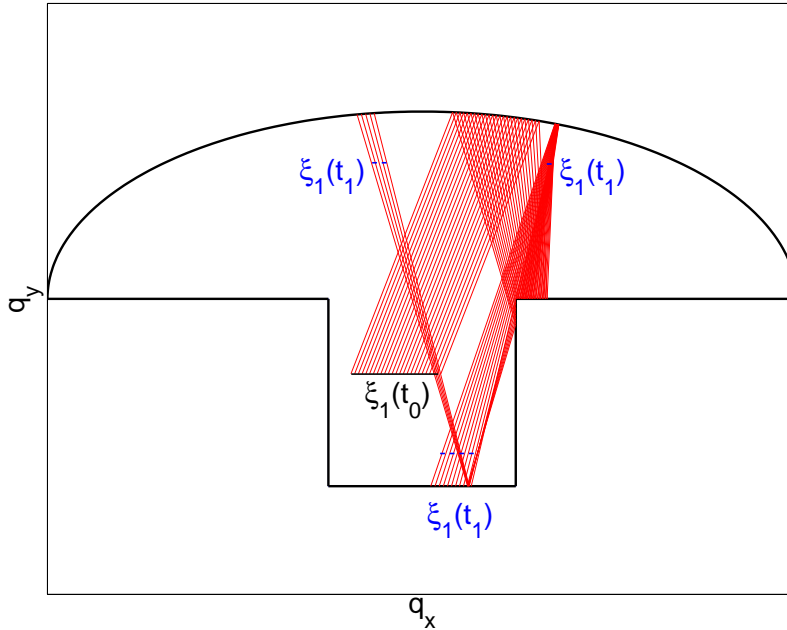


Figure 1: The black/solid interval $\xi_1(t_0)$ represents a range of initial conditions in the coordinates ξ_1 at time t_0 . This is also an interval in the coordinate q_x . After a few bounces, the interval $\xi_1(t_0)$ is transformed into at least three disjoint sets (contained in the three blue/dashed lines labeled by $\xi_1(t_1)$). The figure has been produced with the help of the program made available by Porter and Lansel (2004).

by the function: $H(z) = H(\vec{q}, \vec{p}) = \frac{\vec{p}^2}{2m} + V(\vec{q})$, where m is the mass of the ball, and $V(\vec{q}) = 0$ for all \vec{q} inside the billiard and $V(\vec{q}) = \infty$ outside. These formulae contribute to the length of $\sigma(\mathcal{B}^{(L)})$ with about 35 characters, to which one should add a few more characters to describe the boundary conditions ($B_M(z) = 0$) associated to the mushroom shape of the billiard. Since the BEC of this theory (z_{ref}) already appears among the primitive concepts, no further definition is needed. Finally, the contribution to $\sigma(\mathcal{B}^{(L)})$ due to all the standard psychological, physical, mathematical and logical assumptions, is ignored, since it remains unaltered throughout this discussion.

Following the idea of the trivialization argument, there is a special language (L'), that makes the principles of \mathcal{B} very concise. The trivialization argument does not explain how to build such a language, nor how to connect it to measurable quantities. However, it is not difficult to find a suitable language for the theory \mathcal{B} . In fact, a natural choice for L' is defined by those coordinates $\xi = (\xi_1, \xi_2, \xi_3, \xi_4)$, in which Newton's laws take the exceedingly concise form " $\xi = \text{constant}$ ". Such choice of coordinates can be defined (with respect to the language L) by setting $\xi = \xi(t_0) = z(t_0)$ at a reference time t_0 and then assigning the same value of ξ to all future and past configurations that belong to the same trajectory $z(t)$.

There are now two possibilities. Imagine, first, that we want to keep the original BEC $z_{\text{ref}} = (\vec{q}_{\text{ref}}, \vec{p}_{\text{ref}})$. In this case, the single BEC z_{ref} measured at t_0 does not suffice, because the principles of the theory do not provide the relation between the coordinates ξ and the coordinates z at any time different from t_0 . Hence, we do not know how to perform measurements at times different from t_0 . The BECs (z) at time $t \neq t_0$ can be related to the primitive concepts ξ by using the Hamiltonian $H(z)$, the boundary conditions $B_M(z)$, and computing the evolution of the trajectories from t_0 to t . These are computable but very cumbersome expressions, that becomes more and more complex after each bounce. Since we do not want to include $H(z)$ and $B_M(z)$ among the principles, such expressions are the only link we have between the principles and the BECs, and hence we have to include them in $\sigma(\mathcal{B}^{(L')})$, as

required by the definition of Section 3.1. This implies that $\sigma(\mathcal{B}^{(L')})$ grows indefinitely with the time separation from t_0 , while $\sigma(\mathcal{B}^{(L)})$ remains fixed.

The second possibility is to drop the coordinates z altogether, and use the ξ coordinates not only as primitive concepts in the formulation of the theory, but also directly as BECs. This leads to a theory that we denote $\bar{\mathcal{B}}^{(L')}$, which—apparently—could be much more concise than $\mathcal{B}^{(L)}$ and yet empirically equivalent to it. The problem is that the ξ coordinates, which have a clear interpretation at reference time t_0 , cannot be empirically detected at time t_1 , a few bounces after t_0 , with the same precision they were at t_0 . This is not just *practically difficult* but *intrinsically impossible*, because the system \mathcal{B} displays chaotic dynamics (Rabinovich and Rulkov, 2004), which is characterized by a high sensitivity to the starting conditions. This means that two initially nearby trajectories diverge very fast in time. To illustrate the consequence of this in a simple way, let us restrict the attention to the two coordinates q_x and ξ_1 of the ball. By construction, they coincide at t_0 (i.e., for any interval at t_0 , $[q_x \pm \Delta] = [\xi_1 \pm \Delta]$, where $\Delta = 1\text{mm}$), but at t_1 the trajectories that were close at t_0 have taken many different directions. Consequently, the interval $[q_x \pm \Delta]$ at t_1 corresponds to many disjoint and very small intervals¹⁷ in the coordinate ξ_1 . Conversely, any interval $[\xi_1 \pm \Delta]$ at t_1 corresponds to many disjoint and very small intervals in the coordinate q_x (see Figure 1). But, there is an important difference between the intervals $[q_x \pm \Delta]$ and $[\xi_1 \pm \Delta]$ at t_1 : prototypes for the former are possible, while for the latter are not, as a matter of *principle*, because we have no way to measure the many disjoint pieces that compose $[\xi_1 \pm \Delta]$. Of course, the measurable $[q_x \pm \Delta]$ intervals could be expressed in the ξ coordinates as the union of many extremely small disjoint intervals, but, as required in Section 2, these cannot be associated to valid prototypes, and hence the ξ cannot be ECs at t_1 . In conclusion, the obvious requirement that ECs are associated to connected intervals is sufficient to formally exclude—in agreement with the intuition—the ξ concepts as empirical.

In order to use the ξ coordinates to characterize the system at time t_1 , it would be necessary to introduce a new coordinates system: besides the ξ with reference at t_0 , one would need the $\xi^{(t_1)}$, with reference at t_1 , and the procedure should be repeated for a full sequence of times t_i . But, the measurements of $\xi^{(t_i)}$ cannot be operationally defined from those of $\xi^{(t_0)}$, since, as shown in the previous paragraph, the size of the overlaps of the respective intervals is much below the experimental sensitivity. Hence, new BECs—and corresponding new prototypes—are needed for each different time t_i . In order to keep the same empirical adequacy as the original theory, the new theory should define essentially as many BECs as experimental data, which would make again $\sigma(\bar{\mathcal{B}}^{(L')})$ extremely large.

3.4 Other Scientific Theories

In the previous section we have examined a particular theory, and showed that the tools at our disposal fail to make it more concise. Hence, the theory \mathcal{B} illustrates some obstacles that prevent closing the gap in the general trivialization argument of Section 3.2. In this section we further note that similar obstacles appear quite in general for realistic theories. This should convince the reader that a recovery of some version of the trivialization argument, covering a relevant set of scientific theories, is very unlikely.

One reason is that, as stressed in Section 2, scientific theories are multidisciplinary collections of principles gathered from different domains of science. Because of this, it is sufficient that the mechanism described in the previous section applies in one corner of the theory, to constrain the possible languages in all other sectors. Given that the vast majority of real physical systems admit chaotic phenomena, it is easy to appreciate the effectiveness of this constraint. Another reason, which is less compelling but more general, is the following. If the laws of a theory are expressed in a form that is so concise that no nontrivial result can be deduced, then all the consequences of the theory must be evident in the BECs of

¹⁷Because of the sharp (non-differentiable) corners in the boundaries of the mushroom shaped billiard, the Poincaré map—that associates the coordinates of the initial points to those of the evolved points—is not continuous. Hence, a single interval in the parameter set of the initial conditions is split, after each bounce, into disjoint intervals.

the theory. It follows that, either the theory has very limited consequences, or it needs to introduce a large number of BECs, or—finally—the interpretation of the BECs is very rich. But in this last case, it should not be too difficult to identify not only practical but also *fundamental* obstacles to the measurability of those BECs.

It is clear that this argument applies only to theories with sufficiently complex consequences. Even the idealized solar system, that played a glorious role in the history of science, is not rich enough—alone—to exhibit the idea above. In fact, it may not be impossible to reduce the Ptolemaic model to a very concise theory by using a small set of suitable BECs. After all, the orbital motion of a few idealized celestial bodies is an exactly integrable and periodic system. But, as soon as one considers, for example, Newton’s laws in more general contexts, the amount and the variety of phenomena that can be described becomes arbitrarily large, while the set of laws and BECs remains small. Also the curve fitting problem—which is often employed as a toy-model to discuss simplicity in the philosophical literature—is not rich enough to show any *insuperable* conflict between conciseness and empirical adequacy, as we have already seen. Indeed, *it is only in a sufficiently rich system that the conciseness of the description may come into insurmountable conflict with the accuracy of the description.*

This argument is expected to be relevant not only for highly mathematical sciences, but for all theories that entail many different empirical consequences. An exhaustive analysis of the implications of this idea for all scientific fields is obviously impossible here, but one general conclusion can be drawn: for any theory, no trivial formulation can be assumed to exist (in terms of ECs) unless it is explicitly found. Hence the available most concise formulation acquires an objective cognitive value.

3.5 Nontriviality Of Other Characterizations Of Simplicity

In the previous sections we have seen that the trivialization argument fails—in general—to reduce the value of conciseness, as defined in Section 3.1. Here, we show that the same result holds for any *acceptable* definition of the complexity of the assumptions. In order that a notion of complexity/simplicity be **acceptable**, we require at least the following two properties. First, the complexity of a theory should take into account the cost of defining the BECs of the theory in terms of the concepts appearing in the principles (the primitive concepts). Second, the complexity of an expression must be higher than the complexity of any of its proper sub-expressions¹⁸. These properties are presumably not sufficient to characterize an acceptable notion of complexity/simplicity, but they are certainly necessary. These properties hold, in particular, for our notion of conciseness. They also hold for the notion of parsimony (Baker, 2004), which measures (somehow) the domain of the logical quantifiers that appear in the postulates, or the notion of simplicity of Goodman (1977), that measures the amount and the complexity of the set of primitive predicates¹⁹.

If we re-examine the theory of Section 3.3, the same argument goes through unchanged, except for the points where the complexity of the theories $\mathcal{B}^{(L')}$ and $\overline{\mathcal{B}}^{(L')}$ needs to be computed. The latter obviously depend on the definition of complexity, but both theories contain expressions that grow indefinitely with the number of empirical observations to which the theory can be compared. In fact, the expressions relating the BECs of $\mathcal{B}^{(L')}$ to the principle Σ become more and more cumbersome, with increased time separation of the measurement from the reference point. While, in the case of the theory $\overline{\mathcal{B}}^{(L')}$, it is the number of BECs that grows indefinitely with time. According to the second requirement stated above, the complexity of a growing expression must grow. Therefore, we must conclude that none of those two theories can be simpler than the original theory $\mathcal{B}^{(L)}$, independently of the particular definition of complexity which is used.

¹⁸We also assume that the complexity function takes integer values, so that the increments cannot be infinitesimal.

¹⁹Since the distinction between the BECs and the primitive concepts is usually not stressed, when discussing simplicity, the first property is not apparent from Baker (2004) and Goodman (1977). But it is obvious, once the definitions of the BECs in terms of the primitive concepts are included among the postulates of the theory.

4 Different Notions Of Simplicity And The Goals Of Science

In Section 3 we saw that the general argument for triviality fails, once the empirical content of the theory is properly taken into account. Under these conditions, essentially any acceptable characterization of the simplicity of the assumptions becomes nontrivial.

Thanks to this result, it becomes meaningful to ask whether different characterizations of simplicity also lead to approximately the same theory selection, when applied to a significant set of real scientific theories. Furthermore, do they also lead to the same theory selection that may be defined by other classic values in science? These questions are very important. The consistency of different criteria would strongly support the high cognitive value of any such criterion. Moreover, it would fully justify the scientists' belief that some theories are unambiguously better than other (empirically equivalent) ones.

Such consistency can never be proved conclusively. It is only possible to accumulate evidence in its favor or falsify it²⁰. This can be done by examining different definitions of simplicity (or different virtues) and applying them to a significant set of real scientific theories. Each of these cases clearly requires a dedicated effort, to be duly investigated. In the rest of this paper we only take a small step in this direction, in order to convince the reader that the consistency mentioned above is not at all unlikely.

Section 4.1 presents a general argument in support of the consistency of criteria based on different definitions of the simplicity of the assumptions. As said, this is far from conclusive, but it suggests an interesting challenge for philosophy of science.

In the subsequent sections the concept of conciseness is examined in more detail, in order to show that it captures significant features of the goals of science. First, in Section 4.2 it is shown how conciseness can be estimated in practice. In Section 4.3 the efficacy of conciseness in penalizing theories with many ad-hoc assumptions is emphasized. Section 4.4 offers a brief overview of other virtues.

4.1 Are Different Notions Of Simplicity Equivalent?

We have seen that the formulation of a theory must include the definition of its BECs in terms of its primitive concepts. Under a different characterization of simplicity, the same theory could achieve its simplest formulation by using different BECs. However, the constraints that the BECs should be measurable (ECs) and rather unambiguous (in order to preserve empirical adequacy) make it *very difficult to find formulations that are radically different from the traditional one*, which is often already the result of strong efforts of simplification (according to some intuitive notion of simplicity). If the choice of the possible formulations is practically limited to small variations from the traditional one, then the different definitions of complexity must be applied to the same (or very similar) formulations. Moreover, we typically want to compare theories that differ only by a rather limited set of assumptions (see also Section 4.2). These observations together imply that we typically have to compare different definitions of complexity applied to *very similar* and *rather short* strings. If so, one should expect that simple theories, according to one criterion, be also simple according to the others, since a short formulation has necessarily also few quantifiers, few predicates, and (except for very peculiar cases) also the converse is true. This suggests that all the definitions mentioned in Section 3.5 may lead to essentially the same theory selection, when applied to real cases.

This argument is certainly not conclusive. It is conceivable that some alternative notion of simplicity might exist, which is still legitimate and very much different from the intuitive one, and for this reason it might have been overlooked by the scientists. It is also possible that the scientists might be overlooking alternative formulations of their theories that would reveal the prejudices behind their assessments of simplicity. However, this can be determined only by providing explicit alternatives and not by general arguments. In the lack of valid alternatives, the simplest available formulation retains an objective cognitive value.

²⁰In this sense, philosophical theories are not different from scientific theories.

4.2 Practical Estimate Of Conciseness

The rest of Section 4 examines the notion of conciseness and compares it to other classic cognitive values. The first issue is its practical estimate.

A first remark is that, in order to minimize the conciseness of a theory, it is very hard to use languages that are radically different from the traditional one. In fact, this would correspond to a major new discovery. If we are limited to small departures from the traditional language, then the conciseness can be estimated by simple inspection of the length of the principles expressed in their traditional form.

A second remark is that a precise computation of $\mathcal{C}(T)$ is not realistic, even in a given language, and even for very simple theories as the one analyzed in Section 3.3. But, we are never interested in the absolute value of $\mathcal{C}(T)$. The interesting problem, in practice, is always to compare two theories that share most of the assumptions and are empirically equivalent. In these cases, the difference $\mathcal{C}(T) - \mathcal{C}(T')$ between two theories T and T' is typically easy to estimate—possibly using informal languages—and not impossible to compute exactly.

As an example of how one can estimate the difference $\mathcal{C}(T) - \mathcal{C}(T')$ in an informal language, consider the two theories of special relativity (SR) and classical Galilean relativity (CR)²¹. In their modern most concise formulations, the two theories differ by a single postulate, which is, in the case of CR: *time and space intervals are constant in all inertial frames*, while for SR it reads: *the speed of light is constant in all inertial frames*. A suitable language can make these postulates considerably shorter, but both theories need at least one symbol for each of the concepts of *time*, *space*, *interval*, *velocity*, *light*, etc. This shows that CR cannot be made more concise than SR, without a (presently unknown) radical revision of the formulation of these theories. Consequently, if we had to correct the wrong predictions of CR by adding ad-hoc hypothesis, we would certainly attain a much more complex formulation than SR.

4.3 Conciseness, Ad-hoc Assumptions And Information

This section examines the efficacy of conciseness in penalizing theories that include many ad-hoc assumptions. As stressed in the Introduction, defining a measure for the amount and complexity of the assumptions is a prerequisite for a precise characterization of many classic cognitive values in science. It is well known that the presence of ad-hoc assumptions is difficult to characterize from a strictly logical point of view. For example, adding more assumptions makes a theory more restrictive. But, the property of being restrictive is not a good characterization of having many ad-hoc assumptions, because the best theories are extremely restrictive and admit only what really happens. What is bad in ad-hoc assumptions is not that they introduce restrictions, but that we are unable to express them without adding *more words*, while a good theory manages to be very restrictive with few words. Consideration of the syntax, besides the logical structure, is clearly necessary to represent the intuitive idea of ad-hoc assumptions. If the shortest formulation of the theory T' is not longer than the one of T , then T' cannot be seen as the addition of ad-hoc assumptions on top of T , even if T' implies T . This means that a *nontrivial* measure of conciseness can—at least in some cases—exclude that a new theory is obtained by adding ad-hoc assumptions.

For example, most theories of gravity within the ppN formalism (Will, 2006) are build as modification of Einstein's (or Newton's) theory of gravity. For many of those ppN theories, we cannot imagine a way to express them more concisely than Einstein's (Newton's) theory itself: we know how to formulate them only by formulating Einstein's (Newton's) theory first, and then adding further elements. Moreover, under the reasonable assumption that the Lagrangian formalism and differential geometry are standard tools which are needed anyway, it is hard to imagine a theory as concise as general relativity and empirically equivalent to it. These ppN theories are generally recognized as possessing more ad-hoc assumptions than general relativity, and they actually correspond to longer formulations.

²¹Since the two theories are not empirically equivalent, the comparison is interesting only from the technical point of view of computing their conciseness.

Another example is the following. Assuming that a given thermometer was not working properly on some specific occasions, may explain a few strange results. But, if we try to explain all strange measurements of temperature in this way, we have to add to the general theory a huge list of specific exceptions. Alternatively, assuming that all thermometers of some brand give a wrong answer 10% of the times can contribute to provide a more consistent description of a large set of measurements around the world, with a limited increase of the complexity of the theory. The latter procedure is clearly less ad-hoc and more concise than the former.

The sensitivity to ad-hoc assumptions is a consequence of a more basic virtue of concise theories: out of two theories with the same consequences, the more concise one provides evidence that *the needed information to obtain the same results is less* than it would be expected from less concise theories. This is also confirmed by the following observation. If a scientist is confronted with two formulations of the same theory she would never erase the shorter one, even if it misses other qualities that she might find desirable. In that case she would keep both. This highlights an important *cognitive advantage* of the most concise formulation, which is completely independent from any reference to reality, and hence fits well an *empiricist* view of science.

4.4 Conciseness And The Goals Of Science

This section sketches some connections between the concept of conciseness and other classical criteria for theory appraisal. Again, this is not meant to show any superiority of conciseness with respect to other characterizations of simplicity, but rather to exemplify how a well defined and nontrivial characterization of simplicity gives the chance to establish explicit connections with other cognitive values.

As already mentioned in the introduction, the idea of conciseness may enable a more precise formulation of the idea of *unification*. Two theories are unified when they are substituted by a single theory that answers at least all the questions previously answered by the original two theories. If the unification is not mere juxtaposition, some of the old assumptions should appear as duplicated and be combined in a single one, or be both dropped in favor of another more powerful assumption.²² This suggests that most interesting cases of unification have also produced more concise theories, although a systematic historical analysis would be certainly needed to assess this point conclusively.

A similar argument can be used to interpret many cases of *reduction* of scientific theories as cases of increased conciseness²³. Classic examples are Newton's reduction of Kepler's laws to the laws of mechanics, and the reduction of thermodynamics to statistical mechanics²⁴. In the first case, the laws that describe mechanical phenomena are shown to be sufficient also to explain astronomical phenomena; in the second case the laws of mechanics and probability are sufficient to explain also thermodynamical phenomena. Both cases correspond to the realization that all the phenomena under consideration can be explained with less overall assumptions. Other examples are being provided, currently, by computational sciences, that have achieved tremendous successes in reducing various phenomenological laws to more fundamental ones.

Among the recognized values, that a scientific theory should have, is also that of *coherence* with the other accepted theories. This does not seem to be related to conciseness. But, in our approach (see Section 2), a scientific theory is necessarily a multidisciplinary collection of *all* the assumptions that are needed to derive the results that can be compared to real experiments. In this context, coherence between the different domains of science is not a virtue: it is a necessity, that is assumed at the start.

²²It may be the case that a unifying theory introduces more sophisticated mathematical tools. But, according to the definition in Section 2, a scientific theory is necessarily a multidisciplinary collection of assumptions coming from different fields. Sophisticated mathematical tools—besides being generally very concise—have usually many fields of applicability, which considerably reduce their impact in the overall conciseness.

²³Note that, if T is more empirically adequate than T' , it is not very interesting to compare the conciseness of T to the one of T' , but rather to the one of a theory T'' , which is obtained by adding to T' suitable assumptions able to correct the wrong predictions of T' .

²⁴It is controversial whether the latter is an example of reduction, but it is anyway an example of increased conciseness.

An original application is the explanation of the problem of *fine tuning* in the standard model of elementary particles. This problem lies in the fact that the fundamental parameters of the model need to be known with a very large number of digits, in order to reproduce (even with moderate precision) the experimental values. Since the fundamental parameters must be included in the principles of the theory, this is, effectively, a problem of conciseness.

The idea of conciseness can also explain why *solipsism* is void of interest. Solipsism cannot be excluded neither on logical nor on empirical grounds. The problem with solipsism is rather the unnecessary amount of assumptions that need to be made in order to explain the experience. In fact, the experiences *reported* to the subject by other people require different explanations—and hence additional postulates—from those explaining the *direct* experiences of the subject. What the subject sees can be explained much more *concisely* by assuming a underlying reality, independent of the mind.

Finally, one should also mention that there exist research programs that aim at recognizing signatures of *irreducible complexity* in nature. In such programs, conciseness cannot be a value, by construction. But, this is consistent with the fact that those goals are not recognized by the vast majority of the scientific community, since no evidence can possibly exclude the existence of yet uncovered more concise rules.

5 Conclusions And Perspectives

Scientists often regard simpler assumptions as unambiguously preferable to complex ones. Moreover, most classic standards of progress in science implicitly rely on a characterization of the simplicity of the assumptions, in order to acquire a precise meaning.

Any precise definition of simplicity—which is relevant in this sense—necessarily requires the examination of the principles of the theory. Moreover, in order to evade general arguments for the triviality of any notion of simplicity, it is also necessary to establish a formal connection between the principles and the measurable concepts of the theory (ECs). This paper shows explicitly how the principles and the ECs can be included in a view of scientific theories, which is not in contradiction with modern views, and avoids the pitfalls of the traditional view. Although the ECs are, in general, theory dependent, each theory includes concepts that are empirical by construction of the theory itself.

The ECs are important not only in order to compare the theory with the experiments and with other theories, but also to constraint its possible formulations. In fact, *a theory must be expressed in a language able to represent its empirical content*. The importance of this requirement cannot be appreciated when considering isolated toy-theories, that entail only few consequences. But it becomes crucial for realistic theories, whose consequences are many and complex. In fact, in those cases, improving the simplicity of the formulation may conflict with the need of preserving its accuracy. This is illustrated through the inspection of a specific example of a theory and by employing the precise notion of conciseness. As a result, *the fact that some theories are more concise than others is not purely conventional. It is as objective as the fact that some quantities are measurable and others are not*.

The concept of conciseness introduced in this paper is just one of the many possible characterization of simplicity. Here it is used mainly as an example, showing that a nontrivial characterization of simplicity is possible. Similar arguments can be applied to other definitions of simplicity, that also become nontrivial, once the precise connection to measurable quantities is taken into account.

These observations lead naturally to the important question whether different—nontrivial—definitions of simplicity induce approximatively the same theory selection, when applied to a significant set of real cases. A further question is whether these criteria are also consistent with the other classic standard of progress in science. The availability of a class of nontrivial definitions of simplicity is a crucial prerequisite to address these questions precisely. *A positive answer to these questions would provide a solid philosophical justification on support of the scientists' belief that some theories are unambiguously better than other (empirically equivalent) ones*. This paper cannot support a positive answer conclusively, but it argues, through a few general considerations and some examples, that this possibility is not presently

excluded. In order to prove the scientists wrong, it is necessary to identify a legitimate definition of simplicity that contradicts some of the assessments that are universally held by the scientists. General arguments about its existence are not sufficient.

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A Appendix. Representing And Comparing The Empirical Contents

The main goal of this paper is to illustrate how the empirical content of a theory represents an obstacle to the simplicity that the theory can attain. In Section 2 it was proposed to describe the empirical content of a theory through the ECs. This appendix shows that the ECs are actually well suited for this purpose, in particular they enable the comparison of the empirical statements of different scientific theories. This is not obvious, since the ECs are theory dependent, and the prototypes are subjective. The key ideas are those of Putnam (1975a).

Semantic incommensurability (Feyerabend, 1962; Kuhn, 1996) notoriously challenges the possibility of comparing the statements of two different theories²⁵. Semantic incommensurability may be further distinguished (Scheffler, 1982) into *variation of sense* and *referential discontinuity*. Variation of sense refers to the difficulty of interdefining concepts that are implicitly defined within different axiomatic systems. This problem has been investigated within, e.g., the structuralist program (Schmidt, 2008; Balzer et al., 1987). Interdefinability is only possible after establishing logical relations of reduction between theories, which are possible only in very limited cases. This is not sufficient for our goals.

Referential discontinuity, on the other hand, corresponds to the problem that the ECs of different theories may fail to refer to the same phenomena. Referential continuity is sufficient to ensure the existence of a common ground for the comparison of two scientific theories and the lack of interdefinability is not an obstacle to it.

There are other obstacles to referential continuity. One problem is that different people may have different interpretations of the BECs, since these are based on sets of subjective prototypes. This is considered in Section A.1. A second problem is that two scientific theories may use different ECs and it is not clear how the comparison of the empirical adequacy is possible. This is the subject of Section A.2. Finally, Section A.3 shows that this view is consistent with the semantic one.

A.1 The Relation Between Prototypes And BECs

The relation between prototypes and BECs can be established by pointing our finger to prototypes and by naming them. Everyone taking part to such ceremonies of baptism will then extend, by analogy, the set of her own subjective prototypes. This mechanism is the basis of the causal theory of reference for physical magnitude terms (Putnam, 1975a), which is a special case of the causal theory of reference for natural kinds (Kripke, 1980; Putnam, 1975b). This theory has been extensively studied and its limits are well known. In particular, when we point our finger and assert: “that is the symbol 0 on the display”, misunderstandings can *never* be excluded completely. Even if some people agree that some prototypes are in the extension of a BEC, we can never be sure that the same people will always agree on the assignment of new prototypes that may eventually appear in future. Each ambiguity can be eliminated

²⁵Methodological incommensurability (Bird, 2004), which refers to the incomparability of the *cognitive values* used to judge different theories, is not considered in this appendix. But the conclusions of this paper clearly support the general value of simplicity, next to that of empirical adequacy.

by agreeing on further prototypes, but other ambiguities are always possible (Quine, 1960)²⁶. In fact, the procedures of baptism establish *correlations* between the subjective extensions that different people associate to the same BECs—which are partially tested by the feedback that these people return to each other—but they can never *guarantee a one-to-one correspondence* between these sets. However, correlations are precisely all what science needs. Referential discontinuities, like those envisaged here, can be seen as some of the many unavoidable sources of experimental errors. What is necessary for a scientific theory is not to eliminate them, but to produce an estimate of the probability and magnitude with which they occur. For example, the agreement of different observers on those BECs, whose prototypes are simple pictures, holds with high probability (that can be estimated) under the assumption of a neuropsychological theory connecting the light signals hitting the retina with the formation of pictures in the mind, that can be classified by analogy, and the assumption that such mechanisms are rather similar across humans. The inclusion of these assumptions increases the complexity of the theory (see Section 3), but also protects it from being ruled out by a single experimental oversight.

These assumptions have the same epistemological status as the other principles of the theory and all face together the tribunal of experience (Quine, 1969). (There cannot be truly *non-problematic* assumptions in a modern view of scientific theories.) This makes it harder to identify the assumptions which are responsible for a bad matching between a theoretical prediction and the empirical data. But this is a practical and not an epistemological problem.

In order to maximize the probability of correlation between the different subjective interpretations of the BECs, a scientific theory has the possibility to select a convenient small set of BECs. Modern theories, in particular, tend to reduce every BEC to the reading of the digital displays of suitable experimental devices, for which appropriate theoretical models are assumed²⁷. As a result, modern scientific theories predict a strong (and quantifiable) correlation between the subjective extensions that different people assign to the same BEC. These theories effectively associate each of their BEC to an approximately well identified and observer-independent set of prototypes.

If a theory cannot quantify the correlation between the subjective extensions of its own BECs, this is a problem for the theory, that presumably has a poor empirical adequacy, but not an epistemological problem. In conclusion, *when a scientific theory properly includes all the theoretical assumptions which are necessary to predict the experimental results, a failure of reference is not a problem for the theory of reference (nor for epistemology): it is a potential problem for the scientific theory itself*. This conclusion is completely consistent with the idea of naturalized epistemology (Quine, 1969) for which this section represents nothing more than a concrete exemplification.

A.2 The Relation Between The ECs Of Different Theories

After having identified the assumptions supporting a stable interpretation of the ECs of a single theory, it is necessary to consider the relation between the ECs of different theories. In order to compare their empirical statements, two scientific theories T and T' must provide two sets of ECs \mathcal{E} and \mathcal{E}' that can be identified through a correspondence \mathcal{I} , as required in Section 2.3. The ECs \mathcal{E} and \mathcal{E}' do not have to share the same *meaning*—which may not be possible between concepts belonging to incompatible theories (Feyerabend, 1962)—but should have the same *extension* (Hempel, 1966, p. 103), i.e., in particular, share the same prototypes. In other words, the two theories should stipulate coinciding measuring *procedures*, even though the *justifications* and the *descriptions* of such procedures could be very different.

²⁶The causal-descriptive theory of reference (see e.g., Psillos, 1999, chap. 12) has also been proposed as a way to constrain the possible interpretations. But, on one hand, the BECs have a formal definition in terms of the primitive concepts of the theory, which may be regarded as a descriptive element. On the other hand, no description can ensure a stable reference, even in combination with ostension (Quine, 1960).

²⁷There may still be people who stubbornly refuse to see the difference between the digits 0 and 1 printed on a display. There is no way to *prove* them wrong—without assuming other BECs—if they insist that this is what they see, but there are sociological theories that tell how often such eccentric behaviors may appear.

It is natural to ask whether it is always possible to find two non empty sets \mathcal{E} and \mathcal{E}' with these properties. Answering this question in full generality is well beyond the goals of this paper. For the present purposes, it is enough to remark that this is possible for a wide class of real scientific theories whose comparison is interesting. This is actually the case even for classic examples of syntactic incommensurability. For instance, the concept of *mass* in Newtonian mechanics and the one in relativistic mechanics have different meanings and extensions. But *there exist procedures to measure space-time coordinates*, which are valid for both theories (once the reference frame is specified), and the numerical results are in one-to-one correspondence. This means that the corresponding ECs have the same extension. Moreover, there seem to be no examples of real scientific theories whose comparison would be interesting but it is impossible because of lack of corresponding ECs (Godfrey-Smith, 2003, p. 92), and even those authors that defend the relevance of syntactic incommensurability for real science (e.g., Carrier, 2004) insist that this is only meaningful for theories that do identify some of their ECs.

Note that the set \mathcal{E} , supplied by T to establish a comparison with T' , does not enable a comparison also with any other theory. If, later, we want to compare T with another theory T'' , we may need to choose a new set of ECs $\hat{\mathcal{E}}$, among those that can be defined within T . For example, measurements of absolute time represented legitimate ECs for classical dynamics, and were associated to prototypes where the speed of the clock was disregarded. Such ECs enabled the comparison of pre-relativistic theories, but are not suitable for comparing those theories with special relativity. In fact, as a consequence of the theory-ladenness of the ECs, there is no lingua franca of observations: the empirical languages change with the emergence of new theories, but this does not typically hinder their comparability.

Sometimes, the kind of problems described in Section A.1 are revealed by the introduction of a new theory. For example, the theory of special relativity reveals that measurements of absolute time may give inconsistent results, if taken from different reference frames. However, if an EC (e.g., absolute time), which is expected to be sufficiently unambiguous according to the theory T (Galilean relativity), turns out to be ambiguous after the introduction of T' (special relativity), this is a problem for the empirical adequacy of the assumptions of T , rather than a problem for the comparison of T with T' . In other words, once we have identified two sets of ECs \mathcal{E} and \mathcal{E}' , *the conditions for the comparability of T with T' are already part of the assumptions that both T and T' need to incorporate, in order to formulate their own experimental predictions.*

A.3 Syntactic And Semantic Views

The discussion in Section 2 is formulated in a language that bears many similarities with the one used within the received syntactic view of scientific theories. Although we have stressed the crucial differences, it is also worth comparing with modern semantic approaches (Suppes, 1967; van Fraassen, 1980). Here it is shown that the present view differs from the one of van Fraassen (2008) only in the *emphasis* on some syntactic aspects—which are necessary for the purposes of this paper—and it is otherwise consistent with it.

In any semantic approach a central role is played by the possible *models* of a scientific theory (Suppes, 1967; van Fraassen, 1980). Models are, in general, a combination of results derived from the principles of the theory (e.g., a class of solutions of Newton's differential equations of motion) together with specific initial conditions. Models contain original informations with respect to the principles, because (as stressed in Section 2) the derivation of the results typically requires original ideas. Moreover, models can be used directly to produce theoretical predictions and to perform a comparison with the experiments. It is true that many properties of a scientific theory can be conveniently appreciated by examining its set of models, and consideration of its axiomatic structure is unnecessary for that. But, models typically include many *consequences* of the theory, besides its minimal set of *assumptions*. For this reason, they are not suitable to evaluate the complexity of the assumptions, which is our goal. This is more directly expressed by the principles of the theory. Note that, according to Section 2, the principles really include everything that is necessary to derive measurable predictions. For example, if a general theory admits

different solutions, and if the measurable initial conditions (which are part of the questions) do not allow the complete identification of the relevant solution, suitable assumptions must be added to the principles, in order to select a single solution.

It is interesting to pursue the parallel with van Fraassen (2008) somewhat further. In particular, van Fraassen's *empirical substructures* can be identified with those results of the theory that can be expressed exclusively in terms of ECs. Furthermore, van Fraassen (2008) emphasizes the fact that measurements are *representations*, that need the specification of the context and the experimental setup, in order to allow the interpretation of the outcome. As stressed in Section 2, all these informations must be part of the ECs. Note that the compatibility of the two views is possible because both van Fraassen's empirical substructures and the ECs introduced here are integral parts of the theory, and not above it.

The connection with the phenomena is achieved, in both the present and van Fraassen's approach, via indexicality (and the identification of prototypes). As stressed by van Fraassen (2008), indexical statements plays a central role also in evading Putnam's paradox (Putnam, 1981), which states that almost any theory can be seen as a true theory of the world. In fact, it is almost always possible to find a correspondence between the concepts of a theory and the phenomena in the world, such that—with this *interpretation*—the theory is true. In order to evade this paradox, one needs to fix the correspondence between the ECs and the phenomena in an independent way. Such independent constraints are imposed precisely by indexical statements (with all the caveats already explained): when we point our finger, we insist that *this* is the symbol 0 on the display and not whatever suits the theory in order that the theory is correct.

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