Interpreting the Modal Kochen-Specker Theorem: Possibility and Many Worlds in Quantum Mechanics

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Abstract

In this paper we attempt to physically interpret the Modal Kochen-Specker (MKS) theorem. In order to do so, we analyze the features of the possible properties about quantum systems arising from the elements in an orthomodular lattice and distinguish the use of "possibility" in the classical and quantum formalisms. Taking into account the modal and many worlds non-collapse interpretation of the projection postulate, we discuss how the MKS theorem rules the constrains to actualization, and thus, the relation between actual and possible realms.

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Introduction

In classical physics, every physical system may be described exclusively by means of its *actual properties*, taking 'actuality' as expressing the *preexistent*

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mode of being of the properties themselves, independently of observation — the 'pre' referring to its existence previous to measurement. The evolution of the system may be described by the change of its actual properties. Mathematically, the state is represented by a point $(p,\ q)$ in the correspondent phase space Γ and, given the initial conditions, the equation of motion tells us how this point moves in Γ.[1] Physical magnitudes are represented by real functions over Γ. These functions commute between each other and can be all interpreted as possessing definite values at any time, independently of physical observation. Thus, as mentioned above, each magnitude can be interpreted as being actually preexistent to any possible measurement without conflicting with the mathematical formulation of the theory. In this scheme, speaking about potential or possible properties usually refers to functions of the points in Γ to which the state of the system might arrive to in a future instant of time; these points, in turn are also completely determined by the equations of motion and the initial conditions.

In the orthodox formulation of quantum mechanics (QM), the representation of the state of a system is given by a ray in Hilbert space $\mathcal{H}$. But, contrary to the classical scheme, physical magnitudes are represented by operators on $\mathcal{H}$ that, in general, do not commute. This mathematical fact has extremely problematic interpretational consequences for it is then difficult to affirm that these quantum magnitudes are *simultaneously preexistent*. In order to restrict the discourse to sets of commuting magnitudes, different Complete Sets of Commuting Operators (CSCO) have to be chosen. This choice has not found until today a clear justification and remains problematic. This feature is called in the literature *quantum contextuality* and will be discussed in section 1. Another fundamental feature of QM is due to the linearity of the Schrödinger equation which implies the formal existence of the well known *quantum superpositions*. The path from a superposition of states to the eigenstate corresponding to the measured eigenvalue is given, formally, by an axiom added to the formalism: *the projection postulate*. In section 2 we will discuss the different physical interpretations of this postulate which is, either thought in terms of a "collapse" of the wave function (i.e., as a real physical interaction) or in terms of non-collapse proposals, such as the modal and many worlds interpretations. After having introduced and discussed these two main features of QM —namely, the formal existence of contextuality and superpositions— we will present, in section 3, our formal analysis regarding possibility in orthomodular structures. In

[1]For simplicity, we have in mind a system that is only a material point.

section 4, we shall discuss and analyze the distinction between mathematical formalism and physical interpretation, a distinction which can raise many pseudo-problems if not carefully taken into account. As a consequence of this distinction we will also put forward the difference between 'classical possibility' and 'quantum possibility'. In section 5, we are ready to advance towards a physical interpretation of both of quantum possibility and the MKS theorem —taking into account the specific formal constraints implied by it to modality. In section 6 we will discuss the consequences of the MKS theorem regarding the many worlds interpretation. Finally, in section 7, we provide the conclusions of our work.

1 Quantum Contextuality and Modality

The idea that a *preexistent* set of definite properties constitutes or describes reality is one of the basic ideas which remains the fundament of all classical physical theories and determines the possibility to discuss about an independent objective world, a world which does not depend on our choices or consciousness. Physical reality can be then conceived and analyzed in terms of a theory —which describes a preexistent world— independently of actual observation. But, as it is well known, this description of physical reality faces several inconveniences when presupposed in the interpretation of the quantum formalism. In formal terms, this is demonstrated by the Kochen-Specker (KS) theorem, which states that if we consider three physical magnitudes represented by operators $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$, with $\mathbf{A}$ commuting with $\mathbf{B}$ and $\mathbf{C}$ but $\mathbf{B}$ non-commuting with $\mathbf{C}$, the value of $\mathbf{A}$ depends on the choice of the context of inquiry; i.e. whether $\mathbf{A}$ is considered together with $\mathbf{B}$ or together with $\mathbf{C}$ [30]. From an instrumentalist point of view, this is bypassed by considering the context (in KS sense) as the experimental arrangement —in line with the original idea of N. Bohr. However, if we attempt to go beyond the discourse regarding measurement results and provide some kind of realist representation of what is going on according to QM, we need to make sense of the indeterminateness of definite valued properties. As Chris Isham and Andreas Döring clearly point:

> "When dealing with a closed system, what is needed is a realist interpretation of the theory, not one that is instrumentalist. The exact meaning of 'realist' is infinitely debatable but, when used by physicists, it typically means the following:

1. The idea of 'a property of the system' (i.e., 'the value of a physical quantity') is meaningful, and representable in the theory.
2. Propositions about the system are handled using Boolean logic. This requirement is compelling in so far as we humans think in a Boolean way.
3. There is a space of 'microstates' such that specifying a microstate leads to unequivocal truth values for all propositions about the system. The existence of such a state space is a natural way of ensuring that the first two requirements are satisfied.

> The standard interpretation of classical physics satisfies these requirements, and provides the paradigmatic example of a realist philosophy in science. On the other hand, the existence of such an interpretation in quantum theory is foiled by the famous Kochen-Specker theorem." [22, p. 2]

Contextuality can be directly related to the impossibility to represent a piece of the world as constituted by a set of definite valued properties independently of the choice of the context. This definition makes reference only to the actual realm. But as we know, QM makes probabilistic assertions about measurement results. Therefore, it seems natural to assume that QM does not only deal with actualities but also with possibilities. Then the question arises whether the space of possibilities is subject to the same restrictions as the space of actualities. Formally, on the one hand, the set of actualities is structured as the orthomodular lattice of subspaces of the Hilbert space of the states of the system and, as Michael Dickson remarks in [10], the KS theorem (i.e., the absence of a family of compatible valuations from subalgebras of the orthomodular lattice to the Boolean algebra of two elements **2**) can be understood as a consequence of the failure of the distributive law in the lattice. On the other hand, given an adequate definition of the possibility operator $\Diamond$ —as the one developed in [18, 20]— the set of possibilities is the center of the enlarged structure. Since the elements of the center of a structure are those which commute with all other elements, one might think that the possible propositions defined in this way escape from the constrains arising from the non-commutative character of the algebra of operators. Thus, at first sight one might assume that possibilities behave in a classical manner.

When predicting measuring results the context has been already fixed. However, probability is a measure over the whole lattice and, consequently, the set of events over which the measure is defined is non-distributive, calling the attention on the interpretation of possibility and probability. As noticed by Schrödinger in a letter to Einstein [2, p. 115]: "It seems to me that the concept of probability is terribly mishandled these days. Probability surely has as its substance a statement as to whether something *is* or *is not* the case –of an uncertain statement, to be sure. But nevertheless it has meaning only if one is indeed convinced that the something in question quite definitely *is* or *is not* the case. A probabilistic assertion presupposes the full reality of its subject." Also von Neumann was worried about a sound definition of probability, as it is referred in [37].[2] The difficulties with a rigorous definition of probability made von Neumann abandon the orthodox formalism of QM in Hilbert space which he himself had too much contributed to develop and face the classification of the factors and their dimension functions which led to the subject of von Neumann's algebras.[3]

In order to explicitly verify whether modal propositions escape from KS-type contradictions, in previous works we have developed a mathematical scheme which allowed us to deal with both actual and possible propositions in the same structure.[4] Within this frame we were able to prove a theorem

[2] As Rédei [37, p. 157] states: "To see why von Neumann insisted on the modularity of quantum logic, one has to understand that he wanted quantum logic to be not only the propositional calculus of a quantum mechanical system but also wanted it to serve as the event structure in the sense of probability theory. In other words, what von Neumann aimed at was establishing the quantum analogue of the classical situation, where a Boolean algebra can be interpreted both as the Tarski-Lindenbaum algebra of a classical propositional logic and as the algebraic structure representing the random events of a classical probability theory, with probability being an additive normalized measure on the Boolean algebra."

[3] It might be argued that a complete theory of quantum probability is still lacking. On the one hand, type II_1 factor (the one whose projection lattice is a continuous geometry, and thus an othomodular modular lattice as required by a definition of probability) is not an adequate structure to represent quantum events. On the other hand, there exist different candidates for defining conditional probability and there is not a unique criterium for choosing among them [5]. Moreover, there are situations in which the frequentist interpretation does not apply and consequently it is required to develop new probability structures to account for quantum phenomena [23].

[4] Van Fraassen distinguishes two different isomorphic structures for dealing with possible and actual properties ([42], chapter 9). The main aspects of van Fraassen's modal interpretation in terms of quantum logic are as follows. The probabilities are of events, each describable as 'an observable having a certain value', corresponding to value states. If w is a physical situation in which system X exists, then X has both a *dynamic state* φ and a *value state* λ, i.e. $w =< \varphi, \lambda >$. A *value state* λ is a map of observable $\mathbf{A}$ into non-empty Borel sets σ such that it assigns $\{1\}$ to $1_\sigma \mathbf{A}$. 1_σ is the characteristic function of the set σ of values. So, if the observable $1_\sigma \mathbf{A}$ has value 1, then it is impossible that $\mathbf{A}$ has a value outside σ. The proposition $< \mathbf{A}, \sigma >= \{w : \lambda(w)(\mathbf{A}) \subseteq \sigma\}$ assigns values to physical magnitudes, it is

which describes the algebraic relations between both kinds of propositions. The theorem shows explicitly the formal limits of *possible actualizations*, in short, that no enrichment of the orthomodular lattice with modal propositions allows to circumvent the contextual character of the quantum language. For obvious reasons, we called it the Modal Kochen-Specker (MKS) theorem [18]. As in the case regarding actual propositions, the MKS theorem may be demonstrated with topological tools [19]. It is important to remark that our formalism also provides a formal meaning in an algebraic frame to the Born rule, something that has been discussed by Dieks in relation to the possible derivation of a preferred probability measure [14]. Possible actualizations relate to the path between the possible and the actual realms. As said before, within QM this path remains extremely problematic in itself and needs to be taken into account, contrary to classical physics, through the introduction of an axiom, the projection postulate. In the following section we will analyze its meaning and possible interpretation.

2 Projection Postulate and Quantum Collapse

Classical texts that describe QM axiomatically begin stating that the mathematical interpretation of a quantum system is a Hilbert space, that pure

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a *value-attribution proposition* and is read as '$\mathbf{A}$ (actually) has value in σ'. $\mathcal{V}$ is called the set of value attributions $\mathcal{V} = \{< \mathbf{A}, \sigma >: \mathbf{A}$ an observable and σ a Borel set$\}$. The logic operations among value-attribution propositions are defined as: $< \mathbf{A}, \sigma >^{\perp} = < \mathbf{A}, \Re - \sigma >$, $< \mathbf{A}, \sigma > \wedge < \mathbf{A}, \theta > = < \mathbf{A}, \sigma \cap \theta >$, $< \mathbf{A}, \sigma > \vee < \mathbf{A}, \theta > = < \mathbf{A}, \sigma \cup \theta >$ and $\wedge \{< \mathbf{A}, \sigma_i >: i \in \mathcal{N}\} = < \mathbf{A}, \cap \{\sigma_i : i \in \mathcal{N}\} >$. With all this, $\mathcal{V}$ is the union of a family of Boolean σ-algebras $< \mathbf{A} >$ with common unit and zero equal to $< \mathbf{A}, S(\mathbf{A}) >$ and $< \mathbf{A}, \wedge >$ respectively. The Law of Excluded Middle is satisfied: every situation w belongs to $q \vee q^{\perp}$, but not the Law of Bivalence: situation w may belong neither to q nor to $q^{\perp}$. A *dynamic state* φ is a function from $\mathcal{V}$ into [0, 1], whose restriction to each Boolean σ-algebra $< \mathbf{A} >$ is a probability measure. The relation between dynamic and value states is the following: φ and λ are a dynamic state and a value state respectively, only if there exist possible situations w and w' such that $\varphi = \varphi(w)$, $\lambda = \lambda(w')$. Here, φ is an eigenstate of $\mathbf{A}$, with corresponding eigenvalue a, exactly if $\varphi(< \mathbf{A}, \{a\} >) = 1$. The *state-attribution proposition* $[\mathbf{A}, \sigma]$ is defined as: $[\mathbf{A}, \sigma] = \{w : \varphi(w)(< \mathbf{A}, \sigma >) = 1\}$ and means '$\mathbf{A}$ must have value in σ'. $\mathcal{P}$ denotes the set of state-attribution propositions: $\mathcal{P} = \{[\mathbf{A}, \sigma] : \mathbf{A}$ an observable, σ a Borel set$\}$. Partial order between them is given by $[\mathbf{A}, \sigma] \subseteq [\mathbf{A}', \sigma']$ only if, for all dynamical states φ, $\varphi(< \mathbf{A}, \sigma >) \leq \varphi(< \mathbf{A}', \sigma' >)$ and the logic operations are (well) defined as: $[\mathbf{A}, \sigma]^{\perp} = [\mathbf{A}, \Re - \sigma]$, $[\mathbf{A}, \sigma] \uplus [\mathbf{A}, \theta] = [\mathbf{A}, \sigma \cup \theta]$ and $[\mathbf{A}, \sigma] \cap [\mathbf{A}, \theta] = [\mathbf{A}, \sigma \cap \theta]$. With all this, $< \mathcal{P}, \subseteq, ^{\perp} >$ is an orthoposet, the orthoposet formed by 'pasting together' a family of Boolean algebras in which whole operations coincide in areas of overlap. It may be enriched to approach the lattice of subspaces of Hilbert space.

states are represented by rays in this space, physical magnitudes by self-adjoint operators on the state space and that the evolution of the system is ruled by the Schrödinger equation. Possible results of a given magnitude are the eigenvalues of the corresponding operator obtained with probabilities given by the Born rule. In general the state previous to the measurement is a linear superposition of eigenstates corresponding to different eigenvalues of the measured observable. In order to give an account of the state of the system after the appearance of a particular result a new axiom needs to be added: the *projection postulate*. In von Neunmann's [44, p. 214] words: "Therefore, if the system is initially found in a state in which the values of $\mathcal{R}$ cannot be predicted with certainty, then this state is transformed by a measurement M of $\mathcal{R}$ into another state: namely, into one in which the value of $\mathcal{R}$ is uniquely determined. Moreover, the new state, in which M places the system, depends not only on the arrangement of M, but also on the result of M (which could not be predicted causally in the original state) —because the value of $\mathcal{R}$ in the new state must actually be equal to this M-result".[5] At this point one needs to introduce the so called *eigenstate-eigenvalue link: after the measurement, the state of the system is that (i.e., the eigenstate) which corresponds to the measured eigenvalue.*

There are different ways to give account of the projection postulate. One of them is to consider it as a "collapse" which takes place during measurement, i.e. as a real physical stochastic "jump" from the state previous to the measurement to the eigenstate corresponding to the measured eigenvalue [4, 40]. This interpretation was strongly debated by the founding fathers, Schrödinger himself is quoted to have said: "Had I known that we were not going to get rid of this damned quantum jumping, I never would have involved myself in this business!".[6] Another way of dealing with the postulate is by adding non-linear terms to the equation of evolution that may conduce

[5] Or in Dirac's words: "When we measure a real dynamical variable ξ, the disturbance involved in the act of measurement causes a jump in the state of the dynamical system. From physical continuity, if we make a second measurement of the same dynamical variable ξ immediately after the first, the result of the second measurement must be the same as that of the first" [16, p. 36].

[6] As noticed by Dieks [15, p. 120]: "Collapses constitute a process of evolution that conflicts with the evolution governed by the Schrödinger equation. And this raises the question of exactly when during the measurement process such a collapse could take place or, in other words, of when the Schrödinger equation is suspended. This question has become very urgent in the last couple of decades, during which sophisticated experiments have clearly demonstrated that in interaction processes on the sub-microscopic, microscopic and mesoscopic scales collapses are never encountered."

to the eigenstate in a stochastic manner, as in the case of GRW theory [26].

A different way to approach the problem is to deny the existence of a collapse during measurement but still use the projection postulate as an interpretational rule. A well known non-collapse interpretation is the so called modal interpretation (MI) of QM. This approach states that superpositions remain in the possible realm always intact, independent of the actual observation.[7] Contrary to the orthodox interpretation, MI keeps the complete superposition in the level of possibility independently of the particular actualization. One might say that the eigenstate-eigenvalue link is understood only in one direction, implying that given a state that is an eigenstate there is a definite value of the corresponding magnitude, i.e. its eigenvalue, but not otherwise. "In modal interpretations the state is not updated if a certain state of affairs becomes actual. The non-actualized possibilities are not removed from the description of a system and this state therefor codifies not only what is presently actual but also what was presently possible. These non-actualized possibilities can, as a consequence, in principle still affect the course of later events." [43, p. 295] There are thus, within MI, two independent levels given by the possible and the actual.[8] The passage from the possible realm to the actual realm is given through different interpretational rules according to the different versions of the MI [43]. However, one could argue that within this scheme —even though van Fraassen[9] and Dieks have taken a stance in favor of an empiricist position regarding modality [41, 15]— there is still place to interpret possibility in an ontological fashion [38].

Many worlds interpretation (MWI) of QM is another well known non-collapse interpretation which has become an important line of investigation within the foundations of quantum theory domain. It is considered to be a direct conclusion from Everett's first proposal in terms of 'relative states'

[7]Van Fraassen discusses the problems of the collapse of the quantum wave function in [42], section 7.3. See also [11]. Dieks [12, p. 182] argues that: "[...] there is no need for the projection postulate. On the theoretical level the full superposition of states is always maintained, and the time evolution is unitary. One could say that the 'projection' has been shifted from the level of the theoretical formalism to the semantics: it is only the empirical interpretation of the superposition that the component terms sometimes, and to some extent, receive an independent status."

[8]These levels are explicitly formally accounted for in both van Fraasen and Dieks MI. While van Fraassen distinguishes between the 'dynamical states' and the 'value states', Dieks and Vermaas consider a distinction between 'physical states' and 'mathematical states'.

[9]According to van Fraassen: modalities are in our theories, not in the world.

[24]. Everett's idea was to let QM find its own interpretation, making justice to the symmetries inherent in the Hilbert space formalism in a simple and convincing way [8]. The solution proposed to the measurement problem is provided by assuming that each one of the terms in the superposition is *actual* in its own correspondent world. Thus, it is not only the single value which we see in 'our world' which gets actualized but rather, that a branching of worlds takes place in every measurement, giving rise to a multiplicity of worlds with their corresponding actual values. The possible splits of the worlds are determined by the laws of QM.

> "The whole issue of the transition from 'possible' to 'actual' is taken care of in the theory in a very simple way —there is no such transition, nor is such a transition necessary for the theory to be in accord with our experience. *From the viewpoint of the theory all elements of a superposition (all 'branches') are 'actual', none any more 'real' than the rest.* It is unnecessary to suppose that all but one are somehow destroyed, since all the separate elements of a superposition individually obey the wave equation with complete indifference to the presence or absence ('actuality' or not) of any other elements. This total lack of effect of one branch on another also implies that no observer will ever be aware of any 'splitting' process." [25, pp. 146-147]

In this case, there is no need to conceptually distinguish between possible and actual because each state is actual inside its own branch and the eigenstate-eigenvalue link is maintained in each world.

3 On the Formal Limits of Possibility

After having discussed some interpretational aspects of both modality and actualization we now shortly review our own development and analysis of the notion of possibility inside the formalism. First we recall from [29, 34] some notions about orthomodular lattices. A *lattice with involution* [28] is an algebra $\langle \mathcal{L}, \vee, \wedge, \neg \rangle$ such that $\langle \mathcal{L}, \vee, \wedge \rangle$ is a lattice and $\neg$ is a unary operation on $\mathcal{L}$ that fulfills the following conditions: $\neg\neg x = x$ and $\neg(x \vee y) = \neg x \wedge \neg y$. An *orthomodular lattice* is an algebra $\langle \mathcal{L}, \wedge, \vee, \neg, 0, 1 \rangle \leftarrow$ of type $\langle 2, 2, 1, 0, 0 \rangle$ that satisfies the following conditions:

1. $\langle \mathcal{L}, \wedge, \vee, \neg, 0, 1 \rangle$ is a bounded lattice with involution,
2. $x \wedge \neg x = 0$.

3. $x \vee (\neg x \wedge (x \vee y)) = x \vee y$

We denote by $\mathcal{OML}$ the variety of orthomodular lattices. Let $\mathcal{L}$ be an orthomodular lattice. *Boolean algebras* are orthomodular lattices satisfying the *distributive law* $x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$. We denote by $\mathbf{2}$ the Boolean algebra of two elements. Let $\mathcal{L}$ be an orthomodular lattice. An element $c \in \mathcal{L}$ is said to be a *complement* of a iff $a \wedge c = 0$ and $a \vee c = 1$. Given a, b, c in $\mathcal{L}$, we write: $(a, b, c)D$ iff $(a \vee b) \wedge c = (a \wedge c) \vee (b \wedge c)$; $(a, b, c)D^*$ iff $(a \wedge b) \vee c = (a \vee c) \wedge (b \vee c)$ and $(a, b, c)T$ iff $(a, b, c)D$, (a,b,c)D^* hold for all permutations of a, b, c. An element z of $\mathcal{L}$ is called *central* iff for all elements $a, b \in L$ we have $(a, b, z)T$. We denote by $Z(\mathcal{L})$ the set of all central elements of $\mathcal{L}$ and it is called the *center* of $\mathcal{L}$.

Proposition 3.1 *Let $\mathcal{L}$ be an orthomodular lattice. Then we have:*

1. *$Z(\mathcal{L})$ is a Boolean sublattice of $\mathcal{L}$* [34, Theorem 4.15].
2. *$z \in Z(\mathcal{L})$ iff for each $a \in \mathcal{L}$, $a = (a \wedge z) \vee (a \wedge \neg z)$* [34, Lemma 29.9].

$\square$

In the tradition of the quantum logical research, a property of (or a proposition about) a quantum system is related to a closed subspace of the Hilbert space $\mathcal{H}$ of its (pure) states or, analogously, to the projector operator onto that subspace. Moreover, each projector is associated to a dichotomic question about the actuality of the property [44, p. 247]. A physical magnitude $\mathcal{M}$ is represented by an operator $\mathbf{M}$ acting over the state space. For bounded self-adjoint operators, conditions for the existence of the spectral decomposition $\mathbf{M} = \sum_i a_i \mathbf{P}_i = \sum_i a_i |a_i\rangle\langle a_i|$ are satisfied. The real numbers a_i are related to the outcomes of measurements of the magnitude $\mathcal{M}$ and projectors $|a_i\rangle\langle a_i|$ to the mentioned properties. The physical properties of the system are organized in the lattice of closed subspaces $\mathcal{L}(\mathcal{H})$ that, for the finite dimensional case, is a modular lattice, and an orthomodular one in the infinite case [34]. Moreover, each self-adjoint operator $\mathbf{M}$ has associated a Boolean sublattice $W_{\mathbf{M}}$ of $\mathcal{L}(\mathcal{H})$ which we will refer to as the spectral algebra of the operator $\mathbf{M}$. Assigning values to a physical quantity $\mathcal{M}$ is equivalent to establishing a Boolean homomorphism $v : W_{\mathbf{M}} \to \mathbf{2}$. As it is well known, the KS theorem rules out the non-contextual assignment of definite values to the physical properties of a quantum system. This may be expressed in terms of valuations over $\mathcal{L}(\mathcal{H})$ in the following manner. We first introduce the concept of global valuation. Let $(W_i)_{i\in I}$ be the family of Boolean sublattices of $\mathcal{L}(\mathcal{H})$. Then a *global valuation* of the physical magnitudes over $\mathcal{L}(\mathcal{H})$ is a family of Boolean homomorphisms $(v_i : W_i \rightarrow \mathbf{2})_{i\in I}$ such that $v_i \mid W_i \cap W_j = v_j \mid W_i \cap W_j$ for each $i, j \in I$. If this global valuation existed, it would allow to give values to all magnitudes at the same time maintaining a *compatibility condition* in the sense that whenever two magnitudes shear one or more projectors, the values assigned to those projectors are the same from every context. The KS theorem, in the algebraic terms, rules out the existence of global valuations when $dim(\mathcal{H}) > 2$:

Theorem 3.2 [17, Theorem 3.2] *If $\mathcal{H}$ is a Hilbert space such that $dim(\mathcal{H}) > 2$, then a global valuation, i.e. a family of Boolean homomorphisms over the spectral algebras satisfying the compatibility condition, over $L(\mathcal{H})$ is not possible.* □

In what follows we delineate a modal extension for orthomodular lattices that allows to formally represent, within the same algebraic structure, actual and possible properties of the system. This allows us to discuss the restrictions posed by the theory itself to the *actualization* of possible properties. Given a proposition about the system, it is possible to define a context from which one can predicate with certainty about it together with a set of propositions that are compatible with it and, at the same time, predicate probabilities about the other ones (Born rule). In other words, one may predicate truth or falsity of all possibilities at the same time, i.e., possibilities allow an interpretation in a Boolean algebra. In rigorous terms, let P be a proposition about a system and consider it as an element of an orthomodular lattice $\mathcal{L}$. If we refer with $\Diamond P$ to the possibility of P then, by Proposition 3.1, we assume that $\Diamond P \in Z(\mathcal{L})$.

This interpretation of possibility in terms of the Boolean algebra of central elements of $\mathcal{L}$ reflects the fact that one can simultaneously predicate about all possibilities because Boolean homomorphisms of the form $v : Z(\mathcal{L}) \rightarrow \mathbf{2}$ can be always established. If P is a proposition about the system and P occurs, then it is trivially possible that P occurs. This is expressed as $P \leq \Diamond P$. Classical consequences that are compatible with a given property, for example probability assignments to the actuality of other propositions, shear the classical frame. These consequences are the same ones as those which would be obtained by considering the original actual property as a possible property. This is interpreted as, if P is a property of the system, $\Diamond P$ is the smallest central element greater than P.

This enriched orthomodular structure can be axiomatized by equations conforming a variety denoted by $\mathcal{OML}^\Diamond$ [18, Theorem 4.5]. More precisely, each element of $\mathcal{OML}^\Diamond$ is an algebra $\langle \mathcal{L}, \wedge, \vee, \neg, \Diamond, 0, 1\rangle$ of type $\langle 2, 2, 1, 1, 0, 0\rangle$ such that $\langle \mathcal{L}, \wedge, \vee, \neg, 0, 1\rangle$ is an orthomodular lattice and $\Diamond$ satisfies the following equations:

S1 $x \leq \Diamond x$

S2 $\Diamond 0 = 0$

S3 $\Diamond\Diamond x = \Diamond x$

S4 $\Diamond(x \vee y) = \Diamond x \vee \Diamond y$

S5 $y = (y \wedge \Diamond x) \vee (y \wedge \neg\Diamond x)$

S6 $\Diamond(x \wedge \Diamond y) = \Diamond x \wedge \Diamond y$

S7 $\neg\Diamond x \wedge \Diamond y \leq \Diamond(\neg x \wedge (y \vee x))$

Each algebra of $\mathcal{OML}^\Diamond$ is called *Boolean saturated orthomodular lattice*. Orthomodular complete lattices are examples of Boolean saturated orthomodular lattices. If $\mathcal{L}$ is a Boolean saturated orthomodular lattice, it is not very hard to see that for each $x \in \mathcal{L}$,

$$\Diamond x = Min\{z \in Z(\mathcal{L}) : x \leq z\}$$

We can embed each orthomodular lattice $\mathcal{L}$ in an element $\mathcal{L}^\Diamond \in \mathcal{OML}^\Diamond$ (see [18, Theorem 10]). In general, $\mathcal{L}^\Diamond$ is referred as a *modal extension of* $\mathcal{L}$. This modal extension represents the fact that each orthomodular system can be modally enriched in such a way as to obtain a new propositional system that includes the original propositions in addition to their possibilities. These possibilities are formulated as classical propositions. Let $\mathcal{L}$ be an orthomodular lattice and $\mathcal{L}^\Diamond$ a modal extension of $\mathcal{L}$. We define the possibility space of $\mathcal{L}$ in $\mathcal{L}^\Diamond$ as as the subalgebra of $\mathcal{L}^\Diamond$ generated by the set $\{\Diamond(P) : P \in \mathcal{L}\}$. This algebra is denoted by $\Diamond\mathcal{L}$ and we can prove that it is a Boolean subalgebra of the modal extension.

The possibility space represents the modal content added to the discourse about properties of the system. Within this frame, the actualization of a possible property acquires a rigorous meaning. Let $\mathcal{L}$ be an orthomodular lattice, $(W_i)_{i\in I}$ the family of Boolean sublattices of $\mathcal{L}$ and $\mathcal{L}^\Diamond$ a modal extension of $\mathcal{L}$. If $f : \Diamond\mathcal{L} \rightarrow \mathbf{2}$ is a Boolean homomorphism, an actualization compatible with f is a global valuation $(v_i : W_i \rightarrow \mathbf{2})_{i\in I}$ such that $v_i \mid W_i \cap \Diamond\mathcal{L} = f \mid W_i \cap \Diamond\mathcal{L}$ for each $i \in I$. A kind of converse of this possibility of actualizing properties may be read as an algebraic representation of the Born rule, something that has no place in the orthomodular lattice alone. *Compatible actualizations* represent the passage from possibility to actuality, they may be regarded as formal constrains when applying the interpretational rules proposed in the different modal versions. When taking into account compatible actualizations from different contexts, an analogous of the KS theorem holds for possible properties.

Theorem 3.3 [18, Theorem 6.2] *Let $\mathcal{L}$ be an orthomodular lattice. Then $\mathcal{L}$ admits a global valuation iff for each possibility space there exists a Boolean homomorphism $f : \Diamond\mathcal{L} \to \mathbf{2}$ that admits a compatible actualization.* $\square$

The MKS theorem shows that no enrichment of the orthomodular lattice with modal propositions allows to circumvent the contextual character of the quantum language. Thus, from a formal perspective, one is forced to conclude that quantum possibility is something different from classical possibility.

4 Distinguishing the Mathematical Formalism from its Physical Interpretation

The almost direct relation between classical logic and natural language is not respected within QM. We argue that this fact must be carefully taken into account and might be responsible of many pseudo problems when considering the question "what is QM talking about?" In the following section, we attempt to provide a clear distinction between the algebraic structure, its correspondent formal language and the meta-language used in the theory.

At this point, regarding the question of interpretation, we need to be explicit about the stance we shall take regarding the possibility of going beyond the concepts of classical physics. Following Dieks we argue that one should not demand that classical physics should determine the conceptual tools of new theories, for this "would deny the possibility of really new fundamental theories, conceptually independent of classical physics." [13, p. 1417] Thus, we do not take for granted there is a self evident and univocal interpretation of a mathematical formalism —i.e., a pre-established set of concepts which have to be necessarily applied to interpret mathematical structures. For as Heisenberg [27, p. 264] remarked: "The history of physics is not only a sequence of experimental discoveries and observations, followed by their mathematical description; it is also a history of concepts. For an understanding of the phenomena the first condition is the introduction of adequate concepts. Only with the help of correct concepts can we really know what has been observed."

In mathematical terms, a context is a Boolean subalgebra of the complete lattice. Thus, it may seem that the natural language we use to refer to the compatible magnitudes represented by these commuting operators poses no problem. However, this is not the case, due to the fact it is also necessary to take into account the state of the system (usually a superposition) when interpreting this algebra as the algebra of a set of magnitudes of a physical system. Indeed, we have to consider two very different cases: may be the state is an eigenstate of the CSCO —a trivial case in which the values of all magnitudes represented by the operators in the CSCO are determined, even when not measured— or it may be the case that the state of the system is not an eigenstate —the general case in which the election of the CSCO does not determine anything; in fact it only establishes which magnitudes we are interested in. In this case, when we refer to possible properties we have to keep in mind that the meaning of "posssible" is not the same in the Boolean structure of classical logic and in the Boolean subalgebra of the orthomodular structure of QM. Let us make this point clear. Note that if $\mathcal{L}$ is a Boolean saturated orthomodular lattice in which $\mathcal{L}$ is a Boolean algebra, $\Diamond$ is the identity operator. This can be seen from the fact that, if $\mathcal{L}$ is a Boolean structure, $\mathcal{L} = Z(\mathcal{L})$ and then $\Diamond x = Min\{z \in Z(\mathcal{L}) : x \leq z\} = x$ since $x \leq x$ and $x \in Z(\mathcal{L})$. In other words, the concept of possibility that corresponds to the definition of the previous section becomes a "trivial possibility" in a classical structure. We have indicated with $\Diamond$ the possibility operator related to the Boolean structure and we add a subindex Q for the quantum case. Thus, $\Diamond_Q$ is the $\Diamond$ related to the orthomodular structure. Summing up,

	Algebraic Structure	**Language**	**Meta-Language**
Classical Mechanics	*Boolean lattice,* $\Diamond$	$\Diamond$ *means possibility within a Boolean structure*	*(classical) possibility*
Quantum Mechanics	*orthomodular lattice,* $\Diamond_Q$	$\Diamond_Q$ *means possibility within an orthomodular structure*	*quantum possibility*

In the classical case, the elements $A \in \wp(\Gamma)$ interpreted as the properties of the system are part of a Boolean algebra (with Γ the classical phase space and $\wp(\Gamma)$ its power set). The elements of the corresponding modal structure are constructed by applying the possibility operator $\Diamond$ to the elements A. These new elements $\Diamond A$, that belong to the modal structure, correspond to possible properties as spoken in the natural language. However, in this case, the seemingly larger structure that includes both actual and modal propositions does not enlarge the expressive power of the language. This is due to the fact that there exists a trivial correspondence between any pair of classical valuations v_c and w_c of the actual and the possible structures to truth-falsity. This relation can be written as follows: let $A_k \in \wp(\Gamma)$, k a fix index, then:

$$w_c(\Diamond A_k) = 1 \Leftrightarrow v_c(A_k) = 1$$

$$w_c(\Diamond A_k) = 0 \Leftrightarrow v_c(A_k) = 0$$

Thus, given the state of a classical system, possible properties at a certain time coincide with (simultaneous) actual ones, they may be identified. And the distinction between the two sets of properties is never made. In fact, when referring to possible properties of a classical system in a given state, one is always making reference to *future* possible values of the magnitudes, values that are determined because they are the evaluation of functions at points $(p,\ q)$ in Γ at future times. These points are determined in turn by the equation of motion. Thus, not even future possibilities are classically indeterminate and they coincide with *future actual properties*.

On the contrary, in the quantum case, the projectors $\mathbf{P}_a = |a\rangle\langle a|$ on $\mathcal{H}$, which are interpreted as the properties of a system, belong to an orthomodular structure. As we have mentioned above, the orthomodular lattice is enlarged with its modal content by adding the elements $\Diamond_Q|a\rangle\langle a|$. Due to the fact that there is no trivial relation between the valuations of subsets of the possible and actual elements to truth-falsity, this new structure genuinely enlarges the expressive power of the language. Formally, if $w_q(\Diamond_Q \mathbf{P}_k) = 1$, with $\mathbf{P}_k \in W_i$, then there exists a valuation v_q such that $v_q(\mathbf{P}_k) = 1$ and another v'_q such that $v'_q(\mathbf{P}_k) = 0$. Thus, contrary to the classical case, even at the same instant of time, we may consider two different kind of properties, two different realms, possible and actual, that do not coincide. In order not to misinterpret the $\Diamond_Q$ operator, it is of great importance to clearly distinguish between the formal language and the metalanguage. As a matter of fact, both $\Diamond A$ and $\Diamond_Q|a\rangle\langle a|$ are called *possible* within their own structures even though, at least formally, the meaning of "possible" in each case is extremely different.

5 Quantum Possibility and the Physical Interpretation of the MKS Theorem

In the literature regarding QM many times the classical notion of possibility is self evidently assumed —without any criticism nor analysis— as a tool to interpret the formalism. As we have argued above, there is however no reason why such interpretation of the formalism should be necessarily applied, rather, this is part of an interpretational choice. In this section, we are mainly interested in the physical interpretation of the MKS theorem and the consequences and constraints it might determine, within the formalism, for applying a coherent interpretation to $\Diamond_Q |a\rangle\langle a|$. For this purpose we shall explicitly distinguish between these two notions of possibility, both formally and linguistically. To avoid any misunderstanding we shall use "possibility" in relation to the operator appearing in any Boolean structure ($\Diamond$) and "quantum possibility" when the operator relates to an orthomodular structure ($\Diamond_Q$).

The distance between quantum and classical possibilities is also related to the formal difference between classical and quantum probabilities. As it is well known, the possibility of actualization of a physical property in classical statistical mechanics is given by the *probability weights*. In this case the probability is Kolmogorovian and can be interpreted as epistemic; i. e. as providing information of an unknown —but existent— state of affairs. In the quantum case, the wave function is a linear combination of vectors (each one in correspondence with a projector in the orthomodular lattice, associated to a physical property) with complex coefficients interpreted as *probability amplitudes*. Contrary to the classical case, the probability implied by this structure is a non-Kolmogorovian one, and thus, cannot be interpreted in terms of ignorance. Furthermore, when evaluating quantum probabilities there is an *interference term* which does not appear within the classical probability scheme. So, at least formally, probabilities in QM interfere. The meaning of this interference of possibilities deserves careful attention.[10]

It is important to distinguish here between the interference among the

[10] As noticed by Dieks [15, p. 124-125]: "In classical physics the most fundamental description of a physical system (a point in phase space) reflects only the actual, and nothing that is merely possible. It is true that sometimes states involving probabilities occur in classical physics: think of the probability distributions ρ in statistical mechanics. But the occurrence of possibilities in such cases merely reflects our ignorance about what is actual. The statistical states do not correspond to features of the actual system (unlike the case of the quantum mechanical superpositions), but quantify our lack of knowledge of those actual features. This relates to the essential point of difference between quantum mechanics and classical mechanics [...]: in quantum mechanics the possibilities contained in the superposition state may interfere with each other. There is nothing comparable in classical physics. In statistical mechanics the possibilities contained in ρ evolve separately from each other and do not have any mutual influence. Only one of these possibilities corresponds to the actual situation."

coefficients of the wave function, i.e. a vector in $\mathcal{H}$, and the interference of waves in three-dimensional coordinate space. Both waves belong to and evolve in quite different domains. While the Schrödinger wave function belongs to Hilbert space, a space which like configuration space shifts its dimensions with the considered situation —being in general an infinite dimensional space—, the classical or electromagnetic wave evolves in a three-dimensional coordinate space which, in turn, can be interpreted as physical space. This makes problematic to interpret the wave function, a probability measure over the orthomodular lattice that changes unitarily in $\mathcal{H}$, as 'something existing within physical space-time'. Many times in the literature these two situations are thought as equivalent, leading to inconsistencies. Moreover, as we have already mentioned, while in the classical case at each instant of time there is an isomorphism between the valuations to truth of possible and actual properties, in the Boolean saturated orthomodular lattice this isomorphism disappears, leaving actuality and possibility as two different and separated realms. Finally, it is also important to remark that the fact quantum possibilities interact —through entangled and superposition states— is used today within the latest technical developments in quantum information processing [36, 9, 46, 3, 39, 45, 32, 35, 1]. Finally, we call the attention to the *welcher-weg* type experiments which seem to break down the classical understanding of causal possibility [33, 6].

In order to physically interpret our MKS theorem what we need is an explicit map between the formal language and the meta-language. Instead of presupposing a set of metaphysical principles from which the formalism needs to be developed, our proposal attempts to provide a coherent interpretation starting from what we know about the formalism itself, and the structures it determines. In order to do so, we construct a dictionary relating names to the elements of the different structures:

1. $\Diamond A_i$ with A_i in the Boolean lattice $\wp(\Gamma)$ is called "possibility of A_i".
2. $\Diamond_Q \mathbf{P}_i$ with $\mathbf{P}_i$ in the orthomodular lattice $\mathcal{L}$ is called "quantum possibility of $\mathbf{P}_i$".
3. The set of all the $\Diamond_Q \mathbf{P}_i$ with $\mathbf{P}_i$ in the orthomodular lattice $\mathcal{L}$ is called the "set of quantum possibilities".
4. A Boolean sub-algebra of the orthomodular lattice $\mathcal{L}$ is called a "context".

5. The set of quantum possibilities valuated to $1 \in \mathbf{2}$ is called the "set of existent quantum possibilities".

6. The subset of quantum possibilities in direct relation to a context valuated to $1 \in \mathbf{2}$ is called the "set of existent quantum possibilities in a situation".

7. The subset of projectors of the context valuated to $1 \in \mathbf{2}$ is called the "actual state of affairs".

Physically, it follows from the given definitions that:

1. An "actual state of affairs" provides a physical description in terms of definite valued properties.

2. A "situation" provides a physical description in terms of the quantum possibilities that relate to an actual state of affairs.

3. Formally, to go from the "set of existent quantum possibilities" to one of its subsets (each of which relates to a "context") is to define an application; physically, this path relates to the choice of a particular measurement set up, restricting the expressiveness of the total set of existent possibilities to a specific subset.

4. Formally, to give values to the projectors $\mathbf{P}_i$ in a context is to valuate; physically, the valuation determines the set of properties (in correspondence with the projectors $\mathbf{P}_i$ valuated to $1 \in \mathbf{2}$) which are considered as preexistent.

5. The "situation" expresses an existent set of quantum possibilities (which must not be considered in terms of actuality) while the valuated context expresses an actual state of affairs. This leaves open the opportunity to consider quantum possibility as determining a different mode of existence (independent to that of actuality).

We have distinguished between a 'situation' which makes reference to a definite set of existent possibilities and an 'actual state of affairs' which can be interpreted as a specific measurement set up. The 'context' is the limit in between the actual and the possible and makes reference to the non-commutative formal structure. As we have argued above it is important to notice this distinction is given within a single instant of time. It is the

MKS theorem, understood within this specific scheme, which exposes the fact that the actualization of possible properties cannot be understood in terms of a classical path. Forcing the classical notion of possibility within the quantum structure is a move that contradicts the mathematical formalism, which contemplates the interaction of possibilities (in the possible realm) in the same way as classical physics contemplates the interaction of actualities (in the actual realm). From this standpoint there is no need of invoking the *eigenstate-eigenvalue link* for both realms are independent. Thus, on the one hand, like in MI, only one way of the *if and only if* is required. The particular actualization (i.e., the measurement result) is a singular expression of the relation between the possible and actual realms, and is not considered as a physical interaction. We could say that, like in MI, the projection postulate is accepted but the collapse is denied. On the other hand, like in the MWI, every term in the superposition is interpreted as physically existent, however, there are no multiple (actual) worlds but rather a set of existent possibilities interacting in one single world.

6 The MKS Theorem and Many Worlds

The notion of possibility has been also investigated in relation to the idea of possible worlds [31]. Regarding QM, this logical analysis has found an expression in the many worlds interpretation [7]. In order to discuss this notion of possibility within our own scheme, we have developed an algebraic framework which allows us to analyze the modal aspects of the Many Worlds Interpretation (MWI) from a logical perspective [21].

According to the MWI all possibilities encoded in the wave function take place, but in different worlds. When a measurement of a physical magnitude $\mathbf{M}$ is performed and one of its possible outcomes a_1 occurs, then in another world a_2 occurs, and in some other world a_3 occurs, etc. In modal wording, suppose that $\mathbf{M}$ has associated a Boolean sublattice $W_{\mathbf{M}}$ of $\mathcal{L}(\mathcal{H})$. The projectors of the family $(\mathbf{P}_i)$ are identified as elements of $W_{\mathbf{M}}$. If a measurement is performed and its result is a_i, this means that we can establish a Boolean homomorphism

$$v : W_{\mathbf{M}} \to \mathbf{2} \qquad s.t.\ \ v(\mathbf{P}_i) = 1$$

In a possible world where $v(\mathbf{P}_i) = 1$ we will have classical consequences. Let us make precise the notion of *classical consequence* taking into account modal extensions built from Boolean saturated orthomodular lattices.

Definition 6.1 Let $\mathcal{L}^\Diamond$ be an arbitrary modal extension of $\mathcal{L}(\mathcal{H})$ and $\mathbf{P} \in \mathcal{L}(\mathcal{H})$. Then $x \in \Diamond\mathcal{L}$ is said to be a *classical consequence* of $\mathbf{P}$ iff for each Boolean sublattice W in $\mathcal{L}^\Diamond$ (with $\mathbf{P}_i \in W$) and each Boolean valuation $v : W \to \mathbf{2}$, $v(x) = 1$ whenever $v(\mathbf{P}_i) = 1$.

We denote by $Cons_{\mathcal{L}^\Diamond}(\mathbf{P})$ the set of classical consequences of $\mathcal{L}$. By Proposition [21, 3.5] we have that $Cons_{\mathcal{L}^\Diamond}(\mathbf{P}_i) = \{x \in \Diamond\mathcal{L}(\mathcal{H}) : \Diamond\mathbf{P}_i \leq x\}$. The modal extension *does not depend* on the valuation over the family $(\mathbf{P}_i)$. Thus, it is clear that the modal extension is independent of any possible world. Modal extensions are simple algebraic extensions of an orthomodular structure. Thus, when referring to a property $\mathbf{P}_i$, it is equivalent to consider the classical consequences in the possible world where $v(\mathbf{P}_i) = 1$ or to study the classical consequences of $\Diamond\mathbf{P}_i$ before the splitting.

Formally, MWI *maintains that in each respective i-world,* $v_i(\mathbf{P}_i) = 1$ *for each* i. Thus, a family of valuations $(v_i(\mathbf{P}_i) = 1)_i$ may be simultaneously considered, each member being realized in each different i-world. From an algebraic perspective, this would be equivalent to have a family of pairs $\langle \mathcal{L}(\mathcal{H}), v_i(\mathbf{P}_i) = 1\rangle_i$, each pair being the orthomodular structure $\mathcal{L}(\mathcal{H})$ with a distinguished Boolean valuation v_i over a spectral sub-algebra containing $\mathbf{P}_i$ such that $v_i(\mathbf{P}_i) = 1$. In [21], we have shown that the $\mathcal{OML}^\Diamond$ structure is able to capture this fact in terms of classical consequences. While MWI considers a family of pairs $\langle \mathcal{L}(\mathcal{H}), v_i(\mathbf{P}_i) = 1\rangle_i$ for each possible i-world and the classical consequences of $v_i(\mathbf{P}_i) = 1$ in the i-world, the $\mathcal{OML}^\Diamond$ structure, by Proposition [21, prop. 3.5], considers classical consequences of each $v_i(\mathbf{P}_i) = 1$ coexisting simultaneously in one and the same structure. In fact, as a valuation $v : \Diamond\mathcal{L} \to \mathbf{2}$ exists such that $v(\Diamond\mathbf{P}_i) = 1$ for each i, each element $x \in \Diamond\mathcal{L}$ such that $\mathbf{P}_i \leq x$ necessarily satisfies $v(x) = 1$. In physical terms, this analysis shows that MWI talks about possible propositions based on an orthomodular lattice without taking into account the intrinsic features of the structure itself. This has the consequence that, like in classical physics, in spite of the wording about possibility that is present in the MWI, only actuality plays a role. Rather than discussing about quantum possibilities, MWI restricts their physical discourse to (classical) possibilities. One could say that all possibilities have become all actual in each correspondent world. Thus, the MKS theorem does not restrict the MWI scheme. Furthermore, there is no need of the projection postulate. MWI could be then considered as extending the MI proposed by Dieks to all terms of the multiple superpositions.[11]

7 Conclusions

In this paper we have discussed the characteristics of possible propositions based on the orthomodular lattice to physically interpret the meaning and scope of our MKS theorem. In order to do so, we have distinguished the use of "possibility" in the classical and quantum formalisms. To escape from the improper relation between formalism and language, we have built a dictionary that clearly expresses the link between formal elements and physical concepts. The construction of the dictionary has also led us to the recognition of the independence between the realms of quantum possibility and actuality, in contradistinction to the classical case in which both possibility and actuality collapse. Furthermore, we have understood how the MKS theorem rules, through the constrains to actualization, the relation between both realms. Finally, we have analyzed the use of modality within the MWI, concluding that —due to the fact they are not directly confronted with the interpretation of quantum possibility— they escape both the KS and MKS theorems.

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