Embedding Fundamental Aspects of the Relational Blockworld Interpretation in Geometric (or Clifford) Algebra

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Abstract

I summarize Silberstein, *et. al*'s (2006) discussion of the derivation of the Heisenberg commutators, whose work is based on Kaiser (1981, 1990) and Bohr, *et. al.* (1995, 2004*a*,*b*). I argue that Bohr and Kaiser's treatment is not geometric *enough,* as it still relies on some unexplained residual notions concerning the unitary representation of transformations in a Hilbert space. This calls for a more consistent characterization of the role of *i* than standard QM can offer. I summarize David Hestenes' (1985,1986) major claims concerning the essential role Clifford algebras play in such a fundamental characterization of *i*, and I present a Clifford-algebraic derivation of the Heisenberg commutation relations (taken from Finkelstein, *et. al.* (2001)). I argue that their derivation exhibits a more fundamentally geometrical approach, which unifies geometric and ontological content. I also point out how some of Finkelstein's ontological notions of "chronon dynamics" can give a plausible explanatory account of RBW's "geometric relations."

I. Introduction

Silbertein, *et. al.* (2006) refer to the work of Bohr, *et. al.* (1995, 2004*a*,*b*) and Kaiser (1981, 1990) to show that the fundamental Heisenberg commutation relations[2] of non-relativistic QM (NRQM) 'reside' in a 'Kaiser space' or a 'weakly relativistic' spacetime geometry, wherein the relativity of simultaneity still holds, but $\gamma = 1$ (no time dilation or length contraction). This result obtains from the $c \rightarrow \infty$ limit of the Lie Algebra of the Poincare' group: (for translations[3] X_i and boosts K_i):

$$\left[X_i, K_j\right] = -ic^{-2}\delta_{ij} X_0$$
$$P_0 \equiv \hbar X_0 \Rightarrow \lim_{c\rightarrow\infty}\left[X_i, K_j\right] = -i\hbar^{-1}\delta_{ij} M$$

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[2] I.e., the defining 'product' $[\, p_i, x_j\,] = -i(\,{}^{h}/_{2\pi})\delta_{ij}$ of the Heisenberg Algebra $\mathscr{H}$ (which is a Lie algebra, as the product obeys the Jacobi Identity: $\forall(\varsigma,\xi,\zeta) \in \mathscr{H}$: $[[\varsigma,\xi],\zeta] + [[\xi,\zeta],\varsigma] + [[\zeta,\varsigma],\xi] = 0$.

[3] X_0 is displacement in time.

Identifying $M \equiv mId$ (where Id is the identity operator and m is a scaling factor), along with:

$$P_i = \hbar X_i \qquad\qquad Q_i = \tfrac{\hbar}{m} K_i$$

Then the following result is derived:

$$\lim_{c\to\infty}\left[X_i, K_j\right] = -i\hbar^{-1}\delta_{ij} M \Leftrightarrow \lim_{c\to\infty}\left[P_i, Q_j\right] = -i\hbar\delta_{ij} Id$$

Hence, the Heisenberg commutation relations are recovered.

This geometrically-inspired[4] derivation of the Heisenberg commutation relations of NRQM serves as a useful preliminary device to characterize all fundamental quantum mechanical results and principles in terms of global and kinematic relations. This is indicative of the authors' attempt to substantiate a metaphysical commitment toward a position of "ontological structural realism[5]."(15)

II. The Derivation's Shortcomings

The derivation, however, is not geometric enough, insofar as it automatically incorporates $i = \sqrt{-1}$ into its formalism. Bohr and Kaiser begin with a *unitary* representation of infinitesimal conformal transformation of the Poincare' algebra, or Lie Algebra of the Poincare' group. Weinberg (1995) demonstrates this explicitly, for example, when he derives the fundamental commutation relations of the Poincare' algebra (results 2.4.18-2.4.24, p. 61). He begins his derivations via a unitary representation of infinitesimal Lorentz transformations:

$\Lambda^{\nu}{}_{\nu} = \delta^{\mu}{}_{\nu} + \omega^{\mu}{}_{\nu}, a^{\mu} = \varepsilon^{\mu}$ (where ω, ε are infinitesimal 4-vectors.[6])

So right from the start, unitary representations in Hilbert space essentially constitute some of the derivations of Kaiser and Bohr. But the whole point for an RBW interpretation of QM has to

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[4] As opposed to those driven more by the specific operator-algebraic methods in Hilbert space, loosely based on the Bohr Correspondence Principle. (e.g., Sakurai (1985), 45-47)

[5] "[M]anifestations of spacetime relations distributed among and compos[e] the elements of the experimental configuration per the spacetime symmetries. Such acausal, global determination relations do not respect any (past or future) common cause principle. This fact should not bother anyone who has truly transcended the idea that the dynamical or causal perspective is the most fundamental one." (6-7) In other words, ontological structural realism is pitted against the causal/dynamical view. The latter is guilty of the matter/geometry dualism (35) which RBW transcends in a manner analogous to the reduction of dynamical effects (e.g. the gravity force) within the background of a constant spacetime (in pre-relativistic physics) to purely geometric effects a' la general relativity.

[6] These relations are the infinitesimal versions of the conformal transformations $x'^{\mu} = \Lambda^{\mu}{}_{\nu} + a^{\mu}$ (Weinberg (1995), 56)

do with avoiding any realist commitments to the Hilbert space, opting instead for "spatiotemporal relations provid[ing] the ontological basis for our principle geometric interpretation of quantum theory" (Silberstein *et. al.* (2006) 32). "[E]verything 'transpires' or rather resides in a 4D spacetime…nonetheless, some phenomena, namely quantum phenomena, cannot be modeled with worldlines if one is to do justice to its non-commutative structure." (ibid., 39)

Another way to phrase this shortcoming is that nowhere in such a derivation of the Heisenberg algebra is any account made of the profound *disanalogy* between the symplectic geometry of classical mechanical canonical transformations characterized by the Poisson bracket on the one hand, versus the associated quantum mechanical Heisenberg commutation relations on the other. The central issue is *not* that the commutator of *p* and *x* is non-vanishing in the quantum-mechanical case,[7] it is rather to do with the appearance of *i*:

$$[u,v]_{CM} \leftrightarrow \tfrac{1}{i\hbar}[\hat{u},\hat{v}]_{QM}$$

Granted, the Copenhagen-inspired presentations of QM gloss over this disanalogy completely, invoking for instance Schwinger's maxim concerning formally re-casting classical variables in canonical QM, even if the relation is understood to be purely formal. J.J. Sakurai ((1985), 45-47) for instance derives the Heisenberg commutation relations mainly by straddling the interpretative fences between QM and CM. On the one hand, he presents an operator representing infinitesimal spatial translations dx : $T(dx) = Id - iK\bullet dx$ such that $T(dx)$ is sufficient for the satisfaction of the following NRQM stipulations for operators representing observable processes:[8]

1.) Unicity: $T^{\dagger}(dx)T(dx) = Id$
2.) Additivity: $T(dx)T(dx') = T(dx + dx')$
3.) Invertibility: $T(-dx) = T^{-1}(dx)$
4.) Continuity: $\lim_{dx \to 0} T(dx) = Id$

[7] In the case of the classical Poisson bracket (a bilinear form) of two functions *u*, *v* defined for canonical variables (*q*, *p*): $[u,v]_{(q,p)} = \sum_{i=1}^{N}\left(\frac{\partial u}{\partial q_i}\frac{\partial v}{\partial p_i} - \frac{\partial u}{\partial p_i}\frac{\partial v}{\partial q_i}\right)$. So, for instance: $[q_i, p_j]_{(q,p)} = \delta_{ij}$ is therefore likewise non-vanishing.

[8] Derivable, of course, from von Neumann's axioms of QM.

On the other hand, the characterization of $T(dx) = Id - iK\bullet dx$ is "justified" via Schwinger's prescription: The "formal analogy" is evidenced via the symplectic[9] approach to canonical transformations (Goldstein (1980), 378-437). The generating function for infinitesimal spatial translations:

$$x' \rightarrow x + dx \qquad p' \rightarrow p$$

is of course: $F_1(x, p') = x \cdot p' + p \cdot dx,$ whereas the generating function for the identity transformation:

$$x' \rightarrow x \qquad p' \rightarrow p$$

is: $F_{\mathrm{Id}}(x, p') = x \cdot p'$.

So then: $F_1(x, p') = F_{\mathrm{Id}}(x, p') + p \cdot dx.$ Now in the standard Hilbert space formalism of QM, of course, the trivial identification $F_{\mathrm{Id}}(x, p') \leftrightarrow Id$. So then, it's a seemingly small step to make the identification: $p \cdot dx \leftrightarrow - iK\bullet dx$.

However this begs the question: wherefore *i*? Hestenes (1985,1986) and others (Conte (1993-2000), Finkelstein (1996-2004)) argue that the distinction between CM and QM signaled by the appearance of *i* is fundamentally based on the *geometric* distinction between a symplectic and a Clifford structure.

It is important at this stage to mention in unambiguous terms Hestenes' (and other researchers') worries concerning the uninterpreted or ambiguous role that *i* plays in standard QM Writes Hestenes:

> [In standard QM] [o]ne can distinguish three fundamentally different geometric roles tacitly assigned to the unit imaginary $i = \sqrt{-1}$, namely:

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[9] Algebraically speaking, matrix *M* obeys a *symplectic* condition if: $MJ\tilde{M} = J$ (where: $\tilde{M}$ is M's transpose, and *J* is any antisymmetric matrix of the form: $J = \begin{bmatrix} 0 & Id \\ -Id & 0 \end{bmatrix}$, where 0 and *Id* are the *n*-dimensional zero and identity matrices, respectively (if *J* is 2*n*-dimensional). So, for example, the Poisson brackets has a typical symplectic structure: $[u, v]_\eta = \frac{\partial \tilde{u}}{\partial \eta} J \frac{\partial v}{\partial \eta}$ for (2*n* dimensional row vector $\eta = (q,p)$ and $\frac{\partial \tilde{u}}{\partial \eta}, \frac{\partial v}{\partial \eta}$ are the associated (transposed and nontransposed) transformation matrices with respect to functions *u* and *v*.) If the transformation: $\eta = (q,p) \rightarrow \zeta = (q',p')$ is canonical, then the transformation matrices are sympletic, and the Poisson brackets in this case simplify to: $[\zeta, \zeta]_\eta = \frac{\partial \tilde{\zeta}}{\partial \eta} J \frac{\partial \zeta}{\partial \eta} = J$ (Goldstein (1980), ch.9)

(1) the generator of rotations in a plane [$\tilde{\varphi}(x,t) \mapsto e^{i\theta}\varphi(x,t)$]

(2) the generator of duality transformations [$\psi^{\uparrow}{}_{B} = \left(M_{AB}\psi^{A}\right)^{*}$ [10]]

(3) the indicator of an indefinite metric [$\psi^{\uparrow}\varphi = \left(\psi^{A} M_{AB} \psi^{B}\right)^{*}$, an indefinite sesquilinear form[11]]

> Confusion is difficult to avoid when *i* is required to perform more that one of these roles in a single system. Worse yet, in physics all three possibilities are sometimes realized in a single theory confounded with problems of physical interpretation…[t]he multiplicity of geometric interpretations shows that conventional mathematical formalisms are profoundly deficient in their tacit assumption that there is a unique 'imaginary unit'. Therefore, in the interest of fidelity to geometric interpretation, the convention that complex numbers are scalars should be abandoned in favor of…a system in which each basic geometric distinction has a unique algebraic representation. Geometric [Clifford] Algebra has this property. (1984, xii – xiii).

So, for instance, Kaiser and Bohr, as well as Sakurai, import a notion of *i* as essential for the definition of unicity, which of course must respect the constraints of the duality (or standard adjoint) transformation in a Hilbert space.[12]

III. The Remedy: Deriving the Heisenberg Algebra from an Underlying Clifford Algebra

Finkelstein *et. al.* (2001) however have derived the Heisenberg commutation relations as an algebraic contraction of their Clifford-algebraic characterization of quantum fields on a quantum space-time:

> We hypothesize that the dynamics of a suitably isolated physical system [consists of]…elementary dynamical processes or chronons[13], and that the ambient vacuum breaks its Clifford algebra $\mathscr{C}$ down into…mutually commuting local…Clifford algebras…We construct a simple finite-dimensional Clifford algebra $\mathscr{C} = \mathscr{A}$ that approaches or 'contracts

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[10] "For complex action vectors, ↑ can be expressed as ↑ = M , where is **complex conjugation** and M is a hermitian symmetric form." (Finkelstein (1996), 30.)

[11] This is the definition of the metric ascribed to a *Dirac-space* , which can admit vector space decomposition: $_{U}\oplus$ $_{-}$ in which the Hilbert spaces $_{U}$Ư $_{-}$ are endowed with isometric (norm-preseerving) and anti-isomeric (**reversing the sign of the norm**) projections $_{U}$Ư $_{-}$. (Finkelstein (1996), 552)

[12] Explicit mention above was already made in the case of J.J. Sakurai, with respect to the infinitesimal translation operator *T*(*dx*). In the case of Bohr, Kaiser, etc., the commutation relations for the Poincare' algebra arise fundamentally form the (unitary) representation of infinitesimal conformal transformations: $U(Id + \omega, \varepsilon) = Id + \frac{1}{2} i\omega_{\mu\nu} J^{\mu\nu} - i\varepsilon_{\rho} P^{\rho} + \ldots$ (according to Weinberg ((1995),59).), where J and P are Hermitian, and ω, ε are infinitesimal elements comprising the conformal transformations (see page 3 above) So basically this QFT approach, motivated essentially by perturbation series methods, is essentially no different from the "linear" depictions of *T*(*dx*) according to J.J. Sakurai in NRQM.

[13] "Chronons and the basic Clifford variables…that represent them are prelocal in the extreme, since they all antimcommute. Nevertheless they are the raw material of the universe, we propose." (ibid)

to' the Minkowski manifold algebra… $\mathscr{A}$ may be regarded as a generalization of the Heisenberg algebra of x and p. (Finkelstein *et. al.*, 2001, p. 1496)

In their paper, Finkelstein *et. al* selected to characterize their theory[14] with Clifford algebra primarily because they argue that typically abstract (adjoint-based) algebraic characterizations of quantum dynamics (whether C^*, Heisenberg, etc.) preclude the possibility of a description of a fully quantum mechanical space-time fine structure.[15] A Clifford algebra, on the other hand, can express a quantum space-time, as well as a quantum dynamics. (2001, 1494).

This is a case of an attempted ontological and geometrical unification. The Clifford algebra supports a description of a fully quantum spacetime, in addition to a supervening quantum dynamics. This is an example of the collapse of the matter (and its associated dynamical relations) / geometry dualism, as described in Silberstein *et. al* ((2006), 35).

The "prime variable," in other words, is not the space-time *field*, as Einstein stipulated, but rather the *dynamical law.* "The dynamical law [is] the only dependent variable, on which all others depend."[16] (2001, 6) The "atomic" quantum dynamical unit (represented by a generator γ^{α} of a Clifford algebra) is the *chronon* χ, with a closest classical analogue being the tangent or cotangent vector, (forming an 8-dimensional manifold) and *not* the space-time point (forming a 4-dimensional manifold)[17].

Prima facie this notion of a fundamental 'dynamical' law may appear metaphysically opposed to a fundamentally 'kinematic' commitment toward "a radical ontology of spacetime relations" (Silberstein (2006), 15). However, this 'fundamental dynamic' shares much conceptual common ground with RBW's hypothesized "comprehensive, fundamental spatiotemporal 'extremum' principle Д." (19) The extremum principle Д is reminiscent of the analogy with the *Hilbert action* concerning the fundamental dynamical law discussed by Finkelstein in his 1999 paper (entitled "Matter-space-time-energy-dynamic."[18])

[14] Erstwhile known as quantum network dynamics or QND in (1996).
[15] The space-time structure must are supplied by classical structures, prior to the definition of the dynamical algebra. (2001, 1494)
[16] This comprises a general and central notion in Finkelstein's research, reminiscent in my opinion with the 'ontological structural realism' of RBW. Finkelstein (1996, chs. 1-4) contrasts "praxism," with "ontism." A praxic characterization begins with a (kinematic) notion of the pattern of elementary actions and some dynamical law, and reveals that the epistemic notion of "state" is derivative. "Ontism" works in the opposite direction, taking state as the primitive and deriving elementary dynamics in terms of mappings between states. The 'praxic' metaphysical position appears to share some important resonances with the RBW approach, insofar as *patterns of relations* are assumed to act as explanatory primitives.
[17] The authors show that Clifford statistics for chronons adequately expresses the distinguishability of events as well as the existence of half-integer spin. (2001, 1496)
[18] In the case of general relativity (GR), when varying the Hilbert action, and optimizing in a standard variational approach, one discovers that the extremum principle 'regulates its own regulation.' That is to

I sketched out some conceptual issues that RBW share with Finkelstein *et. al* (1996-2004). Here I summarize some of the relevant results Finkelstein *et. al*. (2001) derived, concerning the Heisenberg algebra: Decomposing the Clifford algebra $\mathscr{C}$ (describing the algebra of all possible transformations on any subsystem Ω of the universe, however "simple"—like in the case of a region of the vacuum , or complex—like in the case of particle(s)) into *N* mutually commuting local Clifford algebras[19] generated by local variables $\gamma_\mu(n)$ the following local anticommutation relations are obtained:

$$[\gamma_\mu(n), \gamma_\nu(m)]_+ = t_{\mu\nu}\delta_{nm}$$

The form: $t_{\mu\nu} = \left(\frac{(N+1)\delta_{\mu\nu} - 2}{(N-1)}\right)$ is interpreted as the inner product between two unit vectors from the center of a *N*-1 simplex to its *N* vertices.[20] Infinitesimal elements $\delta x^\mu = 2^{-\frac{1}{2}}\gamma^\mu, \delta p_\mu = 2^{-\frac{1}{2}}\gamma^\uparrow\gamma_\mu, \delta i = \gamma^\uparrow$ are defined for the respective position, momenta, and i operators, where $\gamma^\uparrow$ represents the 'top' element, or element of maximal grade in the Clifford subalgebra.[21] Then:

1. The Heisenberg commutation relations are recovered as a contraction of the Clifford relations (expressions (24), p. 1497) as shown in Eqns. (27) (p. 1497):

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say, as the curvature of spacetime indicates the presence of mass (and vice versa), so the extremals of the Hilbert Action govern the local behavior of the metric $g_{\mu\nu}$, which in turn constitutes the Hilbert action. Finklestein (1999) extends this analogy to include the very concept of the fundamental dynamical 'law' itself. Echoing the 'Humean' notions (laws describing regularities) in Silbertein *et. al.* (2006, p. 3) Finkelstein borrows Peirce's notion laws are can 'evolve' via some reciprocal relation. A mechanism for how laws can 'evolve' via group simplification and Lie algebraic stabilization (Finkelstein (2004a)) is subsequently leant some greater mathematical clarification.

[19] *N* is the total number of degrees of freedom of the system, represented by the maximal grade of $\mathscr{C}$. It can be very large, but it's *finite.*

[20] For further details concerning this result and the motivations comprising it, see (2001), pp. 1490-1496. This approach essentially generalizes the 'hypercubical lattice' through which a QND action principle was derived in chapter 16, Finkelstein (1996).

[21] For further details concerning top elements in a Clifford algebra, see (Kallfelz (2006a), 21). The unit or psuedoscalar $\gamma^\uparrow$ should *not* be interpreted as a multiplicative identity, i.e. it is certainly *not* the case that for any Clifford element $A \in CL(V)$, $A\gamma^\uparrow = A = \gamma^\uparrow A$ (where $CL(V)$ is the Clifford algebra generated over vector space *V*.) Rather, the unit pseudoscalar is adopted to define an element of dual grade A^* : for any pure Clifford element $A_{k,}$ (where $0 \le k < N$) : the grade of $A\gamma^\uparrow$ (or A^*) is $N - k$, and vice versa. Thus an inverse element A^{-1} can in principle be constructed, for every nonzero $A \in CL(V)$. So the linear equation $AX = B$ has the formal solution $X = A^{-1}B$ in $CL(V)$. "Much of the power of geometric (Clifford) algebra lies in this property of invertibility." (Lasenby, et. al. (2000), 25)

$$\breve{x}^{\mu} = \sum_{n,\beta} \delta x^{\mu}(n,\beta) \qquad \breve{p}_{\mu} = \tfrac{1}{2N}\sum_{n,\beta} \delta p_{\mu}(n,\beta) \quad \breve{i} = \sum_{n,\beta} \delta i(n,\beta)$$

where the breve superscript on the variables above represent their (algebraically contracted) form. Hence: $[\breve{x}^{\mu}, \breve{p}_{\nu}] = \breve{i}\, \delta^{\mu}{}_{\nu}$.[22]

2. Moreover, the first expression (26): $[\delta x^{\mu}, \delta p_{\nu}] = \delta i \delta^{\mu}{}_{\nu}$ expands the notion of i, as Hestenes proposed, in terms of "a system in which each basic geometric distinction has a unique algebraic representation. Geometric [Clifford] Algebra has this property." (1984, xii – xiii).

3. The last two relations of expression (26): $[\delta i, \delta p_{\nu}] = -2\delta x_{\nu}$, $[\delta i, \delta x^{\mu}] = 2\delta p^{\mu}$ indicate the 'expanded i' with infinitesimal representation $\delta i = \gamma^{\uparrow}$ in terms of the unit pseudoscalar or maximal grade element of the Clifford algebra "generates the symplectic symmetry between x and p." (ibid., 1497)

IV. Conclusion

So here is an instance in which a Clifford-algebraic characterization can account for the Heisenberg commutation relations of NRQM (as 1. above describes) in a far more fundamental manner than in Bohr's or Kaiser's approach. An explicitly *geometric* characterization of the role played by i is produced, as opposed to being merely buried in the unitary representations of infinitesimal transformations in the Poincare' algebra (in the case of Kaiser) or inserted by hand in the heuristic correspondence with symplectic transformations (in the case of Sakurai). Moreover, (as 3. above shows) the *symplectic* structure is likewise preserved and demonstrated as constitutive of this Clifford algebraic characterization. This comprises a clear-cut instance of a unification of *geometric* content.

Moreover, the issue of *ontological* unification is suggested by some of the remarks I made above regarding the unification of quantum dynamics and spacetime structure as indicative of the collapse of the matter-geometry dualism. The ontological primitives in Finkelstein's theory are the 'chronons,' or elementary 'units' or quantum processes at the Planck scale. Is there any ontological connection in the form of a simplification or unification with the vacuum chronon 'dynamics' of Finkelstein's theory and the 'pattern of spacetime relations' of RBW? I believe a key connecting concept is that of *information.*[23] In my forthcoming studies I intend to develop

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[22] 'Natural' units are assumed throughout, i.e.: $h/2\pi = c = 1$.

[23] For example, H.S. Green (2000) presents a unified theory, comprising all areas of field theory and gravitation based fundamentally on the notion of a qubit. See Kallfelz (2005a) for more details.

and refine this connection—my sense is that the spacetime relations of RBW are derived as 'contraction' or 'condensation' asymptotic limits of some underlying Clifford-based statistics comprising such entities (chronons) an experimental region.[24]

V. References:

Bohr, A. and Ulfbeck, O. (1995). *Reviews of Modern Physics*, **67**, 1-35.

Bohr, A., Mottelson, B.R. and Ulfbeck, O. (2004a). *Foundations of Physics*, **34**, 405-417.

Bohr, A., Mottelson, B.R. and Ulfbeck, O. (2004b). *Physics Today*, **57**, 15-16.

Conte, Elio (1993). "An Example of Wave Packet Reduction Using Biquaternions." *Physics Essays,* vol. 6, n 4, 532-535.

Conte, Elio (1994). "Wave Function Collapse in Biquaternion Quantum Mechanics." *Physics Essays,* vol. 7, n 4, 429-435.

Conte, Elio (1995). "A Generalization of Schroedinger's Equation Using Biquaternions: The Possibility of Fusion for Particles." *Physics Essays,* vol. 8, n 1, 52-59.

Conte, Elio (1996). "A Criticism of Fine's Solution to the Quantum Measurement Problem: A New Epistemology for Quantum Mechanics." *Physics Essays,* vol. 9, n 1, 141-147.

Conte, Elio (2000). "Biquaternion Quantum Mechanics." (book synopsis) *Advances in Applied Clifford Algebras*, v.10 n.2, 298-299.

Finkelstein, David (1996). *Quantum Relativity: A Synthesis of the Ideas of Einstein and Heisenberg*. Berlin: Springer-Verlag.

Finkelstein, David & Galiautdinov, Andrej (2000) *Clifford Statistics* http://ariXiv:hep-th/0005039v1_4May_2000

Finkelstein et al., (2001) Clifford Algebra as Quantum Language, *J. Math. Phys* 42, 1489. http://www.physics.gatech.edu/people/faculty/finkelstein/CliffLang2001.pdf

Finkelstein, David (2002) *Post Quantum Relativity* (manuscript), School of Physics, Georgia Institute of Technology, Atlanta, Georgia.

Finkelstein, David (2004a) *Finite Quantum Relativity*, URL=http://www.physics.gatech.edu/people/faculty/finkelstein/FQR02.pdf

[24] Reminiscent of Green, Finkelstein (2001) derives an expression of the metric $\gamma^{\mu}(x) \circ \gamma^{\nu}(x) = g^{\mu\nu}(x)$ of the space-time manifold in a singular limit of the Clifford algebra representing the global dynamics of the chronons in an experimental region.

Finkelstein et al.,(2004b) *Quantum Binary Gravity*
URL=http://www.physics.gatech.edu/people/faculty/finkelstein/QBGravity031215.pdf

Finkelstein and Shiri-Garakani (2004c) *Finite Quantum Harmonic Oscillator*
URL=http://www.physics.gatech.edu/people/faculty/finkelstein/FHO0410082.pdf

Goldstein, Herbert (1980). *Classical Mechanics* (2nd edn.) Reading, MA: Addison-Wesley Co.

Green, H. S. (2000) *Information Theory and Quantum Physics: Physical Foundations for Understanding the Conscious Process,* Berlin: Springer-Verlag.

Hestenes and Sobczyk (1984) *Clifford Algebras to Geometric Calculus: A Unified Language for Mathematics and Physics*, (Fundamental Theories of Physics), Dordrecht: D. Reidel.

Hestenes, David. (1985) *Found. Phys.* 15, n63.

Hestenes, David (1986). "Clifford Algebra and the Interpretation of Quantum Mechanics," in Chisholm, J. S. R. & Commons, A. K. (eds). *Clifford Algebras and their Applications in Mathematical Physics.* Dordrecht/Boston: D. Reidel, 321-346.

Hestenes, David (2003). "Reforming the mathematical language of physics." (Oersted Medal Lecture 2002). *Am. J. Phys.* **71** (2) Feb 2003.

Kaiser, Gerald (1981), "Phase-space approach to relativistic quantum mechanics. III. Quantization, relativity, localization and gauge freedom", *Journal of Mathematical Physics* 22 (4): 705 – 714.

Kaiser, Gerald (1990), *Quantum Mechanics, Relativity, and Complex Spacetime: Towards a New Synthesis*. Amsterdam: North-Holland Press. pp. 90-98.

Kallfelz, William (2005b) "Contracting Batterman's 'No-Man's Land:' Reduction Rejoins Explanation" *Philosophy of Science Archives: Physics*
http://philsci-archive.pitt.edu/archive/00002402/

Kallfelz, William (2005c) "Getting Something Out of Nothing: Implications for a Future Information Theory Based on Vacuum Microtopology," *Proceedings of International Association of Nanotechnology*, held in San Francisco, Oct. 28 – Nov. 1, 2005.
http://www.ianano.org

Kallfelz, William (2006a) "Methodological Fundamentalism: Why Batterman's Different Notions of Fundamentalism Don't Make a Difference," *Philosophy of Science Archives: Physics*
http://philsci-archive.pitt.edu/archive/00002402/

Lasenby, J., Lasenby, A., Doran, C. (2000) "A Unified Mathematical Language for Physics and Engineering in the 21st Century," *Phil. Trans. R. Soc. Lond.* A, 358, 21-39.

Sakurai, JJ (1985) *Modern Quantum Mechanics*. Addison-Wesley.

Silberstein, M., Stuckey, W. M., Cifone, M. (2007) "Relational Blockworld: Radically Archimedean Physics," http://philsci-archive.pitt.edu/archive/00003213/

Weinberg, Steven (1995) *The Quantum Theory of Fields* (vol. 1) Cambridge U. Press.