How to reconcile modal interpretations of quantum mechanics with relativity

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1. Introduction[3]

Modal interpretations[4] are hidden-variable, no-collapse (typically) indeterministic interpretations of quantum mechanics that were designed to solve the measurement problem and to reconcile quantum mechanics with relativity. But, as recent no-go theorems by Dickson and Clifton (1998), Arntzenius (1998) and Myrvold (2002) demonstrate, current modal interpretations are not fundamentally relativistic. In this paper, we suggest strategies for how to reconcile the modal interpretation with special relativity. We begin by examining Myrvold's theorem (Section 2), which in a sense is a generalization of the other two no-go theorems. We then suggest two strategies for circumventing these theorems (Section 3), and show how they can be developed into new modal interpretations with a dynamics that picks out no preferred foliation of space-time (Section 4).

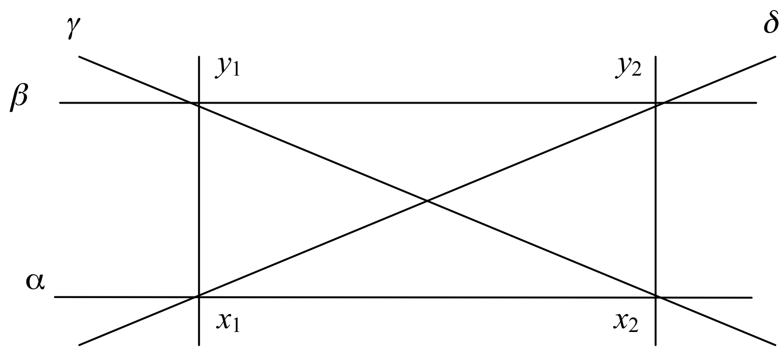

Figure 1. The spacelike hypersurfaces used in Myrvold's theorem.

2. Myrvold's no-go theorem

There are three main types of modal interpretations: The Schmidt-decomposition, the spectral-resolution and the preferred-observables modal interpretations. In the Schmidt-decomposition

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[4]For examples, see the theories proposed by van Fraassen, Kochen, Dieks, Healey, Bub, and Vermaas and Dieks. For overview, evaluation and references, see Dieks and Vermaas (1998).

modal interpretations, the unique biorthogonal decomposition of pure (non-degenerate) quantum-mechanical states of systems picks out the values of some observables as definite. In the spectral-decomposition modal interpretations, which are generalizations of the Schmidt-decomposition interpretations to mixed states, the properties of a system are given by the spectral resolution of its reduced state. The range of possible properties of a system and their single-time probabilities are given by the non-zero diagonal elements of the spectral resolution of the system's reduced state (obtained by partial tracing). Finally, in the preferred-observables modal interpretations, the definite properties of a system are given by some preferred (time-independent) observables.

Myrvold (2002) argues that all the above property assignments are incompatible with special relativity. The main idea of the argument is the following. Myrvold presupposes that if a modal interpretation is to be compatible with relativity, it must satisfy the following condition (Myrvold, ibid., p. 1783):

> *Relativistic Born Rule*. Let q and r be any possible values of the quantities Q_1 and R_2, respectively. For any spacelike hypersurface σ, if the quantum-mechanical state of the composite system S_1+S_2 on σ is $\psi(\sigma)$, and if $Q_1=q$ and $R_2=r$ are local definite properties of S_1 and S_2 respectively, then the probability of $Q_1=q$ and $R_2=r$ on σ is equal to Tr[$P_{Q1}(q)$ $P_{R2}(r)$ $\psi(\sigma)$]; where $P_{Q1}(q)$ and $P_{R2}(r)$ are the projections onto the eigenspaces $Q_1=q$ and $R_2=r$, respectively.

But, argued Myrvold, this condition is incompatible with the above property assignments. The reasoning is as follows. Let α and β be two hyperplanes of simultaneity in some reference frame (see Figure 1 above). Let x_i and y_i be small regions on α and β respectively, in which the system S_i is located. Suppose that x_1 is spacelike separated from y_2 and x_2 is spacelike separated from y_1. Let γ be a spacelike hypersurface containing y_1 and x_2 and let δ be a spacelike hypersurface containing x_1 and y_2. Let R_1 and R_2 be quantities associated with the systems S_1 and S_2 respectively, and let A_1 and A_2 be measurement apparatuses that record the values of R_1 and R_2. Suppose that the states of $S_1+S_2+A_1+A_2$ on α, β γ and δ are the following:

$$|\varphi(\alpha)\rangle = 1/2\surd 3(|p_1+\rangle|r_1+\rangle|r_2+\rangle|p_2+\rangle - |p_1+\rangle|r_1+\rangle|r_2-\rangle|p_2-\rangle - |p_1-\rangle|r_1-\rangle|r_2+\rangle|p_2+\rangle| - 3|p_1-\rangle|r_1-\rangle|r_2-\rangle|p_2-\rangle) \quad (1)$$

$$|\varphi(\beta)\rangle= 1/\surd 3(|p_1+\rangle|r_1+\rangle|r_2-\rangle|p_2-\rangle + |p_1-\rangle|r_1-\rangle|r_2+\rangle|p_2+\rangle| - |p_1+\rangle|r_1+\rangle|r_2+\rangle|p_2+\rangle) \quad (2)$$

$$|\varphi(\gamma)\rangle= 1/\surd 6(|p_1-\rangle|r_1-\rangle|r_2+\rangle|p_2+\rangle| + |p_1-\rangle|r_1-\rangle|r_2-\rangle|p_2-\rangle - 2|p_1+\rangle|r_1+\rangle|r_2-\rangle|p_2-\rangle) \quad (3)$$

$$|\varphi(\delta)\rangle=1/\surd 6(|p_1+\rangle|r_1+\rangle|r_2-\rangle|p_2-\rangle + |p_1-\rangle|r_1-\rangle|r_2-\rangle|p_2-\rangle - 2|p_1-\rangle|r_1-\rangle|r_2+\rangle|p_2+\rangle|); \quad (4)$$

where $|r_i+\rangle$ and $|r_i-\rangle$ are the eigenstates of R_1 and R_2 respectively, and $|p_i+\rangle$ and $|p_i-\rangle$ are the eigenstates of pointer observables P_1 and P_2 associated with the measuring devices A_1 and A_2, respectively. As is easily shown, $|\varphi(\beta)\rangle$, $|\varphi(\gamma)\rangle$ and $|\varphi(\delta)\rangle$ are obtained from $|\varphi(\alpha)\rangle$ by applying the following Hadamard transformations to the eigenstates of $R_i \otimes P_i$:

$$U_i\, |r_i+\rangle|p_i+\rangle= 1/\surd 2 \ \ (|p_i+\rangle|r_i+\rangle + |p_i-\rangle|r_i-\rangle) \quad (5)$$

$U_i |r_i-\rangle|p_i-\rangle = 1/\sqrt{2}\ (|p_i+\rangle|r_i+\rangle - |p_i-\rangle|r_i-\rangle)$.

That is, $\varphi(\beta)=U_1\otimes U_2|\varphi(\alpha)\rangle$, $\varphi(\gamma)=U_1\otimes I_2|\varphi(\alpha)\rangle$ and $\varphi(\delta)=I_1\otimes U_2|\varphi(\alpha)\rangle$; where I is the identity transformation.

According to the spectral-resolution and the Schmidt-decomposition modal interpretations, R_1 and R_2 are definite in the states (1)-(4). Suppose that these observables are also picked out as definite by the preferred-observables modal interpretations. Suppose further that the value of R_i correspond to a local property of the system S_i, so that R_i has the same value on any two space-like hypersurfaces that intersect the spacetime region in which S_i is located. Then, if the probabilities of the values of the R_i are supposed to satisfy the Relativistic Born Rule on all the four hypersurfaces, there must be a joint probability distribution over the values of R_i that yields as marginals the (single-time) Born-rule probabilities for these values on the hypersurfaces α, β, γ and δ. But, such joint probability distribution would satisfy certain Bell-like inequalities, which are violated in the states $\varphi(\alpha)$, $\varphi(\beta)$, $\varphi(\gamma)$ and $\varphi(\delta)$ in (1)-(4). This means that the Relativistic Born Rule fails, and more generally, that the probabilities of local, possessed properties in modal interpretations cannot be given by the Born probabilities along every foliation of spacetime for any arbitrary initial quantum state.[5]

For example, suppose that on hypersurface α, R_1 and R_2 have the values (r_1+,r_2+). Since, by assumption, R_1 is a local property of S_1, it must have the same value on the hypersurface δ. Assuming the Relativistic Born Rule, the state on δ, $|\varphi(\delta)\rangle$, assigns probability zero to the pair of values (r_1+,r_2+) and non-zero probability to the pair of values (r_1+,r_2-). Thus, on δ, the probability that R_2 has the value r_2- is one. Since R_2 is a local property of S_2, if it has the value r_2- on δ, it will also have this value on β. A parallel argument leads to the conclusion that if R_2 has the values r_2+ on α, it will have the value r_2- on β. Thus, if R_1 and R_2 have the values (r_1+,r_2+) on α, the probability that these quantities have the values (r_1-,r_2-) on β is one. By contrast, the Relativistic Born Rule, $|\varphi(\beta)\rangle$ assigns zero probability to these values. Thus, it is impossible to obtain the Born probabilities for the values of R_1 and R_2 on all the four hypersurfaces.

3. **On the nature of properties**

In response to Myrvold's theorem, one may object to the presupposition that the Relativistic Born Rule is a necessary condition for relativistic modal interpretations. This presupposition seems to rely on the natural assumption that the probabilities of possessed properties should invariably be equal to the Born-rule probabilities. But one may reject this assumption and insist that *only* the probabilities of *observable*, possessed properties should be equal to the Born-rule probabilities. In what follows, we shall suggest two other strategies for circumventing this theorem. As is easily seen, Myrvold's theorem also relies on the following two premises:

(I) The values of the observables R_1 and R_2 (P_1 and P_2) are local (intrinsic) properties of the subsystems S_1 and S_2 (A_1 and A_2), respectively.

[5] The scope of Myrvold's theorem is not restricted to modal interpretations. It may also be applicable to other no-collapse theories that admit local properties of the type discussed above.

(II) The joint probabilities of the values of R_1, R_2, P_1 and P_2 are definite on α, β γ and δ (henceforth, Joint Probability).

While current modal interpretations satisfy these assumptions for supposedly local properties, we shall suggest below modal interpretations that violate them, and accordingly circumvent Myrvold's theorem.

To remain general and to simplify things, we shall work with schematic modal interpretations in which the possible properties of a given system and their single-time probabilities are picked out by the nonzero diagonal elements of the reduced state in a given basis. Further, as a working hypothesis, we shall assume that this property assignment is consistent with the Kochen & Specker-type theorems, and that it leads to sensible solution of the measurement problem. (In Berkovitz 2004, 2005), we argue that this working hypothesis can well be motivated in the context of the modal interpretations under consideration.) Finally, similarly to most current modal interpretations, we shall assume that the so-called 'property composition' and 'property decomposition' conditions fail.

> Let H^1 and H^2 be the Hilbert spaces of the systems S_1 and S_2, respectively. Let Q_1 be an observable that pertains to H^1 and q be one of its values, let $P_{Q1}(q)$ be a projection onto the eigenspace $Q_1=q$, and let I_2 be the identity operator for H^2. Then:
>
> *Property Composition*. If S_1 has the property (associated with) $P_{Q1}(q)$, then S_1+S_2 has the property (associated with) $P_{Q1}(q)\otimes I_2$.
>
> *Property Decomposition*. If S_1+S_2 has the property (associated with) $P_{Q1}(q)\otimes I_2$, then S_1 has the property (associated with) $P_{Q1}(q)$.

Note that these violations imply that the properties of composite systems are not decomposable into the properties of their subsystems. For example, the values of R_1 and R_2 may be r_1+ and r_2- respectively, whereas the value of $R_1\otimes R_2$ may be $r_1-\otimes r_2+$. This means that in order to prove that a modal interpretation is compatible with relativity it will be necessary to demonstrate that Myrvold's theorem is inapplicable not only to the values of R_1 and R_2, but also the values of $R_1\otimes R_2$. Note also that the violations of property composition and property decomposition naturally suggest that the composite properties $R_1\otimes R_2$ are non-local, so that in transformations of the eigenstates of say R_2 the value of $R_1\otimes R_2$ may change as a whole: e.g. the value of $R_1\otimes R_2$ may be $r_1+\otimes r_2+$ on α and $r_1-\otimes r_2+$ on δ, even though the reduced state of S_1 does not change by the Hadamard transformation of the eigenstates of $R_2\otimes P_2$ between α and δ.

3.1. The holistic interpretation

In the modal interpretations we have in mind, the possible values of $R_1\otimes R_2$ are $r_1+\otimes r_2+$, $r_1+\otimes r_2-$, $r_1-\otimes r_2+$, and $r_1-\otimes r_2-$ in the state $|\varphi(\alpha)>$, and $r_1+\otimes r_2-$, $r_1-\otimes r_2+$, and $r_1-\otimes r_2-$ in the state $|\varphi(\delta)>$ ($|\varphi(\gamma)>$). Thus, if the value of $R_1\otimes R_2$ on α is $r_1+\otimes r_2+$, it has to be different on δ (γ). Myrvold assumes that the values of R_1 and R_2 are local properties of S_1 and S_2 respectively, and accordingly the value of $R_1 \otimes R_2$ on δ (γ) has to be $r_1+\otimes r_2-$ ($r_1-\otimes r_2+$). But, this assumption will be unwarranted if we construe the value of $R_1 \otimes R_2$ as a holistic property of the composite system S_1+S_2, which is not decomposable into the properties of S_1 and S_2; for on such holistic interpretation, the values of $R_1 \otimes R_2$ on α, γ and δ are all non-local properties, which are independent of the values of R_1 and R_2. Thus, if the value of $R_1 \otimes R_2$ is interpreted as a holistic, non-decomposable property of the composite system S_1+S_2, Myrvold's theorem will be inapplicable to the values of $R_1 \otimes R_2$.

If the values of R_1 and R_2 are unrelated to the values of $R_1 \otimes R_2$, then we can also motivate the violation of Joint Probability. Recall that in the modal interpretations under consideration the single-time probabilities of properties of a system S are given by the on-diagonal elements of the reduced state of S in a given basis. This means that the single-time probabilities of the (possible) values of R_1 are given by the reduced state of S_1, the single-time probabilities of the (possible) values of R_2 are given by the reduced state of S_2 and the single-time probabilities of the (possible) values of $R_1 \otimes R_2$ are given by the reduced state of S_1+S_2. Since the values of $R_1 \otimes R_2$ are unrelated to the values of R_1 and R_2 and the single-time probabilities of the values of $R_1 \otimes R_2$ do not correspond to the joint probabilities of the values of R_1 and R_2, the reduced state of S_1+S_2 cannot be used to assign joint probabilities for the values of R_1 and R_2. Thus, the holistic modal interpretation has no means for assigning joint probabilities for the values of R_1 and R_2. But if the values of R_1 and R_2 have no joint probabilities, the Relativistic Born Rule will be inapplicable to them.

Based on the above reasoning, it is not difficult to show that Myrvold's theorem is also inapplicable to any other properties assigned by the holistic modal interpretation.

The question whether the holistic interpretation is compatible with the Relativistic Born Rule depends on the dynamics of properties. The simplest dynamics that is compatible with this rule is one in which the transition probabilities of the properties that a system has on a hypersurface β given its properties on a hypersurface α are equal to the single-time probabilities of these properties on β. But, as we shall see in the next section, one may develop a more sophisticated dynamics.

The strategy of construing the values of $R_1 \otimes R_2$ as holistic properties can formally be integrated in current modal interpretations. But, the challenge is to explicate the nature of these properties. In current modal interpretations, the nature of these properties, as well as the failures of property composition and property decomposition are largely unexplained (see e.g. Clifton 1996). Further, given that the properties of systems are not related in any obvious way to the properties of their subsystems, there is also the challenge of explaining how such holistic properties are related to our experience.[6]

3.2. The relational interpretation

While one may go along with the formal outline of the holistic interpretation, it is unnecessary to endorse the holistic interpretation of properties. Rather, one may interpret the properties assigned by modal interpretations as relational. Here, the main idea is that the core-property assignment is of

[6]Clifton's (1996) discussion of the nature of properties in modal interpretations may be construed as an attempt to highlight this difficulty.

relational properties, i.e. properties that systems have relative to other systems (rather than intrinsic properties). On this alternative interpretation, the failure of property composition and property decomposition is explained by the fact that the properties that a system has relative to different systems (contexts) may be different and moreover unrelated to each other. For example, the value that R_1 (R_2) has, as a property of S_1 (S_2), relative to $S_2+A_1+A_2$ ($S_1+A_1+A_2$) may be different from the value that R_1 (R_2) has, as a property of S_1+S_2, relative to A_1+A_2; and the values that R_1 has relative to $S_2+A_1+A_2$ and R_2 has relative to $S_1+A_1+A_2$ do not constrain the values that R_1 and R_2 have relative to A_1+A_2, and *vice versa*. By contrast, the value that $R_1 \otimes R_2$ has relative to A_1+A_2 is decomposable into the values that R_1 and R_2 each have relative to A_1+A_2. More generally, properties that are defined relative to the same systems (contexts) are related to each other.

Since properties that are related to different contexts are unrelated, they have no joint probabilities and accordingly Joint Probability fails. But, similarly to the holistic interpretation, this failure can also be motivated on technical grounds. For recall that in the modal interpretations in which the properties of systems and their single-time probabilities are derived from reduced states, there are no means for assigning probabilities of properties that are related to different contexts. The single-time probabilities of the values that R_1 may have, as a property of S_1, relative to $S_2+A_1+A_2$ are determined by the reduced state of S_1, and the single-time probabilities of the values that R_2 may have, as a property of S_2, relative to $S_1+A_1+A_2$ are determined by the reduced state of S_2. But the joint probabilities of these properties are not given by the reduced state of S_1+S_2 (or any other reduced state); the reduced state of S_1+S_2 only prescribes the joint probabilities of the values that R_1 and R_2 each has relative to A_1+A_2 (which are unrelated to the joint probabilities of the values that R_1 may have relative to $S_2+A_1+A_2$ and the values that R_2 may have relative to $S_1+A_1+A_2$).

This failure of Joint Probability means that Myrvold's theorem does not apply to the values of R_1 relative to $S_2+A_1+A_2$ and the values of R_2 relative to $S_1+A_1+A_2$. But, Myrvold's theorem also fails to apply to the values that R_1 and R_2 have relative to A_1+A_2. And again, as in the holistic interpretation, the failure is due to the holistic nature of these properties, though here the holism is due to the relational nature of properties rather than to their non-decomposability. The reasoning is as follows. Myrvold's theorem would not be applicable to these relational properties if for example the value of R_1 (R_2) relative to A_1+A_2 may not be the same on α and on δ (γ). The easiest way to reconcile such interpretation with the Relativistic Born Rule is to postulate that the transition probabilities will be equal to the corresponding single-time probabilities, e.g. that the probability of $R_1 \otimes R_2$ having the value (r_1+,r_2-) on δ given that it has the value δ given that it has the value (r_1+,r_2+) on α is equal to the single-time probability of $R_1 \otimes R_2$ having the value (r_1+,r_2-) on δ. But, as we shall see in the next section, a more sophisticated dynamics, where the chance that the value of an observable of a system is different on two spacelike hypersurfaces that intersect the space-time region in which the system is located depends on the degree of entanglement between the system and other relevant systems, could also be reconciled with the Relativistic Born Rule. In this dynamics, the chance that the value of e.g. R_1 relative to A_1+A_2 will not be the same on α and on δ depends on the entanglement between S_1 and S_2+A_2 and the single-time probabilities of the values of R_1 relative to A_1+A_2 on δ. The higher the degree of entanglement is, the higher is the chance that this relational value of R_1 will not be the same on α and δ. In particular, if there were no entanglement between S_1 and S_2+A_2, the chance that the value of R_1 relative to A_1+A_2 will not be the same on α and δ is zero. This dependence on the degree of entanglement is desirable. For it yields the non-locality in the value of R_1 (R_2) relative to A_1+A_2 required for circumventing Myrvold's theorem, and at the same time entails that this non-locality is virtually impossible to observe.

Based on the above reasoning, it is not difficult to show that Myrvold's theorem will also be inapplicable to other relational values of R_1 and R_2 as well as any other relational properties.

It may be argued that although the relational interpretation evades Myrvold's theorem, it is not genuinely relativistic because it postulates hypersurface-dependent properties, which are frame-dependent properties in disguise. We believe that this objection is misguided. First, there is a conceptual difference between frame-dependent and hypersurface dependent properties (see Aharonov and Albert 1981; Fleming and Bennett 1989; Maudlin 1996). Secondly, the properties postulated by the relational modal interpretation are not hypersurface dependence *per se*. Indeed, in Myrvold's set up the value of e.g. R_1, as a property of S_1+S_2, relative to A_1+A_2 may be not the same on the hypersurfaces α and δ. But while this relational value of R_1 is highly non-local, it is not hypersurface *per se*, at least not if by hypersurface-dependent properties it is meant properties that are defined relative to hypersurfaces; for this value of R_1 is not defined relative to hypersurfaces. In any case, as is not difficult to see this value of R_1 (as well as all the other properties discussed in Myrvold's theorem) are invariant across all inertial reference frames and accordingly are frame independent. Moreover, as is not difficult to see from the dynamics below, the relational modal interpretation does not pick out any preferred reference frame.

Given that supposedly local properties like pointer position are highly non-local according to the relational interpretation, one may wonder how the relational modal interpretation recovers our experience. We discuss this question in Berkovitz and Hemmo (2004; 2005) and argue that given our proposed dynamics below, such non-locality is unobservable. Here, we only remark that it follows from this dynamics that the chance that such non-locality occurs in any experimental situation, where macroscopic systems undergo decoherence with the environment, is virtually zero.

4. The dynamics

In the previous section, we remarked that a trivial dynamics in which the transition probabilities are equal to the corresponding single-time Born probabilities would be compatible with the Relativistic Born Rule. In this section, we outline a more sophisticated dynamics, which is applicable to the holistic and the relational modal interpretations alike, the transition probabilities depend on the degree of entanglement between the relevant systems (a measure that we define below). To simplify matters, we first present the outlines of this dynamics in two extreme cases: (i) the dynamics in cases of no entanglement; and (ii) the dynamics in cases of maximal entanglement. We then propose a generalized dynamics that entails these special cases.

Consider, first, the case of no entanglement. Let S_{I} and S_{II} be a partition of the universe into two systems. Let U be a unitary transformation on the state of S_{I}+S_{II}. If the reduced state of S_{I} does not change under U, the relational properties of S_{I} relative to S_{II} do not change. If the reduced state of S_{I} changes, then the probabilities of the properties that S_{I} may have relative to S_{II} depend on the properties that S_{I} has relative to S_{II} and the transformation U. The properties of S_{I} associated with projectors that commute with U evolve deterministically, so as to return the single-time Born probabilities; and the properties of S_{I} associated with projectors that do not commute with U evolve indeterministically, so as to return the single-time Born probabilities.

While the above transition probabilities resemble the probabilities obtained by a sequential application of the Born rule in a collapse theory, the transition probabilities in cases of maximal entanglement are very different. In these cases, the transition probabilities of the properties of S_{I} in a transformation U on subsystems of $S_{\mathrm{I}}+S_{\mathrm{II}}$ are proportional to the distance between the (reduced) state of S_{I} before and after applying U and the single-time probabilities of the properties of S_{I} after applying U. That is, the probability that S_{I} has the property $Q=q$ after applying U, given that it has the property $R=r$ before applying U, is equal to (the normalized distance between the (reduced) states of S_{I} before and after applying U) times (the single-time probability of $Q=q$ in the (reduced) state of S_{I} after applying U). When the degree of entanglement is less than maximal the dynamics is a weighted average of the two extreme cases (see (6) below).

There are various measures of entanglement. As an example, we shall use Shimony's (1995) measure, where the degree of entanglement between systems S_i and S_j is defined in terms of the minimal (normalized) distance in Hilbert space norm of the state ψ of the composite S_i+S_j from the set of all product states in the Hilbert space of S_i+S_j. In the case of mixed states, the distance between B, a (normalized) mixed state of S_i+S_j, and A, any mixed product state in the convex set $\mathcal{C}$ of all product states of S_i+S_j, is defined in the space of the self adjoint operators as $Tr(B \otimes A)$. The distance between B and the convex set of all the product states of S_i+S_j is defined to be the minimal distance between B and the states in $\mathcal{C}$.

The relevant systems for measuring the degree of entanglement are determined by the properties under consideration. Let S_{I} and S_{II} be a partition of the universe, and let $S_{\mathrm{I}}{}^*$ and $S_{\mathrm{I}}{}^{**}$ be a partition of S_{I}, and $S_{\mathrm{II}}{}^*$ be a subsystem of S_{II}.[7] Let U be any unitary transformation on the state of $S_{\mathrm{I}}{}^*+S_{\mathrm{II}}{}^*$ and the identity transformation on the state of all the other subsystems of $S_{\mathrm{I}}+S_{\mathrm{II}}$. The effect of U on the properties of S_{I} depends on the degree of entanglement between $S_{\mathrm{I}}{}^*+S_{\mathrm{II}}{}^*$ and $S_{\mathrm{I}}{}^{**}$ in the initial state (i.e. before applying U). The higher is the degree of entanglement, the more distanced is the dynamics from the dynamics in the case of no-entanglement. The idea here is that when $S_{\mathrm{I}}{}^{**}$ is entangled with $S_{\mathrm{I}}{}^*+S_{\mathrm{II}}{}^*$, a transformation on $S_{\mathrm{I}}{}^*+S_{\mathrm{II}}{}^*$ may induce stochastic changes in the properties of S_{I} as a whole and not only in the properties of the subsystem $S_{\mathrm{I}}{}^*$, where the probabilities of these changes depend on the degree of entanglement between $S_{\mathrm{I}}{}^{**}$ and $S_{\mathrm{I}}{}^*+S_{\mathrm{II}}{}^*$. For example in the Hadamard transformation on S_2+A_2 between the hypersurfaces α and δ, $S_{\mathrm{I}}{}^{**}$ is S_1, $S_{\mathrm{I}}{}^*$ is S_2, $S_{\mathrm{II}}{}^*$ is A_2, and the relevant degree of entanglement is the one between S_1 and S_2+A_2 in the (reduced) state of $S_1+S_2+A_2$ on α (obtained by partial tracing from $|\varphi(\alpha)\rangle$).

We can now give the universal dynamics for all degrees of entanglement. Let $|\psi\rangle$ *be* the state of $S_{\mathrm{I}}+S_{\mathrm{II}}$, and $d(e)$ be the degree of entanglement between $S_{\mathrm{I}}{}^*+S_{\mathrm{II}}{}^*$ and $S_{\mathrm{I}}{}^{**}$ in the state $|\psi\rangle$. Let $P(Q=q/R=r)$ be the probability that S_{I} has the property $Q=q$ relative to S_{II} in the state $U|\psi\rangle$ given that it has the property $R=r$ relative to S_{II} in the state $|\psi\rangle$, and let $P_{ME}(Q=q/R=r)$, $P_{NE}(Q=q/R=r)$ and $P_U(Q=q/R=r)$ denote respectively the value of $P(Q=q/R=r)$ according to the dynamics in cases of

[7] Here, by "a subsystem of S_{II}," we mean any subsystem of it, including S_{II} itself or the 'null' system.

maximal entanglement, the dynamics in cases of no entanglement and the universal dynamics. Let $d(s)$ be the (normalized) distance between the reduced states of S_1 in the states $|\psi>$ and $U|\psi>$. Then:

$$P_U(Q{=}q/R{=}r) = d(e) \cdot d(s) \cdot P_{ME}(Q{=}q/R{=}r) + \big(1 - d(e) \cdot d(s)\big) \cdot P_{NE}(Q{=}q/R{=}r). \quad (6)$$

If the distribution of properties is given by the single-time Born probabilities on any spacelike hypersurface, then (by construction) the dynamics in cases of no entanglement and maximal entanglement will both reproduce the single time Born probabilities on any other spacelike hypersurface. Accordingly, the universal dynamics (6) will also reproduce the single time Born probabilities.

Let us apply the universal dynamics to Myrvold's set-up. In the relational interpretation, the universal dynamics entails that the chance that the value of R_1 (R_2) relative to A_1+A_2 on α will be different from its value on δ (γ) depends on the degree of entanglement between S_2+A_2 (S_1+A_1) and S_1 (S_2), and the distance between the reduced states of S_1+S_2 on α and on δ (γ). In the holistic interpretation, the universal dynamics entails that the chance that the value of $R_1 \otimes R_2$ is $r_1 - \otimes r_2+$ or $r_1 - \otimes r_2 -$ ($r_1 + \otimes r_2 -$ or $r_1 - \otimes r_2 -$) on γ (δ) given that it is $r_1 + \otimes r_2 +$ on α depends on the degree of entanglement between S_2+A_2 (S_1+A_1) and S_1 (S_2), and the distance between the reduced states of S_1+S_2 on α and on δ (γ). As can be shown, in the state $\psi(\alpha)$ the degree of entanglement between S_1 (S_2) and S_2+A_2 (S_1+A_1) and the distance between the reduced states of S_1+S_2 on α and on δ (γ) are both substantial. Thus, Myrvold's theorem is inapplicable to the values of $R_1 \otimes R_2$, interpreted as either the properties of S_1+S_2 relative to A_1+A_2 or holistic non-decomposable properties of S_1+S_2.

Consider now the value of R_1 as either the property of S_1 or the property of S_1 relative to S_2+A_1+A_2 and the value of R_2 as a property of S_2 relative to S_1+A_1+A_2. In the transition from α to δ (γ), these values of R_1 and R_2 evolve according to the dynamics in case of no-entanglement, and accordingly they do not change. Yet, since these properties have no joint probabilities, Myrvold's theorem is inapplicable to them.

Based on the considerations above, it is not difficult to show that Myrvold's theorem will also be inapplicable to any other properties postulated by the holistic and the relational modal interpretations.

5. Conclusions

Myrvold's theorem demonstrates that current modal interpretations are not genuinely relativistic. We argued that Myrvold's theorem is inapplicable to certain holistic readings of the nature of properties assigned by modal interpretations, and that these readings may serve as the basis for new modal interpretations – the holistic and the relational modal interpretations. We then equipped these interpretations with a dynamics that does not pick out any preferred frame yet reproduces the Born probabilities of properties on any spacelike hypersurface. As see Berkovitz and Hemmo (2005), the holistic and the relational modal interpretations also get around Dickson and Clifton's and Arntzenius' no-go theorems. Thus, we conclude that there are good prospects for a genuinely relativistic modal interpretation.

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