Evolutionary Epistemology: Random Change in Science

Abstract: Scientific thought is generally characterized as methodical and rational. I would like to present here an opposing view which treats science as a non-systematic activity, where serendipity and tinkering, rather than so-called rational thought, characterizes it. All these kinds of acts, which are considered to be a-rational, are related to an evolutionary view of science. I will deal here with a version of evolutionary epistemology as applied to science. Examples for this phenomenon in science are illustrated by some historical cases, in particular by the evolution of theories of internal symmetries in particle physics.

Key words: evolutionary epistemology, serendipity, the history of internal symmetries in particle physics

1 .The Evolution and Spread of Ideas

Evolutionary epistemology deals with the spread of concepts and ideas in the scientific community. Richard Dawkins devised a special version of evolutionary epistemology. He borrowed from the old Greek the concept of "meme" which stands for ideas. He treats memes as units of information which "propagate themselves in the meme-pool by leaping from brain to brain via imitation" (The Selfish Gene p. 192). Memeticians that followed him claim that memes evolve by natural selection in a manner analogous to biological evolution. They evolve through the processes of variation, mutation, competition and inheritance, each of which influences a meme's reproductive success. I will treat "meme" as a synonym for idea or concept. Memes spread through the behaviors that they generate in their carriers. Certain ideas may become extinct while others may survive, spread and mutate. Memes that replicate most effectively enjoy more success. Traits, such as serendipitous behavior, are responsible for the blindness of the mutations. One individual influencing another to adopt a belief is seen as an ideareplicator reproducing itself in a new host. As with genetics, particularly under the Dawkins' interpretation, a meme's success may be due to its contribution to the effectiveness of its host. Memetics replaces the traditional concern with the truth of ideas and beliefs. Instead, it deals with the success of memes, which is expressed in their spread. Memes are successful when they are considered to represent truth, but this does not mean that they are indeed true in the ordinary sense.

I will concentrate on the implications of this theory to science. As we will see, memes move from one scientist to another by imitation or copying. According to memetism, scientific research relies on the ability to imitate: a scientist adopts an idea, changes it, or employs it in a new context. Susan Blackmore defines a meme as a unit of information which is passed on by imitation. Imitation may lead to creative results, since if an idea A is imitated or copied, A is transformed into A', then A' may involve some

change, or appears in a new context, which may be considered as evolutionary change. Thus, creativity does not involve an invention of novel ideas out of nothing.

I will follow Blackmore's approach to the subject. In his introduction to the Hebrew translation of Blackmore's book *The Meme Machine* (1998) Yoni Mizrachi cites Pavlo Picasso who says that great artists copy and even steal ideas from others. He also refers to a similar saying of Albert Einstein that the key to creativity is the ability to hide your sources, This sounds as stealing ideas. They refer to a general trend, where scientists (as well as artists) cover-up the origin of their ideas. In addition to imitation, we find non-heroic behavior, such as making discoveries by serendipity and tinkering (Kantorovich, 1993). The discoverer, who arrived at his ideas un-deliberately, reconstructs the process as if it was generated rationally by calculated steps. Thus, the system of science is like a machine, where memes, such as ideas and theories, are its products.. The products do not have truth-value. Instead, they may be useful or not as in instrumentalism. Therefore, science is a kind of meme-producing machine. This picture sounds too abstract, but I'll give concrete examples from natural sciences - in particular from theoretical particle physics.

Instead of relying on logical analysis of concepts such as "imitation", which we find in the literature on memetics, I will rely on practicing scientists, such as Einstein, which are not guided always by pure reason. They act according to their intuitions, imitating their colleagues and copying their ideas. Issac Newton does not seem to act as a memetician. However, he relied on theoretical assumptions which could not be proved logically and mathematically, such as the law of universal attraction between massive bodies which he treated as natural law. He was influenced by some of his predecessors, such as Kepler and Galileo but he did not imitate anyone deliberately. The kind of behavior which may be called "memetism" started to spread among scientists around the early twenty century when physical theories departed from the classical picture and became more and more abstract and less and less comprehensible intuitively, as are quantum mechanics and the theory of relativity. At this stage, scientists could not rely on their own considerations. Instead, they rely on their colleagues by imitating them, as was hinted by Einstein in his above-mentioned saying.

Karl Popper, one of the founding fathers of evolutionary epistemology, was not engaged with the process of creation of scientific ideas. He treated the process of science as a process of selecting among "bold lies", without taking care about their origin – since scientists can only refute their conjectures but cannot prove them. According to Popper "the way to truth is paved by bold lies". Dawkins and Blackmore, on the other hand, specify the way pieces of information move from one brain to another by acts of imitation. In fact, pieces of information may also move from a brain to itself, when considering earlier ideas, producing creative changes. We may call this a "self-copying/imitation".

One of the first psychologists who studied the concept of imitation was Edward Thorndike (1898). He characterizes imitation as "learning to do an act from seeing it is done". This definition (applied to visual information) refers to the idea that in imitation a new behavior is learned by copying it from someone else. In fact, students can be viewed as imitating their teachers. This is the essence of learning. Blackmore says:

• "Humans are fundamentally unique not because they are especially clever, not just because they have big brains or language, but because they are capable of extensive and generalized imitation. I think we will discover that it is imitation that gave rise to our cleverness, big brains and language – and it is imitation that makes culture possible, for

only imitation gives rise to a new replicator that can propagate from brain to brain, or from brain to artefact and back to brain. For all these reasons I suggest that we stick with the dictionary, and define the meme as that which is passed on by imitation" (Susan Blackmore, 1998).

Thus, she would treat creativity not as a supreme act of inventing novel ideas out of nothing, but of imitating or copying ideas in new contexts or by self imitation. Creativity is generated by finding new contexts in which the old ideas may apply.

2. Memetics vs. Popper's Approach

According to memetics, in science memes are pieces of information, or ideas, which are copied by imitation. Popper does not speak about the manner scientists arrive at their ideas – as the memeticians do. Scientists arrive at their ideas by imitating, copying or stealing. Popper speaks about another non-heroic view of science: he treats scientific ideas as useful "lies". Newtonian mechanics, for instance, was such a "lie". The memetician would not employ the word "falsification" since "truth" does not appear in his vocabulary. Newton's theory does not apply to certain cases however it is still useful since it yields successful predictions and it helps scientists to employ it in various situations, or to use it in different technological contexts. The memetician would not treat it as a "lie".

3. Plato as a Memetician

Now we turn to the examples. The first one is rather a reconstruction Plato's texts. Toulmin and Goodfield describe in their book The Architecture of Matter (1962) a historical case that can be interpreted as exemplifying the "strategy" of the memetician. This case is taken from the Timaeus. It is not science in its modern sense and my interpretation is somewhat too "creative", but it can exemplify my argument. It is the way Plato's theory about the structure of matter was generated. Plato worked within the framework of the Pythagorean tradition that determined the range of materials and ideas at his disposal. He employed a recently discovered mathematical theorem by Theaetetus who found that there are only five regular convex polyhedra (solids): the tetrahedron, the cube, the octahedron, dodecahedron and icosahedron. He seized upon this opportunity and tried to match the four elements of Empedocles (air, fire, earth and water) to the five solids. He devised intricate arguments for choosing the tetrahedron as the atom of fire, the cube as the atom of earth, the octahedron as that of air and the icosahedron as that of water. No earthly element remained to match the dodecahedron, which consists of twelve pentagons. Plato matched this polyhedron with the boundary of the universe. Thus, for solving his problem, Plato used a mathematical theorem that happened to be discovered at his time. He opportunistically exploited an existing tool for constructing his theory. We have here elements of copying and "stealing" ideas which are characteristic of the memetician.

Toulmin and Goodfield add the following remark in brackets to their story: "The close approximation of the dodecahedron to the sphere was well known to the Greeks, who made their footballs from pentagons of leather sewn in sets of twelve". This is presumably an insinuation that in devising his theory, Plato may have drawn upon this piece of knowledge. As a support for this speculation, we may note that among the five solids, it is the icosahedrons, rather than the dodecahedron, which has the largest number of faces – i.e. twenty triangles. So that at first thought, one might think that the icosahedron is the closest approximation to a sphere. That it was not chosen to be the counterpart of the sphere is related to the fact that the sphere was naturally related to the football in the memetician's/tinkerer's mind. If this plausible interpretation reflects Plato's way of

arriving at his ideas, it might further confirm the view that in devising the system of geometrical atomism, Plato acted like a memetician; he did not derive it by pure thought.

The memetician exhibits an opportunistic behavior since he/she employs ideas which were generated in a given context and applies them in a new context, finding them useful in the new situation. The act of copying an idea involves some change or variation; the same idea employed in a slightly different form. Thus, we have here a development of ideas which resembles the memetician pattern.

4. Solutions in Search of Problems

The principle which denies the existence of a certified method of discovery is one of the characteristics of opportunism in science. In the theory of evolution, "evolutionary opportunism" refers to a specific pattern of development of a species. The traits and behaviors of a species that in the past evolved in order to serve a particular purpose may subsequently lend themselves to a *different* purpose that helps the species to survive. Thus, in a new stage of evolution, a long-existing behavior or physical characteristic can respond to a wholly new opportunity and acquires a new role. It turns out to have *new* advantages or potential benefits the species previously never used. Thus, in the ordinary cases scientists have problems and they look for solutions. In memetism scientists are equipped with memes – ideas and tools – and they look for appropriate problems which might be solved by these memes.

Francoise Jacob (1977) offers several examples of evolutionary tinkering and opportunism. He cites, for example, Ernst Mayr's hypothesis (1964) about the formation of lungs of terrestrial vertebrates:

"Its development started in certain fresh water fishes living in stagnant pools with insufficient oxygen. They adopted the habit of swallowing air and absorbing oxygen through the walls of the esophagus. Under these conditions, enlargement of the surface area of the esophagus provided selective advantage. Diverticula of the esophagus appeared, and under a continuous selective presssure, enlarged into lungs. Further evolution of the lung was merely an elaboration of this theme – enlarging the surface for oxygen uptake and vascularization. To make lung with a piece of esophagus sounds very much like tinkering". (Jacob, 1164).

Another example is the evolution of human brain: The neocortex, which controls intellectual and cognitive activity, was added to the rhinencephalon of lower mammals, which controls emotional and visceral activities. This evolutionary process, i.e. the formation of a dominating neocortex coupled with the nervous and hormonal system strongly resembles the tinkerer's procedure. "It is somewhat like adding a jet engine to an old horse cart" (ibid., 1166).

The above two cases exhibit evolutionary opportunism, where an organ which was selected to fulfill a given function is exploited, after it undergoes certain changes, to perform an entirely new function.

Evolutionary tinkering and memetism in science are characterized by the fact that scientists are guided by solutions rather than by problems. They have (abstract or material) tools and they look for problems which might be solved by these tools. This is a typical selfish behavior. In fact, every old piece of knowledge which is a product of previous evolutionary process may serve as such a tool. This is the basis of the phenomenon called in the sociology of science "intellectual migration". This phenomenon is exemplified also in physics. Physicists develop mathematical or experimental tool

that helps solving a certain problem in a given area and then they move to another area and use this tool for solving problems in the new area. The opportunist exploits conceptual tools that are present in the near vicinity ("shortsightedness"); not planning far-reaching theories ("laziness"); jumping from subject to subject and from one problem to another ("confusion"). This phenomenon can be attributed to Plato as well.

Arthur Koestler describes the great discoverers, such as Columbus and Kepler, as "sleepwalkers" (1964), in accordance with the above profile. Kepler himself and other reknown scientists, such as Planck, Poincare and Einstein, (at least partially) approved this phenomenon from their experience. However, there are scientists and historians of science who reconstruct the process so that the facade of ihe scientific activity is represented as methodical and rational,

Although there is no method that guides creative discovery, we can derive practical guiding lines for *cultivating* the intellectual readiness, the associative ability and improvisation needed for exploiting opportunities for solving problems that the scientist did not intend to solve. Participating in meetings, conferences and in the publication system, the scientific prizes and other means of evaluation – all these play a central role in science that encourage the pattern of imitation and copying. Thus, instead of obeying a strict method, scientists follow a certain conduct which is far from being rational in the ordinary sense. Participation in meetings is an opportunity to imitate other scientists by watching them and not only by getting an information from them through reading their work or listening to their lectures without having a live message from them. The non-verbal behavior includes hidden parts of the message. For instance, we can learn from this how much the scientist believes in certain ideas. This is the reason why scientific meetings play such an important role in modern science. Plato behaved like a memetician, but he could not benefit from this social kind of imitation.

Two replicators, memes and genes, co-evolve. Meme-gene co-evolution produced the brain that is especially good at copying certain kinds of memes. A more general process occurs when a replicator and its replication machinery evolve together. According to this conception, the human brain has been designed not just for the benefit of human genes, but also for the replication of memes and their selection. The scientist is connected to the collective brain of the scientific community which produces the achievements of science and technology.

5. Genotype – Memotype

Now we may ask what is the counterpart of the genotype-phenotype distinction in memetics. The genotype represents the genetic makeup of an organism. If we call "memotype" the counterpart of the genotype in memetics, then in science the memotype is the memetic makeup of a scientific theory. The memetic content of a theory is the collection of ideas and patterns of research which are imitated or copied. We may say that the memotype corresponds to the (Lakatosian) hard-core of a theory or to the (Kuhnian) paradigm. Since memes are units of information which propagate themselves in the memo-pool, scientists imitate only ideas restricted to the prevailing paradigm or hard-core. The phenotype corresponds to the theory constructed around these ideas, including the mathematical equations and the detailed descriptions of the system. Predictions can be derived from the latter. Hence, according to this picture the development of the memotype of a given theory is not chaotic; memes/ideas are restricted to the memotype.

According to memetics there are two patterns of behavior of the scientist: opportunism and imitation. However, both are restricted. The opportunist exploits ideas from the present meme-pool and imitates ideas expressed by other scientists who belong to the paradigm. However, occasionally

a scientist may draw ideas from external sources and thus enriches the idea-pool – see the case of Plato, who exploited ideas from his experience with footballs.

6. Ideas which Spread in Particle Physics

There are memeticians who claim that a meme is a kind of an "infectious idea" since it spreads in a given population as a contagious disease. Dawkins' writes in The Selfish Gene that the expression "infectious idea" refers sometimes to the manner a meme spreads in certain populations. If we talk about a scientific community, we may refer to cases such as the theories of "internal symmetry" in particle physics, which spread by imitation. Let us start with isospin symmetry. It attributed a similarity between the proton and the neutron – the nucleons. This is a "broken" symmetry corresponding to the ("slightly") different masses and charges of the nucleons. This theory was considered to be quite successful in yielding good predictions. Therefore the physicists continued to invent bigger symmetries and they arrived at the "eightfold way" where the doublet of nucleons was enlarged into an octet which included six more particles. These eight particles were called "hadrons" The symmetry-breaking between them was much larger. The new theory was considered to be still successful. But it involved many problems. These symmetries were called "internal" symmetries since they were not related to ordinary space-time. These symmetries yielded new particles. The eight particles, including the nucleons were called "hadrons". Then appeared the theory of "quarks": all hadrons are built of three sub-particles – the quarks. Although free quarks have never been observed experimentally, this could be explained by the so-called "quark-confinement" and quark theory was considered to be a success. Following this a new kinds of sub-particles appeared, such as Haim Harari's "rishonims" which remained in their discoverers' head. This were unsuccessful imitations.

The dissemination of these ideas related to internal symmetries and their products the new "atoms" about fifty years ago can be viewed as a spread of infectious disease. At the time these symmetries were regarded very successful, but already in 1965-7 these concepts were replaced by concepts related to the "standard model" with its new internal symmetries. Thus, since the eightfold way, theories of symmetry were popular in particle physics; when one theory of symmetry failed, physicists looked for another one. And if we adopt Dawkins' terminology, it resembled a spread of infectious disease. Indeed, these theories were regarded more and more problematic – certainly not as the final truth in the field. Scientific progress, according to this conception, is no less than a spread of an epidemic.

Thus, a variety of theories of this kind spread in particle physics in the sixties and the seventies of the last century. Was it an epidemic, fashion or pursuit of truth? We can distinguish here between two conceptions of science: searching for truth or exploiting opportunities to find successful theories. In fact, a science can yield accurate predictions, but very problematic in providing a coherent world-picture. This is the case with the standard model and with other physical theories including quantum mechanics itself. In fact, the above-mentioned two conceptions are realism and instrumentalism which engage extensively philosophers of science. Memetics is a kind of instrumentalism. Memeticians do not look after truth. They look after memes that will spread throughout wide communities of scientists. If they indeed spread, scientists with a realistic stance may treat them as approaching truth. They spread since they seem to agree with experimental results and/or with mathematical/theoretical calculations, but this does not imply anything about their truth. The G2 symmetry model (which preceded the eightfold way), the eightfold way (based on the SU(3) symmetry model) SU(6) symmetry (which included spin) and the standard model where but few examples to this epidemic.

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There are two kinds of mechanisms which encourage the generation of serendipitous discoveries. Internal processes that take place inside the discoverer's head and external-social processes that take place in the social-communicational level of a scientific community. Serendipitous discoveries may be explained by an internal mechanism, which generates semi-random combinations of mental elements or ideas – i.e. memes – in the discoverer's head, mainly subconsciously. This is a psychological conjecture about processes taking place in the individual mind. Scientists, such as Poincare and Einstein, tell us about something like this from their experience. One of the results of such a process is the subconscious phenomenon of incubation, where the system of mental elements is expecting a new element for completing the process. An external event can supply the missing element; the discoverer experiences then an Archimedian *eureka* event.

A complementary serendipitous process takes place in the external-social level. The scientific community and its communication network have a decisive role in cultivating this kind of serendipity. When a scientist proposes an idea for solving a certain problem, the idea may spread in the community, undergoes changes and at last solves unintentionally another problem. The idea does not remain under the control of the original person who proposed it; it has autonomic existence and it can develop in directions the initiator did not envisage and can be applied to different areas and finally a different problem is solved. As an example to such an external-social process we may take the development of quantum mechanics from Planck's idea of the quantum of energy, which was intended to solve the problem of black-body radiation. Quantum theory which developed from this idea solved problems and supplied explanations to phenomena Planck have not dreamt about. Therefore, the social medium of the scientific community cultivates serendipity and therefore the evolutionary blindness of scientific development. In these cases, the process of discovery takes place not only in the individual scientist's head; a large portion of the community deals with the subject and participates in the process. The discovery is produced, therefore, by a community of researchers, where scientists contribute ideas and exploit ideas of their colleagues. The process starts with efforts to solve certain problems and ends with solutions to different problems. The final discovery, in these cases, is therefore blind - it is not achieved as a result of a pressure to solve the solved problem. The process suits the model of natural selection, according to which a variation is generated blindly.

7. Memetics: Science or Philosophy of Science?

There are memeticians who attempt treating memetics as an exact science. But, as we saw, the phenomenon of copying and stealing ideas as described by Picasso and Einstein, for instance, is far from being treated by such a science; at most, it can be described by popular psychology and social science. However, it may be treated by evolutionary epistemology, which deals with the manners ideas evolve and disseminate. Evolutionary epistemology (see for instance Kantorovich, A. and Y. Ne'eman, 1989) is a naturalized philosophy which have psychological-social dimension. But it is not a mathematical discipline.

The process of growth of knowledge has three stages: first is the appearance of new conjectures or ideas; second the rejection of those that do not pass observational or theoretical examination and finally a dissemination of the successful ideas that withstood severe tests. In the neo-Darwinean model the variations generating new traits are not generated deliberately in order to match the environmental conditions. Only after they were generated, the environment selects the fittest. The process is blind. However, blind variation is not totally coincidental; in biology, the genetic background of a specific biological species would restrict the possible kinds of variations. Donald Campbell (1974) distinguishes between blind and coincidental variations; the second is totally arbitrary, whereas the first is not. Blind variation must obey, for instance, the laws of molecular

biology. And so is a new theory: a physical theory must obey some minimal conditions, before the scientific community will treat it seriously. Physical memes must be expressed in mathematical language – only then they may be imitated and disseminate in the scientific community.

The dissemination of certain memes indicates that these memes represent elements of reliable information. One known definition of knowledge in traditional epistemology is "justified true belief". In memetics we may characterize knowledge as a collection of memes which disseminate successfully in a given community. Thus, there is no room for the concepts of "truth" and "justification". Instead, scientists act in an instrumentalist fashion and employ the concept of "successful dissemination", which is relative to a specific community.

Memes, as units of information, may replace direct contact with dangerous objects or situations and thus help survival and so they provide an evolutionary advantage. At the lower part of the evolutionary ladder there are the unicellular organisms, such as the ameba which gets the main information about its environment via direct physical contact. This information is very reliable but may be very dangerous. We are located at the top of the ladder; for instance, we construct theories about the big bang, but we do not have to visit there in order to test the theories. If the information we get by theoretical investigation spread effectively, we treat it as reliable. Thus, truth is replaced by effective dissemination of the memes. Human beings, products of biological evolution, continue the process in the level of knowledge and technology, which developed from the biological level.

Memetics treats science as a communication network. Philosophy of science (POS), on the other hand, deals, for instance, with the relations between observational and theoretical statements or with the ontological status of theoretical entities such as elementary particles, i.e. with the content and interpretation of scientific statements. Memetics deals with units of information and the ways they pass between brains, without treating their content. POS deals with justification, refutation and explanation (etc.) of events and phenomena. The memetic aspect and the POS aspect are posibely related: if the POS aspect is accepted by most philosophers in the field, then the dissemination of the relevant concepts or memes represents information which consequently spreads effectively in the community.

The philosophy of rationality is thus replaced by memetics where rationality is replaced by dissemination and imitation; a scientist chooses to copy ideas that have better chance to spread and to imitate his colleagues that are more efficient in disseminating their ideas. Thus, memetics looks like propaganda for ideas. The question is whether the successful scientist is no more than an efficient propagandist for his/her ideas. But current POS cannot establish truth (nor falsity). So we are left with the criteria of memetics. These criteria can be evaluated only by the effectiveness of the dissemination of the relevant memes but not by their "truth-value". Thus, the criterion of truth is replaced by the criterion of dissemination of memes.

Modern physics, in particular quantum physics and relativistic physics, is abundant with paradoxes, riddles and puzzles. It seems that the more abstract is a theory, we encounter more paradoxes. If we adopt a memetic stance, in particular with instrumentalist approach, we would not be bothered by paradoxes, since the only thing which will interest us is the spread of ideas and not their coherence. A modern physical theory has two components – the mathematical part and the interpretation. When physicists concentrate in computations, the theory may provide successful mathematical explanations and predictions and the instrumentalists will be satisfied. When physicists turn to interpretations and verbal explanations, then paradoxes arise. Memetism is interested in the dissemination of ideas and ignore paradoxes.

Niels Bohr, one of the founding fathers of quantum mechanics, is quoted as saying: "If quantum mechanics hasn't profoundly shocked you, you haven't understood it yet". Bohr refers here to the plethora of paradoxes which populates the theory; he admits that quantum mechanics cannot be understood properly even by expert quantum physicists. One of the reasons for the interest in memetism among some physicists is that paradoxes can be ignored in the memetic approach. Memetism ignores the meaning of ideas and theories and deals mainly with their rate of spread, although he latter is affected by the content of the memes. Medical doctors or psychologists who fight an epidemic which is spread by micro-organisms or memes, respectively, are interested in their rate of spread. The latter is influenced by the changes in the ecology of the host population; in science it means – by changes in the prevailing paradigm of the host scientific community. For instance, internal symmetries spread among particle physicists after what was considered to be the great success of the eightfold way.

To sum up, since philosophers of science and scientists, such as Popper, Einstein and Bohr, despair from seeing science, and in particular modern theoretical physics, as representing absolute truth, they view science as dealing with "lies" and "infectious diseases", in the spirit of memetism and instrumentalism. Instead of learning from scholars and experts in a given field of science, they imitate their colleagues and copy their ideas.

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