Leibniz Equivalence. On Leibniz's (Bad) Influence on the Logical Empiricist Interpretation of General Relativity

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Abstract

Einstein's "point-coincidence argument" as a response to the "hole argument" is usually considered as an expression of "Leibniz equivalence", a restatement of indiscernibility in the sense of Leibniz. Through a historical-critical analysis of Logical Empiricists' interpretation of General Relativity, the paper attempts to show that this labeling is misleading. Logical Empiricists tried explicitly to understand the point-coincidence argument as an indiscernibility argument of the Leibnizian kind, such as those formulated in the 19th century debate about geometry, by authors such as Poincaré, Helmholtz or Hausdorff. However, they clearly failed to give a plausible account of General Relativity. Thus the point-coincidence/hole argument cannot be interpreted as Leibnizian indiscernibility argument, but must be considered as an indiscernibility argument of a new kind. Weyl's analysis of Leibniz's and Einstein's indiscernibility arguments is used to support this claim.

Keywords: Gottfried Wilhelm Leibniz; Logical Empiricism; Philosophical interpretations of General Relativity; Point-coincindence argument; Hole Argument; Indiscernibility arguments; Felix Hausdorff; Hermann Weyl

1. Introduction

It has become commonplace in the literature to argue that Einstein's celebrated argument for general covariance, the so called "point-coincidence argument" (well-known from Einstein's review of his general theory of relativity; CPAE 6, Doc. 30; *Die Grundlage der allgemeinen Relativitätstheorie*, 1916), if considered as a response to the also celebrated "hole argument" (known from Einstein's private correspondence with Michele Besso, Paul Ehrenfest, Hendrik Lorentz and others; CPAE 8a, Doc. 173, 178, 180, 183; see Norton 1984), would amount to a defense of what has been famously labeled "Leibniz equivalence" (Earman and Norton, 1987).

According to this line of thought, Einstein's point-coincidence/hole argument would clearly resemble Leibniz's indiscernibility arguments against Newtonian absolute space. In particular the role played by what we now call "diffeomorphisms" (transformations that preserve only smoothness and the uniqueness of the coordinates) in Einstein's argument would recall that of "translations" (which preserve also the distance between any pair of points) in Leibniz's shift argument against Clarke. In both cases worlds which can be transformed into each other by suitable transformations might be considered the same world. John Earman and John D. Norton drew the parallel in detail in their seminal contribution (Earman and Norton, 1987) and the protagonists of the huge debate created by this paper, such as Tim Maudlin (1988), Jeremy Butterfield (1989), John Stachel (1993), Robert Rynasiewicz (1994), Carl Hoefer (1996) and Simon Saunders (2002), endorse the shift argument/hole argument analogy, or at least adopt it as a polemical target. From this perspective the hole argument should be considered as a "New Leibnizian Argument" (Bartels, 1994; Bartels, 1996).

In this article I will try to show that reference to Leibniz in this context is in many respects confusing. It is not a question so much of historical accuracy, but rather of obscuring the radical novelty of Einstein's indiscernibility arguments in comparison to those of Leibniz. I will try to bring out this point through what I may dare to call an "historical-critical" reconstruction of the role of Leibniz's indiscernibility arguments in the Logical Empiricist interpretation of General Relativity. Logical Empiricists actually tried explicitly to understand the point-coincidence argument as an indiscernibility argument of the Leibnizian kind, encouraged by the use of such arguments in the 19th century philosophical debate about geometry. However, they clearly failed to give a plausible account of General Relativity (Friedman, 1983; Ryckman, 1992; Howard, 1999). An analysis of the reasons for and the origins of this failure should show, in my opinion, that the comparison between Leibniz's indiscernibility arguments and Einstein's point-coincidence/hole argument is misleading.

I will proceed as follows: developing an idea of Hermann Weyl's, I will argue that Leibniz's celebrated thought experiments on the impossibility of noticing a universal dilation of the whole universe or its mirroring by changing east into west and so on, can be considered as the first attempt to define the modern concept of "automorphism" or symmetry transformation, a structure-preserving transformation of space into itself, a way of mapping the object onto itself while preserving all of its structure. In my opinion Weyl's suggestion, although surely questionable from a strict philological point of view, is nevertheless helpful in the attempt to grasp the theoretical significance of Leibniz's indiscernibility arguments and most of all to understand the role that "Leibnizian" arguments played in the 19th century debate about geometry. The major protagonists of that debate, such as Hermann von Helmholtz, Henri Poincaré, but also Felix Hausdorff (to whom I will mostly refer), in considering automorphisms (or isomorphisms) that preserve progressively weaker levels of geometrical structure, were able to generalize Leibniz's thought experiments, showing that two worlds would be indistinguishable even if they were mapped onto each other by any continuous deformation whatsoever.

Early Logical Empiricists (Moritz Schlick, Hans Reichenbach, Rudolf Carnap) explicitly interpreted General Relativity in the light of such Leibnizian kinds of arguments. In particular they considered Einstein's "point-coincidence argument" as an expression of indiscernibility in the sense of Leibniz. I will suggest that the inadequacy of Logical Empiricists' interpretation of General Relativity, which is now commonplace in the literature, depends exactly upon their misleading interpretation of Einstein's point-coincidence argument as one of the many Leibniz-style indiscernibility arguments that appeared in 19th century debate about geometry.

The lesson that, in my opinion, we can draw from such a reconstruction is that, even if the point-coincidence argument as a response to the "hole argument" can be considered a sort of indiscernibility argument, it cannot, however, be interpreted simply as a restatement of indiscernibility in the sense of Leibniz. Interestingly enough, as we shall see, it was precisely Weyl himself who already clearly recognized this point. In a first approximation, we can say that whereas "Leibniz's indiscernibility" is an expression of apparent physical differences that cannot find expression in the mathematical apparatus of the theory, what we may call "Einstein's indiscernibility" is the consequence of apparent mathematical differences that cannot find any correspondence in physical reality. In an appendix I will try to provide a formal elucidation of this point.

2. Leibniz's Indiscernibility Arguments and his Conception of Geometry

Hermann Weyl was perhaps the first to emphasize the depth of the "philosophical twist" (Weyl, 1952, p. 127) that Leibniz gave to the simple geometrical notion of "similitude" or "similarity" between two figures: "Leibnitz [*sic*] declared: two figures are similar or equivalent if they cannot be distinguished from each other when each is considered by itself, because they have every imaginable property of objective meaning in common, in spite of being individually different" (Weyl, 1934/2009, p. 21). Leibniz thus exhibited "the true general meaning of similitude" (Weyl, 1939/1997, p. 15). According to Weyl such a "'philosophical' definition of similar figures" can be considered as the first definition of the more general concept of "automorphism" or symmetry transformation: "an automorphism carries a figure into one that in Leibniz's words is 'indiscernible from it if each of the two figures is considered by itself'" (Weyl, 1952, p. 18). Automorphisms are, namely, transformations of space into itself that leave all relevant geometrical structure intact, so that the result is indistinguishable from the original unless one refers to something that does not participate in the transformation, which thus serves as a frame of reference (Rosen, 2008; Kosso, 2000). The fiction of a change that involves the entire universe serves exactly to exclude in principle the possibility of such comparison. Two worlds arising from each other by an "automorphic" transformation, i.e. by a transformation which preserves some geometrical structure, are to be considered the "same world," since there is in principle nothing outside the universe with respect to which the transformation can be referred. Thus, after the transformation, one cannot say if the transformation has taken place or not, and it is impossible to establish whether one is living in the original universe or in its transmogrified copy.

In the next section I will try to show that Weyl's suggestion can find supporting evidence in Leibniz's texts. Of course no claim to provide an exhaustive and philologically precise account of Leibniz's philosophy of geometry can be made in this context (for a recent reconstruction see De Risi, 2007; still relevant for my exposition is Schneider, 1988; see also Freudenthal, 1972; Münzenmayer, 1979; Wallwitz, 1991). The more humble aim of this section is to show that considering Leibniz's indiscernibility arguments in the light of his reflections about geometry throws a different light on the meaning of similar Leibniz-style arguments in the subsequent history of philosophy of space and spacetime. This will be helpful further on, when we will consider the difference between Leibniz indiscernibility and Einstein indiscernibility.

2.1. Leibniz's Definition of Similarity and the "Nocturnal Doubling" Thought Experiment

Leibniz, as is well known, was unsatisfied by the traditional definition of "similarity" of figures (see *De analysi situs*, GM V, pp. 179-80; Leibniz 1976, p. 255). Every figure, according to the period's mathematical parlance, includes besides quantity also quality or form (GM V, p. 179; Leibniz 1976, p. 254). Similar figures were usually considered those that have the same "form" or "quality," but possibly different magnitude or "quantity": for instance two equilateral triangles or two cubes may have the same "form" but different "size." However, such a definition appeared to Leibniz "fully as obscure as the thing defined" (GM V, p. 180; Leibniz 1976, p. 254). Indeed, according to Leibniz, "it is not enough to designate objects as similar whose form is the same, unless a general concept is further given of form" (GM V, p. 180; Leibniz, 1976, p. 254). But the concept of form, far from being obvious, is laden with cloudy metaphysical presuppositions.

To avoid a direct definition of similarity in terms of identity of quality, Leibniz proposed therefore a sort of phenomenological definition of "similarity," considering similar those figures "which cannot be distinguished when observed in isolation from each other" (GM V, p. 180; Leibniz 1976, p. 255). As far as we know, Leibniz provided such a definition for the first time in 1677 (Leibniz to Gallois; GM I, p. 180), and he later repeatedly insisted that the Euclidean definition of similarity was just a particular case of his more general phenomenological definition (*De analysi situs*, GM V, p. 179 and 181-82; *Specimen geometriae luciferae*, GM VII, pp. 281-82). Here is one of the many passages that one can refer to:

> Things are similar in which when they are considered one by one, nothing by which they can be differentiated can be found as two spheres or circles (or two cubes or perfect squares) A and B. For example if the eye alone without the rest of the body is imagined now to be inside sphere A now in sphere B it will not be able to distinguish them but it will be able if it considers both at once, or if it brings with it other organs of the body, or another standard of measure which it applies now to one now to the other. Therefore to distinguish similar things, they must either be present together, or between them a third thing must be present to each successively. [Similia sunt in quibus per se singulatim consideratis inveniri non potest quo discernantur, ut duo sphaerae vel circuli vel duo cubi aut duo quadrata perfecta A et B. Ut si solus oculus sine aliis membris fingantur, nunc esse intra sphaeram A nunc intra sphaeram B, non poterit eas discernere, sed poterit si ambas simul spectet, vel si secum membra alia corporis aliamve mensuram introrsum afferat, quam nunc uni nunc alteri applicet. Itaque ad similia discernenda opus est vel compraesentia eorum inter se, vel tertii cum singulis successive.] (GM VII, 30; tr. Cox 1978, p. 234)

It is possible, for example, to distinguish "an isosceles triangle from a scalene, even if we do not see them together" [ita triangulum isosceles facile discernitur a scaleno, etsi non simul videantur] (GM V, p. 155). But if one wants to determine which is the larger of two equilateral triangles, one "must compare the two triangles" [collatione Triangulorum cum aliis opus habeo] (GM V, p. 155). Then the quality of a figure is what "can be known in a thing separately," while "quantity" is what can be grasped only when the figures "are actually present together" (GM V, p. 180; Leibniz 1976, p. 254).

Leibniz was deeply convinced of the superiority of his own definition of similarity, for "it has not been deduced from the consideration of angles, which represents only one instance of similarity, but from a deeper principle, that is from the *principle of discerning*" (LH XXXV, I, 14, bl. 23-24; tr. De Risi 2005, p. 145). Euclid's definition of similarity based on the congruence of angles is a "special case which does not reveal the nature of similarity in general" (GM V, p. 181; Leibniz, 1976, p. 256). Leibniz explicitly claims that it is possible to deduce the congruence of angles from his perceptual definition of similarity, allowing congruence to be defined from similarity rather than the other way around.

It is not possible here to discuss the geometrical implications of Leibniz's strategy, adopted for instance in the *Specimen geometriae luciferae* (GM VII, pp. 281-82), to provide a definition of similitude based on proportion of sides, rather than on angle congruence (De Risi, 2007, p. 141ff.) However, in my opinion, following Weyl's interpretative suggestion, we should be able to grasp the philosophical intuition behind Leibniz's "perceptual definition" of similarity. According to Weyl, Leibniz seems to glimpse that the important role of similitude in elementary geometry is related to the fact that transformations which change the size of segments, but preserve both the ratio of lengths and the size of angles, cannot change any "geometric" properties of figures. Geometry does not have at its disposal any conceptual resources to establish the difference between, say, a smaller and a bigger circle; every objective statement about the one would hold about the other, for the size of figures cannot be taken into consideration in geometrical theorems: "every theorem, construction, propriety, proportion or relation that can be found in a circle, could be found also in the other" [Omnia theoremata, omnes constructiones, omnes proprietates, proportiones, respectus, qui in uno circulo notari possunt, poterunt etiam in alio notari] (GM VII, 276). Thus similar figures (which have the same shape but possibly different sizes) are clearly the "same" figure for the geometer. Their difference cannot be expressed "conceptually," but emerges merely through an "intuitive comparison" [comparaison intuitive] (Couturat, 1961, p. 412), that is through what Leibniz calls "comperceptio."

Leibniz's celebrated thought experiments serve exactly to show ideal cases, where the possibility of such comparison is fictionally excluded. If, for instance, God were to diminish all appearances in and around us in a closed room ("in aliquo cubiculo"), preserving the proportions ("omnia ... apparentia proportione eadem servata minuere"), everything would appear the same, and we would not be able to distinguish the state before from that after transformation, without exiting our closed room and considering the things that have not been diminished ("nisi sphaera rerum proportionaliter imminutarum, cubiculo scilicet nostro, egrederemur") (GM V, p. 153s). The difference would emerge only through comparison between two situations: one that has been scaled, and one that has not. Something must remain untransformed as a standard against which the transformation is measured. If this comparison would be in principle impossible:

> If God were to change everything conserving the proportion, we would lose all our measures and it would not be possible to know how much the things have changed, because it is impossible to determine a certain definition of measure or to conserve it in the memory. From that I believe I could explain the difference between size and species, between quantity and quality. [At si quemadmodum alibi jam dixi Deus omnia mutaret proportione eadem servata perisset nobis omnis mensura nec possemus scire quantum res mutatae sint, quoniam mensura nulla certa definitione comprehendi adeoque nec memoria retineri potest, sed opus est reali ejus conservatione. Ex quibus omnibus discrimen inter magnitudinem et speciem inter quantitatem et qualitatem elucere arbitror.] (GM VII, 276)

In the well-known terminology due to Leibniz, the original universe and the transformed one would be *indiscernible*.[1] It would not even make sense to speak of a difference. We

[1] An attempt to give an account of the different forms (logical, metaphysical, empirical, etc.) that

cannot ascertain the change of a length in the course of time, but only the change of its ratio "to real standards of measure which are assumed to be unchanging" (GP V, 134; Leibniz 1976, p. 147). Thus if all lengths were diminished or magnified by the same factor, that is conserving the angles and the proportions among lengths, Euclidean geometry would provide us with no tools for ascertaining the difference. Any attempt to establish through geometrical methods whether we are in the original or the scaled universe would be in vain.

2.2. Leibniz's Definition of Congruence and the Static Shift Argument

Thus, Leibniz's "nocturnal doubling"-style thought experiment is simply the counterpart of his definition of similarity. Leibniz himself explicitly establishes this connection for the very first time when he introduced this definition in his never sent letter to Jean Gallois, to which we have already referred.[2]Leibniz was so impressed by his own definition that he repeatedly tried to provide an analogous phenomenological account of the notion of "congruence" (see Schneider 1988 for list of passages). Two figures are congruent if they can be distinguished not only through the simultaneous perception of both of them, but also requiring the perception of a third object: "Congruent are those things that can be distinguished only through the comperception with a third" [Congrua sunt quae sola comperceptione cum tertio discerni possunt] (LA VI.4a 565). Two similar figures that differ in magnitude can be distinguished even if they are in the same place, for one can be part of the other:

> But if two things are not only similar, but also equal, i.e. they are congruent, they cannot be distinguished even if they are perceived together, if not because of the place, that is only when something it is assumed outside them and it is observed that they have a different position respect to this third object. [Si vero duae res non tantum sunt similes sed et aequales, id est si sint congruae, etiam simul perceptas non discernere possum, nisi loco id est, nisi adhuc aliud assumant extra ipsas et observem ipsas diversum habere situm ad tertium assumtum.] (GM V, 155)

Thus congruent figures, in Leibniz's scholastic parlance, are different "solo numero," only by the reference to something external ("solo erga situ ad externa discernuntur"; GM VII, 275), because "one is more on the west or more on the east, more on the north or more on the south, more above or more below, or because some another body is posited outside them" [unum alio orientalius aut occidentalius vel septentrionalius aut meridionalius vel superius aut inferius esse vel alteri alicui corpori extra ipsa posito esse] (GM VII, 276).

Leibniz did not remain satisfied with such a definition of congruence (De Risi, 2007, pp. 143ff.). However, it is plausible that the very famous arguments that Leibniz used in his correspondence with Clarke can be considered the exact counterpart of such a phenomenological definition of congruence, just as, as we have argued, the "nocturnal

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the principle of identity of indiscernibles assumes in Leibniz's work is impossible here and not only for space constraints; on some of the forms that the principle assumes in Leibniz, see (Chernoff, 1981).

[2]"Having thoroughly inquired, I have found that two things are perfectly similar when they cannot be discerned other than by com-presence, for example, two unequal circles of the same matter could not be discerned other than by seeing them together, for in this way we can well see that the one is bigger than the other ... In fact, if all the things of the world affecting us were diminished by one and the same proportion, it is evident that nobody could make out the change." (GM I, p. 180; tr. De Risi (2007, p. 58)).

doubling" thought experiment is the counterpart of Leibniz's definition of similarity. Two congruent figures A and B in the same plane may show many differences, when they are considered in their relations to some fixed frame of reference. However, these differences are not geometrical differences; every objective statement that geometry can make for one figure will hold for the other. If therefore the reference to the third external figure is eliminated, we would be left without any geometrical tools to establish if the figure that we are considering is the figure A or the figure B.

Imagine that the whole universe has been displaced. Since the universe is everything, in principle no external reference frames can be imposed upon it, so that the very concept of spatial displacement would be inapplicable to the universe. Thus according to Leibniz "to suppose that the universe could have had at first another position of time and place, than that which it actually had ... is an impossible fiction" (Leibniz, 1976, p. 667). The details of Leibniz's "static shift argument" (following the nomenclature introduced by Maudlin 1993) have been rehearsed so many times in the literature that I refrain from doing so again here. The only aspect I would like to pinpoint is that it seems clear that the goal of Leibniz's thought experiment is to make the "simultaneous perception" of the original situation and the changed one in principle impossible - showing that difference of position is inessential to geometry, that it is geometrically no difference at all. Geometrically we cannot establish if we are living in the original or in the displaced world.

Similarly, when in the Third Paper, §5, Leibniz so famously argued that if we interchange all matter east to west, or left to right, no difference would emerge, he seems to imply that the inner geometrical structure of Euclidean space does not allow one to distinguish a left from a right-handed screw without reference to some third external asymmetric object. The difference between left and right is not a geometrical difference, as Leibniz seems to admit in this passage: "But it is impossible to distinguish left and right ... if not for the fact itself or the perception, that the human being experiences that a motion is more comfortable on one side that on the other" [Sed dextrum a sinistro discerni non potest ... nisi facto ipso, seu perceptione, dum ab uno latere motum commodiorem quam ab alio homines experiuntur] (C Phil VII, D, II, 2, f. 30).

A passage of Leibniz's summarizes effectively the simple path that we have followed through the rather messy conglomerate of his notes, drafts and manuscripts on geometry: "the quality can be observed in one thing, the quantity in two ... the position in three [qualitas est in uno observabilis, quantitas in duobus ... positio in tribus] (A IV.1, p. 393). If we resort to Weyl's interpretative key, this apparently trivial distinction of the form of a figure from its position and magnitude seems to express the fundamental "difference between conceptual definition and intuitive exhibition" (Weyl, 1927b, p. 73; tr. Weyl, 2009b, p. 11), the idea that is impossible to describe a position or fix a unit of length "in a conceptual way," through geometric methods, and "not by means of a demonstrative this-here" (Weyl, 1934/2009, p. 119). Differences in position and magnitude are geometrically unascertainable and emerge only through an intuitive comparison. Thus Leibniz's indiscernibility arguments seem to correspond exactly to such "perceptual definitions" of congruence and similarity, in as much they serve to fictionally exclude the possibility of such comparison.

However, precisely for this reason, it is easy to see that they do not succeed at all in supporting the fully "relational" conception of space usually attributed to Leibniz, that is to reduce space to the consequence of the "relations among bodies", as in the received view of Leibniz's philosophy of space. On the contrary, the arguments affirm something very precise about the structure that space possesses independently from the "relations among bodies." Leibniz's arguments work because it has been settled in advance that the space, as is said in a recently published manuscript, is not only uniform, i.e. "self-congruent" but also "self-similar" (LH XXXIV, I, p. 14, Bl. 23 retro; tr. De Risi 2005, p. 140), that it is *flat or Euclidean*. Space, as Leibniz explicitly points out, is not like a spherical or a cylindrical surface, i.e. one of the surfaces that Leibniz defined as a "uniform locus," self-congruent, but not self-similar; but it is like a "plane" (LH XXXIV, I, p. 14, Bl. 23 retro; tr. De Risi 2005, p. 145), which is "everywhere internally similar to itself" (GP VII, p. 22; tr. Leibniz 1976, p. 672).

2.3. A Glimpse into the Relationalism vs. Substantialism debate. Leibniz's Kinematic Shift Argument

From this point of view it is not clear how Leibniz could credibly have believed that he challenged Newton's conception of absolute space through his celebrated indiscernibility argument. Newton's absolute space is obviously endowed precisely with the same Euclidean symmetries of Leibniz's space. Leibniz's arguments seem to confuse the problem of absolute position with that of absolute motion, that is the problem of the same position in different times, which was Newton's concern (DiSalle, 2002b). Obviously, Leibniz could extend his indiscernibility arguments to motion itself.

"A ship may go forward, and yet a man, who is in the ship, may not perceive it" (GP VII p. 403; tr. Leibniz 1976, p. 705). As in the other indiscernibility arguments, also in this "Galilean" thought experiment one can perceive the motion of the ship only by reference to something outside the ship, that does not participate in its motion. Once again the fiction of the motion of the whole universe eliminates in principle the possibility of such a comparison:[3] since there is nothing outside the universe, the motion of the whole universe, per definition, cannot be observed, "and when there is no change that can be observed, there is no change at all" (GP VII, p. 404; tr. Leibniz, 1976, p. 705).

Thus the "static shift argument" can be easily transformed into a "kinematic shift argument": "To say that God can cause the whole universe to move forward *in a right line*, or *in any other line*, without making otherwise any alteration in it, is another chimerical supposition." These "two states indiscernible from each other" would be "the same state", it would be "a change without a change" (GP VII, p. 373; tr. Leibniz, 1976, p. 705).

There was a time when the philosophical supremacy of Leibniz's "relationalist" account of motion over Newton's alleged theological-metaphysical "substantialism" appeared unquestionable. However, it has more recently become commonplace to argue that such a generalized indistinguishability between motion and rest, the idea that "there are no

[3]In this sense Galilei's ship experiment is the prototype of every indiscernibility argument. According to Galilei, uniform motion "exists relatively to things that lack it", but for things that participate equally in the motion, motion "is as if it does not exist" (come s'e' non fusse) (Galilei, 1632/2005, I, p. 205; tr. Galilei 1967, p. 116). The indiscernibility emerges from the exclusion of the external reference to "other bodies lacking that motion." Similarly Leibniz's changes that involve the whole universe mean "agendo, nihil agere" (GP VII, p. 396), since the reference to something that has remained unchanged is in principle impossible. There is therefore a connection between global symmetries, indiscernibility and the possibility of subdividing the universe into isolated subsystems (see Brading and Brown, 2003, p. 99-98).

exceptions to the general law of equivalence" (Letter to Huygens, 12/22 June 1694, A III.6, p. 131; tr. Leibniz, 1989, p. 308) is obviously incompatible with Galilean relativity. Galilean relativity is the statement that motion "in a right line" is indistinguishable from stasis, and not that also "motion in any other line" (however zigzagging) is indistinguishable from stasis. Leibniz's indiscernibility arguments, if applied in all their generality to all motions, it is said, are unable to make sense of the privileged status of inertial motion, the uniform motion in straight line, that one should be able to distinguish from an accelerating motion. Thus Newton's bucket experiment correctly shows that circular motion is actually an exception to Leibniz's general law of equivalence. However, Newton's alleged claim (Rynasiewicz, 2000) that this argument would provide evidence for the existence of absolute space is commonly considered puzzling: Newton's laws of motion presuppose absolute time, but not absolute space; absolute acceleration, but not absolute velocity.

It has now become usual (Stein, 1967/1970; DiSalle, 2002a, 2006) to argue that the Leibniz-Newton debate can be better understood if one assumes that it concerns not the geometrical structure of space, but that of spacetime. Let me resort once more to Weyl's account. Not only was Weyl probably the first to make this point clear, but most of all his position will reveal itself particularly significant later on for the purposes of the present paper. As Weyl famously pointed out, "the dynamic inequivalence of different states of motion teaches us that the world bears a structure" (Weyl, 1927a, p. 70 ; tr. Weyl 2009b, p. 101). Since unaccelerated motion is also called "inertial motion," one also refers to such structure as the "inertial structure." But in the concept of "absolute space" this inertial structure is "evidently not sized up correctly; the dividing line does not lie between rest and motion, but between uniform translation and accelerated motion." spacetime of classical mechanics has defined structure, since "straight lines can be objectively distinguished from curves, but in the family of all straight lines one can single out the vertical ones only by a convention based on individual exhibition" (Weyl, 1927a, p. 70 ; tr. Weyl 2009b, p. 101).

Classical mechanics appeals to the action of a background spacetime structure, one may call it "Galilean spacetime," in which the particles are immersed and against which inertial and non-inertial motion can be distinguished (a body that is not moving in a straight line is considered as being acted on by a force). With respect to Galilean spacetime, it is usually argued, "Newtonian spacetime" has, so to speak, "too much structure", since Newton wanted to determine objectively what is a vertical straight line (a body at absolute rest, the objective occurrence of two events in the same place). "Leibnizian spacetime," on the other hand, bears "too little structure"[4]: if Leibniz requires the relativity of all motions, then he could not distinguish straight lines from curved lines (inertial and non-inertial motions). Spacetime would be an amorphous "mass of clay" (Weyl 1927b, p. 57; tr. Weyl 2009a, p. 41), without a real inertial structure. Then, as Weyl points out, the concept of the relative motion of several bodies would have no more foundation than the concept of absolute motion for a single body (Weyl 1927b, p. 57; tr. Weyl, 2009a, p. 105). In fact, if the automorphisms of spacetime do not preserve

[4]A much more sophisticated catalog of classical spacetimes is discussed in John Earman's definitive study *World Enough and spacetime* (Earman, 1989). Leibnizian spacetime in particular is not completely amorphous, because Leibniz seems to admit a foliation in hyperplanes of simultaneity. See Roberts 2003 for an overview and a criticism of the received account of "Leibnizian spacetime."

any structure relational theories of motion would - as we shall see later - be dynamically trivial (Lariviere, 1987).

Needless to say, Weyl's interpretation is dangerously close to that "folk reading" (Huggett and Hoefer, 2009) of Leibniz's philosophy of space and time, which, as recent literature has shown, is far from exhausting Leibniz's much more articulated conception of the relationship between relationalism and absolutism about motion.[5] However, as we shall see later, Weyl's suggestion is useful in order to restate once more what Leibniz's indiscernibility arguments, in my opinion, implicitly presuppose. If the whole universe were to move with constant velocity, we would certainly not notice the difference from a universe at rest: parallel world lines would be mapped into parallel world lines; the affine structure of spacetime gives us no means of picking the vertical lines among the others. All relevant geometrical structure would appear the same, in the universe at rest and in the uniformly moving one. However, classical mechanics affirms that if the whole universe were rotating, parallel lines would be mapped into curved lines. The original universe and the transformed situation would not be indistinguishable at all, for the affine structure gives us the possibility of distinguishing between curved and straight trajectories in spacetime.

This was of course hard to grasp without having the possibility of considering spacetime as a single geometrical structure. However, exactly for this reason, we are in a better position to recognize that it is the geometry, which has been settled in advance, that "decides" the indiscernibility. Any proto-verificationist interpretation of Leibniz's famous remark that motion depends "upon being *possible* to be observed" (GP VII, 403; tr. Leibniz 1976, p. 705) (like Reichenbach's reading, as we shall see) obscures the fact that it is the "inner structure" of Galilean spacetime that does not allow the establishment of the difference between rest and uniform motion, but does allow that between uniform motion and acceleration. From today's standpoint, the fact, taken as obvious, that such a mere geometrical structure is able to exert such an important influence on physical reality appears more than surprising. Not only does spacetime geometry make bodies conspire to move in straight lines at uniform speeds, but it even opposes resistance when one attempts to deviate bodies from such trajectories: "It is probably fair to say that anyone who is not amazed by this conspiracy has not understood it" (Brown, 2005, p. 14).

3. Leibnizian Indiscernibility Arguments in the 19th Century Debate about Geometry.

Weyl's intuition seems to offer a good insight into Leibnizian indiscernibility arguments: in geometry two figures are considered the "same" figure "if one can be carried

[5]As is well known, in many passages Leibniz insists that a distinction between true motion and merely relative motion can be drawn in terms of "force" or *vis viva*, that is mv^2. This claim is usually considered inconsistent or at least circular (measurement of mv^2 depends on the definition a reference of frame); this nevertheless makes clear that Leibniz's spacetime had probably a richer structure than what is now called Leibnizian spacetime (Roberts, 2003, p. 553). In Roberts 2003, for instance, it is argued that Leibniz would have even defended a form of absolutism about motion; for Arthur 1994 on the contrary Leibniz was a full-blooded relativist. According to (Jauernig, 2008) it should be possible to reconcile absolutism and relativism about motion by considering the different ontological levels, the dynamical and the phenomenal levels, that characterize Leibniz's thought. See also (Slowik, 2006).

into the other by an automorphism" (Weyl, 1927a, p. 79; tr. Weyl 2009b, p. 73), i.e. by a structure preserving mapping of the space onto itself. "That is now our interpretation of Leibniz's definition of similar figures as figures that are indiscernible if each is considered by itself"(Weyl, 1927a, p. 79; tr. Weyl 2009b, p. 73). Thus, which worlds count as indiscernible depends on what kind of mathematical structure, we are concerned with. Once this is known, we should be able to pinpoint what mappings preserve the structure, that is the automorphism of this structure. In general transformations to which Leibniz refers - the scaling of the entire world or, in his dispute with Clarke, the interchange east with west or displacement of everything three feet east - are not arbitrary since they preserve some "geometrical structure." The original and the copy are indistinguishable precisely because all the relevant geometrical structure that was found before the transformation will appear the same in the transformed situation. In other words, indiscernibility arises because the geometrical structure we are considering does not allow the expression of differences that we might otherwise consider intuitively evident.

This point becomes particularly clear if one considers the story of indiscernibility arguments in the 19th century debate on the foundations of geometry. The rapid development that geometrical thought experienced from the 1830s opened to the protagonists of this debate, such as Helmholtz, Poincaré and, as we shall see, Hausdorff, the possibility of generalizing Leibniz's thought experiments (even if Leibniz is seldom explicitly mentioned): two worlds will be indistinguishable not only if they are congruent or similar, but even if they are mapped onto each other by any continuous deformation whatsoever, only requiring that points that are close together before the transformation is applied also end up close together. In other words, the 19th century debate seems to be dominated by the tendency to consider transformations of space into itself that preserve progressively weaker levels of geometrical structure. Two worlds arising from each other by a transformation which preserves some geometrical structure are to be considered the "same world."

3.1. Hausdorff's Geographical Maps

Helmholtz's and Poincaré's arguments have been abundantly discussed in the literature (see for instance DiSalle 2006). I will therefore try to reconstruct summarily the 19th century geometrical debate from a significant, but less commonly considered, point of view. The German mathematician Felix Hausdorff, in his major philosophical work *Das Chaos in kosmischer Auslese*[6] (Mongré 1898, now in Hausdorff 2004, with the same pagination), published in 1895 under the pseudonym of Paul Mongré ("Paul to-my-liking"), offered a good overview of all the Leibniz-style indiscernibility arguments that were widespread in the period's debate about geometry. Hausdorff is surely better known for his fundamental contributions to topology and set theory than for his early forays into philosophy of geometry, to which scholars' attention has been attracted only recently (Epple 2006, 2007). Nevertheless Hausdorff's reflections on the foundation of geometry provide a valuable overview into the use of indiscernibility arguments in the 19th century philosophical debate about geometry - a debate that, as is well known, profoundly contributed to shaping the new-born "philosophy of science" at the turn of the century (Friedman, 1999).

[6] *Chaos in Cosmic Selection*, as it might be translated in English

Hausdorff considered first of all translations, rotations, mirroring and scaling of the whole universe showing that a consciousness would necessarily remain unaware of them, as long as all meter sticks used to ascertain any change, also underwent the same deformation (Mongré 1898/Hausdorff 2004, pp. 84-94). But according to Hausdorff, one would consider two worlds as indistinguishable even if the objects of the universe were arbitrarily distorted in arbitrary directions, by any deformation whatsoever, only requiring that, in a first approximation, it is free from "discontinuities and singularities" [Unstetigkeiten und Singularitäten] (Mongré 1898/Hausdorff 2004, pp. 84-94): "We would not notice anything of certain space transformation: this is the refrain of my transcendental dialectic" [Von gewissen Transformationen des Raumes würden wir nichts bemerken: das ist der Refrain meiner, transcendentalen Dialektik] (NL FH Kapsel 49; Fasz. 1079; Bl. 26).

This was also the refrain of the whole period's philosophical-geometrical debate, which was entirely dominated by similar thought experiments, showing "in a popular illuminating way" [in populär einleuchtender Weise], "how a space transformation can elude our empirical perception" [dass eine Raumtransformation sich der empirischen Wahrnehmung entzieht] (Mongré 1898/Hausdorff 2004, p. 100). Hausdorff is aware that the "Helmholtzian convex mirror" [Helmholtzens Convexspiegelbild] (NL FH Kapsel 24; Fasz. 71; Bl. 33) already offered an example of such an approach, even if Helmholtz was more interested in the "possibility of visualizing non-euclidean relationships" [die Anschaubarkeit nicht euklidischer Verhältnisse] (Kapsel 24: Fasz. 71, Bl. 65) than of showing the indeterminacy of the space structure. In an fragment of the *Nachlass,* unfortunately undated, Hausdorff confesses that he found similar reasoning "also by others (Poincaré)" [auch bei Andern (Poincaré)] (NL FH Kapsel 49; Fasz. 1079; Bl. 4), who similarly argued that our actual space does not differ from any space that one can derive from it by any continuous deformation whatsoever.

However, Hausdorff could consider such a type of thought experiment only as a special case of what he called the "principle of transposition, transformation principle, mapping principle; principle of substitutability" [Übertragungsprincip, Transformationsprincip, Abbildungsprincip; Princip der Ersetzbarkeit] (Kapsel 24: Fasz. 71, Bl. bl4 [7]).

> Two spaces that are point-wise coordinated to one another, in such a manner that their whole physical content participates in this point-correspondence, produce the same mental image ... Every space stands for a whole class of spaces, among which no differentiation, and thus also no choice, is possible. [Zwei Räume, die einander punktweise zugeordnet sind, derart dass ihr gesamter physischer Inhalt an dieser Correspondenz der Punkte betheiligt ist, erzeugen dasselbe Bewusstseinsbild. ... jeder Raum Repräsentant einer ganzen Klasse von Räumen, zwischen denen keine Unterscheidung, auch also keine Entscheidung möglich ist] (Kapsel 24: Fasz. 71, Bl. 65)

This approach is surely commonplace for someone who is familiar with the work of Helmholtz or Poincaré. However, completely original in *Chaos in kosmischer Auslese* is Hausdorff's attempt to approach the issue using the modern concepts of Cantorian set theory, then new-born (Cantor, 1874, 1878). This seems surprising at first sight. Only a few mathematicians at that time were interested in this field and probably none attempted to apply set theoretical considerations to philosophical and epistemological problems. However, precisely this first unusual contact with Cantor's theory showed Hausdorff the potentialities of the "Mengenlehre," that he later defined as the "groundwork of all mathematics" [das Fundament der gesamten Mathematik] Hausdorff, 1914/2002, p. 1. In 1904 Hausdorff began to work actively in this area, and in 1914, in his celebrated *Grundzüge der Mengenlehre,* he developed the theory for which he is now rightly famous, the theory of "topological spaces," Hausdorff's labeling for sets endowed with an additional structure in which distinct points have disjoint neighborhoods (actually what we now call Hausdorff spaces; see Hausdorff, 1914/2002, p. 718f.; Hausdorff (1927, 1935/2007, p. 226f.)).

However, already in Hausdorff's geometrical-philosophical speculation, one can see the tendency to consider space as point sets endowed with a structure. In particular, Hausdorff considers the empirical and the absolute space as two "point sets." Between these two, one can apply what "has been newly called a transformation or a mapping of a space onto the other" [was man neuerdings eine Transformation oder Abbildung des einen Raumes auf den anderen nennt] (Hausdorff, 2004, p. 82), that is a transformation that preserves some geometrical structure. That idea that difference between the empirical and absolute space "would not fall in our consciousness" [nicht in unser Bewußtsein fallen würde] (Hausdorff, 1903, p. 17) means now that they can be mapped onto one another by such a structure-preserving mapping. "Thus: geometry is not valid for a particular space (the real one), but for all its univocal mappings" [Also: Geometrie gilt nicht von Einem bestimmten Raum (dem wirklichen), sondern von all seinen eindeutigen Bildern].

Once again, which worlds count as indistinguishable depends upon which geometrical structure one wants to preserve. If all the relevant structure is preserved by the mapping (one-to-one and onto, but Hausdorff considered more general cases), then the original and the deformed world would be regarded as "the same" and difference between them would escape any observation. The sense of the expression "the same" is here intended as mere set-theoretical equivalence, a mere "mapping, coordination, correspondence" [Abbildung, Zuordnung, Correspondenz] between sets:

> Mapping in the sense of correspondence, coordination. The usual parlance assigns to these "pictures" a certain similarity with the original. In this sense one usually says that mind-processes are signs, not "pictures" of the external world, or that the words are signs, and not picture of the concepts. Geographical charts provide already a freer conception of the concept of "mapping." [Abbildung im Sinne von Zuordnung, Correlation. Der gewöhnliche Sprachgebrauch schreibt dem „Bilde" eine gewisse Ähnlichkeit mit dem Original zu; in diesem Sinne sagt man, dass die Bewusstseinsvorgänge Zeichen, nicht Bilder der Aussenwelt oder die Worte Zeichen, nicht Bilder der Begriffe seien. Geographische Karten führen schon zu freierer Auffassung des Begriffs Abbildung. (NL FH Kapsel 49; Fasz. 1079; Bl. 8)

If the relationship between absolute and empirical space is conceived as an "Abbildung," a "mapping" in the set theoretical sense, it becomes clear that it does not make any sense "to demand any congruence or similarity from such a mapping" [von dieser Abbildung Congruenz oder Ähnlichkeit oder derlei zu verlangen] (Kapsel 49: Fasz. 1077, Bl. 4).

In 1903, in his inaugural lecture about "Das Raumproblem" as an extraordinarius at Leipzig University, in order to give an idea of this procedure, Hausdorff effectively compares the empirical space exactly to a geographical map of the absolute space: "If this conception is correct, then the original can undergo every transformation whatsoever, without any change in the mapped copy: exactly as you cannot recognize in a geographical map, if it was drawn from the original or from another geographical map" [Wenn diese Auffassung richtig ist, so muß man das Urbild einer beliebigen Transformation unterwerfen können, ohne daß das Abbild sich verändert: gerade so wie man einer Karte nicht ansehen kann, ob sie nach dem Original oder nach einer anderen Karte gezeichnet ist] (Hausdorff, 1903). Thus in order to know the nature of the absolute space starting from the empirical one, one has to know which kind of projection has been used, that is, what kind of structure has been preserved by the mapping:

> Our empirical space is like a three-dimensional geographical chart, a mapping of the absolute space; but we lack the map legend, we do not know the mapping principle and hence we do not know the original. Between both spaces there is an unknown, and arbitrary relationship or correspondence, a completely arbitrary point-transformation [...] [T]he deformation however does not fall into our consciousness, because not only the objects, but also we, and our measuring instruments are similarly deformed. [Nun, unser empirischer Raum ist solch eine körperliche Karte, ein Abbild des absoluten Raumes; aber es fehlt uns der Eckenvermerk, wir kennen das Projektionsverfahren nicht und kennen folglich auch das Urbild nicht. Zwischen beiden Räumen besteht eine unbekannte, willkürliche Beziehung oder Korrespondenz, eine völlig beliebige Punkttransformation ... die Verzerrung fällt nicht in unser Bewußtsein, weil nicht nur die Objekte, sondern auch wir selbst und unsere Meßinstrumente davon gleichmäßig betroffen werden.] (Hausdorff, 1903, p. 17)

For example, a map of the world shown in Mercator's projection accurately depicts only the equatorial regions of the Earth's surface. As one moves nearer and nearer to the polar regions, so the features of the map become progressively distorted. This distortion is particularly pronounced for Greenland and Antarctica, which become drawn out horizontally far in excess of their true proportions. The reason for this is well known, of course, it being simply due to the fact that the surface of the Earth is spherical, and it is not possible to represent a curved surface on a flat map without distortion. However, since all measuring instruments would be equally distorted, someone living in the distorted situation would not notice the difference. The only way to know the structure of the absolute space from the empirical one, our geographical map, is to know the *Abbildungsverfahren*, the kind of "mapping," that has been used to draw the map (Mercator, Stereographic, etc.). The empirical space is "no faithful copy of the absolute one, but only a mapping according to an arbitrary, indeterminable projection principle" [keine getreue Kopie des absoluten, sondern nur sein Abbild nach einem beliebigen, unbestimmbaren Projektionsverfahren] (RP 17). If we don't know which structure (angles, areas, geodesics, etc.) has been preserved by the mapping, there is no way to infer the "real" geometry of space.

4. The Point-Coincidence Argument as a Leibniz-Style Indiscernibility Argument: the Logical Empiricist Interpretation of General Relativity

Helmholtz and Poincaré, but also Hausdorff/Mongré, are the authors to whom Moritz Schlick refers in his celebrated *Raum und Zeit in der gegenwärtigen Physik* (Schlick, 1922/2006, tr. Schlick, 1978a), which achieved four editions from 1917 to 1922. If Helmholtz's and Poincaré's influence on Schlick is well known and well documented in the literature (Coffa, 1991; Pulte, 2006; Ryckman, 2005; Friedman, 1995), the relationship between Schlick and Hausdorff has only recently been brought to the attention of a larger audience (Epple, 2006). For the aims of this paper, it is interesting in particular that, after an exchange of letters in 1919-20,[7] Schlick, in the fourth edition of his work, felt obliged to add a note to recognize the value of Hausdorff's philosophical reflections.[8] This says a lot about Schlick's position regarding the connection between 19th century Leibnizian indiscernibility arguments and the philosophical interpretation of the newborn theory of General Relativity.

In particular, the third chapter of Schlick's booklet is entirely dominated by the reproduction of Leibniz-style thought experiments. Schlick starts "by considering the case in which the imaginary transformed world is geometrically similar to the original one," then he imagines that "the dimensions of all objects are lengthened or shortened in one direction only," and finally he considers the case where "the objects in the universe are arbitrarily distorted in arbitrary directions" (Schlick 1922/2006, p. 202; tr. Schlick 1978b, p. 227). Schlick's conclusion is invariably the same: as long as we suppose that "all measuring instruments, including our own bodies" share the same deformation, "the whole transformation immediately becomes unascertainable" (Schlick 1922/2006, p. 202; tr. Schlick 1978b, p. 227):

> In mathematical phraseology we can express this result by saying: two worlds, which can be transformed into one another by a perfectly arbitrary (but continuous and one-to-one) point-transformation, are, with respect to their physical reality, identical. That is: if the universe is deformed in any way, so that the points of all physical bodies are displaced to new positions, then [...], no measurable, no "real" change has happened at all, if the co-ordinates of a physical point in the new position are any arbitrary functions whatsoever of the co-ordinates of its old position. (Schlick 1922/2006, p. 204; tr. Schlick 1978b, p. 227)

Moving from the consideration of space to that of spacetime, it was easy for Schlick to use this sort of argument in the context of General Relativity. The connection between the 19th century debate and the new theory can be found in the, as John Stachel has conveniently labeled, "point-coincidence argument"; an argument that Schlick could read in Einstein's 1916 review article *Die Grundlage der allgemeinen Relativitätstheorie*. Einstein, as is well known, maintained that the physical content of a theory is exhausted by the catalog of the "spacetime coincidences" or "verifications of ... meetings of the material points of our measuring instruments with other material points" (CPAE 6, Doc., 30, p. 291f.). According to Schlick, this implies that all worlds that agree on such coincidences are equivalent, and that a choice among them is the result of an arbitrary stipulation (an implication that Schlick called "the geometrical relativity of space"):

> All world pictures which lead to the same laws for these point-coincidences are, from the point of view of physics, in every way equivalent. We saw earlier that it signifies no observable, physically real, change at all, if we imagine the whole world deformed in any arbitrary manner, provided that after the deformation the co-ordinates of every

[7] The letters are preserved in Noord-Hollands Archief in Haarlem (NL): 102/Haus-2 Letter to Schlick (23.2.1919, Greifswald); 102/Haus-2 letter to Schlick (17.7.1920, Greifswald).

[8] "Unfortunately, only after the publication of the second edition of this writing have I learned about the most astute and fascinating book [Das Chaos in kosmischer Auslese]. The fifth chapter of this monograph gives a very perfect presentation of the considerations that follow in the text above. Not only Poincare's reflections, but also the extensions added above have been anticipated there" (Schlick 1922/2006, p. 198, note 1, tr. Schlick 1920, p. 24, note 1).

physical point are continuous, single-valued, but otherwise quite arbitrary, functions of its co-ordinates before the deformation. Now, such a point-transformation actually leaves all spatial coincidences totally unaffected; they are not changed by the distortion, however much all distances and positions may be altered by them. For, if two points A and B, which coincide before the deformation (i.e. are infinitely near one another), ... as a result of the deformation ... must be at the same point (or infinitely near) A. Consequently, all coincidences remain undisturbed by the deformation. (Schlick 1922/2006, p. 232; tr. Schlick 1978b, p. 227)

Schlick's interpretation of the point-coincidence is usually considered the expression of a form of verificationism, as a commitment to a strong observability requirement. In this sense the point-coincidence remark has been famously defined as "the beginnings of the empiricist and verificationist interpretation of science characteristic of later positivism" (Friedman, 1983, p. 24). It is indeed the case that Einstein's point-coincidence argument fascinated contemporary philosophers for its verificationist turn of phrase. But the argument actually offered them much more. It allowed them to exorcise the novelty of Einstein's newborn theory, by simply inserting it in the context of the by then already familiar 19th century debate about geometry. Einstein's point-coincidence argument is just one of those many Leibniz-style philosophical arguments that one can find in Helmholtz, Poincaré or, as we have seen, Hausdorff (Friedman, 1983, p. 47).

Schlick had by that time reached the philosophical stature necessary to transform such an interpretative proposal into the received view. As is well known, he was a trained physicist, having taken his doctorate under Max Planck, who notably singled out Schlick as one of his best students, together with Max von Laue, later awarded the Nobel Prize. But most importantly, Schlick and Einstein were in correspondence by late 1915; Schlick sent Einstein a copy of his paper on the philosophical significance of the theory of relativity (CPAE 8a, Doc. 296; 4 February 1917), which Einstein famously praised for its "unsurpassed clarity and perspicuousness [Übersichtlichkeit]" (CPAE 8a, Doc. 297, p. 389; 6 February 1917; see also Doc. 165; 14 December 1915). When in 1922 Schlick was appointed to the chair in philosophy held earlier by Ernst Mach and by Ludwig Boltzmann at the University of Vienna, he was already the recognized philosophical authority on the subject of relativity.

Schlick's influence on the philosophical debate on geometry and relativity was enormous. The young Rudolf Carnap in his doctoral Dissertation *Der Raum* (Carnap, 1922) drew mostly upon from Schlick's interpretation of Einstein's passage on the geometrical interpretation of the point-coincidence argument - the idea that point-coincidences are the only topological invariant, and are therefore "unambiguous", whereas anything else - projective, affine or metric structure is the result of a stipulation. Later Carnap remained essentially faithful to this approach, even after abandoning the Husserlian/Kantian framework in which it had originally been developed. And it was after having corresponded with Schlick that Hans Reichenbach (1923/24), for whom Einstein later created a chair in the philosophy of science in the physics department at Berlin, quickly modified his early Neo-Kantian interpretation of the theory (Reichenbach, 1920/1977) in the direction of the metric conventionalism for which he later became famous with his classic *Philosophie der Raum-Zeit-Lehre* (Reichenbach 1928/1977 translated as *Philosophy of Space and Time* in Reichenbach 1928/1958).

Reichenbach was perhaps the first among Logical Empiricists to insist on the fact that the indiscernibility arguments that flourished in 19th century philosophy of geometry were nothing but variations of Leibniz's argument in the Leibniz-Clarke correspondence (Reichenbach, 1924). Such arguments, in Reichenbach's "verificationist" reading, express the idea that "it is meaningless to postulate differences in objective existence if they do not correspond to differences in observable phenomena" (Reichenbach 1928/1977, p. 495; tr. Reichenbach 1928/1958, p. 210). Reichenbach felt a profound philosophical affinity with Leibniz. According to Reichenbach, Leibniz "went so far as to recognize the relationship between causal order and time order" (Reichenbach, 1949/1951, p. 300), and most of all with his "principle of the identity of indiscernible, discernible in connection with the verifiability theory of meaning," he laid the foundation of what Reichenbach calls the "theory of equivalent descriptions" (see for instance Lehrer 2004; Klein 2001) where Leibniz's indiscernibility explicitly floods into the period's debate about geometry .

Reichenbach's approach is well known. Only a set of different descriptions, rather than a single description, can correctly describe the geometry of physical space, in as much as these "different geometries can be represented on one another by a one-to-one correspondence" (Reichenbach, 1949/1951, p. 298), so that all objects are assumed to be deformed in such a way that the spatial relations of adjacent bodies remain unchanged:

> In this context belongs the assumption that overnight all things enlarge to the same extent, or that the size of transported objects is uniformly affected by their position. Helmholtz's parable of the spherical mirror comparing the world outside and inside the mirror is also of this kind; if our world were to be so distorted as to correspond to the geometrical relations of the mirror images, we would not notice it, *because all coincidences would be preserved.*" (Reichenbach 1928/1977, p. 38-, tr. Reichenbach, 1928/1958, p. 27; my emphasis)

According to Reichenbach, Einstein's theory of relativity is the result of the recognition of such a "relativity of geometry," the recognition that a different choice of a "coordinate definition" of rigid bodies or straight lines may yield different geometrical descriptions of the world, that are however physically equivalent.

The connection of this strategy with Leibnizian indiscernibility arguments is made particularly clear by Rudolf Carnap in a passage of his *Philosophical Foundations of Physics* (Carnap, 1966), published in 1966:

> Leibniz, the reader may recall, had earlier defended a similar point of view. If there is in principle no way of deciding between two statements, Leibniz declared, we should not say they have different meaning. If all bodies in the universe doubled in size overnight, would the world seem strange to us next morning? Leibniz said it would not. The size of our own bodies would double, so there would be no means by which we could detect a change. Similarly, if the entire universe moved to one side by a distance of ten miles, we could not detect it. To assert that such a change had occurred would, therefore, be meaningless. Poincaré adopted this view of Leibniz's and applied it to the geometrical structure of space. (Carnap, 1966, p. 148)

This passage clearly shows that the strategy adopted by Logical Empiricists to assimilate General Relativity is disarmingly simple: Einstein's point-coincidence argument is simply a Leibniz-style indiscernibility argument, the same kind of argument that dominated the 19th century debate on the philosophical foundation of geometry. If Helmholtz, Poincaré or Hausdorff applied their arguments to space alone, Einstein simply extended them to spacetime, that is to the intersection of world lines: a "coincidence" of two world lines presupposes nothing concerning the metrical relations of space and time, so metrical properties of spacetime are deemed less fundamental than "topological" ones. The "objective" system of coincidences does not depend on an observer, it is therefore "independent of all arbitrariness" and an "ultimate fact of nature," whereas the metric relations are frame-dependent and conventional. Point-coincidences represent a sort of fixed framework in which we can formulate an equivalence class of physically possible geometries mapped onto each other by one-to-one continuous transformations (that is probably what we would called "diffeomorphisms"): "Topological properties turn out to be more constant then the metrical ones", so that the transition from the special theory to the general one should be interpreted as "a renunciation of metrical particularities while the fundamental topological character of space and time remains the same" (Reichenbach 1924, p. 115; tr. Reichenbach, 1924/1969, 195; see Ryckman 2007, 2008).

The verificationist "flavor" of such a reading of the point-coincidence argument, the idea (on which Howard 1999 famously insisted) that point-coincidences are taken to be real because of their observability, and *thus* they qualify as invariant, seems however to induce Logical Empiricists to underestimate the consequences of such an unrestrained use of Leibnizian arguments. In such arguments, as we have seen, indiscernibility does not arise at all from the impossibility of *observing* certain differences *physically*, but from the impossibility of *expressing* them *geometrically*. Just as Euclidean space does not allow one to establish the difference between left or right without a "coordinative definition", a bare "topological" space would not allow one to "observe" the distinction between straight and curved lines. From such a conclusion, it is not hard to prognosticate that Logical Empiricists' interpretation of General Relativity, despite its undisputed historical relevance, was destined to failure from a theoretic point of view. Probably only the implementation of a generalized "Machian" point of view in which "the inertial force can be interpreted … as a dynamic gravitational effect" (Reichenbach 1928/1977, p. 247; tr. Reichenbach, 1928/1958, p. 214), prevented them from seeing that the interpretation they were suggesting would have made General Relativity dynamically empty.

4.1. The Failure of the Logical Empiricist Interpretation of General Relativity

If the interpretation of the point-coincidence argument as Leibniz-style indiscernibility argument can be considered the core of the Logical Empiricist interpretation of General Relativity, at the same time then it is the reason for its substantial inadequacy. As we have tried to show, Leibniz's indiscernibility arguments express the global symmetries of space or spacetime: two universes mapped by some kind of deformation would be indistinguishable, if all relevant structure were preserved, since all that the theory considered geometrically relevant would appear the same in both universes. If the point-coincidence argument were an indiscernibility argument of this kind, its result would be therefore to enlarge the spatio-temporal symmetry group of spacetime to the group of all one-one, bi-continuous point transformations: only the topological features of events are preserved by this group of transformations, that is, the notion of the "coincidence" of two events and the notion of two events being near one another in spacetime.

From a contemporary perspective, the mistake of such an account is easy to recognize: the symmetry group of an arbitrary general relativistic spacetime is not the widest group of all smooth coordinate transformations that preserve only point-coincidences, but the narrowest one consisting of the identity alone. The use of the full group of admissible transformations in General Relativity does not imply that we are working in the context of a very weak, though fixed, geometrical structure, but, rather, that we are working in the context of a highly structured spacetime, endowed with a perfectly definite, although variable, metric. In this context indiscernibility arguments in the sense of Leibniz do not even make any sense. In an inhomogeneous space it really would make a difference if one would shift everything into a region of increasing spatial curvature, and the consequences of doubling would depend on where the doubling were carried out (Nerlich, 1994, p. 152).

When Weyl (1924b), upon whose authority we wish once more to rely, reviewed Reichenbach's *Axiomatik der relativistischen Raum- Zeit-Lehre*, (Reichenbach 1924, tr. Reichenbach, 1924/1969) he found the book "not very satisfactory, too laborious and too obscure" [wenig befriedigend zu umständlich und zu undurchsichtig](Weyl, 1924b). Beyond mathematical technicalities, what Weyl probably felt to be the most philosophically extraneous was Reichenbach's identification of the "philosophical achievement" of General Relativity with the separation between the factual and conventional components, those which are fixed once and for all, and those which result from the stipulations of rigid rods and ideal clocks. Weyl's reliance on the use of trajectories of force-free mass points in the construction of the metric (Ehler, 1988)[9] is, on the contrary, the result of a completely different philosophical attitude (Ryckman, 1995, 2005). As it is well known, according to Weyl, the main novelty of Einstein's theory of gravitation was that it had transformed the "guiding inertial structure," that counts as standard for no-acceleration, from "a rigid geometric property of the world, fixed once for all" (Weyl, 1934/2009, p. 134) to a "guiding field" (*Führungsfeld*, as Weyl famously called it) "a physical reality which is dependent on the state of matter" (Weyl 1921/1968, p. 141 tr. Weyl 1921/2009, p. 21): "The distinction between guidance [Führung] and field is preserved, but *guidance has become a field* (as the electromagnetic field" [An dem Dualismus von Führung und Kraft wird also festgehalten; *aber die Führung ist ein physikalisches Zustandsfeld* (wie das elektromagnetische)] (Weyl, 1924a, p. 198).

The philosophical meaning of General Relativity should be sought not in the distinction between arbitrary and non-arbitrary structures, but in that between dynamical and non-dynamical ones. The difference between Galilean spacetime and Special Relativistic spacetime lies in the difference between their inertial structures. Nevertheless, both structures are non-dynamical in the sense that they are independent of their contents: the unique affine connection (compatible with the spacetime metric), it is said, provides a standard for absolute acceleration and rotation. The radical novelty of General Relativity does not consist in weakening such a fixed background structure, but in transforming the fixed background into a dynamical one. Since all bodies are influenced by gravity in precisely the same way (equivalence principle), there are no physical phenomena independent of gravitation that might serve to measure the background spacetime geometry; on the contrary we can measure the acceleration of a particle in a magnetic field relative to the inertial trajectory of a body that is not affected by magnetism. In other terms, there is no unique decomposition of the affine structure into an inertial structure and the deviation from this structure caused by gravitation.

[9] So Weyl summarizes his position: "the metrical structure of the world is already fully determined by its inertial and causal structure, that therefore mensuration need not depend on clocks and rigid bodies but that light signals and mass points moving under the influence of inertia alone will suffice" (Weyl, 2009b, p. 103).

4.2. The Point-Coincidence Argument as a Response to the Hole Argument

Even if the distinction between dynamical and non-dynamical structures (or dynamical and absolute objects, in James Anderson's terms; Anderson 1967) turned out to be "not sufficiently sharp" (Giulini 2007; see Pitts 2006), nevertheless it is now a widely shared opinion that it better expresses the philosophical spirit of General Relativity than the Logical Empiricists' distinction between arbitrary and non-arbitrary structure. It is not the case that the metric that was non-arbitrary in previous theories has become arbitrary, but that the metric that was non-dynamical has become dynamical - it has become a field among others.

As we argued, the Logical Empiricists' failure to grasp this point seems connected to their interpretation of the point-coincidence argument as a 19th century radicalization of Leibnizian indiscernibility arguments. This mistake is of course at least partly comprehensible. The echo of the widespread use of such arguments in one of the most exciting philosophical debates of the turn of century was still vivid. Moreover Einstein's "Helmholtzian" insistence on the importance of practically rigid rods and clocks (Einstein, 1921, 1925) could be easily seen as the authoritative confirmation that this was the correct interpretative context (Ryckman, 1996; see also Howard, 2005). Only recent scholarship initiated by John Stachel (Stachel, 2002) discovered the key to a proper understanding of the point-coincidence argument, with the help of Einstein's correspondence from that period . The meaning of what we may call (following Rynasiewicz 1999) the *public point-coincidence argument* can be understood only if one knows the *private point-coincidence argument* as a response to the infamous "hole argument." The details of both versions of the argument have been rehearsed many times in recent literature, so it does not seems necessary to repeat here the "Hole Story," as Earman and Norton ingeniously called it. I will attempt anyway a brief exposition.

(I) The *public point-coincidence argument*, as we said, appeared for the first time in Einstein's 1916 review article on General Relativity, and it is used to express the formal request of general covariance, i.e. of coordinate-independent formulation of the laws of nature: "The laws of nature are only propositions about spatio-temporal coincidences; therefore they find their natural expression in generally covariant fields equations" (CPAE 7, Doc. 4, 38; *Prinzipielles zur Relativitätstheorie*, 1918). In this context the argument has nothing to do with indiscernibility, as the Logical Empiricists believed. If coincidences are all that matters physically, then we ought to be able to use any coordinate system, since all coordinate systems necessarily agree on such coincidences (Norton, 1995). The young Wolfgang Pauli (at his third semester), in his celebrated "Enzyklöpedie" article on relativity (Pauli, 1921), sums up this line of reasoning clearly:

> All physical measurements amount to determination of spacetime coincidences; nothing apart from these coincidences is observable. If however two point events correspond to the same coordinates in one Gaussian coordinate system, this must also be the case in every other Gaussian coordinate system. We therefore have to extend the postulate of relativity: The general physical laws have to be brought into such a form that they read the same in every Gaussian coordinate system, i.e. they must be covariant under arbitrary coordinate transformations (Pauli, 1958, p. 149).

In this form the argument is customarily regarded as physically vacuous, since such a requirement can be satisfied by virtually all theories, independently of the content of the laws. The unavoidable reference is to the now familiar argument of the young mathematician Erich Kretchman, who as early as 1917 (Kretschmann, 1917) turned the point-coincidence argument against Einstein: if the physical content of every spacetime theory is exhausted by the catalog of spacetime coincidences, that is by "topological relations," then for this very reason all spacetime theories can be given in a generally covariant formulation (Rynasiewicz, 1999).

(II) The *private point-coincidence argument* is a response to the "hole argument," which probably first occurred to Einstein by November 1913 (Stachel, 1980/2002) at the latest. In this form the argument appears actually as a sort of indiscernibility argument. As it is well known, Einstein was worried that by means of a coordinate transformation, two different solutions of generally covariant field equations would arise at a single point within a hole (a region devoid of matter and energy), whereas the sources outside the hole have not changed. Einstein overcame this difficulty when he came to realize that two sets of field lines that intersect in the same way, that is that agree on point-coincidences, define the same physical situation, since we do not have any means to separate a background system of rigid grids from the field lines. Hence, two metric fields whose geodesics intersect one another in the same way are the "same" metric field.

The nature of the point-coincidence argument as an indiscernibility argument is probably nowhere more in evidence than in Einstein's correspondence with Paul Ehrenfest in late December 1915 and early January 1916 (CPAE 8a, Doc. 173, 26 December 1915, and Doc. 180, 5 January 1916). Ehrenfest, in a letter that no longer exists, presumably asked Einstein to consider a situation in which light from a distant star passes through one of Einstein's holes and then strikes a screen with a pinhole in it that directs the light onto a photographic plate. The question is whether the same point on the photographic plate would have received the light after a coordinate transformation. In fact, the coordinate transformation would change the metric in the hole, determining a different geodesic trajectory of the light rays.

In his answer (CPAE 8a, Doc. 180), Einstein imagines representing the situation described by Ehrenfest "on completely deformable tracing paper [Pauspapier]". If one deforms the tracing paper arbitrarily then one would obtain a solution that "is mathematically a different one from before" (CPAE 8a, Doc. 180, p. 238). Deforming the paper means deforming the coordinate systems, and according to the well-known rules of tensor calculus, one would obtain a different metric field, and therefore a different geodesic trajectory of light rays. Einstein's worries that this would jeopardize the "univocality" of the description of nature disappeared, when he realized that this is only a mathematical difference - "*physically it is exactly the same.*" (CPAE 8a, Doc. 180, p. 239; my emphasis). In fact the background coordinate system (the orthogonal drawing paper coordinate system) with respect to which the situation would have appeared deformed "is only something imaginary [eingebildetes]": "What is essential is this: As long as the drawing paper, i.e., 'space,' has no reality, *the two figures do not differ at all.* It is only a matter of "coincidences," e.g., whether or not the point on the plate is struck by light. Thus, the difference between your solutions A and B becomes a mere difference of representation, with *physical agreement*" (CPAE 8a, Doc. 180, p. 239; my emphasis; tr. from Howard and Norton 1993).

4.3. Weyl on Leibniz's and Einstein's Indiscernibility Arguments.

Logical Empiricists' interpretation of the point-coincidence argument fails to grasp the meaning of both versions of the argument (Ryckman, 1992; Howard, 1999). The public point-coincidence argument is not an indiscernibility argument at all. The private indiscernibility argument is an indiscernibility argument, but, as we shall see, not in the Leibnizian sense. I argued that the reason for such a failure is not so much the consequence of a rather clumsy attempt to find eminent precursors of their "verificationist" point of view. It is rather the fact that, given the philosophical context in which they developed their interpretation, they could hardly resist the temptation to interpret the argument *more geometrico*, as one of the many Leibniz-style argument that dominated the 19th century debate on the foundations of geometry. Only recently has the knowledge of Einstein's correspondence convincingly shown that these remarks of Einstein's linking general covariance and point-coincidences should be understood against a completely different background, one that the Logical Empiricists could not, if not partially, have known (see Einstein's letter to Schlick, CPAE, Doc. 165; 14 December 1915). However, it is significant that it was precisely Hermann Weyl had already clearly seen the difference between Leibniz-style and Einstein-style indiscernibility arguments during roughly the same years when the Logical Positivists were publishing their philosophical reflections on relativity.

Weyl provides on many occasions his own version of a indiscernibility argument *á la* Leibniz applied to spacetime: "Let us imagine the four-dimensional world as a mass of plasticine traversed by individual fibers, the world lines of material particles" (Weyl 1927b, p. 73; tr. Weyl 2009b, p. 105). According to Weyl it is "impossible to *distinguish conceptually* between the system of lines and the system of curves resulting from them by a spatial deformation" (Weyl 1927b, p. 21; tr. Weyl 2009b, p. 24; my emphasis). In fact "only such relations have an objective significance as are *preserved under arbitrary deformations of the plasticine.* The *intersection of two world lines* is, for instance, of this kind" (Weyl 1927b, p. 73; tr. Weyl 2009a, p. 129; my emphasis). In this way, however, the world would be an "amorphous continuum without any structure" (Weyl 1934/2009, p. 129) or better without any "post-differential" structure: "Only statements concerning the *distinctness or coincidence of points* and the continuous connection of point configuration can be made at this stage", but it would be impossible to "distinguish the straight lines from the curved ones" (Weyl, 1932/2009, p. 41; my emphasis). Thus every "guiding structure," every standard for distinguishing inertial motions and deviation from the inertial motion would have been lost. The consequence of the Leibnizian strategy stubbornly pursued by Logical Empiricists is here effectively described by Weyl. Such a strategy clearly destroys the possibility of dealing with the problem of the relativity of motion, since "no solution of the problem is possible" as long as "one disregards the structure of the world" (Weyl 1927b, p. 65; tr. Weyl 2009b, p. 105; see Coleman and Korté 1984).

Einstein's indiscernibility argument, the point-coincidence argument as a response to the hole argument, cannot therefore be confused with an indiscernibility argument in the sense of Leibniz. The great achievement of General Relativity, as Weyl never tires of telling, lies in the fact that "the inertial structure of the world is not rigid, but flexible, and changes under material influences" (Weyl, 1934/2009, p. 133). The standard for distinguishing between inertial and non-inertial motion has itself become dynamical, that is, in Weyl's parlance, a "guiding field": acceleration means deviation from the trajectories of particles subject only to gravitation, trajectories that, however, depend in turn on the contingent distribution of matter. In this context an indiscernibility argument, as Weyl writes in a nice scientific-philosophical dialog, *Massenträgheit und Kosmos,* published in

1924, serves to avoid “the mistake that Einstein committed in 1914” [den gleichen Fehler, den Einstein 1914 machte] (Weyl, 1924a, p. 202).

The reference is clearly to the “hole argument” that was first published in January 1914 (CPAE 4, Doc. 26; January 1914) in the “Bemerkungen” added to the article version of the *Entwurf* paper written with Grossman (first published as separatum CPAE 4, Doc. 13; 1913) and repeated in two papers both dating from early 1914 (CPAE 4, Doc. 25, January 1914; CPAE 6, Doc. 2, May 1914).

In fact Weyl first observed that “*if the matter disappears*, the guiding field must become *undetermined*” [bei verschwindender Materie muss das Führungsfeld unbestimmt werden] (Weyl, 1924a, p. 202; my emphasis). The very same physical state can be realized in infinite possible mathematical ways.[10] In fact, “if the laws of nature are invariant under arbitrary coordinate-transformations, then I will get from a solution [of the field equations] by means of a transformation, infinitely many new ones” [wenn die Naturgesetze invariant sind gegenüber beliebigen Koordinatentransformationen, so erhalte ich aus einer Lösung durch Transformation unendlich viele neue] (Weyl, 1924a, p. 203). Weyl’s reference to “Einstein’s mistake” is particularly significant, since in 1913, as Einstein was working on the *Entwurf* theory, Weyl was his colleague at the ETH in Zürich (Weyl came to Zürich in Fall 1913, whereas Einstein left Zürich for Berlin in Summer 1914).

In *Massenträgheit und Kosmos* Weyl reformulates an indiscernibility argument *à la* Einstein in the following way: “I divide the world in two parts through a three-dimensional cut that separates both its edges” [Teile ich die Welt durch einen dreidimensionalen Querschnitt, welcher ihre beiden Säume, voneinander trennt, ... in zwei Teile]. Weyl’s reformulation of the hole argument is similar to that of David Hilbert in 1917 (Renn and Stachel, 2007), in using a open space-like hypersurface (a Cauchy surface) that separates the future from the past, rather than a closed hypersuface as Einstein. Then, Weyl continues, “if I apply only those [coordinate] transformations that leave unchanged the part ‘below’” [verwende [Ich] nur solche Transformationen, welche die “untere” Hälfte unberührt lassen], but change the metric field in the part above, “then all these solutions [of the field equations] will describe also in the underpart the *same state evolution* as of the original ones” [so stimmen alle diese Lösungen gleichwohl in der unteren Welthälfte mit der ursprünglichen überein] (Weyl, 1924a, p. 203; emphasis mine). According to Weyl, Einstein’s mistake depends on the fact that he initially overlooked “that there was a difference only if the four-dimensional world were a resting medium” [daß ein Unterschied nur bestünde, wenn die vierdimensionale Welt ein stehendes Medium wäre] (Weyl, 1924a, p. 203). However, as Weyl immediately emphasizes “such a resting Medium ... is completely repudiated by the theory of relativity” [Ein solches stehendes Medium wird aber ... von der Relativitätstheorie durchaus geleugnet] (Weyl, 1924a, p. 203).

Einstein’s kind of indiscernibility argument, as Weyl’s exposition shows, does not imply that the inertial structure has been dissolved in a Leibnizian/Machian way, but that it has been, as Weyl wrote in an essay of 1925, “so to speak, *freed from space*” [vom Raume abgelöst]. It has become “an existing field within the remaining structureless space” [sie wird zu einem in dem zurückbleibenden strukturlosen Raume existierenden Feld] (Weyl, 1925/1988, p. 4). Differences that would appear only with respect to such a structureless space are not real differences. On the other hand, Weyl shows that the strategy implied

[10]“So gibt es, doch unendlich viele Möglichkeiten, wie sich dieser Zustand im Weltcontinnum realisieren kann” (Weyl, 1924a, p. 202; my emphasis)

in Leibniz's indiscernibility arguments, to which the Logical Empiricists resorted, has simply, as we may say, *freed space* from the minimum of structure that can play the role of inertial guidance. In these two opposite manners of grasping Einstein's famous remark that spacetime has lost its "last vestige of physical reality," one can recognize most easily the different consequences of Leibniz's and Einstein's indiscernibility arguments.

It may be worth mentioning, even if only in passing, that Weyl's insistence on the fact that, in General Relativity, the "metric field has been freed from the manifold" (Korté, 2006, pp. 193, 201) should probably be understood in the context of his attempt to define mathematically the concept of "manifold" set-theoretically, via the concept of "neighborhood" (*Umgebung*, Weyl 1913). Such an attempt, initiated by Hilbert (1902) in the same years, was, as Weyl recognizes, was pursued "more systematically [Systematischer]" "by Hausdorff in his *Grundzügen der Mengenlehre* (1914)" (Weyl 1925/1988, p. 4; the reference is to Hausdorff 1914/2002). By contrast Einstein's work is based on an older approach, where the "manifold" was considered as a number manifold, as the manifold of all possible values of x, y, z, t. Thus it could be argued, that Einstein was forced to introduce his indiscernibility argument precisely in order to "wash out" this additional structure (Norton, 1989, 1992, 1999).

5. On Learning from the Mistakes of Logical Empiricists. Some Lessons for the Recent Debate

My attempted historical-critical analysis of the Logical Empiricists' misunderstanding of the point-coincidence argument as an indiscernibility argument in the sense of Leibniz is, in my opinion, instructive for the animated debate stirred by Earman and Norton's fundamental paper (1987). Weyl's version of the "hole argument" is astonishingly similar to that of Earman and Norton (much more so than Einstein's original version), and also addresses the same "substantivalist" opponents; but at same time Weyl is also careful to distinguish the hole/point-coincidence argument from a Leibnizian indiscernibility argument.

The point-coincidence argument cannot be simply considered a "stronger version of a famous argument due to Leibniz himself against Newton's substantival ontology of space" (Janssen, 2005, p. 74). The analogy between Leibniz's and Einstein's arguments could actually appear *prima facie* very plausible. Leibniz considered two material universes as indistinguishable or as the "same universe." Einstein, roughly two centuries later, found himself likewise considering two field configurations as empirically indistinguishable and thus physically identical. In both cases there are apparently different "possible worlds" allowed by the theory that actually correspond to the same physical reality. Leibniz referred to alternative worlds that differ from the actual one only in position, orientation or magnitude, but agree in the measure of the angles and proportions of lengths. Einstein could imagine alternative worlds that agree exactly with the actual world outside the hole, while differing within the hole. In both cases what seems at first sight a dramatic difference reveals itself actually as being no difference at all, since the difference is declared irrelevant.

However, the similarity between the two arguments is only apparent. The difference should strike the reader when they simply consider the different structures of the two arguments. Leibniz-style arguments, as we have seen, always presuppose a transformation that affects every physical entity *without exception*; if this condition is violated, the two situations could be easily distinguished. But Einstein's argument explicitly violated this condition, allowing a transformation that leaves everything unchanged *with the exception* of a region devoid of matter. Of course both are indiscernibility arguments. They both aim to fictionally eliminate the reference to a fixed standard against which the change can be measured, so that the apparent change would reveal itself to be no change at all. However, such a standard is clearly different in each case: Leibniz-style arguments dissipate the illusion of a transformation that would appear only with respect to some physical entity; Einstein's argument dissolves the appearance of a transformation that would emerge, only while referring to the rigid geometry of the empty space.

The now usual exposition of physical theories in terms of models, where the models are intended to represent the physically possible worlds that satisfy the laws of the theory, can be useful to see this point. I will provide a slightly more formal presentation in the appendix. Each model comprises a differential manifold and various geometric objects on it, such as metric and matter fields. Indiscernibility arguments in the sense of Leibniz work well in a theory with global symmetries that presupposes that all relevant geometrical structure of space appears "the same" in all universes or models governed by the theory, so that the theory does not have the conceptual resources to distinguish among them. In General Relativity, on the contrary, there are no non-trivial symmetries, except identity. Therefore there is no spacetime background that would look the "same" across all possible universes or models allowed by the theory, so that any reference to such a background cannot be used to distinguish among them.

Thus in theories where spacetime is endowed with global symmetries, it makes perfect sense to apply Leibniz's indiscernibility arguments. Such arguments after all simply postulate a *trivial identity* (Stachel and Iftime, 2005) of all models of the theory: it is, so to speak, the very same model or better - in Leibniz's terms - models that are different *solo numero*. On the contrary, Einstein's indiscernibility arguments make sense in theories without global symmetries, where one has to deal with a plurality of different models that are declared *non-trivially equivalent*, although they show, to resort to Leibniz's parlance again, a "more than numerical" difference.

As we have seen, Leibniz's arguments serve to identify the physically relevant geometrical structure of a theory, so that physical differences that do not find expression in such a structure should not be considered differences. Einstein's argument signals on the contrary the presence of a surplus mathematical structure, so that differences with respect to such a structure cannot be considered physically meaningful. In the first case we have different physical situations expressed by the same mathematical model, in the second case different mathematical models that express the same physical situation (the very same inertio-gravitational field). Thus in Leibniz-style thought experiments worlds that at first sight *physically different* turn out to be *mathematically identical*; in the hole argument apparently *mathematically different* worlds reveal themselves as *physically identical*.

If one can speak in both cases of "indiscernibility," it seems to me that we have to do with different forms of indiscernibility: (1) an indiscernibility that arises because there is *too little* structure to express some alleged *physical differences* - differences that might otherwise be thought to have physical significance are therefore declared *mathematically irrelevant*; (2) an indiscernibility that arises, because there is *too much* structure, from which apparent *mathematical differences* emerge, that are however declared *physically irrelevant*, since they express the same physical situation in reality. In the first case, one might say, "indiscernibility" is the consequence of *underdetermination*, since the theory does not have the tools to express at first sight real differences. In the second case, we find *overdetermination*, because the mathematical apparatus of the theory introduces differences that do not have any correspondence in reality. In my opinion simply regarding the point-coincidence argument as a restatement of "Leibniz's equivalence," as it is ritually repeated in the literature, would miss the difference between these two sorts of indiscernibility arguments: different physical situations that are declared *mathematically indiscernible* and different mathematical objects that are declared *physically indiscernible*.

6. Conclusion. Leibniz Equivalence vs. Einstein Equivalence

The historical reconstruction I have attempted should have shown that neither the public version of the point-coincidence argument, expressing the requirement of general covariance, nor its private version, a response to the hole argument, can be interpreted simply as a restatement of indiscernibility in the sense of Leibniz, without committing the mistakes of the Logical Empiricists. Leibnizian indiscernibility arguments misleadingly induced the Logical Empiricists to declare generally relativistic spacetime "metrically amorphous," as Adolf Grünbaum famously put it. The point-coincidence argument, as a response to the hole argument, shows on the contrary that the main feature of General Relativity is best summarized by John Stachel's celebrated motto: "no metric, no spacetime."

The Logical Empiricists believed that General Relativity was the result of a sort of "Leibniz's equivalence" *stricto sensu*: in the Logical Empiricist interpretation of the point-coincidence argument "diffeomorphisms" play exactly the same role that "translations" or "scaling" play in Leibniz's arguments, that is they expresses a *global symmetry* of space or spacetime. However, such an approach clearly failed to give a plausible account of General Relativity. "Einstein's equivalence" (as we may call it) must therefore have a different meaning and diffeomorphism must play a completely different role, a role that it is more similar to that which it is usually called *gauge freedom*, akin to that of electrodynamics (Giulini and Straumann 2006, p. 151). Just as in electrodynamics where the same physically measurable field strengths can be expressed by several potentials,[11] similarly in General Relativity an entire equivalence class of diffeomorphically-related solutions to the field equations should correspond to one inertio-gravitational field.

[11]"The freedom to choose among gauge-equivalent potentials is not a physical degree of freedom: it rather results from the fact that we have many *distinct mathematical objects* all of which represent the *same physical state of affairs*" (Maudlin, 2002, p. 2, my emphasis; see also Belot, 1998). Another usual example of the role of surplus structure in physical theories is to be found in modern particle physics. Let a single, free non-relativistic particle be described by the wave function $\psi(\overrightarrow{x})$. Multiplying this wave function by a complex number of unit modulus, a phase factor of the form $e^{\iota\theta}$, gives a wave function $\psi'(\overrightarrow{x}) = e^{\iota\theta}\psi(\overrightarrow{x})$: $\psi(\overrightarrow{x})$ and $\psi'(\overrightarrow{x})$ differ *mathematically* by an overall global phase. However, they represent *physically* the same quantum state: the probability distribution for position and momentum and the time evolution of probability distribution would be the same. As these well-known examples show gauge freedom arises because the mathematical formalism introduces differences that are physically meaningless. At the contrary, global spacetime symmetries arise because certain empirical differences (of position, velocity etc.) are not allowed to appear in the mathematical expression of any physical law (see also Brading and Brown, 2004).

We cannot enter the discussion on the legitimacy of such an analogy between General Relativity and gauge theory (Weinstein, 1999), on which Norton himself, has insisted (Norton, 2003). However, the simple possibility of establishing such an analogy, shows the difference between "Leibniz equivalence" and "Einstein equivalence". Leibniz equivalence erases differences: indiscernibility results from the acknowledgment of a *lack of mathematical structure* that serves to express such differences. On the contrary, Einstein equivalence declares differences redundant; indiscernibility follows from the acknowledgment of an *excess of mathematical structure*. Leibniz indiscernibility arguments, we may say, try to convince us that are no differences at all. It is more a kind of "Leibniz identity" than a "Leibniz equivalence". The Einstein indiscernibility argument wants to persuade us not to worry if the differences appear to be too many.

Appendix. Leibniz Equivalence vs. Einstein Equivalence in Terms of Spacetime Models

The use of spacetime models offers a very simple way to grasp what I think it is the theoretical core of the historical reconstruction I suggested: the difference between Leibniz and Einstein indiscernibility, or between Leibniz and Einstein equivalence. Even if the symbolic apparatus is now more than commonplace in philosophical discussion, I will provide at least a rapid overview. Following the current parlance stemming from (Hawking and Ellis, 1974, ch. 3; see also Wald) "models" of a spacetime theory consist of a manifold, a metric with Lorentz signature, and optionally one or more matter fields (electromagnetic field, neutrino field etc.) that can be regarded as material content of spacetime. Each of such fields is assumed to satisfy the field equations. The models of a theory are those that satisfy such partial differential equations.

From this point of view, one can easily see the difference between a theory like Special Relativity (SR) and General Relativity (GR), and the reason why Leibniz indiscernibility arguments apply only to the former and Einstein indiscernibility argument only to the latter kind of theory. In pre-general-relativistic theories one always has an *a priori* chrono-geometrical structure, that is one always knows what the geometry is, independent of obtaining any solution to the equations of motion. In General Relativity, on the other hand, the relevant geometrical structure has no *a priori* prescribed values, but rather obeys the equations of motion.

A model of SR has the form $\langle \mathcal{M}, \eta_{\mu\nu,}, \mathcal{F}_{1,}, \mathcal{F}_{2,...,} \mathcal{F}_{n,} \rangle$ where $\mathcal{M}$, $\eta_{\mu\nu}$, and $\mathcal{F}_i$, represent the spacetime manifold, the metric field and the other fields (gravitational, electromagnetic ...) respectively. SR exhibits global symmetries, because of the invariance of the $\eta_{\mu\nu}$: in all models allowed by the theory $\eta_{\mu\nu} = (-1, 1, 1, 1)$. Thus there is a fixed spacetime structure, the metric structure $\eta_{\mu\nu,}$, whose affine structure $\Gamma^{\lambda}{}_{\mu\nu} = 0$ represents a fixed standard of non-acceleration. This structure will appear the same in all possible worlds.

A model of GR is given by a triple $M = \langle \mathcal{M}, g_{\mu\nu}, T_{\mu\nu} \rangle$ where $\mathcal{M}$, $g_{\mu\nu}$, and $T_{\mu\nu}$ represent the spacetime manifold, the metric field, and the stress-energy field respectively. Such models are taken to represent the physically possible worlds of General Relativity when they satisfy Einstein's field equations $G_{\mu\nu} = \kappa T_{\mu\nu}$ (where $G_{\mu\nu}$ is the Einstein tensor describing the curvature of spacetime, and κ the coupling constant, proportional to Newton's universal constant of gravitation). In contrast to SR, the $g_{\mu\nu}$ are then subjected to the equations of the theory: they have become a field among others. In

GR the symmetry group of $\langle \mathcal{M},\ g_{\mu\nu}\rangle$ is the identity group, since there is in general no transformation that leaves $g_{\mu\nu}$ invariant. Hence there will be in general different models of the theory: $\langle \mathcal{M},\ g_{\mu\nu}\rangle, \langle \mathcal{M},\ g'_{\mu\nu}\rangle, \langle \mathcal{M},\ g''_{\mu\nu}\rangle$, ... The Christoffel symbols $\Gamma^{\lambda}{}_{\mu\nu}$ figure as the components of the gravitational-inertial field (so they are in general $\neq 0$), whereas the Curvature tensor or Riemann-Christoffel tensor ($B^{\rho}_{\mu\sigma\tau}$) represents the field gradient, or tidal field. The fact that $\Gamma^{\lambda}{}_{\mu\nu}$ is not a tensor is the formal expression of the non-unique decomposition of the affine connection into an inertial and a gravitational part.

The analogy of the "Leibniz shift" can be found only in pre-relativistic theories, such as SR: it can be represented by a map h between worlds or models (spatial or temporal displacements, rotations, or boosts up to the speed of light) that *preserves all relevant geometrical* structure (in the case of SR the metric structure uniquely determining the inertial structure), that is $h * \eta_{\mu\nu} = \eta_{\mu\nu}$ (an isometry). If such a structure would not be preserved, that is if $h * \eta_{\mu\nu} \neq \eta_{\mu\nu}$, the two universes or models would not be indistinguishable. The indistinguishability arises because the transformation produces *the very same model* (it is a trivial identity). The original and the transformed situations are indiscernible simply because they are *mathematically exactly the same.* The $\eta_{\mu\nu}$ are the only quantities pertaining to spacetime structure which can appear in any physical law. Thus differences that cannot be encoded in such a spacetime structure are not differences. In order to introduce further distinctions, for instance of spatial orientation, one has to allow further aspects of spacetime structure to appear in physical laws (Wald, 1984, p. 60).

Einstein's hole/point-coincidence argument makes sense only in GR[12]: it implies a transformation h ($diff(\mathcal{M})$) that *does not preserve the relevant geometrical structure* (the metric structure determining the geodesics), that is $h*g_{\mu\nu} \neq g_{\mu\nu}$. An indiscernibility argument in the sense of Einstein is needed because the theory introduces *different models* that, however, are declared *physically exactly the same*: the same gravitational field corresponds to an equivalence class of $\langle \mathcal{M}, g_{\mu\nu}\rangle$- it is so to speak, a non-trivial equivalence.

The Logical Empiricists clearly confused Leibniz and Einstein indiscernibility (or better, had at their disposal only Leibniz indiscernibility): noticing that the group of transformations of GR, does not preserve the metric $g_{\mu\nu}$, ($h * g_{\mu\nu} \neq g_{\mu\nu}$), they declared the metric "conventional" and identified spacetime with the "invariant" $\mathcal{M}$ held across models. Differences that cannot be expressed in $\mathcal{M}$, like in a Hegelian night, are not differences: that is they identify $diff(\mathcal{M})$ with a *global spacetime symmetry.* Einstein's indiscernibility argument moving from $h*g_{\mu\nu} \neq g_{\mu\nu}$ concludes that spacetime *is* the equivalence class (up to diffeomorphism) of all $\langle \mathcal{M},\ g_{\mu\nu}\rangle$,$\langle \mathcal{M},\ g'_{\mu\nu}\rangle$,..., so that differences with respect to $\mathcal{M}$ are not differences, they are a mere mathematical redundancy: $diff(\mathcal{M})$ resembles more closely an expression of *gauge freedom* (a whole family of gauge-related solutions of the field equations represent the same physical situation) (Wald, 1984, p. 438).

References

A: Leibniz, G. W., 1923. Sämtliche Schriften und Briefe. O. Reichl, Darmstadt.

[12] For sake of comparison I follow here John Stachel in arguing that the hole argument cannot be taken over to the pregeneral-relativistic case (Stachel, 1993). This opinion is however controversial. I cannot however address this issue here. My aim is rather to emphasize the opposite issue that Leibniz arguments work only in pregeneral-relativistic theories.

Anderson, J. L., 1967. Principles of Relativity Physics. Acad. Pr., New York [et al.].
Arthur, R., 1994. Space and Relativity in Newton and Leibniz. The British Journal for the Philosophy of Science 45 (1), pp. 219–240.
Bartels, A., 1994. What is Spacetime, if not a Substance? Conclusions from the New Leibnizian Argument. In: Majer, U., Schmidt, H.-J. (Eds.), Semantical Aspects of Spacetime Theories. BI Wissenschaftsverlag (1994), Mannheim, pp. 41–51.
Bartels, A., 1996. Modern Essentialism and the Problem of Individuation of Spacetime Points. Erkenntnis 45, 25–43.
Belot, G., 1998. Understanding Electromagnetism. The British Journal for the Philosophy of Science 49 (4), pp. 531–555.
Brading, K., Brown, H. R., 2003. Symmetries and Noether's theorems. In: Symmetries in Physics. Philosophical Reflections. Cambridge University Press, Cambridge, U.K.; New York, pp. 87–109.
Brading, K., Brown, H. R., 2004. Are Gauge Symmetry Transformations Observable? British Journal for the Philosophy of Science 55, 645–665.
Brown, H. R., 2005. Physical relativity : space-time structure from a dynamical perspective, 1st Edition. Clarendon, Oxford.
Butterfield, J., 1989. The Hole Truth. The British Journal for the Philosophy of Science 40 (1), 1–28.
Cantor, G., 1874. Ueber eine Eigenschaft des Inbegriffs aller reellen algebraischen Zahlen. Journal für die reine und angewandte Mathematik (Crelles Journal) 1874 (77), 258–262.
Cantor, G., 1878. Ein Beitrag zur Mannigfaltigkeitslehre. In: Zermelo, E. (Ed.), Gesammelte Abhandlungen mathematischen und philosophischen Inhalts. No. 119-133. Olms, Hildesheim.
Carnap, R., 1922. Der Raum. Ein Beitrag zur Wissenschaftslehre. Reuther & Reichard, Berlin.
Carnap, R., 1966. Philosophical Foundations of Physics: An Introduction to the Philosophy of Science. Basic Books, inc., New York.
Chernoff, F., 1981. Leibniz's Principle of the Identity of Indiscernibles. The Philosophical Quarterly 31 (123), 126–138.
Coffa, A. J., 1991. The Semantic Tradition from Kant to Carnap: To the Vienna Station. Cambridge University Press, Cambridge.
Coleman, R. A., Korté, H., 1984. Constraints on the Nature of Inertial Motion Arising from the Universality of Free Fall and the Conformal Causal Structure of Space-Time. Journal of Mathematical Physics 25, 3513–3527.
Couturat, L., 1961. La logique de Leibniz d'après des documents inédits. Olms, Hildesheim.
Cox, C. B., 1978. Leibniz's Philosophy of Space. Diss. - Columbia University, New York.
CPAE: Einstein, A., 1996-. The Collected Papers of Albert Einstein. Princeton University Press, Princeton.
De Risi, V., 2005. Leibniz on Geometry: Two Unpublished Texts with Translation and Commentary. The Leibniz Review 15, 127–132.
De Risi, V., 2007. Geometry and Monadology: Leibniz's analysis situs and Philosophy of Space. Birkhäuser, Basel; Boston.
DiSalle, R., 2002a. Conventionalism and Modern Physics. A Re-Assessment. Noûs 36 (2), 169–200.
DiSalle, R., 2002b. Newton's Philosophical Analysis of Space and Time. In: Cohen, I. B. (Ed.), The Cambridge Companion to Newton. Cambridge University Press, Cambridge, pp. 33–54.
DiSalle, R., 2006. Understanding Space-Time: The Philosophical Development of Physics from Newton to Einstein. Cambridge University Press, Cambridge, UK ; New York.
Earman, J., 1989. World Enough and Space-Time: Absolute vs. Relational Theories of Space and Time. MIT Press, Cambridge, Mass.
Earman, J., Norton, J. D., 1987. What Price Substantivalism. The Hole Story. British Journal for the Philosophy of Science 38, 515–525.
Ehler, J., 1988. Hermann Weyl's Contributions to the General Theory of Relativity. In: Deppert, W. H. (Ed.), Exact sciences and their philosophical foundations: Vorträge des Internationalen Hermann-Weyl-Kongresses, Kiel 1985 = Exakte Wissenschaften und ihre philosophische Grundlegung. Lang, Frankfurt am Main [et.al.].
Einstein, A., 1921. Geometrie und Erfahrung. Springer, Berlin.
Einstein, A., 1925. Nichteuklidische Geometrie und Physik. Die neue Rundschau 36, 16–20.
Epple, M., 2006. Felix Hausdorff's Considered Empiricism. In: Ferreirós, J., Gray, J. (Eds.), The Architecture of Modern Mathematics: Essays in History and Philosophy. Oxford University Press, Oxford.
Epple, M., 2007. An Unusual Career between Cultural and Mathematical Modernism: Felix Hausdorff, 1868-1942. In: Charpa, U., Deichmann, U. (Eds.), Jews and Sciences in German Contexts: Case Studies from the 19th and 20th centuries. Mohr Siebeck, Tübingen, pp. 77–100.

Freudenthal, H., 1972. Leibniz und die analysis situs. Studia Leibnitiana 4 (2), 61–60.
Friedman, M., 1983. Foundations of Space-Time Theories: Relativistic Physics and Philosophy of Science. Princeton University Press, Princeton.
Friedman, M., 1995. Poincaré's Conventionalism and the Logical Positivists. Foundations of Science 2, 299–314, now in [Friedman 1999].
Friedman, M., 1999. Reconsidering Logical Positivism. Cambridge University Press, New York.
Galilei, G., 1632/2005. Discorso intorno a due nuove scienze. In: Brunetti, F. (Ed.), Opere. UTET, Torino.
Galilei, G., 1967. Dialogue Concerning the Two Chief World Systems, Ptolemaic and Copernican. University of California Press, Berkeley.
Giulini, D., 2007. Remarks on the Notions of General Covariance and Background Independence. Lecture Notes in Physics 721, 105–120.
Giulini, D., Straumann, N., 2006. Einstein's Impact on the Physics of the Twentieth Century. Studies In History and Philosophy of Science Part B: Studies In History and Philosophy of Modern Physics 37 (1), 115 – 173, 2005: The Centenary of Einstein's Annus Mirabilis.
GM: Leibniz, G. W., 1850. Leibnizens mathematische Schriften. Schmidt, Halle.
GP: Leibniz, G. W., 1875. Die philosophischen Schriften von Gottfried Wilhelm Leibniz. Weidmann, Berlin.
Hausdorff, F., 1903. Das Raumproblem. Annalen der Naturphilosophie 3, 1–23.
Hausdorff, F., 1914/2002. Grundzüge der Mengenlehre. In: Brieskorn, E., Chatterji, S. D., Epple, M. (Eds.), Gesammelte Werke. "Grundzüge der Mengenlehre". Vol. II. Springer, Berlin, Heidelberg etc.
Hausdorff, F., 1927, 1935/2007. Mengenlehre. In: Brieskorn, E., Hirzebruch, F., Remmert, R., Purkert, W., Scholz, E. (Eds.), Gesammelte Werke: Deskriptive Mengenlehre und Topologie. Vol. III. Springer, Berlin, Heidelberg etc.
Hausdorff, F., 2004. Das Chaos in kosmischer Auslese. Ein erkenntnisskritischer Versuch. In: Stegmaier, W. (Ed.), Gesammelte Werke: Philosophisches Werk. Vol. VII. Springer, Berlin.
Hausdorff, F., 2010. Gesammelte Werke: Geometrie, Raum und Zeit. Vol. VI. Springer, Berlin, Heidelberg etc.
Hawking, S. W., Ellis, G. F. R., 1974. The large scale structure of space-time. Cambridge monographs on mathematical physics ; 1. Cambridge Univ. Press, Cambridge.
Hilbert, D., 1902. Über die Grundlagen der Geometrie. Mathematische Annalen 56, 381–422.
Hoefer, C., 1996. The Metaphysics of Space-Time Substantivalism. The Journal of Philosophy 93 (1), pp. 5–27.
Howard, D., 1999. Point Coincidences and Pointer Coincidences. Einstein on the Invariant Content of Space-Time Theories. In: Goenner, H. (Ed.), The Expanding Worlds of General Relativity. Birkhäuser, Basel, pp. 463–500.
Howard, D., 2005. 'No Crude Surfeit': A Critical Appreciation of The Reign of Relativity"., delivered at 2005 Pacific meeting of the American Philosophical Assocation, manuscript.
Howard, D., Norton, J. D., 1993. Out of the Labyrinth. Einstein, Hertz, and the Gottingen Answer to the Hole Argument. In: The attraction of gravitation : new studies in the history of general relativity. Birkhäuser, Basel, pp. 30–62.
Huggett, N., Hoefer, C., 2009. Absolute and Relational Theories of Space and Motion. In: Zalta, E. N. (Ed.), The Stanford Encyclopedia of Philosophy, fall 2009 Edition.
Janssen, M., 2005. Of pots and holes: Einstein's bumpy road to general relativity. Annalen der Physik 14, 58–85.
Jauernig, A., 2008. Leibniz on Motion and the Equivalence of Hypotheses. The Leibniz Review 18, 1–40.
Klein, C., 2001. Conventionalism and Realism in Hans Reichenbach's Philosophy of Geometry. International Studies in the Philosophy of Science 15 (3), 243–251.
Korté, H., 2006. Einstein's Hole Argument and Weyl's Field-body Relationalism. In: Demopoulos, W., Pitowsky, I. (Eds.), Physical Theory and its Interpretation. Vol. 72 of The Western Ontario Series in Philosophy of Science. Springer Netherlands, pp. 183–212.
Kosso, P., 2000. The Empirical Status of Symmetries in Physics. The British Journal for the Philosophy of Science 51 (1), 81–98.
Kretschmann, E., 1917. Über den physikalischen Sinn der Relativitätspostulate. A. Einsteins neue und seine ursprüngliche Relativitätstheorie. Annalen der Physik 53, 575–614.
Lariviere, B., 1987. Leibnizian Relationalism and the Problem of Inertia. Canadian Journal of Philosophy 17 (2), 437–448.
Lehrer, K., 2004. Reichenbach on Convention. Synthese 34/2, 237–248.
Leibniz, G. W., 1976. Philosophical Papers and Letters: A Selection, 2nd Edition. Synthese historical

library ; 2. Riedel, Dordrecht.
Leibniz, G. W., 1989. Philosophical Essays. Hackett Pub. Co., Indianapolis, IN.
Maudlin, T., 1988. The Essence of Space-Time. SA: Proceedings of the Biennial Meeting of the Philosophy of Science Association 2, 2–91.
Maudlin, T., 1993. Buckets of Water and Waves of Space. Why Spacetime Is Probably a Substance. Philosophy of Science 60 (2), 183–203.
Maudlin, T., 2002. Thoroughly Muddled McTaggart. Or, How to Abuse Gauge Freedom to Create Metaphysical Monostrosities. Philosophers' Imprint 2 (4), 1–19.
Münzenmayer, H. P., 1979. Der Calculus Situs und die Grundlagen der Geometrie bei Leibniz. Studia Leibnitiana 11, 274–300.
Mongré, P., 1898. Das Chaos in kosmischer Auslese. Ein erkenntnisskritischer Versuch. Naumann, Leipzig, now in Hausdorff [2004].
Nerlich, G., 1994. What Spacetime Explains: Metaphysical Essays on Space and Time. Cambridge Univ. Press, Cambridge [et al.].
NL FH: Hausdorff, F., -. Nachlass, the Nachlass is located at the Main Library of the University of Bonn (D). An online catalogue can be found at http://www.aic.uni-wuppertal.de/fb7/hausdorff/findbuch.asp. The papers on philosophy of geometry will be published in Hausdorff [2010].
Norton, J. D., 1984. How Einstein found his Field Equations: 1912-1915. Historical Studies in the Physical Sciences 14, 253–316.
Norton, J. D., 1989. Coordinates and Covariance: Einstein's View of Spacetime and the Modern View. Foundations of Physics 19, 1215–1263.
Norton, J. D., 1992. The Physical Content of General Covariance. In: Studies in the History of General Relativity: Einstein Studies. Birkhauser, Boston, pp. 281–315.
Norton, J. D., 1995. Did Einstein Stumble? The Debate over General Covariance. Erkenntnis 42 (2), 223–245.
Norton, J. D., 1999. Geometries in Collision: Einstein, Klein and Riemann. In: Gray, J. (Ed.), The Symbolic Universe. Oxford University Press, Oxford; New York, pp. 128–144.
Norton, J. D., 2003. General covariance, gauge theories, and the Kretschmann objection. In: Brading, K., Castellani, E. (Eds.), Symmetries in Physics. Philosophical Reflections. Cambridge University Press, Cambridge, pp. 110–123.
Pauli, W., 1921. Relativitätstheorie. In: Enzyklopädie der mathematischen Wissenschaften. Vol. 5. Teubner, pp. 539–775.
Pauli, W., 1958. Theory of Relativity. Pergamon Press, London.
Pitts, J. B., 2006. Absolute Objects and Counterexamples: Jones-Geroch dust, Torretti Constant Curvature, Tetrad-Spinor, and Scalar Density. Studies In History and Philosophy of Science Part B: Studies In History and Philosophy of Modern Physics 37 (2), 347 – 371.
Pulte, H., 2006. The Space between Helmholtz and Einstein: Moritz Schlick on Spatial Intuition and the Foundations of Geometry. In: Hendricks, V. F., Jørgensen, K. F., Lützen, J., Pedersen, S. A. (Eds.), Interactions Mathematics, Physics and Philosophy, 1860-1930. Springer, Dordrecht.
Reichenbach, H., 1920/1977. Relativitätstheorie und Erkenntnis a priori. In: Kamlah, A., Reichenbach, M. (Eds.), Gesammelte Werke in 9 Bänden. Vol. 3. Braunschweig; Wiesbaden, Vieweg.
Reichenbach, H., 1924. Axiomatik der relativistischen Raum-Zeit-Lehre. Braunschweig, Vieweg.
Reichenbach, H., 1924/1969. Axiomatization of the Theory of Relativity. University of California Press, Berkeley,.
Reichenbach, H., 1928/1958. The Philosophy of Space and Time. Dover Publ., New York.
Reichenbach, H., 1928/1977. Die Philosophie der Raum-Zeit-Lehre. In: Kamlah, A., Reichenbach, M. (Eds.), Gesammelte Werke in 9 Bänden. Vol. 2. Vieweg, Braunschweig; Wiesbaden.
Reichenbach, H., 1949/1951. The Philosophical Significance of the Theory of Relativity. In: Schilpp, P. A. (Ed.), Albert Einstein. Philosopher-Scientist. Tudor, New York, pp. 289–311.
Renn, J., Stachel, J., 2007. Hilbert's Foundation of Physics: From a Theory of Everything to a Constituent of General Relativity. In: Renn, J., Janssen, M. (Eds.), The Genesis of General Relativity. Springer Netherlands, pp. 857–972.
Roberts, J. T., 2003. Leibniz on Force and Absolute Motion. Philosophy of Science 70 (3), pp. 553–573.
Rosen, J., 2008. Symmetry Rules: How Science and Nature Are Founded on Symmetry. Springer, Berlin; Heidelberg.
Ryckman, T., 1992. (P)oint-(C)oincidence Thinking. The Ironical Attachment of Logical Empiricism to General Relativity. Studies in History and Philosophy of Modern Physics 23 (3), 471–497.
Ryckman, T., 1995. Weyl, Reichenbach and the Epistemology of Geometry. Studies in History and

Philosophy of Science 25 (6), 831–870.
Ryckman, T., 1996. Einstein Agonists: Weyl and Reichenbach on Geometrv and the General Theory of Relativity. In: Giere, R. N. (Ed.), Origins of Logical Empiricism. University of Minnesota Press, Minneapolis, pp. 165–209.
Ryckman, T., 2005. The Reign of Relativity. Philosophy in Physics 1915-1925. Oxford University Press, Oxford, New York.
Ryckman, T., 2007. Logical Empiricism and Philosophy of Physics. In: Richardson, A. W., Uebel, T. (Eds.), The Cambridge Companion to Logical Empiricism, 1st Edition. Cambridge University Press, Cambridge [et al.], pp. XI, 430 S.
Ryckman, T., 2008. Early Philosophical Interpretations of General Relativity. In: Zalta, E. N. (Ed.), The Stanford Encyclopedia of Philosophy: Fall 2008 Edition. plato.stanford.edu/entries/genrel-early/.
Rynasiewicz, R., 1994. The Lessons of the Hole Argument. The British Journal for the Philosophy of Science 45 (2), 407–436.
Rynasiewicz, R., 1999. Kretschmann's Analysis of Covariance and Relativity Principles. In: Goenner, H. (Ed.), The Expanding Worlds of General Relativity. Birkhäuser, Basel, pp. 431–462.
Rynasiewicz, R., 2000. On the Distinction between Absolute and Relative Motion. Philosophy of Science 67 (1), 70–93.
Saunders, S., 2002. Indiscernibles, General Covariance, and other Symmetries. In: Ashtekar, A. V., Renn, J., Howard, D. (Eds.), Revisiting the Foundations of Relativistic Physics: Festschrift in Honour of John Stachel. Kluwer, Dordrecht/Boston/London, pp. 151–173.
Schlick, M., 1920. Space and Time in Contemporary Physics, an Introduction to the Theory of Relativity and Gravitation. Oxford University Press, Oxford/New York.
Schlick, M., 1922/2006. Raum und Zeit in der gegenwärtigen Physik. In: Neuber, M., Engler, O. (Eds.), Gesamtausgabe. Vol. I,2. Springer, New York/Berlin.
Schlick, M., 1978a. Philosophical Papers. Vienna Circle collection ; 11. Reidel, Dordrecht.
Schlick, M., 1978b. Space and Time in Contemporary Physics. In: Mulder, H. L. (Ed.), Philosophical Papers. Vol. 1 of Vienna Circle collection ; 11. Reidel, Dordrecht.
Schneider, M., 1988. Funktion und Grundlegung der Mathesis Universalis. Studia Leibnitiana Sonderheft 15, 162–182, (Leibniz: Questions de Logique).
Slowik, E., 2006. The "dynamics" of Leibnizian relationism: Reference frames and force in Leibniz's plenum. Studies In History and Philosophy of Science Part B: Studies In History and Philosophy of Modern Physics 37 (4), 617 – 634.
Stachel, J., 1980/2002. Einstein's Search for General Covariance, 1912-1915. In: Einstein from "B" to "Z". Birkhäuser, Boston, pp. 301–337, the paper was first paper read at the Ninth International Conference on General Relativity and Gravitation, Jena in 1980.
Stachel, J., 1993. The Meaning of General Covariance: The Hole Story. In: Earman, J. (Ed.), Philosophical problems of the internal and external worlds : essays on the philosophy of Adolf Grünbaum. Pittsburgh-Konstanz series in the philosophy and history of science ; [1]. Univ. of Pittsburgh Pr., Pittsburgh, Pa., festschrift Adolf Grünbaum.
Stachel, J., Iftime, M., 2005. Fibered Manifolds, Geometric Objects, Structured Sets, G-Sets and all that: The Hole Story from Space Time to Elementary Particles. arxiv.org.
Stachel, J. J., 2002. Einstein from B to Z. Einstein studies ; 9. Birkhäuser, Boston.
Stein, H., 1967/1970. Newtonian Space-Time. In: Palter, R. (Ed.), The Annus mirabilis of Sir Isaac Newton, 1666-1966. The MIT Press, Cambridge (Ma.)/London, pp. 254–284.
Wald, R. M., 1984. General relativity. Univ. of Chicago Pr., Chicago [et al.].
Wallwitz, G. G., 1991. Strukturelle Probleme in Leibniz Analysis Situs. Studia Leibnitiana 23, 111–118.
Weinstein, S., 1999. Gravity and Gauge Theory. Philosophy of Science 66, 146–155.
Weyl, H., 1913. Die Idee der Riemannschen Fläche, 2nd Edition. Mathematische Vorlesungen an der Universität Göttingen ; 5. Teubner, Leipzig [et al.].
Weyl, H., 1921/1968. Elektrizität und Gravitation. In: Chandrasekharan, K. (Ed.), Gesammelte Abhandlungen. Vol. 2. Springer, Berlin, pp. 141–142.
Weyl, H., 1921/2009. Electricity and Gravitation. In: Pesic, P. (Ed.), Mind and Nature: Selected Writings on Philosophy, Mathematics, and Physics. Princeton Univerity Press, Princeton.
Weyl, H., 1924a. Massenträgheit und Kosmos. Ein Dialog. Naturwissenschaften 12, 11.
Weyl, H., 1924b. Rezension von: H. Reichenbach: Axiomatik der relativistischen Raum- Zeit Lehre. Deutsche Literaturzeitung. 45, 2122–2128.
Weyl, H., 1925/1988. Riemanns geometrische Ideen, ihre Auswirkung und ihre Verknüpfung mit der Gruppentheorie. Springer, New York; Berlin; Heidelberg.
Weyl, H., 1927a. Philosophie der Mathematik und Naturwissenschaft. In: Bäumler, A., Schröter, M.

(Eds.), Handbuch der Philosophie. Olde, München/Berlin.
Weyl, H., 1927b. Zeitverhältnisse im Kosmos, Eigenzeit, gelebte Zeit und metaphysische Zeit. In: Proceedings of the Sixth International Congress of Philosophy. Harvard University, Cambridge, Massachusetts, United States of America, September 13-17, 1926. Longmans, Green and Co., Cambridge, Ma., pp. 54–58.
Weyl, H., 1932/2009. The Open World. In: Pesic, P. (Ed.), Mind and Nature: Selected Writings on Philosophy, Mathematics, and Physics. Princeton Univerity Press, Princeton.
Weyl, H., 1934/2009. Mind and Nature. In: Pesic, P. (Ed.), Mind and Nature: Selected Writings on Philosophy, Mathematics, and Physics. Princeton University Press, Princeton, pp. 83–150.
Weyl, H., 1939/1997. The Classical Groups: Their Invariants and Representations. Princeton University Press, Princeton.
Weyl, H., 1952. Symmetry. Princeton Univ. Pr., Princeton, NJ.
Weyl, H., 2009a. Mind and Nature: Selected Writings on Philosophy, Mathematics, and Physics. Princeton Univerity Press, Princeton.
Weyl, H., 2009b. Philosophy of Mathematics and Natural Science. Princeton University Press, Princeton.