Absolute Objects, Counterexamples and General Covariance

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Abstract

The Anderson-Friedman absolute objects program has been a favorite analysis of the substantive general covariance that supposedly characterizes Einstein's General Theory of Relativity (GTR). Absolute objects are the same locally in all models (modulo gauge freedom). Substantive general covariance is the lack of absolute objects. Several counterexamples have been proposed, however, including the Jones-Geroch dust

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and Torretti constant curvature spaces counterexamples. The Jones-Geroch dust case, ostensibly a false positive, is resolved by noting that holes in the dust in some models ensure that no physically relevant nonvanishing timelike vector field exists there, so no absolute object exists. The Torretti constant curvature spaces case, allegedly a false negative, is resolved by testing an irreducible piece of the metric, the conformal metric density of weight -2/3, for absoluteness; this geometric object is absolute. A new counterexample is proposed involving the orthonormal tetrad said to be necessary to couple spinors to a curved metric. The threat of finding an absolute object in GTR + spinors is overcome by the use of an alternative spinor formalism that takes a symmetric square root of the metric (with the help of the matrix diag(-1,1,1,1)), eliminating 6 of the 16 tetrad components as irrelevant. The importance of eliminating irrelevant structures, as Anderson emphasized, is clear. The importance of the choice of physical fields is also evident. A new counterexample due to Robert Geroch and Domenico Giulini, however, finds an absolute object in vacuum GTR itself, namely the scalar density g given by the metric components' determinant. Thus either the definition of absoluteness or its use to analyze GTR's substantive general covariance is flawed. Anderson's belief that all absolute objects are nonvariational (that is, not varied in a suitable action principle) and *vice versa* is also falsified by the Geroch-Giulini counterexample. However, it remains plausible that all nonvariational fields are absolute, so adding nonvariationality as a necessary condition for absoluteness, as Hiskes once suggested, would likely leave no useful work to the Anderson-Friedman condition of sameness in all models. Simply having only variational fields in an action principle (suitably free of irrelevant fields) might be a satisfactory analysis of substantive general covariance, if one exists. This proposal also resembles the suggestion that GTR is "already parameterized," if one decides to parameterize theories by defining the nonvariational fields in terms of preferred coordinates called clock fields. More questions need to be addressed. Which fields should be tested for absoluteness: only primitive fields (which ones?), or all or some (which?) of their concomitants also? Geroch observes that some kinds of geometric objects, such as tangent vectors, scalar densities, and tangent vector densities of non-unit weight, satisfy the condition of sameness in all models if they merely fail to vanish. If these "susceptible" geometric objects can hardly help being absolute, to what degree are they, or the theories harboring them, responsible for this absoluteness? The answer to this question helps to determine the significance of the Geroch-Giulini counterexample.

1 Introduction

James L. Anderson analyzed the novelty of Einstein's so-called General Theory of Relativity (GTR) as its lacking "absolute objects" (Anderson, 1967; Anderson, 1971). Metaphorically, absolute objects are often described as a fixed stage on which the dynamical actors play their parts. A review of Anderson's definitions will be useful.

Absolute objects are to be contrasted with dynamical objects. The values of the absolute objects do not depend on the values of the dynamical objects, but the values of the dynamical objects do depend on the values of the absolute objects (Anderson, 1967, p. 83). Both absolute objects and dynamical objects are, mathematically speaking, geometric objects or parts thereof. Trautman defines geometric objects as follows:

> Let X be an n-dimensional differentiable manifold....
>
> Let $p \in X$ be an arbitrary point of X and let $\{x^a\}, \{x^{a'}\}$ be two systems of local coordinates around p. A geometric object field y is a correspondence
>
> $$y : (p, \{x^a\}) \rightarrow (y_1, y_2, \cdots y_N) \in R^N$$
>
> which associates with every point $p \in X$ and every system of local coordinates $\{x^a\}$ around p, a set of N real numbers, together with a rule which determines $(y_{1'}, \cdots y_{N'})$, given by
>
> $$y : (p, \{x^{a'}\}) \rightarrow (y_{1'}, \cdots y_{N'}) \in R^N$$
>
> in terms of the $(y_1, y_2, \cdots y_N)$ and the values of [*sic*] p of the functions and their partial derivatives which relate the coordinate systems $\{x^a\}$ and $\{x^{a'}\}$.... The N numbers $(y_1, \cdots y_N)$ are called the components of y at p with respect to the coordinates $\{x^a\}$. (Trautman, 1965, pp. 84, 85)

Geometric objects were considered with great thoroughness by Albert Nijenhuis (Nijenhuis, 1952) and by Kucharzewski and Kuczma (Kucharzewski and Kuczma, 1964).

Before absolute objects can be defined, the notion of a covariance group must be outlined. Here it will prove helpful to draw upon the unjustly neglected work of Kip Thorne, Alan Lightman, and David Lee (TLL) (Thorne et al., 1973); a useful companion paper (LLN) was written by Lee, Lightman and W.-T. Ni (Lee et al., 1974). According to TLL,

> A group $\mathcal{G}$ is a covariance group of a representation if (i) $\mathcal{G}$ maps [kinematically possible trajectories] of that representation into [kinematically possible trajectories]; (ii) the [kinematically possible trajectories] constitute "the basis of a faithful representation of $\mathcal{G}$" (i.e., no two elements of $\mathcal{G}$ produce identical mappings of the [kinematically possible trajectories]); (iii) $\mathcal{G}$ maps [dynamically possible trajectories] into [dynamically possible trajectories]. (Thorne et al., 1973, p. 3567)

One can now define absolute objects. They are, according to Anderson, objects with components ϕ_α such that

> (1) The ϕ_α constitute the basis of a faithful realization of the covariance group of the theory. (2) Any ϕ_α that satisfies the equations of motion of the theory appears, together with all its transforms under the covariance group, in every equivalence class of [dynamically possible trajectories]. (Anderson, 1967, p. 83)

Thus the components of the absolute objects are the same, up to equivalence under the covariance group, in every model of the theory. It is the dynamical objects that distinguish the different equivalence classes of the dynamically possible trajectories (Anderson, 1967, p. 84).

It has been asserted that there is a sense in which GTR is nontrivially or strongly generally covariant, and that this is its lack of absolute objects (Anderson, 1967) or "prior geometry" (Misner et al., 1973, pp. 429-431). John Norton discusses this claim with some sympathy (Norton, 1992; Norton, 1993; Norton, 1995), though technical problems such as the Jones-Geroch dust and Torretti constant spatial curvature counterexamples are among his worries (Norton, 1993; Norton, 1995). Anderson and Ronald Gautreau encapsulate the definition of an absolute object as an object that "affects the behavior of other objects but is not affected by these objects in turn." (Anderson and Gautreau, 1969, p. 1657) Anderson claims that absolute objects violate what he calls a "generalized principle of action and reaction" (Anderson, 1967, p. 339) (Anderson, 1971, p. 169). Norton has argued, rightly I think, that such a principle is hopelessly vague and arbitrary and that it should not be invoked to impart a spurious necessity to the contingent truth that our best current physical theory lacks them (Norton, 1993, pp. 848, 849)—except that the scalar density counterexample will show that our best current physical theory *has* one!

In Anderson's framework, an important subgroup of a theory's covariance group is its symmetry group (Anderson, 1967, pp. 84-88). One first defines the symmetry group of a *geometrical object* as those transformations that leave the object unchanged. The symmetry group of a physical theory is

> the largest subgroup of the covariance group of this theory, which is simultaneously the symmetry group of its absolute objects. In particular, if the theory has no absolute objects, then the symmetry group of the physical system under consideration is just the covariance group of this theory. (Anderson, 1967, p. 87)

Thus having fewer absolute objects leaves a larger symmetry group.

Finding Anderson's definition obscure, Michael Friedman amended it in the interest of clarity (Friedman, 1973; Friedman, 1983). As it turns out, Friedman has made a number of changes to Anderson's definitions, not all for the better. First, Friedman's equivalence relation, which he calls *d*-equivalence, comprises only diffeomorphism freedom (Friedman, 1983, pp. 58-60), not other kinds of gauge freedom such as "internal groups" (Anderson, 1967, pp. 35, 36) like local Lorentz freedom or electromagnetic or Yang-Mills gauge freedom, or combined internal-external supersymmetry transformations. Second, Friedman's mathematical language is less general than Anderson's and fails to accommodate some useful mathematical entities that Anderson's permits. Anderson knows what sorts of mathematical structures physicists need, while

Friedman restricts his attention to that narrower collection of entities that all modern coordinate-free treatments of gravitation or (pseudo-)Riemannian geometry presently discuss, namely tensors and connections, but not, for example, tensor densities, which are important for two examples below. Tensor densities, even of fractional or irrational weight, are useful or crucial in a variety of applications, including the modern canonical quantum gravity project, the conformal-traceless decomposition of the spatial metric in numerical work in general relativity, and massive theories of gravity. Accidentally restricting one's vocabulary in this way also prevents one from using irreducible geometric objects, thus dooming one to wrong answers for the Torretti and Geroch-Giulini cases. Friedman's mathematical language also excludes spinors, whether of the usual orthonormal tetrad formalism or the less common formalism of V. I. Ogievetskiĭ and I. V. Polubarinov (Ogievetskiĭ and Polubarinov, 1965), to be discussed below. A third difference pertains to the notion of standard formulations of a theory. Anderson (somewhat confusingly) and TLL require that theories should be coordinate-covariant under arbitrary manifold mappings. Friedman, by contrast, takes as standard a form in which the absolute objects, if possible, have *constant components* (Friedman, 1983, p. 60). Friedman implies that one can always choose coordinates such that the absolute objects (a) have constant components and (b) thus drop out of the theory's differential equations. However, these claims both suffer from counterexamples. Concerning (a), (anti-)de Sitter background metrics of a single value of curvature are absolute but do not have constant components. Concerning (b), absolute objects can appear algebraically in the field equations, not just differently, so their components need not drop out even if constant (Freund et al., 1969). Thus the Thorne-Lee-Lightman fully reduced generally covariant formulation is therefore preferable to Friedman's standard formulation. Friedman's expectation that the components of absolute objects could be reduced to constants in general, though incorrect, usefully calls attention to the role (or lack thereof) of Killing vector fields and the like in analyzing absolute objects. TLL's additional category of "confined" objects is a useful supplement to geometric objects and can accommodate various structures that savor of absoluteness without satisfying a definition of absolute objects designed for geometric objects.

2 Jones-Geroch counterexample and Friedman's reply

With a clear grasp of absolute objects in hand, one can now consider the Jones-Geroch counterexample that claims that the 4-velocity of cosmic dust counts, absurdly, as an absolute object by Friedman's or Anderson's standards. Friedman concedes some force to this objection made by Robert Geroch and amplified by Roger Jones, here related by Friedman:

> ... [A]s Robert Geroch has observed, since any two timelike, nowhere-vanishing

> vector fields defined on a relativistic space-time are d-equivalent, it follows that any such vector field counts as an absolute object according to [Friedman's criterion]; and this is surely counter-intuitive. Fortunately, however, this problem does not arise in the context of any of the space-time theories I discuss. It could arise in the general relativistic theory of "dust" if we formulate the theory in terms of a quintuple $\langle M, D, g, \rho, U \rangle$, where ρ is the density of the "dust" and U is its velocity field. U is nonvanishing and thus would count as an absolute object by my definition. But here it seems more natural to formulate the theory as a quadruple $\langle M, D, g, \rho U \rangle$ where ρU is the momentum field of the "dust." Since ρU does vanish in some models, it will not be absolute. (Geroch's observation was conveyed to me by Roger Jones, who also suggested the example of the general relativistic theory of "dust."...) (Friedman, 1983, p. 59)

Friedman's response is nearly satisfactory, though it has two weaknesses as he expressed it. I will discuss the more serious one. He states that ρU, the mass density times the 4-velocity, does vanish in some models, but he should have said that "ρU does vanish in some neighborhoods in some models" to show that he is considering only genuine models of GTR + dust, in which dust vanishes in some neighborhoods in some models, rather than some models with (omnipresent?) dust and some degenerate models which nominally have dust but actually have no dust anywhere. Clearly some models with dust have neighborhoods lacking dust, and it is these models which will prevent the dust 4-velocity from constituting an absolute object. The Jones-Geroch counterexample fails because there is no physically meaningful everywhere (nonvanishing) timelike vector field in the set of solutions of GTR + dust, because there is none where the dust has holes in some models. Not just globally irrelevant fields, but locally irrelevant portions of fields should be excluded before testing a theory for absolute objects.

3 Hiskes's redefinition of absoluteness, Maidens's worry, and Rosen's answer in advance

To address the Jones-Geroch dust counterexample, Anne Hiskes proposed amending the definition of absolute objects so that no field varied in a theory's action principle would be regarded as absolute (Hiskes, 1984). Such a move makes use of what seemed to be a true generalization about absolute and dynamical objects to both Anderson (Anderson, 1967, pp. 88, 89) and Thorne, Lee, Lightman and Ni (Thorne et al., 1973; Lee et al., 1974). Let use call objects "(non)variational" if they are (not) varied in an action principle (Gotay et al., 2004). We have seen that Hiskes's amendment is not necessary to resolve the Jones-Geroch dust counterexample. Anna Maidens has suggested that there might be some way to reformulate *special* relativistic theories such that the flat metric, which surely ought to count as absolute, is varied in the action principle. If that could be done, then Hiskes's definition of absolute objects would prove to be too strict (the opposite problem from what the Jones-Geroch example suggests about Friedman's), because it fails to count the metric tensor of special relativity as an absolute object in some formulations. Maidens's conjecture is correct that one can derive the flatness of a metric from a variational principle with the help of Lagrange multipliers, as was shown long ago by Nathan Rosen and again more recently by Rafael Sorkin (Rosen, 1966; Rosen, 1973; Sorkin, 2002). So Hiskes's move seems unpromising. However, one might argue that the Rosen-Sorkin Lagrange multiplier fields are irrelevant fields and so should not be used. Given the qualification that irrelevant variables should be excluded, Hiskes's proposal might yet have some use in addressing other counterexamples.

Does it follow that Anderson's and others' intuition that fields are absolute iff nonvariational is vindicated? Before accepting such a claim, one must address parameterized theories (Sundermeyer, 1982; Kuchař, 1973; Arkani-Hamed et al., 2003; Norton, 2003; Earman, 2003), in which preferred coordinates are rendered variational. Because the resulting "clock fields" X^A are scalars and their gradients are linearly independent, the Noether-Bianchi identities ensure that $\frac{\delta S}{\delta X^A} = 0$ due to the other fields' Euler-Lagrange equations, even if X^A are nonvariational. If we stipulate that fields should only be varied only there is some benefit to doing so, then preferred coordinates usually should not be varied.

4 Torretti's and Norton's examples have absolute objects

A second long-standing worry concerning the Anderson-Friedman absolute objects project was suggested by Roberto Torretti (Torretti, 1984). He considered a theory of modified Newtonian kinematics in which each model's space has constant curvature, but different models have different values of that curvature. Because every model's space has constant curvature, such a theory surely has something rather like an absolute object in it, Torretti's intuition suggests. Though contrived, this example is relevantly like the cases of de Sitter or anti-de Sitter background metrics of constant curvature that are sometimes discussed in the physics literature, where one often lumps together space-times with different values of constant curvature. The failure of the metrics to be locally diffeomorphically equivalent for distinct curvature values entails that the metric tensor does not satisfy Anderson's or Friedman's definition of an absolute object (or TLL's, for that matter). But it seems intuitively clear to Torretti that his theory has an absolute object, so he infers that Friedman's analysis is wrong.

I observe that the Anderson-Friedman analysis, when applied to Torretti's example, actually does yield a very specific and reasonable conclusion involving an absolute object. Though the spatial metric is not absolute, the conformal spatial metric density, a symmetric $(0, 2)$ tensor density of weight $-\frac{2}{3}$ (or its $(2, 0)$ weight $\frac{2}{3}$ inverse) is an absolute object. This entity, when its components are expressed as a matrix, has unit determinant. It appears routinely in the conformal-traceless decomposition used in finding initial data in numerical studies of GTR. It defines angles and relative lengths of vectors at a point, but permits no comparison of lengths of vectors at different points. In three dimensions, conformal flatness of a metric is expressed by the vanishing of the Cotton tensor (Aldersley, 1979; Garcia et al., 2004), not the Weyl tensor, which vanishes identically. That the conformal metric density is an absolute object is shown in the following way. Every space with constant curvature is conformally flat (Wolf, 1967). For conformally flat spatial metrics, manifestly the conformal parts are equal in a neighborhood up to diffeomorphisms. The conformal part just is the conformal metric density. Concerning Norton's modification of Torretti's example to Robertson-Walker space-time metrics (Norton, 1993, p. 848), analogous comments could be made: these space-*times* are conformally flat (Infeld and Schild, 1945) and so have as an absolute object the space-time conformal metric density. In neither case is the conformal metric density the only absolute objects present, but it suffices to observe that it is present.

5 Tetrad-spinor: Avoiding absolute object by eliminating irrelevant fields

One potential counterexample to the Anderson-Friedman project that seems not to have been noticed until now (Pitts, 2006) arises from the use of an orthonormal tetrad formalism, in which the metric tensor (or its inverse) is built out of four orthonormal vector fields e^{μ}_{A} by the formula $g^{\mu\nu} = e^{\mu}_{A}\eta^{AB}e^{\nu}_{B}$ or the like. Four vector fields have among them 16 components, rather more than the 10 components of the metric, so there is some redundancy that leaves a new local Lorentz gauge freedom to make arbitrary position-dependent boosts and rotations of the tetrad. It is unnecessary to use a tetrad instead of a metric as the fundamental field when gravity (as described by GTR) is coupled to bosonic matter (represented by tensors, tensor densities or perhaps connections). However, it is widely believed to be necessary to use an orthonormal tetrad to couple gravity to the spinor fields that represent electrons, protons, and the like (Weinberg, 1972; Deser and Isham, 1976). The threat of a counterintuitive absolute object then arises. Given both local Lorentz and coordinate freedom, one can certainly bring the timelike tetrad leg into the component form $(1, 0, 0, 0)$ at least in a neighborhood about any point. Unlike the dust case, there cannot be any spacetime region in any model such that the timelike leg of the tetrad vanishes. Thus GTR coupled to a spinor field using an orthonormal tetrad gives an example of a Gerochian vector field: nowhere vanishing, everywhere timelike, gauge-equivalent to $(1, 0, 0, 0)$, and (allegedly) required to couple the spinor and gravity and thus not irrelevant. If it is true that coupling spinors to gravity requires an orthonormal tetrad and that an orthonormal tetrad formalism for GTR yields an absolute object, then the intuitively absurd conclusion that GTR + spinors has an absolute object follows.

The tetrad-spinor example seems rather more serious a problem for definitions of absolute objects than the Jones-Geroch cosmological dust example was, because the spinor field is surely closer to being a fundamental field than is dust or any other perfect fluid. The solution seems to be the following: one can remove irrelevant fields here and thus avoid this unexpected absolute object. This removal is achieved using the alternative spinor formalism of V. I. Ogievetskiĭ and I. V. Polubarinov (Ogievetskiĭ and Polubarinov, 1965) to eliminate "enough" of the orthonormal tetrad as irrelevant that the timelike nowhere vanishing vector field disappears from the theory. A brief summary suffices here. Their formalism's symmetric "square root of the metric" resembles an orthonormal tetrad gauge-fixed to form a symmetric matrix by sacrificing the local Lorentz freedom while preserving diffeomorphism freedom. The square root of the metric has only ten components rather than sixteen and can be computed using a binomial series expansion or generalized eigenvector formalism. This work was followed among high energy physicists with further discussion of nonlinear group representations. To handle the double-valuedness of spinors, I suggest treating spinors as equivalence classes defined only up to a sign. Geometric objects have been generalized to admit equivalence classes by Siwek (Siwek, 1965), who called the results "geometric pseudoobjects."

6 Geroch-Giulini scalar density example: Does GTR have an absolute object?

Unimodular GTR was invented by Einstein and was discussed by Anderson along with David Finkelstein (Anderson and Finkelstein, 1971). Though it is rather well known today (Earman, 2003), still it turns out that consideration of unimodular GTR helps one to reach the startling conclusion that not only it, but GTR itself, has an absolute object on Friedman's definition. This fact was pointed out in 2005 by Robert Geroch (Pitts, 2006) and in 2006 by Domenico Giulini (Giulini, 2006). Unimodular GTR comes in two flavors: the coordinate-restricted version in which only coordinates that fix the determinant of the metric components matrix to -1, and the weakly generally covariant version that admits any coordinates with the help of a nonvariational scalar density of some nonzero weight and a dynamical conformal metric density, which is a $(0, 2)$ tensor density of weight $-\frac{2}{n}$ or a $(2, 0)$ tensor density of weight $\frac{2}{n}$ in n space-time dimensions. As Anderson and Finkelstein observe, a metric tensor as a geometric object is reducible into a conformal metric density and a scalar density. They further observe that this scalar density is an absolute object in unimodular GTR. This observation seems unremarkable because that scalar density is not variational. For comparison, recall that one can write the Lagrangian density for GTR in terms of the conformal metric density and a scalar density (Peres, 1963). Surely the result is still GTR and not some other theory. To my knowledge, no one (prior to Geroch, in effect) has ever considered whether the scalar density, even if variational, might still count as an absolute object. Once the question is raised about GTR with the Peres-type variables, a positive answer seems obvious: GTR has an absolute object, on Friedman's definition of local diffeomorphic equivalence. This absolute object is a scalar density of nonzero weight, because every neighborhood in every model space-time admits coordinates (at least locally) in which the component of the scalar density has a value of -1. Thus variationality and absoluteness by Friedman's standards have come apart for GTR. Thus either Anderson's claim that GTR's novelty lay in its lack of absolute objects, or his analysis of absolute objects (as modified by Friedman to require only local diffeomorphic equivalence), is flawed.

7 Which Fields to Test for Absoluteness?

We have seen that the Torretti and Norton theories were thought to lack absolute objects on the Anderson-Friedman analysis but to have them intuitively; these alleged false negatives were used to criticize the analysis. We have also seen that GTR was thought to lack absolute objects both on the Anderson-Friedman analysis and intuitively; this alleged fact was the best advertised application of the analysis. However, these conclusions are both wrong, as one notices once one pays attention to tensor densities and irreducible geometric objects. It is tempting (Pitts, 2006) to conclude that *only* irreducible geometric objects should be tested for absoluteness, but that is also wrong. Consider some field in STR on the one hand, and the conformally flat space-times of the Nordström-Einstein-Fokker scalar theory of gravity on the other hand. Both theories have a conformal metric density with vanishing Weyl tensor, so the conformal metric density is absolute. Both theories have $\sqrt{-g}$ as an absolute object because nonvanishing scalar densities are automatically absolute. But only STR has an entire metric tensor that is absolute. If one tests only irreducible geometric objects for absoluteness, then one cannot distinguish the absoluteness of the metric in STR from the non-absoluteness of the metric in Nordström's theory. The absoluteness of the metric in STR comes not from any flatness *property* of $\sqrt{-g}$, but from a *relation* between $\sqrt{-g}$ and the conformal metric density $\hat{g}_{\mu\nu}$. This relation is expressed in Ricci-flatness $R_{\mu\nu} = 0$, which is 10 equations at each space-time point, algebraically. More strictly, perhaps one should count $9\infty^4 + 1$ equations, nine at each spacetime point and one further global equation, given the identity $\nabla_\mu G^{\mu\nu} = 0$, the automatic vanishing of the covariant divergence of the Einstein tensor $G^{\mu\nu} = R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R$. One could divide this relation into $9\infty^4$ equations expressing the constancy of the curvature and one more expressing the vanishing (or not) of that constant. While one must test all the irreducible geometric objects for absoluteness, one must also test some reducible geometric objects, such as metric tensors, because they can display absoluteness that is a relation between two irreducible geometric objects rather than a property of one irreducible field.

8 Geometric Objects Susceptible to Absoluteness

As Robert Geroch has pointed out, some kinds of geometric objects, such as tangent vectors and scalar densities, satisfy the condition of local sameness in all models by merely failing to vanish. All nonvanishing scalar densities are alike, as are all nonvanishing tangent vector fields, modulo coordinate freedom. I note in passing that Ted Jacobson's Einstein-Aether theory (Jacobson and Mattingly, 2001),has an absolute object in its timelike unit vector field, though he calls the theory generally covariant. I call the behavior of being the same in all models (locally, modulo coordinate freedom) simply by virtue of failing to vanish, "susceptibility to absoluteness," and the fields that exhibit it "susceptible."

One might then ask further questions. Here is one question: are there any other susceptible geometric objects? When I posed that question to Robert Geroch in the summer of 2006, he showed that all nonvanishing tangent vector densities, except weight 1, are susceptible. On the other hand, Geroch showed that no covector density of any weight (including covectors, weight 0) is susceptible, because $w_{[\mu}\partial_{\nu}w_{\alpha]}$ is a concomitant that can vanish or fail to vanish invariantly. For weight 1 tangent vector densities, the coordinate divergence is tensorial, so this case is special. I find that if one considers those with vanishing divergence and those with nonvanishing divergence separately, then those with nonvanishing divergence are susceptible, as are those with vanishing divergence. In fact the question of geometric objects that are susceptible was solved almost in its entirely by Andrzej Zajtz (Zajtz, 1988). Not surprisingly, every susceptible geometric object has no more components m than there are space-time dimensions n. Having enough coordinate freedom to achieve the same form locally intuitively seems like a necessary condition for susceptibility. It is not sufficient: some but not all geometric objects with no more components m than there are space-time dimensions n are susceptible to absoluteness.

One also wonders what the significance of the absoluteness of susceptible objects is. Does their inability to avoid absoluteness excuse them entirely, so that having susceptible geometric objects does not make a theory guilty of violating substantive general covariance? Or is that inability to avoid absoluteness the strongest confirmation of their violation of substantive general covariance? The question resembles a standard puzzle in the free will & determinism literature; unfortunately this parallel sheds no light on the answer. Perhaps different kinds of susceptible objects should be evaluated differently. For example, scalar densities like $\sqrt{-g}$, lacking directionality, intuitively might seem less contrary to strong general covariance than does an object with a direction, such as a tangent vector or vector density. The difficulty here is to give principled answers rather than writing in by hand the desired result. Perhaps having susceptible objects is a milder violation of strong general covariance than is having nonsusceptible ones? Unfortunately I do not have compelling answers to these questions. That is especially unfortunate given that the significance of the Geroch-Giulini $\sqrt{-g}$ counterexample in GTR is at stake. Perhaps the phenomenon of susceptibility, which evidently was not anticipated by Anderson, Thorne-Lee-Lightman, Friedman, or Hiskes, suggests that absolute objects do not form a natural kind the presence of which points to some deeper meaning such as strong general covariance.

The widespread belief (Anderson, 1967; Thorne et al., 1973) that all absolute objects are nonvariational and *vice versa* is falsified by the Geroch-Giulini counterexample: $\sqrt{-g}$ must be varied if Einstein's equations are to be obtained, but $\sqrt{-g} = 1$ can be achieved in any neighborhood by a coordinate choice. While at least this one absolute object is variational, the converse remains plausible (to my knowledge): for theories that putatively describe the whole physical world (with no externally applied forces), all nonvariational fields are absolute in the sense of local sameness in all models. One might consider redefining absoluteness by adding nonvariationality as a further necessary condition for absoluteness, much as Anne Hiskes once suggested (Hiskes, 1984), but with the Anderson-TLL ban on irrelevant fields enforced. In that case $\sqrt{-g}$ would not be absolute, and GTR would have no absolute objects after all, and one might call that result strong general covariance. But notice that if all nonvariational fields are locally the same in all models (modulo coordinate freedom), then the core Anderson-Friedman notion of absoluteness is largely idle; nonvariationality does the interesting work.

Simply having only variational fields in an action principle (suitably free of irrelevant fields) might be a satisfactory analysis of substantive general covariance, if one exists—though clearly it applies only to theories with action principles. That suggestion is not new, but the motivation in terms of the apparent failure of the Anderson-Friedman analysis on the grounds given above provides it a fresh urgency. This proposal also resembles the suggestion that GTR is "already parameterized," if one decides to parameterize theories by defining the nonvariational fields in terms of preferred coordinates called clock fields. Clock fields just are preferred coordinates, so Einstein's rejection of preferred coordinates might hold the key to strong general covariance after all.

Alternatively, one might accept that GTR has an absolute object and infer, *pace* Einstein, that absolute objects are just fine. In any case, the phenomenon of susceptibility makes it difficult to use the Anderson-Friedman analysis to identify some allegedly virtuous strong general covariance of GTR that earlier theories lacked.

9 Conclusion

Reviewing the Anderson-Friedman absolute objects program and various possible counterexamples yields a number of lessons. Anderson's and TLL's demand that irrelevant descriptive fluff be removed needs even more attention that they gave it. This demand as written helps to address the tetrad-spinor case. Irrelevance comes in even more varieties than they imagined, such as local irrelevance for the Jones-Geroch dust case and irrelevant variationality for clock fields. Furthermore, one's mathematical vocabulary should be chosen by the demands of physics, not the accidental fashions of contemporary differential geometry. Thus spinor fields and tensor densities should be considered. Otherwise it is difficult or impossible to discuss the tetrad-spinor and Geroch scalar-density examples, while the Torretti counterexample and Norton's variant are misjudged as serious. Reducible geometric objects such as metric tensors should be expressed as concomitants of irreducible ones such as certain scalar and tensor densities. However, one also needs to test some reducible geometric objects for absoluteness, such as metric tensors, because some cases of absoluteness are *relations* between irreducible geometric objects rather than properties of an irreducible geometric object. Accommodating spinors without irrelevant fields appears to require using *nonlinear* geometric objects, or perhaps nonlinear geometric pseudoobjects.

The scalar density counterexample, which arguably is the only serious problem for the Anderson-Friedman framework, shows that either GTR has an absolute object or the Anderson-Friedman definition of absolute objects is flawed. This case points to the more general phenomenon of susceptibility to absoluteness for certain geometric objects with no more components than there are space-time dimensions. It is unclear whether susceptible objects should be regarded as especially contrary to strong general covariance, not contrary to it at all, mildly contrary to it, or contrary to it in some cases but not others. This very profusion of options perhaps suggests that absoluteness in the Anderson-Friedman sense of sameness in all models is not the right criterion, or not all of the right criterion, for the violation of strong general covariance. If strong general covariance is a clear concept that admits analysis, the absence of nonvariational fields might be it. Nonvariational fields also apparently can be analyzed in terms of clock fields, so perhaps strong general covariance really is just the lack of preferred coordinates.

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