The Geometry of Conventionality

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Abstract

Hans Reichenbach famously argued that the geometry of spacetime is conventional in relativity theory, in the sense that one can freely choose the spacetime metric so long as one is willing to postulate a "universal force field". Here we make precise a sense in which the field Reichenbach defines fails to represent a "force field". We then argue that there is an interesting and perhaps tenable sense in which geometry is conventional in classical spacetimes. We conclude with a no-go result showing that the variety of conventionalism available in classical spacetimes does not extend to relativistic spacetimes.

Keywords: Reichenbach, conventionality of geometry, general relativity, Newton-Cartan theory, geometrized Newtonian gravitation

Reichenbach (1958) famously argued that spacetime geometry in relativity theory is conventional, in the following precise sense. Suppose that the geometry of spacetime is given by a model of general relativity, (M, g_{ab}).[1] Reichenbach claimed that one could equally well represent spacetime by any other (conformally equivalent) model,[2] $(M, \tilde{g}_{ab})$, so long as one

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[1]A *relativistic spacetime* is an ordered pair (M, g_{ab}), where M is a smooth, connected, paracompact, Hausdorff 4-manifold and g_{ab} is a smooth Lorentzian metric. Relativistic spacetimes are models of general relativity. For background, including details of the "abstract index" notation used here, see Malament (2012) or Wald (1984).

[2]Two metrics g_{ab} and $\tilde{g}_{ab}$ are said to be *conformally equivalent* if there is some non-vanishing scalar field Ω such that $g_{ab} = \Omega^2 \tilde{g}_{ab}$. Two spacetime metrics are conformally equivalent just in case they agree on causal structure, i.e., they agree with regard to which vectors at a point are timelike or null. Reichenbach did not insist on conformal equivalence when he originally stated his conventionality thesis, but, as Malament (1986) argues, given that he argued elsewhere that the causal structure of spacetime was non-conventional, to make his views consistent it seems one needs to insist that metric structure is conventional only up to a conformal transformation.

was willing to postulate a "universal force field" G_{ab}, defined by $g_{ab} = \tilde{g}_{ab} + G_{ab}$. Various commentators have had the intuition that this universal force field is "funny"—i.e., that it is not a "force field" in any standard sense.[3] We will begin by presenting a concrete example that, we believe, undermines the interpretation of G_{ab} as representing a "force field" at all. We will next show that in classical spacetimes there *is* a robust sense in which arbitrary choices of spacetime geometry can be accommodated by postulating a universal force field, albeit with a rather different trade-off equation from the one Reichenbach proposed. Indeed, the force field one needs to postulate in that context is not so funny at all: in certain ways, it is strikingly similar to familiar force fields, such as the electromagnetic field. Turning back to relativity theory, we will prove a no-go result to the effect that the trade-off equation we describe for classical spacetimes does not have a relativistic analog. The upshot is that there is an interesting and perhaps tenable sense in which geometry is conventional in classical spacetimes, but in the relativistic setting Reichenbach's position seems much less appealing.[4]

It will be helpful to begin with a few preliminary remarks about "forces" and "force fields".[5] By "force" we mean some physical quantity acting on a massive body (or, for present purposes, a massive point particle). Forces are represented by vectors at a point and the total force acting on a particle at a point (computed by taking the vector sum of all of the individual forces acting at that point) must be proportional to the acceleration of the particle at that point. We understand forces to give rise to acceleration, and so we

[3]We get the term "funny force" from Malament (1986), though it may predate him. Other classic discussions of Reichenbach's conventionality thesis, including various expressions of skepticism, can be found in Sklar (1977), Glymour (1977), and Norton (1994).

[4]Of course, there are many reasons why one might be skeptical about claims concerning the conventionality of geometry, aside from the character of the force law. (See Sklar (1977) for a detailed discussion.) Our point here is to clarify just how a conventionality thesis would go if one were serious about postulating a universal force field in any recognizable sense.

[5]What follows should not be construed as a full account or explication of either "force" or "force field". Instead, our aim is to explain how we are using the terms below. That said, we believe that any reasonable account of "force" or "force field" in a Newtonian or relativistic framework would need to agree on at least this much, and so when we refer to forces/force fields "in the standard sense," we have in mind forces or force fields that have the character we describe here.

expect the total force at a point to vanish just in case the acceleration vanishes. Since the acceleration of a curve at a point, as determined relative to some derivative operator, is always orthogonal to the tangent vector of the curve at that point, it follows that the total force on a particle at a point must always be orthogonal to the tangent vector of the particle's worldline at that point.

A "force field," meanwhile, is a field on spacetime that may give rise to forces on particles/bodies at a given point, where the force produced by a given force field may depend on factors such as the charge or velocity of a body.[6] We understand force field to generate forces on bodies, and so there can be a force associated with a given force field at a point just in case the force field is non-vanishing at that point. (The converse need not hold: a force field may be non-vanishing at a point and yet give rise to forces for only some particles at that point.) A canonical example of a force field is the electromagnetic field in relativity theory. Fix a relativistic spacetime (M, g_{ab}). Then the electromagnetic field is represented by the Faraday tensor, which is an anti-symmetric rank 2 tensor field F_{ab} on M. Given a particle of charge q, the force experienced by the particle at a point p of its worldline is given by $F^a{}_b\xi^b$, where ξ^a is the unit tangent vector to the particle's worldline at p. Note that since F_{ab} is anti-symmetric, this force is always orthogonal to the worldline of the particle, because $F_{ab}\xi^a\xi^b = 0$.[7]

Given this background, one can immediately identify several troubling features of Reichenbach's proposal for a "universal force field". For one, Reichenbach does not give a prescription for how the force field he defines gives rise to forces on particles or bodies. That is, he gives no relationship between the value of his field G_{ab} at a point and a vector

[6]Note that there is a possible ambiguity here between a "force field" in the present sense, which may be represented by a tensor field and which gives rise to forces on particles at each point of spacetime, and a vector field that directly assigns a force to each point of spacetime. We will always use the term in the former, more general sense.

[7]In general one can understand the other so-called "fundamental forces" as acting on particles via a force field in much the same way, though we are not limiting attention to force fields that arise in this way.

quantity, except to say that the force field is "universal", which we take to mean that the relationship between the force field and the force experienced by a particle at a point does not depend on features of the particle such as its charge or species. One might imagine that the relationship is assumed to be analogous to that between other force fields represented by a rank 2 tensor field, such as the electromagnetic field, and their associated forces at a point. But this does not work. Given Reichenbach's definition, it is immediate that G_{ab} must be *symmetric*, and thus the vector $G^a{}_b\xi^b$ can be orthogonal to ξ^a at a point p for all timelike vectors ξ^a at p—i.e., for all vectors tangent to possible worldlines of massive particles through p—only if G_{ab} vanishes at p. For these reasons, it is difficult to directly evaluate Reichenbach's proposal.

That said, there is a way to see that Reichenbach's "universal force field" is problematic even without an account of how it relates to the force on a particle. Consider the following example. Let (M, η_{ab}) be Minkowski spacetime and let ∇ be the Levi-Civita derivative operator compatible with η_{ab}.[8] Choose a coordinate system t, x, y, z such that $\eta_{ab} = \nabla_a t \nabla_b t - \nabla_a x \nabla_b x - \nabla_a y \nabla_b y - \nabla_a z \nabla_b z$. Now consider a second spacetime $(M, \tilde{g}_{ab})$, where $\tilde{g}_{ab} = \Omega^2 \eta_{ab}$ for $\Omega(t, x, y, z) = x^2 + 1/2$, and let $\tilde{\nabla}$ be the Levi-Civita derivative operator compatible with $\tilde{g}_{ab}$. Then $\tilde{\xi}^a = \Omega^{-1} \left(\frac{\partial}{\partial t}\right)^a$ is a smooth timelike vector field on M with unit length relative to $\tilde{g}_{ab}$. Let γ be the maximal integral curve of $\tilde{\xi}^a$ through the point $(0, 1/\sqrt{2}, 0, 0)$. The acceleration of this curve, relative to $\tilde{\nabla}$, is $\tilde{\xi}^n \tilde{\nabla}_n \tilde{\xi}^a = 2\sqrt{2} \left(\frac{\partial}{\partial x}\right)^a$ for all points on $\gamma[I]$. Meanwhile, γ is a geodesic (up to reparameterization) of ∇, the Levi-Civita derivative operator compatible with g_{ab}. According to Reichenbach, it would seem to be a matter of convention whether (1) $\gamma[I]$ is the worldline of a free massive point particle in (M, η_{ab}) or (2) $\gamma[I]$ is the worldline of a massive point particle in $(M, \tilde{g}_{ab})$, accelerating due to the universal force field $G_{ab} = \eta_{ab} - \tilde{g}_{ab}$. But now observe: along $\gamma[I]$, the conformal factor Ω is equal to

[8] Minkowski spacetime is the relativistic spacetime (M, η_{ab}) where M is $\mathbb{R}^4$ and (M, η_{ab}) is flat and geodesically complete.

1—which means that along $\gamma[I]$, $g_{ab} = \tilde{g}_{ab}$ and thus $G_{ab} = \mathbf{0}$. And so, if one adopts option (2) above, one is committed to the view that the universal force field can accelerate particles even where G_{ab} vanishes.

This example shows that G_{ab} cannot be a force field in the standard sense (i.e., as described above), since a force field cannot vanish if the force it is meant to give rise to is non-vanishing (or, equivalently, the acceleration associated with that force is non-vanishing). It appears to follow that, whatever else may be the case about Reichenbachian conventionalism about geometry in relativity theory, the universal force field Reichenbach defines is unacceptable. This example is especially striking because, as we will presently argue, there is a natural sense in which classical spacetimes *do* support a kind of Reichenbachian conventionalism about geometry, though the construction is quite different from what Reichenbach describes. To motivate our construction, we will begin by considering (an analog of) Reichenbach's trade-off equation in classical spacetimes. Suppose the geometry of spacetime is given by a classical spacetime (M, t_a, h^{ab}, ∇).[9] Direct analogy with Reichenbach's trade-off equation would have us consider classical metrics $\tilde{t}_a$ and $\tilde{h}^{ab}$ and universal force fields F^a and G_{ab} satisfying $t_a = \tilde{t}_a + F_a$ and $h^{ab} = \tilde{h}^{ab} + G^{ab}$. We might want to assume that G^{ab} must be symmetric, since $\tilde{h}^{ab}$ is assumed to be a classical spatial metric. And as in the relativistic case, we might insist that these new metrics preserve causal structure—which here would mean that the compatibility condition $\tilde{t}_a \tilde{h}^{ab} = \mathbf{0}$ must be met, and that simultaneity relations between points must be preserved by the transformation, which means that $t_a \tilde{h}^{ab} = \mathbf{0}$ and $\tilde{t}_a h^{ab} = \mathbf{0}$. Together, these imply that $G^{ab} F_b = \mathbf{0}$.

Given these trade-off equations, Reichenbachian conventionalism about classical space-

[9] A classical spacetime is an ordered quadruple (M, t_a, h^{ab}, ∇), where M is a smooth, connected, paracompact, Hausdorff 4-manifold, t_a and h^{ab} are smooth fields with signatures $(1, 0, 0, 0)$ and $(0, 1, 1, 1)$ respectively, which together satisfy $t_a h^{ab} = \mathbf{0}$, and ∇ is a smooth derivative operator satisfying the compatibility conditions $\nabla_a t_b = \mathbf{0}$ and $\nabla_a h^{ab} = \mathbf{0}$. The fields t_a and h^{ab} may be interpreted as a (degenerate) "temporal metric" and a (degenerate) "spatial metric", respectively. Classical spacetimes are models of Newtonian gravitation and geometrized Newtonian gravitation (sometimes, Newton-Cartan theory). For more on classical spacetimes, see Malament (2012).

time geometry might go something like this: the metrics (t_a, h^{ab}) are merely conventional since we could always use $(\tilde{t}_a, \tilde{h}^{ab})$ instead, so long as we also postulate universal forces F_a and G^{ab}. One could perhaps investigate this proposal to see how changes in the classical metrics affect the associated families of compatible derivative operators, or even just to understand what the degrees of freedom are. But there is an immediate sense in which this proposal is ill-formed. The issue is that the metrical structure of a classical spacetime does not have a close relationship to the acceleration of curves or to the motion of bodies. Acceleration is determined relative to a choice of derivative operator and in general there are infinitely many derivative operators compatible with any pair of classical metrics. All of these give rise to different standards of acceleration. And so it is not clear that the fields F_a and G^{ab} bear any relation to the acceleration of a body. As in the relativistic example given above, this counts against interpreting them as force fields at all.

These considerations suggest that Reichenbach's force field does not do any better in Newtonian gravitation than it does in general relativity. But it also points in the direction of a different route to conventionalism about classical spacetime geometry. The proposal above failed because acceleration is determined relative to a choice of derivative operator, not classical metrics. Could it be that the choice of derivative operator in a classical spacetime is a matter of convention, so long as the choice is appropriately accommodated by some sort of universal force field? We claim that the answer is "yes".

Proposition 1. *Fix a classical spacetime (M, t_a, h^{ab}, ∇) and consider an arbitrary torsion-free derivative operator on M, $\tilde{\nabla}$, which we assume to be compatible with t_a and h^{ab}. Then there exists a unique anti-symmetric field G_{ab} such that given any timelike curve γ with unit tangent vector field ξ^a, $\xi^n \nabla_n \xi^a = \mathbf{0}$ if and only if $\xi^n \tilde{\nabla}_n \xi^a = G^a{}_n \xi^n$, where $G^a{}_n \xi^n = h^{am} G_{mn} \xi^n$.*

Proof. If such a field exists, then it is necessarily unique, since the defining relation determines its action on all vectors (because the space of vectors at a point is spanned by the timelike vectors). So it suffices to prove existence. Since $\tilde{\nabla}$ is compatible with t_a and

h^{ab}, it follows from Prop. 4.1.3 of Malament (2012) that the $C^a{}_{bc}$ field relating it to ∇ must be of the form $C^a{}_{bc} = 2h^{an}t_{(b}\kappa_{c)n}$, for some anti-symmetric field κ_{ab}.[10] Pick some timelike geodesic γ of ∇, and suppose that ξ^a is its unit tangent vector field. Then the acceleration relative to $\tilde{\nabla}$ is given by $\xi^n\tilde{\nabla}_n\xi^a = \xi^n\nabla_n\xi^a - C^a{}_{nm}\xi^n\xi^m = -2h^{ar}t_{(n}\kappa_{m)r}\xi^n\xi^m = -2h^{ar}\kappa_{mr}\xi^m$. So we can take $G_{ab} = 2\kappa_{ab}$ and we have existence. $\square$

This proposition means that one is free to choose any derivative operator one likes (compatible with the fixed classical metrics) and, by postulating a universal force field, one can recover all of the allowed trajectories of either a model of standard Newtonian gravitation or a model of geometrized Newtonian gravitation. Thus, since the derivative operator determines both the collection of geodesics—i.e., non-accelerating curves—and the curvature of spacetime, there is a Reichenbachian sense in which both acceleration and curvature are conventional in classical spacetimes. Most importantly, the field G_{ab} makes good geometrical sense as a force field. Like the Faraday tensor, the field defined in Prop. 1 is an anti-symmetric, rank 2 tensor field; moreover, this field is related to the acceleration of a body in precisely the same way that the Faraday tensor is (except that all particle have the same "charge"), which means that the force generated by the field G_{ab} on a particle at some point is always orthogonal to the worldline of the particle at that point. Thus G_{ab} as defined in Prop. 1 is not a "funny" force field at all.[11]

It is interesting to note that from this perspective, geometrized Newtonian gravitation and standard Newtonian gravitation are just special cases of a much more general phenomenon. Specifically, one can always choose the derivative operator associated with a

[10] The notation of $C^a{}_{bc}$ fields used here is explained in Malament (2012, Ch. 1.7) and Wald (1984, Ch. 3). Briefly, fix a derivative operator ∇ on a smooth manifold M. Then any other derivative operator $\tilde{\nabla}$ can be written as $\tilde{\nabla} = (\nabla, C^a{}_{bc})$, where $C^a{}_{bc}$ is a smooth, symmetric (in the lower indices) tensor field that allows one to express the action of $\tilde{\nabla}$ on an arbitrary tensor field in terms of the action of ∇ on that field.

[11] Of course, we have not provided any field equation(s) for G_{ab}, and so some readers might object that they cannot evaluate whether G_{ab} is "funny" or not. At very least, the analogy with the Faraday tensor is limited, since one cannot expect G_{ab} to satisfy Maxwell's equations. This is a fair objection to the specific claim we make here—though it applies equally well to Reichenbach's own proposal.

classical spacetime in such a way that the curvature satisfies the geometrized Poisson equation and the allowed trajectories of bodies are geodesics (yielding geometrized Newtonian gravitation), or one can choose the derivative operator so that the curvature vanishes—and when one makes this second choice, if other background geometrical constraints are met, the force field takes on the particularly simple form $G_{ab} = 2\nabla_{[a}\varphi t_{b]}$, for some scalar field φ that satisfies Poisson's equation (yielding standard Newtonian gravitation). These are non-trivial facts, but they arguably indicate that some choices of derivative operator are more convenient to work with than others (because the associated G_{ab} fields take simple forms), and not that these choices are canonical.[12]

Now let us return to the original question, concerning conventionality about geometry in relativity theory. We have seen that in classical spacetimes, there is a trade-off between choice of derivative operator and a not-so-funny universal force field that does yield a kind of Reichenbachian conventionality. Does a similar result hold in relativity? The analogous proposal would go as follows. Fix a relativistic spacetime (M, g_{ab}), and let ∇ be the Levi-Civita derivative operator associated with g_{ab}. Now consider another torsion-free derivative operator $\tilde{\nabla}$.[13] We know that $\tilde{\nabla}$ cannot be compatible with g_{ab}, but we can insist that causal structure is preserved, and so we can require that there is some metric $\tilde{g}_{ab} = \Omega^2 g_{ab}$ such that $\tilde{\nabla}$ is compatible with $\tilde{g}_{ab}$. The question we want to ask is this. Is there some rank 2 tensor field G_{ab} such that, given a curve γ, γ is a geodesic (up to reparameterization) relative to

[12]There is certainly more to say here regarding what, if anything, makes the classes of derivative operators associated with standard Newtonian gravitation and geometrized Newtonian gravitation "special", in light of Prop. 1. Several arguments in the literature might be taken to apply. For instance, though he does not show anything as general as Prop. 1, Glymour (1977) has observed that one can think of the gravitational force in Newtonian gravitation as a Reichenbachian universal force. He goes on to resist conventionalism by arguing that geometrized Newtonian gravitation is better confirmed, since it is empirically equivalent to Newtonian gravitation (with the funny force), but postulates strictly less. For an alternative perspective on the relationship between Newtonian gravitation and geometrized Newtonian gravitation, see Weatherall (2013). A second argument for why geometrized Newtonian gravitation should be preferred to standard Newtonian gravitation—one that can likely be extended to the present context—has recently been offered by Knox (2013). But we will not address this question further in the present paper.

[13]An interesting question that we do not address here is whether the torsion of the derivative operator can be seen as conventional in a Reichenbachian sense.

∇ just in case its acceleration relative to $\tilde{\nabla}$ is given by $G^a{}_n\tilde{\xi}^n$, where $\tilde{\xi}^a$ is the tangent field to γ with unit length relative to $\tilde{g}_{ab}$? The answer is "no", as can be seen from the following proposition.

Proposition 2. *Let (M, g_{ab}) be a relativistic spacetime, let $\tilde{g}_{ab} = \Omega^2 g_{ab}$ be a metric conformally equivalent to g_{ab}, and let ∇ and $\tilde{\nabla}$ be the Levi-Civita derivative operators compatible with g_{ab} and $\tilde{g}_{ab}$, respectively. Suppose Ω is non-constant.*[14] *Then there is no tensor field G_{ab} such that an arbitrary curve γ is a geodesic relative to ∇ if and only if its acceleration relative to $\tilde{\nabla}$ is given by $G^a{}_n\tilde{\xi}^n$, where $\tilde{\xi}^n$ is the tangent field to γ with unit length relative to $\tilde{g}_{ab}$.*

Proof. Since g_{ab} and $\tilde{g}_{ab}$ are conformally equivalent, their associated derivative operators are related by $\tilde{\nabla} = (\nabla, C^a{}_{bc})$, where $C^a{}_{bc} = -1/(2\Omega^2)\,(\delta^a{}_b\nabla_c\Omega^2 + \delta^a{}_c\nabla_b\Omega^2 - g_{bc}g^{ar}\nabla_r\Omega^2)$. Moreover, given any smooth timelike curve γ, if ξ^a is the tangent field to γ with unit length relative to g_{ab}, then $\tilde{\xi}^a = \Omega^{-1}\xi^a$ is the tangent field to γ with unit length relative to $\tilde{g}_{ab}$. A brief calculation reveals that if γ is a geodesic relative to ∇, then the acceleration of γ relative to $\tilde{\nabla}$ is given by $\tilde{\xi}^n\tilde{\nabla}_n\tilde{\xi}^a = \tilde{\xi}^n\nabla_n\tilde{\xi}^a - C^a{}_{nm}\tilde{\xi}^n\tilde{\xi}^m = \Omega^{-3}\,(\xi^a\xi^n\nabla_n\Omega - g^{ar}\nabla_r\Omega)$. Now suppose that a tensor field G_{ab} as described in the proposition existed. It would have to satisfy $\Omega^{-1}\tilde{g}^{an}G_{nm}\xi^m = \Omega^{-3}\,(\xi^a\xi^n\nabla_n\Omega - g^{ar}\nabla_r\Omega)$ for every unit (relative to g_{ab}) vector field ξ^a tangent to a geodesic (relative to ∇). Note in particular that G_{ab} must be well-defined as a tensor at each point, and so this relation must hold for *all* unit timelike vectors at any point p, since any vector at a point can be extended to be the tangent field of a geodesic passing through that point. Pick a point p where $\nabla_a\Omega$ is non-vanishing (which must exist, since we assume Ω is non-constant), and consider an arbitrary pair of distinct, co-oriented unit (relative to g_{ab}) timelike vectors at that point, μ^a and η^a. Note that there always exists some number α such that $\zeta^a = \alpha(\mu^a + \eta^a)$ is also a unit timelike vector. Then it follows

[14] If Ω were constant, then the force field $G_{ab} = \mathbf{0}$ would meet the requirements of the proposition. But metrics related by a constant conformal factor are usually taken to be physically equivalent, since they differ only by an overall choice of units.

that,

$$\tilde{g}^{an}G_{nm}\zeta^m = \frac{1}{\Omega^2}\left(\zeta^a\zeta^n\nabla_n\Omega - g^{ar}\nabla_r\Omega\right) = \frac{1}{\Omega^2}\left(\alpha^2\left(\mu^a\mu^n + \mu^a\eta^n + \eta^a\mu^n + \eta^a\eta^n\right)\nabla_n\Omega - g^{ar}\nabla_r\Omega\right).$$

But since G_{ab} is a linear map, we also have

$$\tilde{g}^{an}G_{nm}\zeta^m = \alpha\tilde{g}^{an}G_{nm}\mu^m + \alpha\tilde{g}^{an}G_{nm}\eta^m = \frac{\alpha}{\Omega^2}\left(\mu^a\mu^n\nabla_n\Omega - g^{ar}\nabla_r\Omega\right) + \frac{\alpha}{\Omega^2}\left(\eta^a\eta^n\nabla_n\Omega - g^{ar}\nabla_r\Omega\right).$$

These two expressions must be equal, which, with some rearrangement of terms, implies that

$$(2\alpha - 1)g^{ar}\nabla_r\Omega = \alpha\left[(1-\alpha)(\mu^a\mu^n + \eta^a\eta^n) - 2\alpha\eta^{(a}\mu^{n)}\right]\nabla_n\Omega.$$

But this expression yields a contradiction, since the left hand side is a vector with fixed orientation, independent of the choice of μ^a and η^a, whereas the orientation of the right hand side will vary with μ^a and η^a, which were arbitrary. Thus G_{ab} cannot be a tensor at p. $\square$

So it would seem that we do not have the same freedom to choose between derivative operators in general relativity that we have in classical spacetimes—at least not if we want the "universal force field" to be represented by a rank 2 tensor field.

Of course, the considerations raised here do not *refute* Reichenbachian conventionalism. For instance, one might argue that the senses of "force" and "force field" that we described above, which play an important role in our objection to Reichenbach's trade-off equation, are too limiting, and that there is some generalized notion of force field that can accommodate the field Reichenbach defines. Perhaps a more appealing option would be to argue that the force field need not be represented by a rank 2 tensor field. And indeed, given a relativistic spacetime (M, g_{ab}), a conformally equivalent metric $\tilde{g}_{ab}$, and their respective derivative operators, ∇ and $\tilde{\nabla}$, there is always *some* tensor field such that we can get a "funny force field" trade-off. Specifically, a curve γ will be a geodesic relative to ∇ just in case its acceleration relative to $\tilde{\nabla}$ is $\tilde{\xi}^n \tilde{\nabla}_n \tilde{\xi}^a = G^a{}_{nm} \tilde{\xi}^n \tilde{\xi}^m$, where $\tilde{\xi}^a$ is the unit (relative to $\tilde{g}_{ab}$) vector field tangent to γ, and $G^a{}_{bc} = -(\Omega^{-1} \delta^a{}_b \nabla_c \Omega + C^a{}_{bc})$, with $C^a{}_{bc}$ the field relating $\tilde{\nabla}$ to ∇. That the field $G^a{}_{bc}$ exists should be no surprise—it merely reflects the fact that the action of one derivative operator can always be expressed in terms of any other derivative operator and a rank three tensor. This $G^a{}_{bc}$ field presents a more compelling force field than the one Reichenbach defines, since $G^a{}_{bc}$ will always be proportional (in a generalized sense) to the acceleration of a body, just as one should expect. In particular, it will vanish precisely when the acceleration of the body does, which as we have seen is not the case for Reichenbach's force field.

Ultimately, though, the attractiveness of a conventionalist thesis turns on how much one needs to postulate in order to accommodate alternative conventions. In some sense, one can be a conventionalist about *anything*, if one is willing to postulate enough—an evil demon, say. The considerations we have raised here should be understood in this light. We have shown that in the Newtonian context, one does not need to postulate very much to support a kind of conventionalism about spacetime geometry: one can accommodate any torsion-free derivative operator compatible with the classical metrics so long as one is willing to postulate a force field that acts in many ways like familiar force fields, such as the electromagnetic field. Of course, one may still resist conventionalism about classical spacetime geometry by arguing that even this is too much. But whatever else is the case, it seems the costs of accepting conventionalism about geometry in relativity theory are higher still. As we have shown, Reichenbach's own proposal requires a non-standard sense of "force/force field"; meanwhile, if one wants to maintain the standard notion of "force field," then the universal force field one needs to postulate cannot be represented by a rank 2 tensor field. So it must be something more exotic than we are accustomed to—which, it seems to us, counts against the appeal of Reichenbach's view in relativity theory.

Acknowledgments

The authors would like to thank David Malament, Erik Curiel, Arthur Fine, Thomas Ryckman, J. Brian Pitts, Jeremy Butterfield, Adam Caulton, Eleanor Knox, and an anonymous referee for helpful comments on previous drafts of this paper. Versions of this work have been presented to the Hungarian Academy of Science, to a seminar at the University of Pittsburgh, and at the 17th UK and European Meeting on the Foundations of Physics; we are grateful to the organizers of these events and for the insightful discussions that followed the talks.

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