Observations on Hyperplane: II.
Dynamical Variables and Localization Observables

by

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Abstract

This is the second of two papers responding (somewhat belatedly) to 'recent' commentary on various aspects of hyperplane dependence (HD) by several authors. In this paper I focus on the issues of the general need for HD dynamical variables, the identification of physically meaningful localizable *properties*, the basis vectors representing such properties and the relationship between the concepts of 'localizable within' and 'measureable within'. The authors responded to here are de Koning, Halvorson, Clifton and Wallace. In the first paper of this set (Fleming 2003b) I focused on the issues of the relations of HD to state reduction and unitary evolution and addressed comments of Maudlin and Myrvold. The central conclusion argued for in this second paper (**§§ 5, 7**) is **the non-existence of strictly localizable objects or measurement processes and the consequent undermining of the principle of universal microcausality**. This contrasts with the existence of **strictly localizable properties** and results in the consequent priority of the concept of 'localizable within' over 'measureable within'. The paper opens with discussions of the need for and status of HD dynamical variables which are responses to anonymous queries.

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1: Introduction

This is the second of two papers (The first is Fleming 2003b) in which I try to clarify some of my views on Lorentz covariant quantum theory (LCQT) and the role of hyperplane dependence (HD) therein. The main body of the discussion is structured around a detailed response to 'recent' arguments in the literature concerning HD, state reduction, dynamical evolution and localization.

In this second paper of the set the topics addressed are as follows: In **§ 2** the need for HD over ordinary time dependence of dynamical variables and states and the relationship between classical and quantum examples of HD dynamical variables is discussed in the form of responses to anonymous queries. In **§ 3,** as a response to de Koning's (2001), I consider issues of frame dependence and frame independence that are resolved by HD and the relationships between the localization of observables and the localizability of measurements. In a longish **§ 4**, responding to Halvorson's (2001), the relationship between putative localization schemes, the comparison of local subluminal dynamics over a superentangled vacuum on one hand and non-local HD superluminal dynamics over a product vacuum on the other, the distinction between localizing entities and localizing *properties* of entities and the status of universal microcausality and, again, the localizability of measurements are all considered. In **§ 5**, responding to Halvorson and Clifton's (2002), some of the same issues as in **§ 4** as well as *the non-existence of strictly localizable objects* and, consequently, strictly localizable measurements and, once again, the status of universal microcausality are addressed. In **§ 6** I consider Wallace's (2001a) and comment on the status of Newton-Wigner (NW) localization in his analysis. Section **7** contains a detailed quantitative comparison between HD NW localization and a variant of Halvorson's 'standard' localization scheme and *a proof of the unbounded space-like extension of single spinless quantons in QFT*. I devote the last section, **8**, to considerations related to the speculative conjectures concerning spatio-temporal ontology proposed at the end of the preceding paper (Fleming 2003b).

The specific topics addressed in the first paper of this set, in the order in which they arise, were: (**§1**) general considerations on the status and nature of the particles or quanta of quantum field theory (QFT) (here I tried to shed a reputation for championing views I, in fact, do not hold), (**§2**) the causal analysis of statistical correlations between state reductions on intersecting hyperplanes (response to Maudlin's 1996), (**§3**) the part-whole relations that can hold within composite and/or spatially extended quantum systems and the different forms they can take on intersecting hyperplanes (response to Maudlin's 1998), (**§4**) the limiting case of hyperplane dependent state vector assignments in the presence of composite systems with constituents space-like separated by distances greatly in excess of their own space-like dimensions (response to Myrvold's 2002) and (**§5**) dynamical evolution outside of foliations and spin as a non-local observable (response to Myrvold's 2003). In the last section, **6**, I entertained some speculative conjectures as to where the preceding considerations may be pushing regarding the ontology of the spatio-temporal framework.

1a: the term 'quanton'.

In the first paper I proposed (in place of the misleading term 'particle' or the overused term 'quanta') as the name of the family to which photon, electron, muon, tauon, lepton, gluon, neutron, proton, nucleon, pion, kaon, meson, hadron, and yes - - - quark and neutrino, belong as instances or subsets – the term '**quanton'**. The term has been suggested before (Levy-Leblond 1988) (von Bayer 1997). Continuing the effort to generate familiarity, I will use that term throughout this paper.

To firm up the definition, let's reserve 'quanton' for any particle-like quanta of a quantized field or any bound state of such quanta due to interactions which have, themselves, no macroscopic classical field manifestation, i.e., so far as we know, interactions other than electromagnetism or gravitation. Thus, atoms, molecules, bricks and planets are not quantons while nucleons and the nuclei of atoms, such as α-particles, are.

2: The need for hyperplane dependence (HD).

2a: motivating HD over time dependence

Some years ago I was asked by a prominent member of the philosophy of physics community (who shall remain nameless as I don't remember his exact words) an interesting question. He asked whether it was not the case, that any observable on a given hyperplane could always be expressed as a function of observables at a definite time, and whether, therefore, any measurement of an observable on a given hyperplane could not be implemented by measuring appropriately chosen observable(s) at a definite time? I took the purpose of the question to be that of undermining the notion that we need HD observables at all. For strictly speaking, the answer to the question is yes. But I asked my questioner if he agreed that his question was analogous to asking, within non-relativistic quantum theory, another question. That question is whether an observable at a given time could always be expressed as a function of observables at the time, t = 0, and whether, therefore, any measurement of an observable at a given time could be implemented by measuring appropriately chosen observable(s) at the time, t = 0? This question also has the answer, yes, and my questioner agreed to the analogy. But that (replacing observables at definite times by observables at time, t = 0) would be a crazy way to do physics, said I! No it wouldn't, said he. Circumstances then intervened and the discussion ended, never to be resumed.

I will here explain why I claimed it would be a crazy way to do physics. Why, in fact, it would be an *impossible* way to actually do physics! The explanation will be given, first, for the non-relativistic question.

To know the functional dependence of observables at a given time on observables at t = 0, and to, therefore, know what must be measured at t = 0 in order to measure a given observable at a given time – we must know the solutions of the equations of motion of the physical system. And for realistic systems, we *never* know those solutions! At best we have approximations to those solutions, which, if exploited in the manner suggested by the question and to the degree required, would put us far off the mark, indeed. We need direct access, albeit always approximate access, to observables at arbitrary times, in order to examine and test and improve the best ideas we can muster concerning the solutions of the equations of motion. How else to determine whether we've got the equations of motion, themselves, reasonably correct?

In the Lorentz covariant domain, relatively moving inertial observers associate their observables at definite times with distinct families of hyperplanes. So we can not know how the *general*, time dependent, observables of one inertial observer are related to those of a relatively moving inertial observer unless we know the solutions of the equations of motion, which, again, for realistic systems, we never do.

The counter argument that is sometimes offered, that we don't need to know the solutions because the Lorentz transformations between inertial observers are purely kinematical, is erroneous. It fails to take into consideration the dependence of Lorentz boosts on the presence and nature of interactions (Fleming 2003a); in short, on dynamics. While non-relativistic Galilean boosts were interaction free, Lorentz boosts are not.

If *all* the observables we ever measured were local fields at points, this would not be a problem, for a point is a point in any inertial frame. But we never actually measure local fields at points. Nor does the local algebra approach avoid this problem. For although it employs functionals of the fields over extended space-time domains, it fails to come to grips with the fact that relatively moving inertial observers tend to favor different observables within a given local algebra. Each inertial observer tends to favor, for measurement, observables that approximate observables at a definite time for that observer. Furthermore, as I've argued in the first paper (2003b), local observables associated with space-like *bounded* domains are insufficient for physical conceptual adequacy. There are natural physical observables with space-like unbounded functional dependence on the local fields. The total energy, momentum, angular momentum and the center of energy (CE) and Newton-Wigner (NW) positions, for arbitrary systems, are among them. The argument that in real measurements we can approximate these global observables adequately well by local observables presumes the ability to prejudge that space-time domain beyond which we need not consider.

Since one inertial observers' definite time is another inertial observers tilted hyperplane the favored observables can not be related without recourse to solving the dynamics of the system. Instead, for unfettered comparison of observables and measurements, without recourse to dynamical solutions, each inertial observer must have direct access to the definite time observables for all inertial observers. In other words, *each inertial observer must have direct access to observables on arbitrary hyperplanes*, and thus, to the HD version of observables.

If my argument holds water, how is it that contemporary high energy physics makes no explicit use of the HD formalism? The answer lies in the limited character of the observables actually measured in this branch of physics. They are all scattering observables, i.e., properties of individual quantons or collections of quantons moving freely ‘long’ before or ‘long’ after participating in interactions. The dynamical and transformational aspects of these observables, themselves, are trivial. But if we are ever to go beyond this restricted set of observables, the HD approach, or something very much like it, will be required. Some interesting examples of recent practical *use* of the HD formalism have appeared in contexts out side of scattering theory. The examples occur in the ongoing research into relativistic quantum state diffusion, (Breuer et al 1998, 2002), where one is trying to build a covariant dynamical theory of state reduction, and in the field of relativistic plasma physics, (Hoell et al 2001a, b, 2002), where the quantons are always immersed in intense electromagnetic interactions.

But the philosophically interesting issue is whether the HD operators that I've been arguing for are, in principle, genuine physical observables or not. I claim they are and that they are required for a conceptually adequate formulation of LCQT.

2b: functional dependence and fundamentality

More recently, another prominent member of the history and philosophy of physics community told me that he had heard expressions of puzzlement about why I spent so much time, in some of my articles and presentations, on the classical instances of HD dynamical variables (e.g., Fleming 2000). Since all of the classical instances are functionals of more fundamental non HD fields (integrals over hyperplanes of non HD or trivially HD integrands), the examples seemed to emphasize the, at best, secondary, derived and non-fundamental character of the HD dynamical variables and thereby acted to depress interest in HD dynamical variables in LCQT.

My response was three-fold:

(1) The classical examples make clear that HD is *not novel* with Lorentz covariant QM and help to render more clear (and, hopefully, more palatable) some seemingly odd features of HD such as the violation of world-line invariance by HD position variables.

(2) In the context of local QFT the HD dynamical variables are also functionals (often exactly the same functional form as in the classical analogue) of non-HD local field structures. But these functionals play the role of collective coordinates for the systems they refer to (which latter can

be physically quite arbitrary) and, in my view, are more closely related to what we actually measure than the so-called more fundamental fields.

(3) Unlike the classical case, the quantum HD dynamical variables are usually incompatible (i.e., non-commuting) with the non-HD dynamical variables of which they are functionals. Thus one can never, even in principle, measure the HD quantities by measuring the local fields and then calculating the HD quantities. Thus they offer genuinely alternative modes of description and their formally secondary character is somewhat less diminishing than in the classical domain.

Perhaps the most elementary instance of this third feature is provided by the total 4-momentum of a system, S, which, if the system is open, is HD,

$$\hat{P}_S{}^{\mu}(\eta, \tau) = \int d^4x \, \delta(\eta x - \tau) \, \hat{\theta}_S{}^{\mu\nu}(x)\eta_\nu \quad , \tag{2.1}$$

where $\hat{\theta}_S{}^{\mu\nu}(x)$ is the stress-energy-momentum tensor field for the system, S.

We are accustomed to thinking of the total 3-momentum of a composite system as commuting with the contributions to it of the constituents of the system. The same with the total energy if interactions between the constituents are absent. But at the field theoretic level this notion must be qualified. Quite generally, for any bounded region, R, of the (η, τ) hyperplane, the total 4-momentum of the system on the hyperplane will not commute with that portion of the system 4-momentum contained within R, i.e.,

$$[\, \hat{P}_S{}^{\mu}(\eta, \tau), \int_{x \in R} d^4x \, \delta(\eta x - \tau) \, \hat{\theta}_S{}^{\nu\lambda}(x)\eta_\lambda \,] \neq 0 \quad . \tag{2.2}$$

When the system, S, is closed we can calculate this commutator explicitly since then the total 4-momentum of the system becomes the generator of space-time translations for the system. The commutator is then equal to the hyperplane integral,

$$-\, i \int_{x \in R} d^4x \, \delta(\eta x - \tau) \; \partial_\mu \, \hat{\theta}_S{}^{\nu\lambda}(x)\eta_\lambda \quad . \tag{2.3}$$

Similar, albeit usually more complicated, results hold for most HD quantities.

For the preceding complex of reasons, then, the functional dependence of HD dynamical variables on non-HD local fields does not seem to me to undermine the interest or importance of the former in LCQT.

3. Response to de Koning (2001) on hyperplane dependent localization

In a wide ranging and interesting dissertation, entitled, *"Particles out of place: The feasibility of a localizable particle concept in relativistic quantum theory"*, Henk de Koning (2001) devotes several pages to a discussion and assessment of the ideas of HD localization. Unique, so far as I know, among commentators, he has learned the HD formalism and provides a useful introduction to some aspects of it. Unfortunately, there are a number of early passages, not intended to be critical, in which I must disagree with his account of the purpose of the HD formalism and what it achieves. I can't quite shake the feeling that our differences here may be primarily a matter of terminology, but it is precisely here that careful choice is all important! After these early passages de Koning's main focus is on the HD extension of NW localization and in one particular he criticizes an aspect of my views on that localization in a manner duplicated, as we will later see, by Halvorson and Clifton.

3a: interpreting the purpose of HD

The first interpretive comments requiring response occur on p. 44 of the dissertation where de Koning writes,

"Already in 1965 Gordon Fleming noted the problem involving 'non-objective' localization associated with the NW-operator as well as the superluminal propagation by an initially NW-localized particle. In order to safeguard the localization concept in the context of relativistic quantum theory he introduced the notion of hyperplane dependence. Among other things hyperplane dependence implies that the (non-spatiotemporal) properties depend on the inertial reference frame you have chosen. So, judging whether a system is localized depends on the inertial reference frame you happen to be in."

He returns to the same points on p. 96 with,

> "First of all the [HD]-version of Newton-Wigner localized states are completely delocalized under a passive Lorentz boost as we can see from Eq. (4.31). This is evidently not a problem anymore if we adopt the [HD] framework; localization is hyperplane dependent and a passive Lorentz boost corresponds to shifting perspective from one to another hyperplane. Combining these two features makes it clear that it is in general not to be expected that localization in a bounded region is an *invariant* property under passive Lorentz boosts."

De Koning is correct that prior to the HD generalization of NW localization, that localization is non-objective since NW localization *at a definite time* (the only kind available prior to the HD generalization) is lost under a passive Lorentz boost. NW localization at a definite time is dependent upon the inertial frame perspective adopted. However, the HD generalization *removes* that inertial frame dependence. The fact that generalized NW localization occurs *on a definite hyperplane* is a fact that, itself, holds equally from the perspective of all inertial frames. It is not lost under a passive boost! The hyperplane in question 'looks' different from the perspective of different frames, but it is one hyperplane that is in question and *localization on a specific hyperplane or within a region of a specific hyperplane holds or does not hold, uniformly for all inertial frames*. The non-objective character of the original NW localization is completely absent in the HD generalization. That removal of non-objective character was, for myself, the motivating purpose for introducing the HD generalization.

Passive Lorentz boosts shift the perspective from one *inertial frame* to another, not from one hyperplane to another, i.e. from one *system of description* to another, not from one *object of description* – the hyperplane and the physical state of affairs on it – to another. Localization on a hyperplane is *invariant* under all passive Lorentz boosts. Any observer, even one not moving inertially, can choose to examine any aspect of the physical situation on any hyperplane. And he can retain his focus on that one hyperplane even as his state of motion changes. Two differently moving observers, examining the physical situation on one and the same hyperplane, will agree on what that physical situation is, however much they may describe it in different terms. They will, for example, agree on the probability for finding the NW position of a given system to lie within a given region of that hyperplane. They will also agree, for example, on *the point of the hyperplane* designated by the expectation values of the Minkowski components for the CE position operator for that system on that hyperplane.

On p. 92 de Koning misjudges the intended applicability of HD; "The central idea in this approach is that all physical variables or observables are hyperplane dependent." But it is not the case that *all* physical variables are hyperplane dependent. The electric charge-current density 4-vector field is not. The electromagnetic field tensor is not. The stress-energy-momentum tensor density field is not. No local fields are. The total 4-momenta and total generalized angular momenta of closed systems are not.

Roughly speaking, the physical variables that are hyperplane dependent include all physical variables representing non-constant or non-conserved global or collective properties of space-like extended systems. If there are no physical point entities, and I believe there are not (see **§1** in my 2003b and **§7** below), then most position operators, at least, are essentially (as opposed to trivially) HD. The NW operator valued *fields* also have a covariant HD generalization (Fleming 2000), but they are non-local fields.

3b: localizable properties

On p. 94 de Koning, referring to Fleming and Butterfield (1999), writes, " The localization operator Fleming and Butterfield use is the Newton Wigner (NW) operator, - - - ." But in that paper, *"Strange Positions"*, we discussed both the HD CE position operator and the HD NW position operator and, briefly, in pp. 151-153, the general relationship between them. There is no sense in which we *chose* between them or in which it would make sense to choose between them. They each represent the locations on hyperplanes of a particular collective physical property of space-like extended systems. For the CE position operator, the property in question is indicated by the name, *center of energy*.[1] For the NW position operator the property located is, in a very precise sense, **the center of spin** (CS)(Fleming 1965a, b).[2a] We should all start calling the HD NW position operator *the center of spin*, or the HD CS position operator. A calculational advantage to the HD CS comes from it having commuting Minkowski components. A calculational advantage of the HD CE comes from the simple way it is formed for a composite system in terms of the HD CE's for the constituent subsystems[2b], especially in the absence of interactions between the subsystems. For systems with zero internal angular momentum, but only for such systems, the HD CE and the HD CS coincide.

3c: 'localized within' vs. 'measureable within'

The most important comments by de Koning regarding HD concern the meaning of localization. On p. 98 de Koning conjectures an interpretation of comments made in *'Strange Positions'* to undermine the demand for the space-like separated spectral projectors for the HD CE or HD CS to be commuting operators. To our (p.160), " - - the inference to causal anomalies assumes that *association* of the projectors with spacelike separated regions involves precise measurability via operations *performed within* those regions. But this assumption is questionable.", de Koning responds with,

"- - - what is the operational meaning of associating an operator to a region, if this operator can't be measured in that region? In particular in the case of a position operator, the rejection of [this association] is very counterintuitive. It sounds as if one has to look also in a region Δ to figure out if, for instance, a particle is located in region Δ', spacelike related to Δ. Besides, it is not only very counterintuitive, it is also at odds with current experimental practice in which detection and localization experiments are always local."

While I admit that we did not write clearly enough to rule out de Koning's interpretation of our comments as a possible one (Halvorson (2001) and Halvorson and Clifton (2002) also adopted this interpretation), that interpretation was not what we intended! The suggestion that observables *associated* with a space-time region may not be measurable within that region was <u>not</u> intended to mean that we need to examine <u>other</u> regions to determine the observable of interest in the *associated* region. What the suggestion intended was that to determine the observable in the *associated* region *we may need to deploy instruments and initiate processes that **occupy** regions that are not confined to the associated region.*

Contrary to the spirit of de Koning's last quoted sentence, one need only consider the historical examples of Geiger counters, cloud chambers, spark chambers, bubble chambers, electron microscopes and scanning tunneling microscopes, to suspect that the smaller the region of interest, Δ, the more vastly larger, <u>by comparison</u>, the deployment of apparatus might need to be! The instrumentation and physical processes of these devices are hardly strictly confined to the space-time regions in which one expects the phenomena or properties of interest to occur! The same can be said for ordinary unaided human observation. And if one wishes to split hairs and include *ALL* correlated features of the measurement, then the instrumentation and process domain may well be unbounded! Indeed, the real possibility of there not being any precise boundary at all within which all the apparatus was strictly confined is a consideration that will be forced upon us in section **5**! In any case, once the existence is granted, of apparatus and physical processes germane to the measurements but extending outside the regions to which the measurements refer, it is not so easy to justify compatibility of the measurements (commutativity of the corresponding operators) merely on the basis of space-like separation of those regions. For in such a case one could not so easily rule out time-like influences between sets of apparatus deployed to examine space-like separated regions, and thus interfering with commutativity (**Fig. 1**). In this connection it is important to

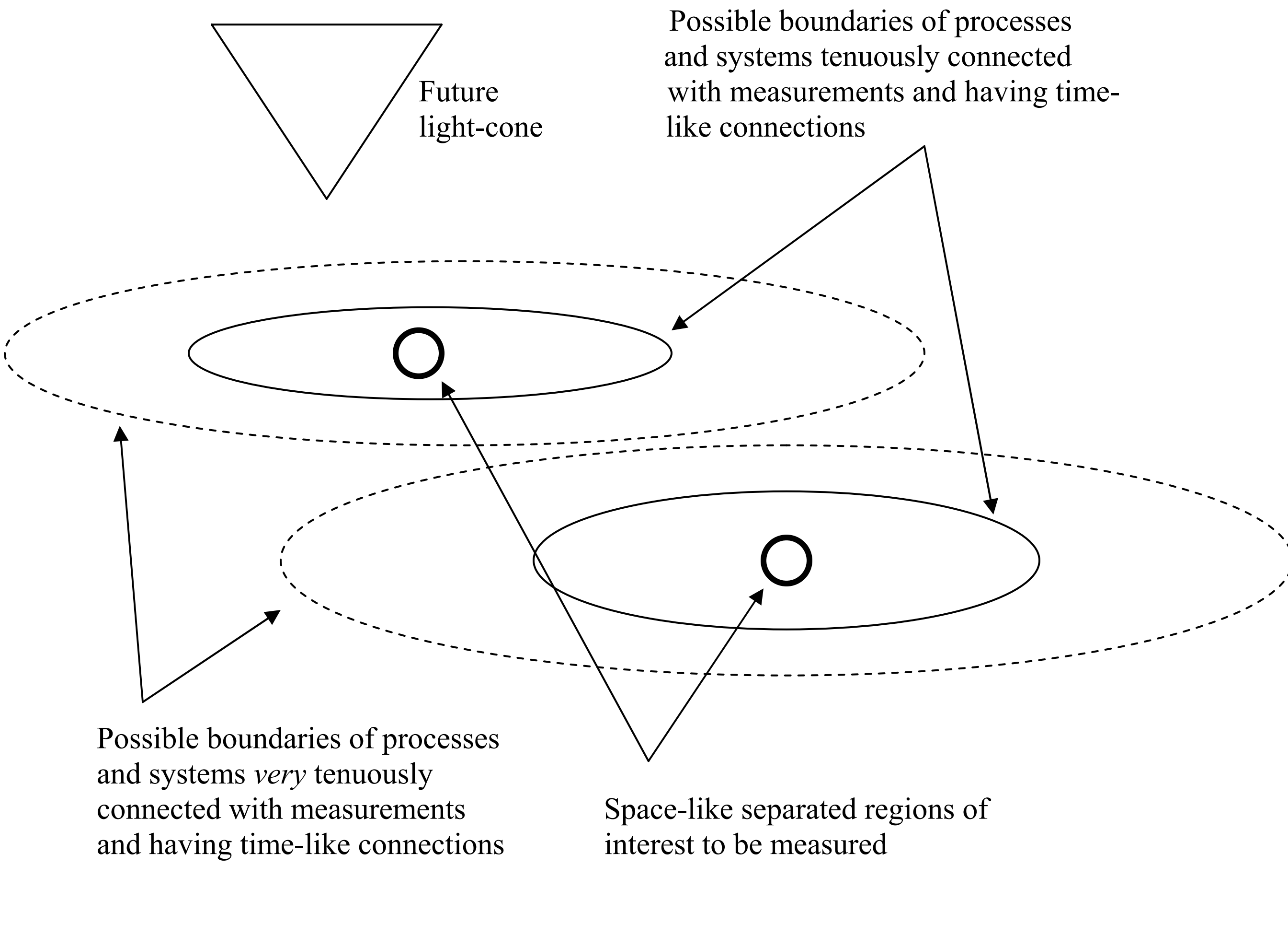

Fig.1: Possible source of incompatibility of measurements of observables associated with space-like separated regions

remember that the microcausality violating non-vanishing commutators in question (commutators between space-like separated spectral projectors for the HD CE or HD CS position operators) are invariably exponentially damped with increasing space-like separation.

3d: the Hegerfeldt theorems

Finally I will take this opportunity to respond to a query of de Koning's. On p. 99 he writes, "Curiously, Fleming and Butterfield don't even mention Hegerfeldt's theorem, so it [is] not so clear how they appreciate it."

While we did *mention* Hegerfeldt's (1974, 1985) on pp. 108, 116 and in the references, we did not *discuss* Hegerfeldt's theorem in *'Strange Positions'* because we accept the theorem completely. In keeping with the conclusion of the theorem both the HD CE and HD CS localized states display HD constrained superluminal components to their evolution (Fleming 1965a, b). Unlike Hegerfeldt, however, I believe this superluminal evolution to be a real physical process, not something to be eliminated from the theory. We suggested, and I continue to expect, that in the context of our views on non-local measureability the HD constrained superluminal evolution does not lead to causal anomalies. Admittedly, all that is certain thus far is that *no one has shown that it does lead to causal anomalies*. If my expectation is correct, demonstrating that it doesn't is a matter to be worked out in the future.

De Koning's dissertation contains much more of interest and worthy of consideration, but as the rest does not bear directly on HD, I leave it here.

4: Response to Halvorson (2001) on the relationship between the Reeh-Schlieder theorem and Newton-Wigner localization.

This section addresses the conflicting claims made in my (2000) '*Reeh-Schlieder meets Newton-Wigner'* and Halvorson's (2001) '*Reeh-Schlieder defeats Newton-Wigner'*. In this instance misinterpretation is, I think, a minor issue, being replaced by genuine disagreement on several points.

Halvorson and I agree that the NW fields, which create or annihilate quantons in NW position eigenstates, satisfy the 4-dimensional version of the Reeh-Schlieder (RS) theorem and that it is only the 3-dimensional version of the RS theorem that the NW fields avoid. We disagree on whether the avoidance of the 3-dimensional RS theorem coupled with the display of superluminal evolution makes the NW field's relation to the vacuum state any less counterintuitive than the local field's relation to the vacuum state. We also disagree on whether NW localization is more susceptible to a coherent interpretation than what Halvorson calls the "standard localization scheme". *I will, in fact, argue that the 'standard' localization scheme is not a* ***quantum*** *localization scheme of any kind at all, i.e., it does not correspond to the identification of any possessable, observable property for the quantum system in question. I will suggest a variant that does so and will compare the variant approximate localization scheme to HD NW localization in §7.* We disagree on the physical acceptability of superluminal evolution, which NW representation state functions and HD NW quantized fields display as a part of their time-like evolution. Finally, we disagree on the acceptability of violations of universal microcausality. I challenge the implication of act-outcome correlations over space-like intervals for NW localization by interpreting the localized states as not confining the quanton itself but rather as confining a localizable property that exists within the (infinitely) extended quanton (See **§5** and **§7**.) and by denying (as stated in **§3c**) the interpretation of *'localized within'* in terms of *'measureable within'*.

4a. cyclic states and superluminal evolution

Let me provide some background to this tangled web.

The RS theorem concerns the breadth of results that can be obtained by applying the members of a local algebra of fields to states of bounded energy. For any 4-dimensional bounded, open region of space-time, O, or any 3-dimensional bounded, open region, $G(\eta,\tau)$, of the (η,τ) space-like hyperplane, the local algebras, $A(O)$ and $A(G(\eta,\tau))$, contain the operator valued functionals of the quantized fields with support in O or $G(\eta,\tau)$, respectively. A state vector, Ψ, in the quantum state space, is said to be cyclic with respect to an algebra, A, of operators acting within the state space iff the set of state vectors, $A\Psi$, obtained by applying all the operators of the algebra to the original state vector, is dense in the state space. The RS theorem asserts that for free quantized local fields, the state vectors with bounded energy spectrum (which includes the vacuum) are all cyclic with respect to any $A(G(\eta,\tau))$ and for both free or interacting quantized local fields, the same state vectors are cyclic with respect to any $A(O)$.

This result was, at first, quite surprising to the Physics community as it seemed to suggest that by the performance of operations confined to bounded regions of space-time or hyperplanes (certain operators within the algebras were interpreted as representing such operations) one could produce results at arbitrary space-like separation from the operations (the generated set of state vectors included members arbitrarily close to those representing such results). With time, however, the majority of field theorists came to accept the RS theorem as just another of the many counterintuitive features of quantum theory with which we must live.

I. E. Segal (1964) pointed out that a non-local transform at definite time of local free fields, which transform creates and annihilates NW position eigenvectors, fails to satisfy the RS theorem for instantaneous regions of the type, G, i.e., if one forms the algebra, $A_{NW}(G((1,\mathbf{0}),x^0))$, of the transformed field with support confined to the bounded, open region of 3-space at the time x^0, $G((1,\mathbf{0}),x^0)$, then the states of bounded energy spectrum are not cyclic under that algebra. At the time the superluminal evolution and non-covariant transformation properties of the new NW field defined by Segal stood in the way of a physically satisfying interpretation of these results.

I introduced the covariant HD generalization of the NW fields in my (1966) but was not then concerned with the RS theorem (by virtue of being wholly unaware of it), which I did not address until my (2000), pointing out there that the lack of cyclicity of the energy bounded states under any of the HD NW-local algebras, $A_{NW}(G(\eta,\tau))$, over the HD NW fields on the (η,τ) hyperplane was a Lorentz invariant feature. I then claimed this feature to provide an *additional perspective* on the structure of the quantum states relative to quantized fields and that the HD NW fields had a less counterintuitive relation to those states than the local fields.

Halvorson (2001) showed that the HD NW fields do still satisfy the 4-dimensional version of the RS theorem for space-time regions, *O*, of *arbitrarily small but non-vanishing time-like dimension*. He then claimed that the failure of the 3-dimensional RS theorem to hold for the $A_{NW}(G(\eta,\tau))$ algebras, with zero time-like dimension, can hardly do much to alleviate the putative counterintuitive character of the RS theorem. He further claimed that the counterintuitive character itself was effectively removed by attention to the distinction between *selective* and *non-selective* operations within the algebras concerned.

On page 125 of his (2001) Halvorson writes "Thus, if worry about the RS theorem is about cyclicity in general, adopting the NW localization scheme does nothing to alleviate this worry." And then on page 127 he writes,

"- - - the RS theorem already has 'counterintuitive' consequences for the fixed time NW localization scheme. In particular, although the vacuum Ω is not cyclic under operations NW-localized in some spatial region G at a single time, Ω *is* cyclic under operators NW localized in G *within an arbitrarily short time interval* . - - - In Fleming's language, then, the NW-local fields 'allow the possibility of arbitrary space-like distant effects' from actions localized in an arbitrarily small region of space over an arbitrarily short period of time. Is this any less 'counterintuitive' than the instantaneous version of the RS theorem for the standard localization scheme?"

And then on page 132, "while their may be very good reasons for seeing the NW fields as covariant structures, avoiding the RS theorem is *not* one of them."

But the *worry*, if 'worry' is the right word, is not about cyclicity per se. It is, as Halvorson realizes is a possibility, about the cyclicity of the vacuum , and states of bounded energy generally, relative to the local algebras of a strictly subluminally evolving, microcausal field. We come to 'understand' the RS theorem, for the local fields, by learning that *relative to local fields* the vacuum is neither empty nor unstructured (see my 2000 and Halvorson's 2001 for details). Indeed, *relative to local fields*, the vacuum is superentangled. *Relative to the NW fields*, however, the vacuum is both empty and 'unstructured'. We then come to 'understand' the 4-dim. RS theorem, *relative to the NW fields*, by recognizing the superluminal aspect of the evolution of the NW fields and their violation of microcausality. By virtue of superluminal evolution the NW fields can propagate the consequences of operations NW localized within G throughout space *within an arbitrarily short time-like interval*. But this can constitute real understanding of the RS theorem, relative to the NW fields, only if the NW fields can be understood as covariant structures, i.e., as structures compatible with the principles of Lorentz covariance. So while we do not *avoid* the four dimensional version of the RS theorem by seeing the NW fields as covariant structures, we do achieve some *dynamical understanding* of the theorem thereby. And yes, while it may reduce to a matter of personal psychology and so be devoid of objective significance, I have to say that a 4-dim. RS theorem springing from HD, covariant, superluminal evolution and microcausality violation is, by virtue of its dynamical character, less counterintuitive to me than the 3 or 4-dim. RS theorem for local fields due to global features, i.e., superentanglement, of the cyclic states.

Ultimately, the difference between the subluminal evolution of microcausal local fields and the superluminal evolution of microcausality violating NW fields has nothing fundamental to do with HD. Instead it has to do with the separation of local fields into their positive and negative frequency parts, or, put another way, into their creation and annihilation parts. The NW field is just an integral transform of the annihilation part of the local field. Even without the integral transform that part of the local field evolves superluminally and violates microcausality. The integral transform is then brought to bear to extract the annihilation operators for HD NW position eigenvectors and this is done to give the Minkowski coordinate that labels the field physical significance as an eigenvalue of an observable, the HD NW position operator.

Finally, for this subsection, a word on the claimed removal of counterintuitive character from any version of the RS theorem via the distinction between *selective* and *nonselective* operations. This distinction was introduced in (Clifton and Halvorson 2000), a paper I'm not formally responding to here as it did not address any claims concerning HD. Halvorson employed the distinction in the paper under consideration, however, where, on page 119 he writes, "- - once one makes the crucial distinction between selective and nonselective local operations, local cyclicity does not obviously conflict with relativistic causality." Without claiming a conflict with relativistic causality, I fail to see how the distinction mentioned does anything to lessen the counterintuitive character of the RS theorem. The argument appears to be that of the two kinds of operations, only the nonselective ones can be construed as consisting solely of *physical* operations and it is not possible for a nonselective local operation associated (in the local algebra sense) with one space-time region *O*, to change the expectation value of any observable associated (in the same sense) with a space-like separated region, *O'*. A selective operation, however, consists of a nonselective operation that yields distinguishable subensembles, a mixed state, in effect, followed by a selection of a subset of those subensembles and it is only the selected subset that can yield altered expectation values within *O'*. But altering expectation values by selecting, i.e., focusing ones attention, on a subset of distinguishable subensembles is hardly counterintuitive.

Now I certainly agree that there is nothing counterintuitive about changing distant expectation values by selecting within distinguishable subensembles for states of the joint *O* & *O'* system. But it was the initial, nonselective, purely physical operation that *produced* the distinguishable subensembles, within which the altered expectation values occur. It is *this* purely physical process which is counterintuitive and yes "amazing". The fact that we don't get to see the effect until we make the selection does nothing, it seems to me, to diminish the surprise that the effect is there to be seen if we choose.

As for being counterintuitive; quantum theory has been characterized by counterintuitive features from the beginning and continues to be so. I suspect we will not see the end of such revelations for some time yet. But the revelations need not mean, and usually don't mean, that anything is wrong. Only that, being counterintuitive, our understanding of them can benefit from a variety of perspectives on how they come about. The local algebra perspective *explains* the counterintuitive features under discussion here via a highly entangled structure of all states of bounded energy, including the vacuum. The NW approach *explains* the same counterintuitive features by eliminating the 3-dimensional version and bringing the 4-dimensional version about via superluminal evolution, all rendered consistent with Lorentz covariance by HD.

4b: superluminal evolution itself

Of course the superluminal aspect of the evolution of the HD NW fields or position eigenvectors or state functions is, itself, troublesome to some. Halvorson, in effect, challenges the idea that superluminal evolution can be physically acceptable when he writes, on page 125, "From a physical point of view, the spectrum condition corresponds to the assumption that (a) all physical effects propagate at velocities at most the speed of light, and (b) energy is positive." The *spectrum condition* referred to is that the joint eigenvalue spectra of physical 4-momentum operators lie within the forward light cone of 4-momentum space. But the first part, (a), of the correspondence referred to is strictly demonstrable only in classical relativity and while it carries over into QFT for the dynamical evolution of local fields, it does not carry over for the HD NW fields, even though *both kinds of field satisfy the spectrum condition*.

Furthermore, it is risky to equate possible *velocities* with possible propagation effects. For while NW position representation state functions have a space-like propagation aspect (which we call superluminal) the time-like derivative of any HD NW position operator, i.e., an HD NW velocity operator, has a strictly subluminal eigenvalue spectrum which does emerge directly from the *spectrum condition*. Thus, for any closed system, S, we have,

$$\frac{\partial \hat{X}_{NW,S}{}^{\mu}(\eta, \tau)}{\partial \tau} = \frac{\hat{P}_S{}^{\mu}}{\eta \hat{P}_S} \quad , \tag{4.1a}$$

strictly subluminal, even though, $< \eta, x; NW| \Psi)$, the HD NW representation state function for a single spinless quanton, evolves, via the free quanton HD Schroedinger equation,

$$i\, \eta\partial < \eta, x; NW| \Psi) = R_\eta < \eta, x; NW| \Psi) \,, \tag{4.1b}$$

partly superluminally, where,

$$< \eta, x; NW\, | \hat{X}_{NW}{}^{\mu}(\eta, \tau = \eta x\,) \;=\; x^{\mu} < \eta, x; NW\, |. \tag{4.2a}$$

and

$$R_\eta : = [\kappa^2 + \partial^\mu\partial_\mu - (\eta\partial)^2]^{1/2} \,, \tag{4.2b}$$

is anti-local (Segal et al 1965).[3]

What is guaranteed by the subluminal velocity operator for the state function evolution is that *no superluminal group velocities can be generated*. But mere space-like contributions to the evolution of fields and/or state functions is compatible with Lorentz covariance and to demand, beyond that, compatibility with classical relativistic causality is, it seems to me, an unwarranted and probably unworkable imposition in the quantum domain.

4c: Halvorson's 'standard' localization scheme and a variant

As an alternative to the HD NW localization scheme, Halvorson contrasts what he calls the 'standard' localization scheme, based on a canonical mapping of classical Cauchy data (CCD) of a real classical Klein-Gordon field onto the space of single quanton states. Avoiding the mildly prejudicial term, 'standard', I will call this the CCD representation of the single quanton state space. Prior to erecting the multi-quanton Fock space and operator algebras on this basis, Halvorson displays the relation between the CCD and the NW representation in the single quanton state space accompanied with the comment, “Thus the one particle [representations, CCD and NW] are mathematically, and hence physically, equivalent. On the other hand, the two [representations] certainly *suggest* different notions of localization.”(my paraphrasing). Putting aside the issue of whether mathematically equivalent structure of distinct formalisms entails physically equivalent content of their interpretations, I want to claim that this comment *suggests* a notion of localization in the CCD representation that can not be sustained.

The displayed relation between the CCD and NW representations is,

$$\psi_{NW} = 2^{-1/2} \left(R^{1/2}u_0 + i\, R^{-1/2}u_1 \right) , \tag{4.3}$$

where u_0 is the real Cauchy data for the field, ϕ, and u_1 is the real Cauchy data for the time derivative, $\partial\phi/\partial t$. I take Halvorson’s comment to imply that the notion of localization *suggested* by the CCD representation is that if u_0 and u_1 have support confined to a region, G, then the quanton or some localizable property of the quanton is confined, or at least approximately confined to G. But suppose we multiply the NW state function by the imaginary unit, i. This can change nothing in the physical content of the state being represented. Yet,

$$\begin{aligned} i\, \psi_{NW} &= 2^{-1/2} \left(i\, R^{1/2}u_0 - R^{-1/2}u_1 \right) \\ &= 2^{-1/2} \left(R^{1/2}(-R^{-1}u_1) + i\, R^{-1/2}(Ru_0)\right), \end{aligned} \tag{4.4}$$

and the corresponding CCD is now $(-R^{-1}u_1, Ru_0)$ rather than (u_0, u_1). In particular, if the original CCD was confined to a bounded G, then the new CCD has unbounded support.

I am not certain Halvorson did intend the notion of CCD localization I’m criticizing here. He never declares it explicitly and he is very aware that CCD confined to a bounded region defines only a real linear space, which is the feature that leads to my example. But he also never cautions against this interpretation which his comment that I quoted lends itself to. If I have misunderstood what he meant, I have no idea what that might be.

The reality of u_0 and u_1 is due to Halvorson working with a self conjugate field, i.e., a field for which the quantons are identical to their anti-quantons.

Such quantons are comparitively rare in the actual quanton zoo and I would prefer to abandon that assumption, thus allowing u_0 and u_1 to be complex. This will not reinstate the questioned CCD localization scheme, however, since the CCD in this case will yield a superposition of a single quanton and a distinguishable anti-quanton (a superposition which usually violates a superselection rule) unless we have (positive frequency condition),

$$i\, u_1 = R u_0\,, \tag{4.5a}$$

for a single quanton state and (negative frequency condition),

$$i\, u_1 = -\, R u_0\,, \tag{4.5b}$$

for a single anti-quanton state. We notice that in these cases at most one of u_0 and u_1 can have bounded support. The NW representation state function for single quanton or single anti-quanton states then are,

$$\psi_{NW} = 2^{-1/2}\,(\, R^{1/2} u_0 + i\, R^{-1/2} u_1\,) = (2R)^{1/2} u_0\,, \tag{4.6a}$$

and

$$\psi^{\wedge}{}_{NW} = 2^{-1/2}\,(\, R^{1/2} u_0 - i\, R^{-1/2} u_1\,)^* = (2R)^{1/2} u_0^*\,, \tag{4.6b}$$

respectively. We will designate the functions, u_0, that satisfy (4.5a, b) in the customary way as $u^{(+)}$ and $u^{(-)}$, for positive and negative frequency, respectively. We then have, quite generally,

$$\psi_{NW} = (2R)^{1/2} u^{(+)}, \qquad \psi^{\wedge}{}_{NW} = (2R)^{1/2} u^{(-)*}. \tag{4.7}$$

With these last results we are again offered a possible alternative localization scheme to compare with the NW scheme and this time it's a scheme that survives arbitrary complex linear superposition. I will call it the definite frequency (DF) localization scheme.

If $F^{(+)}(G)$ is the set of NW state functions with positive frequency data confined to G, then, being a subspace closed under complex linear superposition, the subspace represents <u>some</u> possessed property of the single quanton.[4] The property in question, however, is <u>not</u> *precise* localization of <u>anything</u> within the designated bounded regions! The reason is that even if G and G' are disjoint the state functions in $F^{(+)}(G)$ are not all orthogonal to those in $F^{(+)}(G')$, respectively. In particular, the point localized basis functions for the positive frequency data, i.e.,

$$u_{\mathbf{y}}^{(+)}(\mathbf{x}) := \delta^3(\mathbf{x} - \mathbf{y}) \tag{4.8}$$

are not orthogonal for distinct **y** and **y**'. Thus, within $F^{(+)}(G)$, putting,

$$\psi_{NW,\,\mathbf{y}}(\mathbf{x}) := (2R_{\mathbf{x}})^{1/2} u_{\mathbf{y}}^{(+)}(\mathbf{x}), \tag{4.9}$$

we have,

$$< \psi_{NW,\,\mathbf{y}'} \mid \psi_{NW,\,\mathbf{y}} > = \int d^3x\, \psi_{NW,\,\mathbf{y}'}(\mathbf{x})^* \, \psi_{NW,\,\mathbf{y}}(\mathbf{x})$$

$$= \int d^3x\, [(2R)^{1/2} u_{\mathbf{y}'}^{(+)}(\mathbf{x})]^*[(2R)^{1/2} u_{\mathbf{y}}^{(+)}(\mathbf{x})] = \int d^3x\, u_{\mathbf{y}'}^{(+)}(\mathbf{x})^* 2R\, u_{\mathbf{y}}^{(+)}(\mathbf{x})$$

$$= 2R_{\mathbf{y}'}\delta^3(\mathbf{y}' - \mathbf{y}) \neq 0. \tag{4.10}$$

Consequently the subscript, **y**, labeling these basis functions is not an eigenvalue of <u>any</u> self adjoint operator and does not represent a definitely possessed property.[4] Nevertheless, these non-orthogonal basis functions <u>can</u> be interpreted as ‘states’ <u>approximately</u> localized around the coordinate, **y**, in some sense, and the sense of the approximation can be made precise by analyzing the relationship of these ‘states’ to the NW position operator and the NW position eigenfunctions. This will be done in §**7**.

The discussion of the last few pages has not considered HD in any detail and the reader may suspect that the modified localization scheme based on definite frequency initial data, by virtue of (4.9), generalizes to an HD scheme just as the NW scheme does. This is, however, not the case. To see this we note that the definite frequency parts of the local field operator are given by formally the same relations that define the definite frequency parts of the initial data. Thus, just as we have,

$$u_0 = u^{(+)} + u^{(-)} \qquad \text{and} \qquad u_1 = iR(u^{(-)} - u^{(+)}), \tag{4.11}$$

we also have (allowing for possible HD),

$$\hat{\phi}(x) = \hat{\phi}^{(+)}(\eta, x) + \hat{\phi}^{(-)}(\eta, x) \tag{4.12a}$$

and

$$(\eta\partial)\hat{\phi}(x) = iR_\eta(\hat{\phi}^{(-)}(\eta, x) - \hat{\phi}^{(+)}(\eta, x)) \qquad (4.12b)$$

This yields,

$$\hat{\phi}^{(\pm)}(\eta, x) := \frac{1}{2}[\hat{\phi}(x) \pm i\, R_\eta{}^{-1}(\eta\partial)\hat{\phi}(x)]. \qquad (4.13)$$

But now, if we use,

$$\partial\, (\eta\partial)\, /\partial\eta^\mu = \partial_\mu - \eta_\mu(\eta\partial) : = D_{\eta;\,\mu}\,, \qquad (4.14a)$$

$$\partial\, R_\eta\, /\partial\eta^\mu = (\partial/\partial\eta^\mu)\, [\kappa^2 + \partial^\alpha\partial_\alpha - (\eta\partial)^2\,]^{1/2} = -\, R_\eta{}^{-1} D_{\eta;\,\mu}\, \eta\partial\,, \qquad (4.14b)$$

and the KG field equation,

$$(\eta\partial)^2\,\hat{\phi}(x) = -\, R_\eta{}^2\, \hat{\phi}(x), \qquad (4.14c)$$

it follows that

$$\partial\, \hat{\phi}^{(\pm)}(\eta, x)\, /\partial\eta^\mu = 0\,, \qquad (4.15a)$$

or

$$\hat{\phi}^{(\pm)}(\eta, x) \;=\; \hat{\phi}^{(\pm)}(x). \qquad (4.15b)$$

Thus, for a single spinless quanton state vector, $|\,\Psi\,)$,

$$<x\,|\,\Psi\,) : = (\,\Omega\,|\,\hat{\phi}^{(+)}(x)\,|\,\Psi\,), \qquad (4.16a)$$

is the DF representation state function and

$$<\eta,\, x;\, NW|\,\Psi\,) = (\,2R_\eta\,)^{1/2} <x\,|\,\Psi\,)\,, \qquad (4.16b)$$

is the corresponding HD NW representation state function.

Halvorson spells out in some detail the construction of the operator algebras associated with 3-dimensional compact space-like regions by virtue of being functions and weak limits of functions of field operators that, in turn, are linear functionals of CCD with support confined to the compact regions.

This makes all the operators in such an algebra, themselves, functionals of the corresponding quantized local field with support confined to the region in question and this latter, slightly looser characterization suffices for our purposes. Inside the algebra for a bounded region, G, nothing like the above problem for single quanton states occurs since complex linear superposition is permitted. Accordingly, Halvorson mounts the 'standard' or CCD localization scheme here by identifying the operators in such an algebra with the observables that can be *measured within* the region in question. I turn to that interpretation next.

4d: again, 'measureable within' vs 'localized within'

Just as with de Koning above, Halvorson grounds the relation 'is localized in' on the claimed more fundamental relation 'is measurable in' and is puzzled that (p. 130) "- - Fleming appears to take the localization relation to be primitive. However, if localization is a primitive relation, it is not obvious why we should think it coincides with the assignments made by the NW localization scheme."

In the first place, how would one know that a measurement was a 'measurement within' a given bounded region unless one had reason to accept that either the apparatus or at least the processes involved had, themselves, been 'localized within' that region? Would this assessment require yet another 'measurement within' and thus put us on the road to infinite regress? But we know that this is not required. While an observation is required to confirm localization, that observation need not itself be similarly localized.

In the second place, localization, *per se*, does not , even in my approach, always coincide with NW localization. As was mentioned in p. 11, above, and spelled out in more detail in notes 2a and 2b, in a system with non-zero internal angular momentum NW localization is distinct from localization of the CE and one might very well be concerned with the latter rather than the former. (CE localization could only be *sharply* bounded, however, for one Minkowski component of the CE since the Minkowski components of the CE do not commute in the presence of internal angular momentum)[2b]. In field theoretic systems with non-zero total charge – of the electric or other varieties – we can define a center of charge (CC) position operator and examine the localization of the CC in various quantum states. For general systems in QFT, many different localizable properties can coexist. And this is a crucial point:

It is not some unique kind of localization of objects or systems, themselves, that is at issue, but the localization of various ***properties****, all of which are carried by a great variety of kinds of objects and systems.*

Elsewhere on page 130 Halvorson writes,

"How then *does* Fleming interpret the association of an observable with a region of space? That is, what does he mean by saying that an observable is localized in a region of space? - - - the NW position operator is not contained in any NW local algebra, and there is no natural correspondence between the spectral projections of the NW position operator and the NW local algebras. Thus, even if we were to concede that the NW position operator has 'unequivocal physical significance', this would not appear to clarify the physical significance of NW local algebras."

To further emphasize the ambiguity resulting from taking localization as primitive relative to measurement, Halvorson considers (p.131) applying a unitary transformation, that leaves a hyperplane invariant, to a NW local algebra associated with a bounded region of the hyperplane. Because of the preservation of formal properties under the unitary transformation, he asks how I would decide whether the original algebra or the transformed one represents observables associated with the region.

I will address these comments in reverse order.

While one can always subject <u>every</u> operator and state vector to arbitrary unitary transformations without changing physically significant formal properties, the NW position operator(s) and NW local algebras are not, *by themselves alone*, susceptible to such transformations. In particular, once the assignment of total energy, total momentum, total angular momentum and Lorentz boost operators for *arbitrary* systems is made, *the corresponding assignments of the CE position and the NW position for arbitrary systems is determined.*[5]

The NW local algebras are then <u>defined</u> in terms of these global operators since the NW fields, themselves, are <u>defined</u> as those linear functionals of the local fields and their time-like derivatives that create and annihilate field quantons in NW position eigenstates. An NW local observable is associated with a given region of a hyperplane iff it is a functional of the NW fields with support confined to the points on the hyperplane belonging to that region. The correspondence between the spectral projectors of NW position and the NW local algebras is not as simple as Halvorson seeks and so he may choose not to dub it "natural", but it's the same kind of correspondence that exists in non-relativistic QFT between the spectral projectors of the center of mass position operator and the field theoretic 'local' algebras. In both cases the *center* in question for a system can be sharply confined to a region without all of the parts of that system having their corresponding *centers* similarly confined. But so what?! What's so unphysical about that?

Agreed, none of the global operators, from the total energy to the NW position operator(s) or their spectral projectors can be found within any NW local algebra (or any standard local algebra for that matter). Nor can their analogues be in the non-relativistic case. This is one reason I eschew working solely within the local algebra framework. It offers the sanctuary of the mathematically well behaved at the expense, I think, of physical conceptual adequacy.

In particular, I think a strong motivation for making 'measurement within' a primitive concept is the protection thereby conveyed to the principle of universal microcausality. But notwithstanding the received attitude towards the microcausality condition satisfied by the local algebras based on local fields, it is worth noting that that condition does not require the observables of local algebras to be strictly measureable within the associated space-time region. It, by itself, merely allows for that possibility, in principle. But, in fact, we never measure observables that are localized within a space-time region by deploying apparatus or executing actions strictly confined to the smallest region within which the observable is localized (see **§3 c** above and **§5** below for elaboration). So if, in accordance with Halvorson's CCD scheme, the operators within local algebras are measureable within the regions they are associated with, then they are not the observables we actually measure. In any given instance the situation in this regard may be improved with time by technological developments that will diminish, without eliminating, the dependence of the measurement process on physical processes and apparatus lying outside the minimal domain of the local observable. But the fact that we can, and invariably do, measure properties within a space-time domain by executing actions not confined to that domain means that 'is measurable within' has nothing essential to do with the meaning of 'is localized within' – except to depend, in part, on the latter, for its meaning!

Consequently, the failure of the NW local algebras to satisfy the microcausality condition does not imply that they give rise to act-outcome correlations over space-like intervals. Rather it implies, or can be regarded as implying, that (at least some of) the essential measurement processes lying outside the associated region of the observable have (from the perspective of local fields) time-like influences that undermine commutativity (**Fig. 1**). Indeed, given that any observable within an NW local algebra is a functional of the local fields with space-like unbounded support, there is no reason to expect that such observables can be precisely measured with actions strictly confined to any space-like bounded region at all.

On the other hand, and in the spirit of what I will call the Halvorson and Clifton FAPP argument to be discussed below, the fact that the non-vanishing commutators of space-like separated observables within NW local algebras are 'small' and exponentially decreasing with increasing space-like separation helps to account for the FAPP success we have in discussing measurement processes as if they were confined to space-like bounded domains.

5: Response to Halvorson and Clifton (2002) on localization and measurement

In Halvorson and Clifton's (2002) *"No place for particles in relativistic quantum theory"*, the authors reach some important novel conclusions concerning relativistic quantum theory (RQT). For my purposes, the *central conclusion*, is that ***there are no strictly localized particles or objects*** in such a theory. This result is obtained from some of the premises of the local algebra approach to RQT and from strengthened versions, which the authors provide, of the theorems of Malament (1996) and Hegerfeldt (1998). Throughout the stages of their discussion, the one unchanging presumption, upon which the conclusions reached come more and more to depend, is **universal microcausality** (the principle that any two observables 'referring' to properties localized in space-like separated regions of space-time must commute). This principle is, in turn, justified via the insistence, encountered above in de Koning (2001) and Halvorson (2001), that the concept of 'being localized within a region Δ' receives its meaning from the supposed more primitive concept of 'being measureable within a region Δ'.

With regard to their central conclusion I am completely in agreement! While my own arguments in the past, as mentioned in section **1** of my (2003b), have only been against the point-like character of quantons and have offered some minimal extension diameter estimates, I will here (**§7**) offer additional quantitative arguments in support of the impossibility of strict localization of quantons within any bounded space-like region. And from the strict non-localizability of quantons to the strict non-localizability of any objects whatsoever is, in the context of quantum field theory, a very plausible path.

Unfortunately, I see the arguments used by Halvorson and Clifton to reach their central conclusion to be confused and inconsistent. I find no problem with the purely mathematical aspect of their arguments. It is the physical interpretation of their mathematics that I will argue is, at best, ad hoc and sometimes inconsistent. The upshot, as I will show, is to undermine the support of the principle of universal microcausality by virtue of rendering the concept of **being measurable (strictly) within a region Δ** devoid of reference. These conceptual collapses then open the world of non-localizable objects to description, in part, by hyperplane dependent dynamical variables with their (exponentially damped) violations of universal microcausality.

5a: opening claims of Halvorson and Clifton

Already in the first page of their paper, referring to the implications of the theorems of Hegerfeldt and Malament, Halvorson and Clifton write, "Thus it appears that quantum theory engenders a fundamental conflict between relativistic causality and localizability." The phrase, "relativistic causality" includes the proscriptions against superluminal evolution of position representation state functions (which I discussed above in **§4b**) and non-vanishing commutators of space-like separated observables, particularly space-like separated spectral projectors of position operators. By "localizability" is intended a theory that admits point quantons or at least, *quantons susceptible to total confinement within finite volumes*.

On their p. 2 these judgements are reinforced with two statements. First,

"- - - if we believe that the assumptions of Malament's theorem must hold for any empirically adequate theory, then it follows that our world can not be described by a particle theory."

Second, after a description of some results they will derive in an appendix,

"While these results show that there is no position observable that satisfies relativistic constraints, quantum field theories – both relativistic *and* non-relativistic – already reject the notion of a position observable in favor of localized field observables."

In the first statement, the phrase "a particle theory" means not merely a theory that admits localizable quantons, but one that takes such quantons as fundamental ingredients from which the theory is built up. In the next sentences they promise additional results that will also exclude the former option. In the second statement, the phrase, "already reject the notion of a position observable", must mean rejects the <u>fundamental</u> <u>status</u> of position observables since position observables certainly <u>exist</u> in the non-relativistic case, albeit as derived concepts, e.g. the total center of mass and the center of mass of all electrons, say. Also, I take the "relativistic constraints" of the second statement to mean the same as the "relativistic causality" that I commented upon in the preceding paragraph.

Halvorson and Clifton correctly understand that Butterfield and I do not agree that "the assumptions of Malament's theorem must hold for any empirically adequate theory" and that we do not regard as binding on all observables what they called "relativistic causality". However, when they come to assess the argument that we (1999, see p. 160.) gave on behalf of the violation of space-like commutativity of position operator spectral projectors, they repeat the interpretations of de Koning (2001) and Halvorson (2001).

5b: (and yet again) 'localized within' vs 'measureable within'

Their interpretation appears on their page 7, where they write,

"According to Fleming, the property 'localized in Δ' (represented by E_{Δ}) *need not be detectable within* Δ [my emphasis]. As a result, $[E_{\Delta}, E_{\Delta'}] \neq 0$ does not entail that it is possible to send a signal from Δ to Δ'. However, by claiming that local *beables* need not be local *observables*, Fleming undercuts the primary utility of the notion of localization, which is to indicate those physical quantities that are operationally accessible in a given region of space-time. Indeed, it is not clear what motivation there could be – aside from indicating what is locally measureable – for assigning observables to spatial regions."

Aside from the stridently positivistic slant of these assertions (<u>I</u> would have said the utility of the notion of localization is to indicate what *exists* in a given region of space-time.), the motivation for assigning observables to spatial (or space-like) regions, in those instances where we do so, is provided by, (1) our recognition of the way those observables behave under spatial (or space-like) translations and rotations and, yes, (2) our recognition that measuring those observables requires a form of *focusing* on the spatial (or space-like) region in question. But this *focusing* never requires *confinement* of the whole measurement apparatus and process to that same region. As mentioned above (**§4d**), to so require would lead us into an infinite regress.

For, perhaps unneeded, emphasis I will repeat here some comments made in response to de Koning's related queries. When Butterfield and I wrote (p. 160), " - - the inference to causal anomalies assumes that *association* of the projectors with spacelike separated regions involves precise measurability via operations *performed within* those regions. But this assumption is questionable." we meant to refer to *all the operations of the deployed apparatus for the measurements and to* ***question the confinability of the apparatus to the regions of interest***. One need only consider the historical examples of Geiger counters, cloud chambers, spark chambers, bubble chambers, electron microscopes and scanning tunneling microscopes, to suspect that the smaller the region of interest, Δ, the more vastly larger, by comparison, the deployment of apparatus might need to be. But if this is the case, then one could not so easily rule out time-like influences between sets of apparatus deployed to examine space-like separated regions, and thus interfering with commutativity (**Fig. 1**). This is especially so if one could not determine any precise boundary at all within which all the apparatus was strictly confined.

5c: consequences of the non-existence of strictly localizable objects

Now while Butterfield and I raised no explicit question of the precise boundary of the deployed apparatus, Halvorson and Clifton have reached conclusions that imply no such boundary can exist! For by page 20 of their paper they write,

> " The argument for localizable particles appears to be very simple: Our experience shows us that objects (particles) occupy finite regions of space. But the reply to this argument is just as simple: These experiences are illusory! ***Although no object is strictly localized in a bounded region of space*** [my emphasis], an object can be well-enough localized to give the appearance to us (finite observers) that it is strictly localized."

I will refer to the phrase following the last comma as Halvorson and Clifton's FAPP argument.

But if there are no strictly localizable objects, then **there are no strictly localizable measuring instruments**. Furthermore, **there are no strictly localizable parts of measuring instruments that could be regarded as objects in their own right**. This being the case, it is hard to see how there could be designed any strictly localized measurement processes. But *if there are no strictly localizable measurement instruments or processes, then it is impossible to measure an observable associated with a bounded spatial region, Δ, by the deployment of apparatus or the execution of processes strictly confined to that region, or, for that matter, strictly confined to any region!* The conjecture by Butterfield and myself is thus raised to the level of a deduced conclusion, the concept of "being strictly measurable within a bounded region" is rendered devoid of referance and the argument for universal microcausality, undermined.

Technically, the FAPP utility of the common talk of localizable objects emerges, as Halvorson and Clifton argue, from the existence, in any local algebra, of **effects** [6] that are arbitrarily close in norm to effects that could represent strict localization if it existed. These latter effects do not exist in any local algebra (and, in that sense, are *non-local* effects) and need not satisfy space-like commutativity. But from our inferred non-existence of localizable measurements, there is no longer any justification for excluding these latter effects, and other non-local effects as well, from the domain of the measureable. There is no longer any justification for denying the non-local effects to be just as much observables, as the effects belonging to local algebras. For in the absence of strictly localizable measuring apparatus and measurement processes how could one possibly rule out the measureability, in principle, of the non-local effects?! The non-local effects include the spectral projectors of the generalized Center of Energy and Newton-Wigner position operators which describe ***sharply localized properties*** that can (in principle) be precisely measured but not by strictly localizable apparatus or processes. As mentioned before, it is important to remember here that the microcausality violating non-vanishing commutators in question are invariably exponentially damped with increasing space-like separation.

The reader may feel I have engaged in a vicious circle here, by undermining the internal consistency of the position of Halvorson and Clifton (their position relying on universal microcausality and the grounding of 'localized within' in terms of 'measurable within'), and therefore undermining their derivation of what I have called their central conclusion – the non-existence of strictly localizable objects – which conclusion I have then used to my advantage by ruling out strictly localizable measurements and undermining thereby universal microcausality. Be that as it may, I will provide my own argument on behalf of the central conclusion in section **7**.

6: Wallace on the connection between quantons and fields

In "*Emergence of particles from bosonic quantum field theories*" David Wallace (2001a) analyses the concept of quantons that are understood as excitations or special states of quantized fields. For simplicity he confines himself to scalar fields, just as I have concentrated on spinless quantons here, and, except for a few general remarks, to fields satisfying linear field equations. Needless to say, I welcome his approaching the subject via the canonical Lagrangian/Hamiltonian formalism, which the majority of working quantum field physicists use, rather than the more abstract algebraic approach which, I think, extracts a high price in conceptual adequacy for the easy access to mathematical rigor. Wallace has very nicely defended the status of the Lagrangian/Hamiltonian formalism in the quantum field context in his (2001b).

As I have not seen a published version of Wallace's paper, my comments refer to the electronic posted version.

Compared to my previous sections my comments on Wallace's (2001a) are not a response to criticism as he does not address in detail my views on HD or localization. But his interesting and somewhat unorthodox approach to the quanton concept and his views of the localization of quantons warrant some response here, especially as he interprets NW localization very differently than I do.

It also appears that Wallace harbors (see the last paragraph of his p. 6) some of the widespread misconceptions of my views that I commented upon in **§1** of my (2003b). I will not re-discuss those matters here.

Two aspects of Wallace's paper that seemed puzzling to me were (1) the absence of any discussion of number operators and the countability of quantons as persistent entities (at least when free or weakly interacting) and

(2) the superficial discussion, as if in passing, of the energy-momentum relations displayed by quantons. For while localizability, to some degree or another, of quantons has been an important aspect of the evolution of our concept of quantum particles; historically, the facts that *quantons could be counted* as, at least, semi-persistent entities in appropriate environments and that *the energy and momentum of quantons are functionally related* have been at least as important as localization in forming our concept of the quanton.

From the photo-electric effect through the Compton effect to the contemporary measurements of collision events in high energy physics we have employed, at best, if at all, only mesoscopically precise localization to obtain sharp quanton counts and microscopically precise energy-momentum values. The situation is most striking when Wallace refers, as he does on several occasions for comparative and illustrative purposes, to phonons, the uniquely condensed matter physics version of a field quanton. Phonons are almost never localized and rarely studied in localized states (Wallace alludes to the role of localized phonons in heat transport in his p.7) and their definitions in condensed matter physics texts are as countable excitations displaying specific energy-momentum relations. It seemed to me that had number operators and energy-momentum relations been given their proper due, the discussion could have been noticeably simpler and shorter. It was as if Wallace's real goal, and not an unreasonable goal at that, was to examine how much of the quanton concept could be obtained from the requirement of localizability alone.

6a: effective L-localization

But now to localization. In his section **3** Wallace discusses the problem of constructing a definition of a quantum particle and reaches the conclusion, with which we are in complete accord, that single quanton states of a quantized field must comprise a linear subspace of the entire field theory state space and the subspace must be spanned by a collection of (possibly overcomplete and non-orthogonal) what Wallace calls **effectively L-localized** states. It then turns out that for relativistic quantons of a given rest mass the scale, L, of the effectively L-localized basis states can not be smaller than the Compton wavelength of the quantons. By defining effective L-localization through the exponential drop off of differences between field operator expectation values within the subspace and in the vacuum state when the field operators are evaluated a distance of order L or more away from the localization region of the subspace state, effective L-localization does not require strict confinement of the entire quanton within any ball of finite radius. Thus far I have no problem with these considerations.

But jumping ahead to Wallace's section **7**, where he discusses Newton–Wigner localization, we learn that the NW basis vectors (or, more properly, normalized superpositions of them with support confined to sub Compton wavelength radius balls) are also merely effectively L-localized with L $\geq$ Compton wavelength, as, of course, they would have to be. So long as one interprets, as Wallace does, the NW basis as a construction designed to grant access to quanton states of arbitrarily small spatial confinement *of the entire quanton*, the design must be judged a failure. And Wallace so judges it (p. 36):

The [NW basis vectors] are certainly formally equivalent to position eigenstates, being perfectly localized in configuration space and forming an (improper) basis for the one-particle Hilbert space. But obviously they are not precisely localized in *real space*: [my emphasis] if $|\mathbf{x}_{NW}>$ is the abstract ket corresponding to a delta function at **x**, then - - - it is easy to verify that (for instance)

$$< \mathbf{x}_{NW} | \ \hat{\varphi}(\mathbf{x})\hat{\varphi}(\mathbf{y}) \,|\, \mathbf{y}_{NW} > - < \ \Omega \,|\, \hat{\varphi}(\mathbf{x})\hat{\varphi}(\mathbf{y}) \,|\, \Omega > \qquad (87)$$

is formally equal to

$$(1/2)[\ R^{1/4}\ \delta(\ \mathbf{x} - \mathbf{y}\ )\ ]^2 \qquad (88)$$

and hence is localized only within a region of size ~ L_c . [Wallace's *R* is equivalent to my R^2].

He then goes on to note that this is not a real problem, one just has to avoid taking superpositions of the $|\ \mathbf{x}_{NW} >$ with support smaller than Compton wavelength dimensions too literally since such superpositions will not *really* be so localized. Among his summarizing remarks in this section he declares (p. 37),

As such, the Newton-Wigner representation is a perfectly legitimate, and often very convenient, way of describing states in the one-particle subspace --- but it doesn't give the exact truth of the matter as to where particles are localized, because there isn't one: particles are superpositions of field excitations with finite size, so any attempt to give a wave function description down to arbitrarily small scales is inevitably going to be arbitrary at those scales.

As the reader well knows by now, I view these comments as involving a misunderstanding of the physical significance of the NW eigenvectors. $| \mathbf{x}_{NW} >$ is, indeed, an improper state in which the quanton is only effectively L_c-localised around the point, **x**, but the point, **x**, is a physically significant point and very *real*! It is, in this case of a spinless quanton, the location of the center of the energy distribution, *the quantized field energy distribution*, of the quanton. Thus superpositions of these basis vectors with support confined to balls with sub Compton wavelength radii can be taken quite literally. They yield single quanton states with the center of energy precisely confined within the ball!

There is a widespread tendency to argue against any possibility of sub Compton wavelength localization of *anything* in relativistic quantum theory on the grounds that the resolution required to corroborate or generate any such localization would entail the use of de Broglie wavelengths, and thus the transfer of momenta and energies that would undermine the whole endeavor via quantum fluctuations, particle anti-particle creation, etc. While it's clear that these considerations give reason to expect difficulties and complications to confront any simple minded efforts at such localizations, I see no reasons of principle to justify the conclusion that they are impossible. One must rather analyse what theory, with interactions included, has to say about specific proposals for such measurements. Even if it became clear that no such localization procedure could be implemented on a single quanton that would preserve the single quanton nature in the final state, examination of the final state that did emerge could still corroborate the suitability of the localization interpretation being proposed.

6b: quantum states from coherent states

Let us go back now to Wallace's section **5** where he first identifies operators that create single quanton states effectively L-localised within a region. He approaches this identification via the construction of coherent states that are closely associated with classical phase space data for the field. The motivation for coherent states is the well known way in which the coherent states of quantum mechanical harmonic oscillators mimic classical oscillatory behaviour without dispersion over time. Fourier analyzing the classical field data, $\phi(\mathbf{x})$ and $\pi(\mathbf{x})$, into their harmonic components,[7]

$$\phi(\mathbf{x}) = (2\pi)^{-3/2} \int d^3k \, (2\omega_{\mathbf{k}})^{-1/2} \{\alpha(\mathbf{k})e^{i\mathbf{kx}} + \alpha(\mathbf{k})^{*e-i\mathbf{kx}}\} , \qquad (6.1a)$$

$$\pi(\mathbf{x}) = i\,(2\pi)^{-3/2} \int d^3k\, (\omega_{\mathbf{k}}/2)^{1/2}\, \{\alpha(\mathbf{k})^* e^{-i\mathbf{kx}} - \alpha(\mathbf{k}) e^{i\mathbf{kx}}\}\ , \qquad (6.1b)$$

Wallace builds the coherent state that is a simultaneous eigenstate of the momentum space annihilation operator, $\hat{a}(\mathbf{k})$, for all **k**, with eigenvalues, $\alpha(\mathbf{k})$, i.e.,

$$|\, C(\phi, \pi) > := \exp[-\, \|\alpha\|^2 / 2]\, \exp[\hat{a}^\dagger(\phi, \pi)]\, |\, \Omega > , \qquad (6.2)$$

where,

$$\|\alpha\|^2 = \int d^3k\, |\alpha(\mathbf{k})|^2 \qquad (6.3a)$$

and

$$\hat{a}^\dagger(\phi, \pi) = \int d^3k\, \alpha(\mathbf{k})\, \hat{a}^\dagger(\mathbf{k})\ . \qquad (6.3b)$$

Wallace then argues that the reason this state, notwithstanding it's close connection to classical data, can not represent a single quanton state, lies in the discrepancy between superpositions of such coherent states and the single coherent state for the corresponding superposition of classical data. By considering the limit of vanishingly small classical data he finally settles on the state,

$$|\, \phi, \pi > := \hat{a}^\dagger(\phi, \pi)\, |\, \Omega > , \qquad (6.4)$$

as best representing the single quanton state most closely associated with the classical data (ϕ, π) via effective L_c- localization.[8] In particular, these states do not suffer from the superposition discrepancy infecting the previous coherent states. Needless to say, Wallace is aware that the reader always knows in advance, modulo some details, where the analysis is going to end up. His motivation is to display the insights that his procedure offers and the discussion of the status of his coherent states is valuable. But the absence, in this discussion, of any reference to conserved number operators or energy-momentum relationships is, to repeat myself, very artificial.

Next, to enable the calculation of various field theoretic expectation values of the states (6.4), Wallace considers the commutation relations of $\hat{a}^\dagger(\phi, \pi)$ with the field operators. He finds,

$$[\hat{\varphi}(\mathbf{x}), \hat{a}^\dagger(\phi, \pi)] = (1/2)\{\phi(\mathbf{x}) + i\,(R^{-1/2}\pi)(\mathbf{x})\}, \tag{6.5a}$$

$$[\hat{\pi}(\mathbf{x}), \hat{a}^\dagger(\phi, \pi)] = (1/2)\{\pi(\mathbf{x}) - i\,(R^{1/2}\phi)(\mathbf{x})\}, \tag{6.5b}$$

and proceeds to calculate to show that the quanton is not confined within dimensions of the order of L_c.[9]

But wait, these are pregnant commutators! What must jump out from these equations to many readers but is not mentioned is that by judicious linear combination we obtain,

$$[(1/2)^{1/2}\{R^{1/4}\hat{\varphi} + i\,R^{-1/4}\hat{\pi}\}(\mathbf{x}), \hat{a}^\dagger(\phi, \pi)] = (1/2)^{1/2}\{R^{1/4}\phi + i\,R^{-1/4}\pi\}(\mathbf{x})\,, \tag{6.6a}$$

and
$$[(1/2)^{1/2}\{R^{1/4}\hat{\varphi} - i\,R^{-1/4}\hat{\pi}\}(\mathbf{x}), \hat{a}^\dagger(\phi, \pi)] = 0. \tag{6.6b}$$

Furthermore, we recognize (from our **§4c** if not elsewhere) these linear combinations of the operator fields and of the classical data. They are just the NW operator fields and state functions, i.e.,

$$\hat{\psi}_{NW}(\mathbf{x}) = (1/2)^{1/2}\{R^{1/4}\hat{\varphi} + i\,R^{-1/4}\hat{\pi}\}(\mathbf{x})\,, \tag{6.7a}$$

and
$$\psi_{NW}(\mathbf{x}) = (1/2)^{1/2}\{R^{1/4}\phi + R^{-1/4}\pi\}(\mathbf{x}). \tag{6.7b}$$

In terms of them (6.6a,b) become,

$$[\hat{\psi}_{NW}(x), \hat{a}^\dagger(\phi, \pi)] = \psi_{NW}(\mathbf{x})\,, \tag{6.8a}$$

and
$$[\hat{\psi}_{NW}{}^\dagger(\mathbf{x}), \hat{a}^\dagger(\phi, \pi)] = 0\,, \tag{6.8b}$$

and we confirm
$$[\hat{\psi}_{NW}(\mathbf{x}), \hat{\psi}_{NW}{}^\dagger(\mathbf{y})] = \delta(\mathbf{x} - \mathbf{y})\,, \tag{6.8c}$$

and
$$[\hat{\psi}_{NW}(\mathbf{x}), \hat{\psi}_{NW}(\mathbf{y})] = 0. \tag{6.8d}$$

This, then tells us that

$$\hat{a}^\dagger(\phi, \pi) = \int d^3x\, \psi_{NW}(\mathbf{x})\, \hat{\psi}_{NW}{}^\dagger(\mathbf{x})\,, \tag{6.9}$$

and, consequently, Wallace's coherent states are simultaneous eigenstates of the Newton-Wigner annihilation field operator evaluated at all the points of space, the eigenvalues at any point being the NW state function evaluated at that point and corresponding to the classical data, i.e., for all **x**,

$$\hat{\psi}_{NW}(\mathbf{x}) \,|\, C(\phi, \pi) > \,=\, |\, C(\phi, \pi) > \psi_{NW}(\mathbf{x}) \,. \qquad (6.10a)$$

This preferred status of the NW field, relative to Wallace's coherent states, is shared by any linear transform of the NW field such as the positive frequency part of the local field, $\hat{\varphi}^{(+)}(\mathbf{x})$, or the momentum space annihilation operator, $\hat{a}(\mathbf{k})$, as we began with (see eqs. (6.2) and (6.3)), but it is not shared by any local field. The corresponding relation for the single quanton state, $|\, \phi, \pi >$, is given by,

$$\hat{\psi}_{NW}(\mathbf{x}) \,|\, \phi, \pi > \,=\, |\, \Omega > \psi_{NW}(\mathbf{x}) \,. \qquad (6.10b)$$

Once one recognizes that NW localization has nothing to do with the issue of the *size* of a quanton but, rather, concerns the location of a localizable property *within* a quanton, the preceding relationships can be seen to convey directly precise physical significance regardless of how small the support of the NW state function may be.

7: Localization schemes and the space-like extension of quantons

For the sake of easy reference we gather here the basic relationships we will need in this section. The relation between NW representation state functions and DF representation state functions for single quantons was given, in Dirac notation , by[10]

$$< \eta, x; NW \,|\, \Psi \,) = (\, 2R_\eta \,)^{1/2} < x \,|\, \Psi \,), \qquad (4.16b)$$

where, from (4.2b) and (4.14a), we have,

$$R_\eta := [\, \kappa^2 + \partial^\mu \partial_\mu - (\eta\partial)^2 \,]^{1/2} = [\, \kappa^2 + D_\eta^{\ 2} \,]^{1/2} \,. \qquad (7.1)$$

The bra vectors, $< \eta, x; NW|$, for the orthogonal NW basis, and $< x \,|$, for the non-orthogonal DF basis, are created from the vacuum bra by application of the NW field operator and the positive frequency field operator, respectively, i.e.,

$$< \eta, x; NW | = (\Omega | \hat{\psi}_{NW}(\eta, x) , \tag{7.2a}$$

and,

$$< x | = (\Omega | \hat{\phi}^{(+)}(x) . \tag{7.2b}$$

Accordingly, the relation between the basis bras and the field operators are given by,

$$< \eta, x; NW | = (2R_\eta)^{1/2} < x |, \tag{7.3}$$

and

$$\hat{\psi}_{NW}(\eta, x) = (2R_\eta)^{1/2} \hat{\phi}^{(+)}(x)$$

$$= \frac{1}{\sqrt{2}} [R_\eta^{1/2} \hat{\phi}(x) + i R_\eta^{-1/2} \eta \partial \hat{\phi}(x)] . \tag{7.4}$$

We will also, in **§7b**, need the action of $\hat{\phi}^{(+)}(x)$ on an arbitrary single quanton ket. Since that action must yield a multiple of the vacuum state, it follows from (7.2b) that,

$$\hat{\phi}^{(+)}(x) | \Psi) = | \Omega) < x | \Psi) . \tag{7.5}$$

7a: comparing the NW and DF localization schemes for spinless quantons

Now just what is the nature of the approximate localization of the quanton represented by the basis bra, < x |, compared to the precise localization of the NW position within the quanton represented by < x, η; NW | ? At the risk of belaboring the obvious I reiterate two points expressed earlier in this paper.

(1) It is important to realize that it would make no sense to admit the members of one of these two kinds of bases to physical interpretation and dismiss the others. Both kinds of bases span the same state space. No doubt the members of the two kinds of bases represent different aspects or

different conditions for the quanton, but they *each* represent *some* aspect and *some* condition.

(2) We know that the DF basis vectors, from their definition, (7.3b), are labeled by the coordinates, x, in such a way that under the inhomogeneous Lorentz group the ket change is given by the corresponding classical change in x (the positive frequency part of $\hat{\phi}(x)$ in (7.2b) is a hyperplane independent Lorentz scalar). At the same time we know that the labeling x *can not be an eigenvalue of any position operator* because the DF basis vectors are non-orthogonal for any two distinct values of x. From this alone we can conclude that the quanton is, in no sense, *confined* to the point x but, at best, *localized around* x.

But the *sine qua non* of degree of spatial localization in a quantum state (of any feature of any system – not just of quanton positions) is the rapidity with which the state approaches orthogonality with itself under space-like translation. It is only due to just such behaviour of the inner products of the DF basis vectors, themselves, with space-like separated coordinates, i.e.[11],

$$\delta(\eta x - \eta y) < x \mid y > = \frac{\delta(\eta x - \eta y)}{4\pi^2 \mid x - y \mid} \int_{\kappa}^{\infty} \frac{\mu \, d\mu}{\sqrt{\mu^2 - \kappa^2}} e^{-\mu \mid x - y \mid} , \qquad (7.6)$$

that it is no doubt correct that | x > represents a quanton localized ‘around’ x. But how are we to obtain observational access to the ‘center’ of localization, x?

For this it is important to ask if there are other coordinate labeled kets that approach self orthogonality under space-like translation more rapidly. And, of course, there are! The NW position eigenvectors were originally deliberately constructed (Newton et al 1949) to go orthogonal to themselves under arbitrarily small displacements along instantaneous hyperplanes. The HD generalization of them extends that feature to the corresponding non-instantaneous hyperplanes.

Clearly something is *precisely* localized in an NW position eigenvector and the DF localized vectors are, themselves, superpositions of such NW-vectors. For spinless quantons the precisely localized point in the NW basis vector can be identified as the center of the energy distribution in the quanton. *This interpretation does not require one to think of the quanton* *itself as a point entity* and in the next subsection we will see vividly that it is not. Nevertheless, the quanton as a whole appears more tightly localized in the NW basis vectors than in the DF basis vectors as comparison between (7.8) and the inner product,[12]

$$\delta(\eta y-\eta x) < \eta, y; \mathrm{NW} \,|\, x> = \frac{\delta(\eta y-\eta x)}{4\pi^2\,|\,y-x\,|}\int_\kappa^\infty \frac{\mu\ d\mu}{[\mu^2-\kappa^2]^{1/4}}\, e^{-\mu|\,y-x\,|} \quad , \qquad (7.7)$$

suggests. Note that the integral representations on the right sides of (7.6) and (7.7) are such that the more slowly damped exponentials (low values of μ) are favored in (7.6) over (7.7) while the more strongly damped exponentials (high values of μ) are favored in (7.7) over (7.6), an indication of the more localized nature of the NW-kets.

From the Fourier representation of (7.7)[12] and the one quanton state space dyadic representation of the NW position operator,

$$\hat{X}_{\mathrm{NW}}{}^{\mu}(\eta,\tau) = \int d^4x\ \delta(\eta x-\tau)\,|\,x,\eta;\, \mathrm{NW} > x^{\mu} < x,\eta;\, \mathrm{NW}\,| , \qquad (7.8)$$

we can calculate the ‘matrix elements’ of $\hat{X}_{\mathrm{NW}}{}^{\mu}(\eta,\tau)$ in the DF basis. The result, for x' and x space-like separated and both lying on the (η, τ) hyperplane (i.e., $\eta x = \eta x' = \tau$), is,

$$< x'|\,\hat{X}_{\mathrm{NW}}{}^{\mu}(\eta,\tau)\,|\,x> \;= \frac{1}{2}(\,x'^{\mu} + x^{\mu}\,) < x'|\,x> \;. \qquad (7.9)$$

We see that the x of a DF basis vector behaves like an ‘expectation value’ (loosely understood since we’re dealing with infinite norm kets and bras) of the NW position operator, the (similarly loosely understood) ‘probability amplitude’ for which, (7.7), is symmetrically and monotonically decreasing as one moves away from x on the hyperplane.

Continuing in this vein we obtain a sense of the magnitude of the ‘rms deviation’ of the NW position in the DF basis vectors from,

$$< x'|\,(\,\hat{X}_{\mathrm{NW}}{}^{\mu}(\eta,\tau) - \frac{1}{2}(\,x'^{\mu}+x^{\mu}\,))(\,\hat{X}_{\mathrm{NW}}{}^{\nu}(\eta,\tau) - \frac{1}{2}(\,x'^{\nu}+x^{\nu}\,))\,|\,x>$$

$$= \frac{1}{6}(x'^{\mu} - x^{\mu})(x'^{\nu} - x^{\nu}) < x'| x >$$

$$- \frac{\eta^{\mu\nu} - \eta^{\mu}\eta^{\nu}}{3} \int d^4y' d^4y \delta(\eta y' - \tau)\delta(\eta y - \tau) < x'| y' >< y'| y >< y | x >. \quad (7.10)$$

For our last comparison between the NW basis and the DF basis we will obtain a relation between the expectation values of the total energy-like operator, $\eta\hat{P}$, in any two normalizable states, $| \Psi, NW; \eta, \tau)$ and $| \Psi, DF; \eta, \tau)$, which have identical expansion coefficients in terms of the NW basis and DF basis, respectively, i.e.,

$$| \psi, NW; \eta, \tau) := \int d^4x \, \delta(\eta x - \tau) \, | \eta, x; NW > \psi(x), \quad (7.11a)$$

$$| \psi, DF; \eta, \tau) := \int d^4x \, \delta(\eta x - \tau) \, | x > \psi(x) . \quad (7.11b)$$

From (7.4) we have,

$$| \eta, x; NW > = (2R_{\eta, x})^{1/2} \, | x > = (2\eta\hat{P})^{1/2} \, | x > , \quad (7.12a)$$

and, consequently,

$$| \psi, NW; \eta, \tau) = (2\eta\hat{P})^{1/2} \, | \psi, DF; \eta, \tau) . \quad (7.12b)$$

From this it immediately follows that,

$$< \eta P >_{\psi, NW} = \frac{(\psi, NW; \eta, \tau \,|\, \eta\hat{P} \,|\, \psi, NW; \eta, \tau)}{(\psi, NW; \eta, \tau \,|\, \psi, NW; \eta, \tau)}$$

$$= \frac{(\psi, DF; \eta, \tau \,|\, 2(\eta\hat{P})^2 \,|\, \psi, DF; \eta, \tau)}{(\psi, DF; \eta, \tau \,|\, 2\eta\hat{P} \,|\, \psi, DF; \eta, \tau)} = < \eta P >_{\psi, DF} + \frac{{(\Delta\eta P)_{\psi, DF}}^2}{< \eta P >_{\psi, DF}} , \quad (7.13)$$

the expectation value, $< \eta P >_{\psi, NW}$, is always equal to or larger than $< \eta P >_{\psi, DF}$.

Now what is the upshot of all of this? It is, first, that if one accepts the interpretation of the NW position operator as locating some property within the space-like extended quanton, then the ket, | x >, can not be understood as a maximally localized ‘state’ of the quanton that fails the orthogonality condition for distinct x because of the quanton's extension.

The upshot is, second, that without the foregoing analysis of the physical meaning of the x in the DF ket, | x >, in terms of ‘expectation values’, ‘probability amplitudes’ and ‘rms deviations’ of the NW position operator, that coordinate, x, is just a mathematical symbol, uninvested with physical content. Due to the failure of orthogonality of the | x > , the label, x, is not an eigenvalue of any single observable and, within standard quantum theory, can acquire physical content only by the display of statistical relationships with genuine observables. As we have seen, the NW position observable is eminently suitable for that purpose.

The upshot is, finally, that if we invoke the identification (which holds only for spinless systems) of the NW position with the CE position, then we strengthen the assertions of the first upshot. For we are not concerned here with the distribution of the quanton's energy density (which I will later show extends to space-like infinity in all single quanton states) but with the distribution, in the sense of quantum superposition, of the center, or average position, of differently centered energy densities. The DF ket, | x >, must be understood as an unbounded superposition of NW kets, in each of which the point within the extended quanton localized by the NW operator has a precise value. Thus it must be understood as a superposition of differently located, extended quantons. The, or at least a, maximally localized ket for the quanton is precisely the NW ket which satisfies the orthogonality condition for distinct x on the designated hyperplane, not because the localized quanton is a point entity, but because a particular point within the quanton is precisely localized in these eigenvectors and differently so for distinct x. In particular, any normalizable superposition of NW basis vectors yields a higher energy expectation value, presumably because of greater space-like localization of the quanton, than the identical superposition of DF basis vectors. Thus, to repeat, we are confronted, in the ket, | x >, with a superposition of differently centered extended quantons rather than with a maximally localized extended quanton.

7b: the space-like extension of spinless quantons

The instantaneous energy density field operator for a free scalar field with spinless quanton and anti-quanton excitations is

$$\hat{\theta}^{00}(x^0, \mathbf{x}) = \frac{hc}{2} :\{ \frac{\partial}{\partial x^0} \hat{\phi}^\dagger(x) \frac{\partial}{\partial x^0} \hat{\phi}(x) +$$

$$\frac{\partial}{\partial \mathbf{x}} \hat{\phi}^\dagger(x) \cdot \frac{\partial}{\partial \mathbf{x}} \hat{\phi}(x) + \kappa^2 \hat{\phi}^\dagger(x) \hat{\phi}(x) \}: \quad (7.14a)$$

where $\hat{\phi}(x)$ is the (charged) scalar local field operator and the bracketing colons indicate normal ordering of the creation and annihilation parts of the field. For an arbitrary space-like hyperplane with time-like unit normal, η^μ, the corresponding energy-like density is,

$$\eta_\mu \hat{\theta}^{\mu\nu}(x) \eta_\nu = \frac{hc}{2} :\{ \eta\partial \hat{\phi}^\dagger(x)\, \eta\partial \hat{\phi}(x) -$$

$$D_{\eta,\mu} \hat{\phi}^\dagger(x)\, D_\eta{}^\mu \hat{\phi}(x) + \kappa^2 \hat{\phi}^\dagger(x)\, \hat{\phi}(x) \}: \ , \quad (7.14b)$$

where $D_{\eta,\mu} := \partial_\mu - \eta_\mu (\eta\partial)$.

This quantity becomes the instantaneous energy density in any reference frame for which the hyperplane is instantaneous. In the general case we will call (7.14b) the **hyperplane energy density** or simply, the energy density. As a minimal indicator of the space-like extension of a single spinless quanton on a hyperplane, I propose the support over the hyperplane of the expectation value of the energy density in single quanton states. To disentangle the contribution to energy density distributions due to the *dispersal* of the quanton from the contribution due to the *extension* of the quanton we would expect to need to focus on features of the energy density distribution that persist in all candidates for highly localized states. But in fact, there is no need for that focus since we can immediately establish that ***for all normalizable, single, spinless, quanton states, the energy density expectation value over any hyperplane has unbounded support.*** This is a clear sense in which the quanton, itself has unbounded space-like extension.

In obtaining this result the DF basis vectors play a central role since, for any normalizable, single quanton state vector, $|\,\Psi\,)$, we have,

$$(\Psi|\; x >< x\,|\Psi) = (\Psi|{:}\hat{\phi}^{\dagger}(x)\hat{\phi}(x){:}|\Psi)\;. \tag{7.15}$$

Consequently,

$$(\,\Psi\,|\,\eta_{\mu}\hat{\theta}^{\mu\nu}(x)\eta_{\nu}\,|\,\Psi\,) = \frac{hc}{2}\{R_{\eta,y}\,R_{\eta,x} - D_{\eta,y}\cdot D_{\eta,x}$$

$$+\,\kappa^{2}\,\}(\,\Psi\,|\,y >< x\,|\,\Psi\,)\,|_{y=x}\,. \tag{7.16}$$

where we have used

$$i\,\eta\partial < x\,|\,\Psi\,) = R_{\eta} < x\,|\,\Psi\,) \tag{7.17}$$

Remembering the Lorentz metric we see that the three terms, R^2, $-\,D^2$ and κ^2, in (7.18), are each non-negative after setting $y = x$. But from the *anti-local* property[3] of the psuedo differential operator, R_η, it follows that $< x\,|\,\Psi\,)$ and $R_\eta < x\,|\,\Psi\,)$ cannot both have bounded support. Therefore *the energy density expectation value*, (7.16), *has unbounded support for any* $|\,\Psi\,)$. Since states exist with bounded support for $< x\,|\,\Psi\,)$ and states exist with bounded support for $< \eta, x; NW|\,\Psi\,)$, the impossibility of bounded support in (7.16) can only come from the unbounded space-like extension of the quanton, itself, and not from the unbounded dispersal of some center of the quanton over the hyperplane. Therefore, as indicated by the energy density expectation value, **the single quanton has unbounded space-like extension**. Clearly the same result holds for the anti-quanton and, while considerations of space do not permit the demonstration here, the same holds for quantons of any spin.

It is worth noting that the situation is different for the momentum density-energy flux,

$$\hat{\pi}^{\mu}(\eta, x) := (g^{\mu}{}_{\lambda} - \eta^{\mu}\eta_{\lambda})\hat{\theta}^{\lambda\rho}(x)\eta_{\rho}$$

$$= \frac{hc}{2} : \{(\eta\partial)\hat{\phi}^{\dagger}(x)\,D_{\eta}{}^{\mu}\hat{\phi}(x) + D_{\eta}{}^{\mu}\hat{\phi}^{\dagger}(x)(\eta\partial)\hat{\phi}(x)\} :. \tag{7.18}$$

In this case the single quanton expectation value is given by,

$$(\Psi | \hat{\pi}^{\mu}(\eta, x) | \Psi) = \text{hc Im}[D_{\eta}{}^{\mu} (\Psi | x > R_{\eta} < x | \Psi)], \qquad (7.19)$$

with the result that it has bounded support under circumstances that include when <u>either</u> $D_{\eta}{}^{\mu} < x | \Psi)$ <u>or</u> $R_{\eta} < x | \Psi)$ does. This suggests that the support of these latter functions is *associated* with the absence of fluctuations strong enough to wipe out the expectation value of dynamical quantities equally capable of positive and negative values. The single quanton energy density is intrinsically non-negative and fluctuations can not completely wipe out the expectation value anywhere. Still, it is striking that the energy density expectation value receives additive contributions from the absolute squares of mutually anti-local quantities that contribute only multiplicatively to the momentum density/energy flux!

Notwithstanding the fact that the preceding argument did not require us to compare the energy density distribution for states of varying degrees of localization, that comparison is edifying. To that end consider the state function, $\psi(y)$, over the $(\eta, 0)$ hyperplane with unit norm and centered on the origin, i.e.,

$$\int d^4y\, \delta(\eta y) | \psi(y)|^2 = 1 \quad \text{and} \quad \int d^4y\, \delta(\eta y)\, y^{\mu}\, | \psi(y)|^2 = 0\,. \qquad (7.20)$$

It then follows that, for $1 \geq \lambda > 0$, and $\eta z = 0$,

$$\psi_{\lambda, z}(y) := \lambda^{-3/2}\, \psi(\frac{y - z}{\lambda}), \qquad (7.21)$$

also has unit norm, is centered on z and has been 'squeezed' around z by a factor of λ compared to $\psi(y)$ around the origin. We then define the unit norm state vector, that, on the (η, τ) hyperplane, has its superposition of NW eigenvectors centered on $z + \eta\tau$ and 'squeezed' by λ,

$$| \eta, \tau, \kappa; \psi_{\lambda, z}, \text{NW}) := \int d^4y\, \delta(\eta y)\, | \eta, y + \eta\tau, \kappa; \text{NW} > \psi_{\lambda, z}(y). \qquad (7.22a)$$[13]

A similar construction can be implemented with the DF basis vectors, i.e.,

$$| \eta, \tau, \kappa; \psi_{\lambda, z}, \text{DF}) := \int d^4y\, \delta(\eta y)\, | y + \eta\tau, \kappa > \psi_{\lambda, z}(y)\,, \qquad (7.22b)$$

with the difference that, as a consequence of the specific way in which the DF basis is non-orthogonal (see (7.6)), we have,

$$\| \, | \, \eta, \tau, \kappa; \psi_{\lambda, z}, \mathrm{DF} \,) \, \|^{2} = \lambda \, \| \, | \, \eta, \tau, \lambda\kappa; \psi_{1, 0}, \mathrm{DF} \,) \, \|^{2}, \qquad (7.23)$$

and we must carefully divide by this variable squared norm in calculating any expectation value.

Defining the expectation values of the energy density field in these states by

$$(\, \eta, \tau, \kappa; \psi_{\lambda, z}, \mathrm{NW} \,|\, \eta_\mu \hat{\theta}^{\mu\nu}(x + \eta\tau)\eta_\nu \,|\, \eta, \tau, \kappa; \psi_{\lambda, z}, \mathrm{NW} \,) .$$

$$:= u_{\mathrm{NW}}(\eta, x + \eta\tau \,; \psi_{\lambda, z}, \kappa) \;, \qquad (7.24a)$$

and,

$$\frac{(\, \eta, \tau, \kappa; \psi_{\lambda, z}, \mathrm{DF} \,|\, \eta_\mu \hat{\theta}^{\mu\nu}(x + \eta\tau)\eta_\nu \,|\, \eta, \tau, \kappa; \psi_{\lambda, z}, \mathrm{DF} \,)}{(\, \eta, \tau, \kappa; \psi_{\lambda, z}, \mathrm{DF} \,|\, \eta, \tau, \kappa; \psi_{\lambda, z}, \mathrm{DF} \,)}$$

$$:= u_{\mathrm{DF}}(\eta, x + \eta\tau \,; \psi_{\lambda, z}, \kappa) \;, \qquad (7.24b)$$

we find that,

$$u_{\mathrm{NW}}(\eta, x + \eta\tau \,; \psi_{\lambda, z}, \kappa) \;=\; \lambda^{-4} \, u_{\mathrm{NW}}(\eta, \frac{x - z}{\lambda} + \eta\tau \,; \psi_{1, 0}, \lambda\kappa) , \qquad (7.25a)$$

and,

$$u_{\mathrm{DF}}(\eta, x + \eta\tau \,; \psi_{\lambda, z}, \kappa) \;=\; \lambda^{-4} \, u_{\mathrm{DF}}(\eta, \frac{x - z}{\lambda} + \eta\tau \,; \psi_{1, 0}, \lambda\kappa) . \qquad (7.25b)$$

The λ dividing $x - z$ in the u functions produces the narrowing of the energy distribution as the state is ‘squeezed’, i.e., as λ gets smaller. The λ^{-4} factor out front raises the value of the energy density everywhere as λ decreases and in such a way as to increase the total energy by a factor of λ^{-1} in accordance with the requirements of the uncertainty relation. The factor of λ multiplying the rest mass parameter, κ, depresses the relative contribution from rest energy as compared to kinetic energy with the ‘squeezing’ of the state, also as the uncertainty relation would suggest.

The fact that both states, (7.22a,b), produce the same response, (7.25a,b), to 'squeezing' in the energy density is a further indication, if any were needed, that both the NW and DF basis vectors represent 'localized states' of one kind or another. The orthogonal nature of the NW basis, the statistical results, (7.9, 10), and the relation, (7.13), which guarantees that the state, (7.22a), will always have a larger energy expectation value than (7.22b) then tell us that the NW basis vectors are 'more localized' than the DF basis vectors.

8: Remarks on HD and the ontology of Minkowski space-time

I have very little to add to the discussion of this topic that ended the first paper of this set, my (2003b). The arguments considered here, which are regarded as having secured the *central conclusion* of the **nonexistence of strictly localizable entities**, as opposed to the existence, on hyperplanes, of strictly localizable properties, certainly supports the doubts expressed in the first paper about the fundamental status of the points of Minkowski space-time relative either to *points on hyperplanes* or to *hyperplanes taken holistically*. One question that arises is that since all of the detailed analysis that has been undertaken here focuses on the free quantons of free quantized fields, and while one would not expect the presence of interactions to completely reverse the qualitative nature of our conclusions, is there any reason to expect interactions to either support or divert the directions into which our considerations have led us?

I see two reasons to expect support. First, consider the fact that most of the quantons of contemporary high energy physics are unstable against spontaneous decay, whether as a consequence of strong, weak or electromagnetic interactions. Suppose one then adopts the natural, but possibly naïve, hypothesis that all such unstable quantons can, in principle, exist (perhaps even be prepared) 'alone', in nearly a 3-momentum eigenstate, at any definite time. At any other time a superposition of parent quanton and decay products would be expected. If this is so, then a novel source of HD emerges since the preceding characterization is not covariant under the Lorentz group. The covariant generalization is provided by the parent quanton being 'prepared' on any given hyperplane. A natural consequence of this HD of unstable quanton states is a spin spectrum for the quanton with non-vanishing width to accompany the standard non-sharp spectrum for the rest mass. Sharp spin spectra for unstable quantons, which is standardly assumed but never tested, can be imposed, but only through constraints on the HD.

The second reason I expect support comes from the consideration of introducing primordial HD into the 'basic' fields of theory. By virtue of the fact that the seven dimensional space of points-on-hyperplanes (parameterized by (η, x)) is a homogeneous space of the Poincare' group, many possible field equations (involving partial derivatives with respect to η as well as x) for such fields yield an infinite family of quanton excitations with a precise functional relationship between the rest masses and spins of the quantons. By turning on a single self coupling for such an HD field, all the quantons of the family are assigned a consistent complex of interactions with one another.

I and collaborators examined both of the preceding issues in a few papers (Fleming 1970, 1971, 1979) (Boyer et al 1974) (Ardalan et al 1975) long ago in pre-Standard Model antiquity. The atmosphere was sufficiently desperate in those days to encourage the examination of wild ideas. Sometimes it seems such an atmosphere may be returning and the examination of suitable variants of these ideas in the midst of non-Abelian gauge fields, Superstrings and Loop Quantum Gravity may not be amiss.

Notes

1. (from p. 11) Strictly speaking, it would be more appropriate to call the CE the 'center of hyperplane energy' since, if $\theta^{\mu\nu}(x)$ is the local stress-energy-momentum field, then on a hyperplane normal to the time-like unit vector, η^{μ}, the CE position locates the 'center' of the distribution over the hyperplane of the energy density-like quantity, $\eta_{\mu}\theta^{\mu\nu}(x)\eta_{\nu}$. Nevertheless, in the context of a discussion of hyperplanes in general, we will call $\eta_{\mu}\theta^{\mu\nu}(x)\eta_{\nu}$ simply the energy density.

2a. (from p. 11) Let $M_{\mu\nu}$ be the generator of homogeneous Lorentz transformations for a closed complex of systems and let S be an interacting subsystem of that complex and let $M_{S,\mu\nu}$ be the HD contribution from S, alone, to $M_{\mu\nu}$. Then (understanding that, in what follows, every operator associated with the subsystem, S, is HD, i.e., $\forall A$, $A_S = A_S(\eta,\tau)$, and that all commutators are *equal hyperplane* commutators), $J_S^{\mu} := (1/2)\, \varepsilon^{\mu\alpha\beta\gamma} M_{S,\alpha\beta}\eta_{\gamma}$ is the HD angular momentum due to S, alone, which satisfies the commutation relation,

$[J_S^\mu, J_S^\nu] = i\hbar\, \varepsilon^{\mu\nu\alpha\beta} J_{S,\alpha}\eta_\beta$. The HD NW position operator for S is that unique position operator, X_S^μ, constructed from the dynamical variables of S, alone, with commuting Minkowski components. It decomposes J_S^μ into mutually commuting 'orbital', $L_S^\mu = \varepsilon^{\mu\alpha\beta\gamma} X_{S,\alpha} P_{S,\beta}\eta_\gamma$, and 'internal' angular momenta, S_S^μ, satisfying, $[S_S^\mu, S_S^\nu] = i\hbar\, \varepsilon^{\mu\nu\alpha\beta} S_{S,\alpha}\eta_\beta$. This S_S^μ is the HD spin of S and, in this sense, X_S^μ, is the **center of spin**. Decomposing J_S^μ using the HD CE position operator, Y_S^μ, would yield a different and unfamiliar 'internal' angular momentum, viz., $S_S^{\|,\mu} + (M_S c/\eta P_S) S_S^{\perp,\mu}$, where $S_S^{\|,\mu} := K_S^\mu (K_S S_S)/K_S^2$, $S_S^{\perp,\mu} := S_S^\mu - S_S^{\|,\mu}$, $K_S^\mu := P_S^\mu - \eta^\mu(\eta P_S)$ and $M_S c := [P_S^2]^{1/2}$.

2b. (from p. 11) Let the subsystem, S, be itself composed of subsystems S' and S'' and any direct interaction between them. Now lump the interaction energies, momenta, etc. with S'' to form the environment, E', of S' within S. Then $M_S c Y_S^\mu = M_{S'} c Y_{S'}^{\,\mu} + M_{E'} c Y_{E'}^{\,\mu}$. The commutator between Minkowski components of Y_S^μ is given by, $[Y_S^\mu, Y_S^\nu] = -\, i\hbar\, \varepsilon^{\mu\nu\alpha\beta}(S_S^{\|,\mu} + (M_S c/\eta P_S) S_S^{\perp,\mu})\eta_\beta / (\eta P_S)^2$. The relationship between the HD CE position and the HD NW position for S is given by, $Y_S^\mu - X_S^\mu = \varepsilon^{\mu\alpha\beta\gamma} S_{S,\alpha} K_{S,\beta}\eta_\gamma / [\eta P_S(\eta P_S + M_S c)]$.

3. (from p. 20) The Fourier transform of a function, f(x), of bounded support on an η-hyperplane enjoys certain analyticity properties in the complex momentum plane. Application of the operator R_η to such a function destroys the analyticity properties of the Fourier transform due to the branch points at $k = \underline{+}\, i\kappa$ in $[k^2 + \kappa^2]^{1/2}$. In general, at least one of f(x) and R_η f(x) must have unbounded support over the hyperplane.

4. (from pp. 22, 23) Lest the reader think I am embracing here an outmoded conception of property possession representable only by projection valued measures (PVM), let me deflect that concern. Even in the case of properties represented by the more general positive operator valued measures (POVM), <u>definite possession</u> of a property corresponds to the expectation value of the associated positive operator being unity, in which case the state is an <u>eigenstate</u> of the positive operator with <u>eigenvalue</u> unity. The preservation of this feature under arbitrary linear superposition of such states holds for positive operators in POVMs as well as for projection operators.

5. (from p. 26) Using the notation of notes 2a and 2b, we have $Y_S^\mu := (\eta P_S)^{-1}:(M_S^{\mu\nu}\eta_\nu + \tau P_S^\mu)$, where the colon on the right hand side indicates a symmetrized product. The corresponding expression for X_S^μ is then obtained from the relationships in notes 2a and 2b.

6. (from p. 32) Effects are generalizations of projection operators used to represent measurements that may not yield definite yes/no answers to measurement questions. They are positive operators belonging to families of such which upon summing over all members of a family yield the identity operator. The family members need not commute.

7. (from p. 36) My discussion is narrower than Wallace's in that I confine myself to free fields rather than just linear fields.

8. (from p. 37) Wallace's single quanton states associated with specified classical data are the same states Halvorson referred to in his proposed 'standard localization' scheme that I discussed in **§4**. Unlike Halvorson, however, Wallace never suggests that the support of the classical data strictly limits the domain of localization. Thus my criticism of Halvorson's 'standard localization', on p. 21, does not apply to Wallace's account.

9. (from p. 38) Among several calculations Wallace shows that the expectation value of the field theory energy density is never more sharply confined than via exponential damping on the scale of the Compton wavelength of the quanton, regardless of the classical data. This is very close in spirit to my discussion below in **§7**.

10. (from p. 39) In this section I will use parentheses rather than angle brackets to denote, with emphasis, kets, $| \Psi)$, and bras, $(\Psi |$, of finite norm.

11. (from p. 41) This expression for the inner product can be obtained from the Fourier representation, $< x \mid y > = (2\pi)^{-3}\int[d^4k\theta(\eta k)\delta(k^2 - \kappa^2)/2\eta k]\, e^{ik(y-x)}$, by the use of analytic continuation and contour integration in the complex momentum plane.

12. (from p. 42) This expression for the inner product can be obtained from the Fourier representation, $<\eta, y; NW \mid x > = (2\pi)^{-3}\int[d^4k\, \theta(\eta k)\delta(k^2 - \kappa^2)/(2\eta k)^{1/2}]\, e^{ik(x-y)}$, by analytic continuation and contour integration in the complex momentum plane.

13. (from p. 47) The notation from this point on is complicated by the addition of the symbol for rest mass, κ, appearing in the state vectors. This is required, not because scaling or 'squeezing' the state function changes the rest mass, which is absurd, but because the comparison obtained is between scaled states of quantons with one rest mass and unscaled states of quantons with a scaled rest mass.

References: (Apologies to readers of my (2003b) for the oversight of failing to list there several references referred to in the text of that paper. All the omitted references can be found here.)

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