

# A farewell to IIA

Aki Lehtinen  
Department of Social and Moral Philosophy,  
University of Helsinki  
P.O. Box 9  
00014 University of Helsinki  
Finland  
aki.lehtinen@helsinki.fi  
tel. +358-9-19129284  
fax. +358-9-19129273

19. June 2007

## Abstract

Arrow's Independence of Irrelevant Alternatives (IIA) has been under criticism for decades for not taking account of preference intensities. Computer-simulation results by Aki Lehtinen concerning strategic voting under various voting rules show that this intensity argument does not need to rest on mere intuition. Voters may express intensities by voting strategically, and that this has beneficial aggregate-level consequences: utilitarian efficiency is higher if voters engage in strategic behaviour than if they always vote sincerely. Strategic voting is thus unambiguously beneficial under a utilitarian evaluation of outcomes. What has been considered the main argument for IIA turns out to be one against it. This paper assesses the implications of these results for interpretations of Arrow's theorem and the Gibbard-Satterthwaite theorem in a discussion on the methodological and philosophical arguments concerning preference intensities and IIA.

(JEL classification numbers: D71, D81)

keywords: strategic voting, IIA, utilitarian winner, observability, strategy-proofness

## 1 Introduction

The normative and descriptive relevance of preference intensities and the normative validity of Arrow's (1963) Independence of Irrelevant Alternatives (IIA) have been under debate for decades in social choice theory. It has been argued since its inception that IIA does not take preference intensities into account.

A choice-theoretical definition of IIA is as follows. Let  $\mathbf{p}$  and  $\mathbf{p}'$  denote profiles of individual preferences:  $\mathbf{p}$  assigns a preference ordering  $\succ_i$  for each voter  $i \in I$ :  $\mathbf{p} = (\succ_1, \succ_2, \dots, \succ_N)$ . Let  $\mathbf{p}|_Y$  denote the restriction of the profile  $\mathbf{p}$  to the subset  $Y$  of  $X$ . Let  $C(\succ, S)$  denote the social choice from profile  $\mathbf{p}$  on  $S$ .

*Independence of Irrelevant Alternatives:* If for all  $x, y \in S$  and all individuals,

$$\mathbf{p}|_{\{x,y\}} = \mathbf{p}'|_{\{x,y\}} \rightarrow C(\succ, S) = C(\succ', S). \quad (1)$$

In other words, if the two profiles  $\mathbf{p}$  and  $\mathbf{p}'$  rank each pair of alternatives in the same way, then the social choice from the two profiles should be the same.

Donald Saari (1998, 2001, 2003a) has recently argued that IIA is not normatively acceptable because voting rules that satisfy this condition fail to respect the rationality of voters. His proposal is to replace it with a condition called *binary intensity IIA* (see also Saari 1995). This requires that the aggregate ranking of each pair of alternatives is to be determined by each voter's relative ranking of that pair, and by the intensity of this ranking. The latter is determined by how many other alternatives are ranked between them. Saari argues that the Borda count (BC) satisfies this condition, and respects the rationality of voters by asking them to report a full preference ordering. Naturally, the BC does not satisfy IIA. Those who have not been willing to abandon IIA tend to emphasise its close link with strategic voting.<sup>1</sup> The BC is commonly considered to be highly manipulable.<sup>2</sup>

If there remain proponents of IIA, and if they view the debate related to it as an inevitable trade-off between rationality and intensities on the one hand, and susceptibility to strategic manipulation on the other, they have embarked on an enterprise that is doomed to failure. The simulation results in Lehtinen (2006, forthcoming, 2007) suggest that utilitarian efficiency (the frequency with which the alternative with the highest sum of utility is selected) is higher if voters engage in strategic behaviour than if they always vote sincerely. Strategic voting is thus unambiguously beneficial under a utilitarian evaluation of outcomes. What has been considered the main argument for IIA thus turns out to be one against it. These results show that the traditional intensity argument against it does not need to rest on the mere intuition that it rules out intensity information. They illustrate how all of the voting rules studied take intensity information into account if and when IIA is violated through strategic voting, and *this has beneficial aggregate-level consequences*.

The discussion in this paper focuses on two interrelated topics concerning IIA and preference intensities. The first relates to the fact that Lehtinen's results, as well as the intensity arguments against IIA that were presented before Saari's contributions, were based on a cardinal notion of preference intensity.<sup>3</sup> Saari's

<sup>1</sup>See, e.g., McLean (1987, p. 154). Arrow (1983 [1977]a, p. 168) also makes this argument, and Saari (2001, pp. 45, 137) acknowledges it.

<sup>2</sup>Saari (1990, 2003b) disagrees with this judgment, however.

<sup>3</sup>I mean contributions by Hildreth (1953), Rothenberg (1961, pp. 132-136), Coleman (1966), Campbell (1973), Ng (1979) and Mackay (1980), for example.

notion of the *intensity level* upon which the binary intensity IIA is based is best characterised as an ordinal notion (Risse 2001, Dowding 2006). Given that Lehtinen's results are based on the cardinal notion, what they show is that cardinal intensities affect the results under all voting rules.

I will show that IIA is also violated in amendment agendas by analysing an example taken from Lehtinen (2007). This example is used to show that majority-rule agendas also take preference intensities into account if voters engage in strategic behaviour. It follows that Saari's arguments concerning the transitivity of preferences and the intensity level should not be understood as providing support for the Borda rule and against the majority rule, even though they are convincing *qua* arguments against IIA.

Secondly, I will draw on the methodological and philosophical implications of Lehtinen's results on strategic voting in my interpretation of Arrow's theorem and the Gibbard-Satterthwaite theorem. Hence I will discuss the methodological and philosophical arguments concerning preference intensities and IIA.<sup>4</sup>

The traditional criticisms against preference intensities can be formulated in terms of three arguments for IIA. The *strategic-voting argument* states that strategic voting is to be avoided, and a voting rule that satisfies IIA precludes strategic voting. The *observability argument* states that since it is possible to observe preference orderings, but not preference intensities or interpersonal comparisons of utilities, allowable information must be restricted to preferences for pairs of alternatives, and this is what IIA does: 'Modern economic theory has insisted on the ordinal concept of utility; that is, only orderings can be observed, and therefore no measurement of utility independent of these orderings has any significance [...] The condition of IIA extends the requirement of observability one step farther.' (Arrow 1983 [1967], pp. 75-6).<sup>5</sup> The *epistemological-moral argument* against preference intensities and for IIA states that cardinal von Neumann-Morgenstern (vNM) utilities should not be used in social-welfare judgements because they "reflect only individuals' attitudes towards gambling" (Arrow 1951, p. 9-11).<sup>6</sup> The idea here is that vNM utilities are not appropriate in this context because they inevitably contain attitudes towards risk.<sup>7</sup>

I will respond to these arguments as follows. My main argument against the observability argument is a *tu quoque*: I will show that ordinal utility is not observable in voting contexts either. I will thus not attempt to show that preference intensities are observable, or that we have particularly precise information

---

<sup>4</sup>The question of interpersonal comparisons of preference intensities is not discussed in this paper. Lehtinen (2007) shows that utilitarian efficiencies are higher under strategic than sincere behaviour for a broad range of different interpersonal comparisons.

<sup>5</sup>See also Arrow (1963, p. 110).

<sup>6</sup>Arrow no longer puts forward this argument in the second edition of *Social Choice and Individual Values* (1963). See also Rawls (1971, pp. 172, 323) and Pattanaik (1968).

<sup>7</sup>Arrow has also justified the IIA condition by referring to its 'intuitive appeal' (Arrow 1983[1952], p. 51), arguing that it has 'strong pragmatic justification' (Arrow 1983 [1967], pp. 70, 76), and that it restricts the available information to feasible outcomes (1983[1952], p. 51, 1983 [1967], p. 76, 1983 [1967], p. 164). The first two are not discussed here because they merely appeal to intuition and thus do not seem to be genuine arguments. The third is discussed in connection with the observability and strategic-voting arguments.

on interpersonal comparisons. I will rather establish that observability cannot be used as an argument for ordinal and against intensities in evaluating voting rules. The epistemological-moral argument suffers from a similar shortcoming: voting choices reflect attitudes towards risk under all voting rules whether we like it or not. A reasonable voting model should explicitly take this into account rather than try to avoid the problem by using only ordinal utilities. Furthermore, risk attitudes are not measured in terms of utility functions in Lehtinen's models, but are formalised merely in terms of voters' beliefs.

I will take it for granted that intensities of preference are intrinsically relevant for evaluating voting outcomes normatively. I believe that voting theorists agree with this judgment<sup>8</sup>, and I will thus not attempt to argue for it. This being the case, rebutting the observability and epistemological-moral arguments should suffice to establish that voting outcomes ought to be evaluated on the basis of utilitarian criteria. Then, given a utilitarian evaluation of voting outcomes, strategic voting should be considered an argument against IIA rather than for it, because strategic voting typically increases utilitarian efficiency (or average utility) as compared to sincere voting under all commonly used voting rules. The main reason for this is that voters' behaviour depends on preference intensities when they vote strategically but not when they vote sincerely: the utilitarian winner is likely to get more strategic votes than other alternatives. Information on preference intensities can only be obtained through strategic voting (cf. Coleman 1966).

## 2 The observability argument

Social choice theory has been criticised from the outset for ignoring preference intensities. Dahl (1956, p. 90) provides a typical example of such criticisms:

By making "most preferred" equivalent to "preferred by the most" we deliberately bypass a crucial problem: What if the minority prefers its alternative much more passionately than the majority prefers a contrary alternative? Does the majority principle still make sense?

Those who have opposed the use of preference intensities and vNM utilities in social-welfare judgements have based their criticism on epistemological considerations. Here are Arrow's reasons for not incorporating preference intensities into social choice theory.

The oldest critique of social choice theory ... is that it disregards intensity of preference. Even with two alternatives, it would be argued that a majority with weak preferences should not necessarily prevail against a minority with strong feelings ... The problem in accepting this criticism is that of making it operational. Theoretically,

---

<sup>8</sup>But see Plott (1976, pp. 541-2).

is there any meaning to the interpersonal comparison of preference intensities? Practically, is there any way of measuring them, that is, is there any form of individual behavior from which the interpersonal comparisons can be inferred? (Arrow 1977*b*)

Arrow introduced IIA in order to impose an observational requirement on social choices. The idea was that the available information has to be restricted to ordinal utilities because preference orderings are observable but intensity is not. Indeed, he makes it perfectly clear that cardinal utilities (preference intensities) would be important for social choice and welfare if we could observe them directly (Arrow 1987).

Arrow (1973*a*) argued that ‘In a voting context, the ordinalist-cardinalist controversy becomes irrelevant, for voting is intrinsically an ordinal comparison and no more’ (see also Frohlich & Oppenheimer 1999). Strasnick (1976, p. 243) formulates the difficulty of observing preference intensity in a voting context as follows: ‘There is no sense in which the magnitude or degree or intensity of a choice is observable in the choice itself’.

This, however, does not mean that voters’ choices are unaffected by preference intensities. An example in which the outcomes *depend on preference intensities* even under a voting rule (the majority rule with an amendment agenda) in which voters may express a preference directly only for pairs of alternatives is given in section 4. It shows that voting is intrinsically an ordinal phenomenon only in the sense that voters can merely state whether one alternative is better than another in pair-wise contests. However, if voters engage in strategic behaviour, preference intensities inevitably reflect their choices, and *affect the outcomes* even under a rule that seemingly collects only ordinal information. Before discussing this example I will present a rudimentary version of the Enelow’s (1981) model of strategic voting, and discuss Saari’s arguments concerning IIA.

### 3 Does the majority rule lose information on rationality and preference intensities?

It used to be common to distinguish between different aspects of the IIA condition.<sup>9</sup> The *independence* (or irrelevance) *aspect* refers to the fact that the social ordering (or choice) between any two alternatives must depend only on individual preferences for those alternatives, and not on individual preferences for other irrelevant alternatives. The *ordering aspect* requires that the social ordering (or choice) of any two alternatives must be based only on individual orderings of these alternatives and on nothing else. This aspect explicitly rules out preference intensities.<sup>10</sup>

---

<sup>9</sup>See Sen (1970, p. 89), Mackay (1980, p. 79), and Kemp & Ng (1987).

<sup>10</sup>If IIA is formulated in such a way that it refers to cardinal-utility profiles, we end up with an impossibility result because cardinal utility without interpersonal comparisons does not make the impossibility result vanish (Sen 1970, Kalai & Schmeidler 1977). Accordingly, the

It is generally acknowledged that if relative intensities of preferences are somehow available then the ordering aspect of IIA need not be accepted, but the irrelevance aspect has been considered unassailable (Ng 1979, p. 115). However, it is not quite compelling either because the ‘irrelevant’ alternatives are not, strictly speaking, irrelevant. IIA does not distinguish between alternatives that are not even included in the set of available alternatives and those that belong to that set but do not seem to be under explicit consideration at a given stage of voting. The truly irrelevant alternatives belong to the former set (cf. Hansson 1973, Bordes & Tideman 1991). Fortunately, voters’ choices do take into account preferences for all the alternatives they consider. Even preferences for alternatives outside the pair for which they are voting at a given stage affect their decisions if they maximise expected utility, as in Enelow’s (1981) model.

Saari’s claims concerning IIA are perfectly justifiable. What is not so clear is whether these arguments can be used for defending the BC against the majority rule used in agenda voting, or any other voting rule for that matter. As Thomas Risse (2001, 2005) points out, arguments concerning IIA do not settle his dispute with Saari (2003a, 2006) concerning the BC and the Kemeny rule (see also Saari & Merlin 2000) because both violate IIA. Given, however, that Saari’s target of criticism, at least previously (1995), was agenda voting and the majority rule, what I have to say about agendas may be relevant to this debate as well.

Saari’s IIA criticisms cannot be viewed as arguments for the BC because IIA *also* is violated in agenda voting, and the majority rule in agenda voting takes cardinal as well as ordinal preference intensities into account. In order to illustrate this, let us consider Enelow’s (1981) model of strategic voting under amendment agendas.

Let  $X = \{x, y, z\}$  denote a set of available alternatives and  $U^i$  voter  $i$ ’s utility function. Table 1 shows the possible preference orderings and a normalisation convention for voters’ utilities:  $v^i$  denotes voter  $i$ ’s *intensity* of preference.

type of voter						
t <sub>1</sub>	t <sub>2</sub>	t <sub>3</sub>	t <sub>4</sub>	t <sub>5</sub>	t <sub>6</sub>	$U^i$
x	y	z	x	y	z	$U^i(\cdot) = 1$
y	z	x	z	x	y	$U^i(\cdot) = v^i$
z	x	y	y	z	x	$U^i(\cdot) = 0$

Table 1: Voter types and utility normalisations with three alternatives

Alternatives are put to a sequence of pair-wise majority comparisons in *an amendment agenda*.<sup>11</sup> Two alternatives are put to a majority vote against each other in the first round of voting. The winner of this first contest is then put to vote against the third alternative in the second round. Figure 1 presents the three possible amendment agendas.

---

standard view is that the most reasonable way to eschew Arrowian impossibility is to make interpersonal comparisons.

<sup>11</sup>See Miller (1995) for a discussion on different agendas.

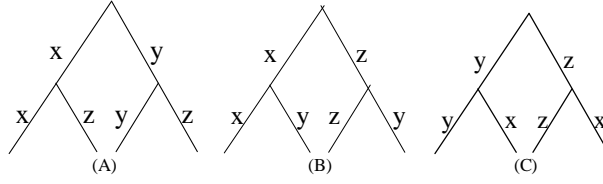


Figure 1: Amendment agendas with three alternatives

Voter  $i$ 's subjective probability that a given alternative  $j$  beats another alternative  $k$  ( $j, k \in X$ ) in a pair-wise second-round contest is denoted  $p_{jk}^i$ . In agenda A, and in the first round of voting, voters choose a branch in the voting tree, by comparing expected utilities for lotteries  $(x, z; p_{xz}^i, 1 - p_{xz}^i)$  and  $(y, z; p_{yz}^i, 1 - p_{yz}^i)$ . Note that merely formulating the voters' choice situation under incomplete information shows that they are making a choice not between the pair  $\{x, y\}$ , but rather between two lotteries that *also involve the third alternative*  $z$ . It follows immediately that their 'choice between  $x$  and  $y$ ' in the first round *may* provide information concerning their preference intensity between this pair of alternatives. Expected-utility expressions need to be formulated in order to show this.

Maximising expected utility implies giving one's vote to the branch in the voting tree that has the greatest expected utility. A voter will vote for the left-hand branch under agenda (A) if

$$p_{xz}^i \cdot U^i(x) + (1 - p_{xz}^i) \cdot U^i(z) \geq p_{yz}^i \cdot U^i(y) + (1 - p_{yz}^i) \cdot U^i(z). \quad (2)$$

Consider now voter types one and four. Both prefer  $x$  to  $y$ , but the preferences of type-four voters are ordinally more intensive because they separate the preferences between these alternatives with  $z$  by preferring  $x$  to  $z$  to  $y$ , whereas type-one voters prefer  $x$  to  $y$  to  $z$ .<sup>12</sup> Type-four voters have a dominant strategy to vote sincerely for  $x$  under agenda A. Applying utility normalisation for a type-one voter to equation 2 yields:

$$p_{xz}^i \cdot 1 + (1 - p_{xz}^i) \cdot 0 \geq p_{yz}^i \cdot v_1^i + (1 - p_{yz}^i) \cdot 0, \quad (3)$$

Type-one voters will thus vote strategically for the right-hand branch ( $y$ ) if:

$$v_1^i > \frac{p_{xz}^i}{p_{yz}^i}.$$

When they do, they are effectively expressing a cardinally strong intensity for  $x$  and  $y$  over  $z$ , and a cardinally weak intensity between  $x$  and  $y$ . Hence, they are able to express their preference intensity between  $x$  and  $y$  by deciding whether

<sup>12</sup>If indifference is ruled out by assuming that  $0 < v^i < 1$  for all voters, this intensity must also be cardinally stronger.

to vote strategically or not. In contrast, type-four voters never vote for  $y$  in the first round, and thereby reveal a strong intensity of preference for  $x$  over  $y$ . Voters thus express their ordinal and cardinal intensities under agenda voting, but they do this only in a probabilistic sense.

One might be willing to argue that since the BC always collects information on ordinal intensity, it should be preferred to majority voting under agendas. However, the normatively important intensity information is cardinal rather than ordinal. What is thus really relevant is the question of which voting rule best reflects cardinal preference intensities on the aggregate level.<sup>13</sup> If the judgment that only aggregate-level cardinal information matters for normative evaluations is defensible, it is normatively irrelevant whether it is possible to obtain information on ordinal intensity from each person separately or not. From this perspective, trying to replace IIA with a condition like Saari's *binary intensity IIA*, or any condition that fails to take voters' behaviour explicitly into account, is not fully satisfactory as a criterion for the choice between various voting rules.

One possible reason why individual-level ordinal intensity information should be collected is that it is intrinsically related to rationality. It is simultaneously information concerning the transitivity of preferences. Saari's rationality argument against IIA could be formulated as follows. Since IIA restricts the relevant information to preferences for pairs of alternatives, a voting rule that satisfies it does not allow for taking into account connecting information between the different pairs. Thus, even though one condition for Arrow's theorem explicitly requires voters to have transitive preferences, this transitivity is trumped by IIA.<sup>14</sup>

Again, Saari's argument is entirely convincing as a criticism of IIA: if there were a voting rule that satisfied IIA, such a rule would lose the information on voters' rationality. There are no such rules, however, because voters *are rational* when they engage in strategic voting, and they will violate IIA under all voting rules if they maximise expected utility.<sup>15</sup> To put it differently, if voters are rational, it is not possible to distinguish between different voting rules by applying the rationality argument against IIA. Indeed, I presume that Saari agrees with me here because his point has been that voting rules that satisfy IIA are incapable of distinguishing the cyclic preferences of (non-existent) irrational individuals from cyclic preference profiles. If there is a rationality-related argument that could distinguish between different voting rules, it must concern the recognition of irrational voters from rational ones when not all voters are rational.

The important case is thus one in which at least some voters are irrational. It would be best to ignore their votes, but as this will probably be impossible under all voting rules, the task becomes one of trying to determine how these

---

<sup>13</sup>Lehtinen (forthcoming a, b) provides results that are relevant to a comparison between the BC and agendas. I will not summarise those results here because my main goal is not to compare particular voting rules.

<sup>14</sup>In addition to previous references to Saari, see Saari and Sieberg (2001, 2004).

<sup>15</sup>Saari agreed in a private conversation (April 2006) that this characterisation of the relationship between IIA, rationality and strategic voting was apt.



voters will affect the voting outcomes. One might argue that the BC forces voters to be rational because it requires them to provide a full ranking of the alternatives. It might have such edifying aspects, but it is just as plausible that irrational voters will provide truncated ballots. Nevertheless, there may be perfectly rational voters, rational in the sense of having transitive preferences, who also provide truncated ballots. If, for example, a voter prefers  $x$  strongly to  $y$  and  $z$ , and is not willing to distinguish between the latter two, he or she may just provide a Borda score for  $x$  and ignore the rest. If and when it is not possible to distinguish rational from irrational voters in practice, I conclude that the rationality argument is valid against IIA but cannot be used as an argument for BC against any other voting rule.

## 4 IIA is violated in amendment agendas

The Condorcet winner (the alternative that is preferred by a majority to all other alternatives) is always selected under the majority rule if the voters vote sincerely (Farquharson 1969) and if they vote strategically under complete information (McKelvey & Niemi 1978, Moulin 1979). It would thus seem that the Condorcet winner is observed since it will be the outcome under the majority rule. I will now use a simple example (from Lehtinen 2007) to show that this result does not hold under incomplete information (see also Ordeshook & Palfrey 1988). It follows that IIA is violated even under the majority rule and an amendment agenda when voters maximise their expected utility.

In fact, the idea that IIA is incompatible with expected utility maximisation has already been acknowledged by all those who have argued that it does not allow voters to express their intensities of preference or cardinal utilities (see in particular Hammond 1991). Assume that the preferences of three voters  $A$ ,  $B$ , and  $C$ , can be described as in the following table:

A	B	C
y (1)	y (1)	x (1)
x (0.9)	x (0.9)	z (0.9)
z (0)	z (0)	y (0)

Table 2: Example 1

The numbers in parentheses denote voters' utilities. The sum of utility for the utilitarian winner  $x$  is  $0.9+0.9+1=2.8$ , and for the Condorcet winner  $y$  it is  $1+1+0=2$ . Thus,  $x$  should be selected according to the utilitarian criterion instead of the Condorcet winner  $y$ . If all voters vote sincerely, the Condorcet winner  $y$  will beat the utilitarian winner  $x$  in the first round and  $z$  in the second round, and emerges as the final outcome.

Let us now see what would happen if the voters maximise expected utility under incomplete information. Assume that all three voters have identical beliefs such that  $p_{yz} = 0.7$ , and  $p_{xz} = 0.9$ . Voters  $A$  and  $B$  are of type five. They will

vote strategically for  $x$  in the first round because  $v_5 < \frac{p_{yz}}{p_{xz}}$  ( $0.9 < \frac{0.7}{0.9} = 0.7778$ ). Voter  $C$  has a weakly dominant strategy to vote for  $x$  in the first round of voting. Thus,  $x$  is the outcome if the voters maximise expected utility because it beats  $y$  in the first round and  $z$  in the second round. The utilitarian winner  $x$  is chosen if they maximise expected utility but the Condorcet winner  $y$  is chosen if they vote sincerely. A Condorcet winner is thus not necessarily chosen under the majority rule.<sup>16</sup>

Let us now proceed to show that IIA is violated in this example. Whether it is defined in terms of pairs of alternatives or in terms of a set of relevant alternatives<sup>17</sup> is not particularly important here, as long as it is formulated in choice-functional terms and the social choice function refers to *actually chosen* alternatives. Let us thus assume that  $C(S)$  denotes a *choice* made by society in voting from a set of alternatives  $S \subset X$ . Arrow's (1963) treatment assumes that all voters vote sincerely so that each one chooses the alternative that he or she prefers the most. Let  $C_i(S)$  denote individual  $i$ 's choice from a set of alternatives  $S$ , and  $\succ_i$  his or her preference ordering, and let  $n(j \succ k)$  denote the number of voters who prefer alternative  $j$  to  $k$ . Arrow requires that the individual choices fulfil equation (4):<sup>18</sup>

$$C_i(S) = \{x | x \in S : \forall y \in S : x \succ_i y\}. \quad (4)$$

The method of majority decision is defined by

$$\forall x, y \in S : C(S) = x \leftrightarrow \forall y \in S : n(x \succ y) > n(x \prec y). \quad (5)$$

A Condorcet winner (CW) is defined by

$$CW = \{x | x \in S : \forall y \in S : n(x \succ y) > n(x \prec y)\}. \quad (6)$$

A Condorcet winner is always chosen in the method of majority decision because (4) guarantees that the social choice according to (5) always selects it.

Consider now example 1. It is easy to see that IIA is violated:  $y$  was chosen when all voters voted sincerely, but  $x$  was chosen if some voted strategically. Two different outcomes emerged from a single preference profile from the two different behavioural assumptions. It is also possible, of course, to obtain two different outcomes from a single profile and only the behavioural assumption of expected utility maximisation if the voters' beliefs are different in two different cases.

The example is of importance for three reasons. First, it shows that attitudes towards risk and preference intensities will *inevitably* affect voting choices if voters maximise expected utility under incomplete information. Secondly, IIA is violated even under the majority rule if voters maximise expected utility and

<sup>16</sup>If satisfying IIA is desirable, selecting a Condorcet winner is desirable because 'IIA implies the Condorcet criterion' (Arrow 1997, p. 5). An argument against the plausibility of IIA is thus simultaneously an argument against the normative appeal of Condorcet winners.

<sup>17</sup>See Ray (1973), Sen (1986), and Bordes & Tideman (1991).

<sup>18</sup>The assumption that all preferences are strict is used here.

if there are at least three alternatives. It is well known that many commonly used voting rules (plurality, runoff, Borda, etc.) may fail to select a Condorcet winner. This means that insofar as Arrow's theorem is considered a theorem about voting rules, the IIA condition *is violated under all democratic voting rules* that consider at least three alternatives (cf. Hansson 1973).

Finally, ordinal utility is not observable either in the sense that the selected alternative need not be the Condorcet winner under the majority rule and amendment agendas. The sum-of-utility criterion has been criticised for not being observable (e.g., Arrow 1973*b*). Preference orderings would be observable if the Condorcet winners were always selected under the majority rule, but this is not the case. The possibility of strategic voting thus undermines the observability argument. Therefore, preference orderings are not observable either, and observability is not a valid argument for ordinal utility and against intensities in a voting context. The claim that preference orderings are scientifically respectable because they can be observed is an invalid argument against intensities *in voting theory* even though this argument may have some weight in other contexts.

It would, of course, be easier to collect information on preference orderings than on intensities by other means than voting. We could, for example, simply ask the voters about their preference orderings. The problem with any procedure other than voting itself, however, is that insofar as the results are used for making decisions, the individuals have an incentive to misrepresent their preferences. If, on the other hand, the results are not used for making decisions, voters, particularly representatives in parliaments, have an incentive to misrepresent their preferences in order to give signals to their constituencies. Collecting information on preference orderings is thus easier than collecting information on preference intensities, but it is ultimately not possible to obtain fully reliable information on either of them.

It is not possible to prove the general claim that intensities will affect the results under all voting rules here. However, it is clear that insofar as an expected-utility model can be formulated for any voting rule, it can be shown that preference intensities will affect the outcomes under this rule. What follows from this is that if the epistemological-moral argument is to be effective against using intensities in voting theory, one has to deny that voting is characterised by decision-making under uncertainty. Surely, however, nobody is willing to argue that voters have complete information on other voters' preferences in an electorate of dozens, thousands or millions. Real-world voting is clearly characterised by decision-making under uncertainty, as Coleman (1966) argued long ago.

Some authors would perhaps be willing to argue that one reason why strategy-proofness is to be imposed is that the possibility of strategic voting makes it impossible to check the legitimacy and/or paradoxical nature of voting outcomes.<sup>19</sup> Indeed, Kelly (1988, p. 103) provides a list of explicit arguments upon which intuitive judgments rely, and which is applicable to the strategy-proofness

---

<sup>19</sup>See e.g., Riker (1982), and for counterarguments see Mackie (2003).

condition.<sup>20</sup>

1. Manipulation introduces an element of randomness into collective decisions.
2. Unequal manipulative skills may lead to the destruction of our efforts to design rules with an equal treatment of individuals.
3. Voters are led to waste resources in manipulation calculations.
4. We are led to try to reduce manipulation by others of us by concealing our preferences, thus reducing the flow of information that might help in collective decision-making.
5. Manipulation by representatives blurs their voting record and makes it difficult for us to determine if they are really representing our interests.

None of these arguments refers to how well individual preferences are satisfied when people vote strategically rather than sincerely. The arguments are not welfarist. It is clear that points 2, 3, 4 and 5 are more relevant to the voting behaviour of parliamentary representatives than to citizens' voting behaviour in mass elections. Therefore, in order to evaluate whether strategic voting is beneficial or harmful in parliaments or courts, we need some further knowledge about the relevance of the five arguments compared to a utilitarian evaluation of its consequences. In the case of mass elections, none of the arguments seems to be compelling. An increasing number of scholars have stated that they do not consider strategic voting morally questionable.<sup>21</sup> This may signal the fact that Satterthwaite's arguments are no longer fully accepted.

Rather than taking these arguments as straightforward criticisms of strategic voting, I propose that they should be turned into open research questions. With the exception of the fifth one, it is possible to study them in a welfarist and empirical way (see e.g., Lehtinen's papers).

## 5 The epistemological-moral argument

Since I am arguing for a utilitarian evaluation of outcomes in voting theory, it would seem natural to take Harsanyi's (1953, 1955, 1977) theorems as a decision-theoretic justification for a utilitarian position. Harsanyi claims that the theorems show that von Neumann-Morgenstern (vNM) utilities represent preference intensities, and that they can be used to provide an argument for utilitarianism. I do not draw on these theorems because I fully accept the criticism that Harsanyi's utilitarianism is 'utilitarianism in name only'<sup>22</sup>: the theorems do not really provide an argument for utilitarianism.

Arrow and Rawls first presented what I call the epistemological-moral argument as a criticism of Harsanyi's position. According to the argument, vNM

---

<sup>20</sup>These arguments were originally presented in Mark Satterthwaite's PhD dissertation: *The Existence of a Strategy Proof Voting Procedure* (University of Wisconsin, 1973). See Van Hees & Dowding (forthcoming) for a comprehensive discussion.

<sup>21</sup>See Kolm (1996), Buchanan & Yoon (2006), and for a more full-blown argument Van Hees & Dowding (forthcoming).

<sup>22</sup>See Sen (1976, 1977, 1986), Weymark (1991), Roemer (1996, pp. 138-150), and Mongin (2001).

utilities should not be used in social-welfare judgements because they inevitably contain morally irrelevant information on attitudes towards risk. The *moral* part is that attitudes towards risk are irrelevant to social-welfare judgments and they should therefore not be taken into account, and the *epistemological* part is that von Neumann-Morgenstern utility functions can only be constructed from choices involving risk.<sup>23</sup> Hence, attitudes towards risk inevitably affect social-welfare judgements if these judgements are based on von Neumann-Morgenstern utilities.

Harsanyi has persistently argued that vNM utility functions may be used for social-welfare judgements: they functions express a willingness to take risks in order to obtain some particular alternative (Harsanyi 1987). Hence, they express the relative intensity with which a person prefers one alternative to another (see also Harsanyi 1978, 1979, and Ng 1999).

Harsanyi (1992, pp. 682-684) argues that Arrow and Rawls confuse ‘process utility’ and ‘outcome utility’ (see also Harsanyi 1993). Process utility, or ‘utility from gambling’, refers to the enjoyment from playing a game that involves risk, while outcome utility relates to the prizes one may obtain. Harsanyi is right in that the reduction of the compound-lotteries axiom precludes process utilities and thereby ‘utility from gambling’. The vNM theory thus rules out attitudes towards enjoyment from gambling by assumption. Harsanyi is also right in pointing out that outcome utilities are ethically important. His arguments could be taken to account for why we think preference intensities are morally relevant. The problem with his argument about process utility and outcome utility is that it does not really provide a response to the criticism: attitudes towards process utilities are not what a carefully stated epistemological-moral argument should be all about. Arrow (1983 [1973]c, p. 107), for example, argues that vNM utilities incorporate attitudes towards risk. The epistemological-moral argument also concerns attitudes towards risk that are related to voters’ willingness to engage in strategic behaviour, not just attitudes towards enjoyment from gambling, and these attitudes are *also* irrelevant to social-welfare judgements.<sup>24</sup>

Arrow and Rawls’ position is buttressed by a well-known decision-theoretical epistemological consideration: standard expected utility theory does not provide any way of distinguishing between the psychological sensations of diminishing marginal utility (or diminishing intensity of satisfaction) and risk aversion, if all we are given are a person’s *choices* under uncertainty. Indeed, Harsanyi (1992, p. 685) admits this. Choices under risk do *reflect* preference intensities, just as he claims, but this argument does not change the fact that attitudes towards risk *also* affect these choices. Hence, while vNM utilities incorporate ethically relevant information concerning preference intensities, they also incorporate ethically irrelevant information concerning attitudes towards risk.

Harsanyi has successfully shown that choices under uncertainty reflect preference intensities, and that these intensities are morally relevant. Neverthe-

---

<sup>23</sup>See e.g., Alchian (1953), Baumol (1958), or Fishburn (1989).

<sup>24</sup>Here I am disregarding the entirely different question of whether the riskiness of the *choice alternatives* in an election should be taken into account.

less, the epistemological-moral argument remains valid because vNM utilities inevitably reflect morally irrelevant attitudes towards risk. However, this argument can be used against using preference intensities in social choice theory only if it is possible to collect reliable information on ordinal utilities that *do not* reflect attitudes towards risk. As shown in the previous section, this is not possible.

## 6 Conclusions

Social choice theorists have not been willing to abandon IIA mainly because it is closely related to excluding strategic voting. However, strategic voting is desirable rather than undesirable under most commonly used voting rules. The basic reason for this is that it reflects preference intensities, and sincere voting does not allow for this under most voting rules. This is why strategic voting should be taken as an argument against IIA rather than for it. By the same token, the strategy-proofness condition is also normatively questionable.

Arrow's impossibility result and the closely related theorems given by Gibbard (1973) and Satterthwaite (1975) are unassailable as deductive proofs. However, we should not be concerned about these results because their most crucial conditions are not justifiable. Fortunately, IIA and strategy-proofness are violated under all democratic voting rules, including the majority rule in agenda voting. Given that Arrow's theorem crucially depends on IIA, its importance is called into question. Saari (1995, p. 196) once formulated the meaning of the Arrow theorem as follows: it asserts that the ignored information is vital and that it is impossible to construct a procedure that systematically discards information on preference intensities. But why should anyone want such procedures in the first place?

None of the arguments discussed in this paper, the epistemological-moral, the observability, and the strategic-voting argument, is successful *as* an argument for using only preference orderings and Condorcet winners in voting theory. There seems to be no good reason for evaluating voting outcomes on the basis of Condorcet winners rather than utilitarian winners. Utilitarian winners are to be preferred on genuine ethical grounds, however, because they take preference intensities into account.

The above arguments thus give rise to three methodological conclusions. First, given that the three main arguments for IIA and against intensities fail, there is no reason to favour Condorcet winners over utilitarian winners in welfare evaluations of voting rules. Secondly, the notion of cyclic preferences and the absence of a Condorcet winner have been given an all too prevalent role in voting theory. The possibility that the preferences are cyclic is only one among many factors that may influence voting outcomes. Beliefs, information and preference intensities are also important. Models that take into account only preference orderings provide a misleading picture of voting rules because such models are based on the false empirical assumption that voting is characterised by choice under certainty.

The third methodological conclusion is that the theory of strategic voting has not addressed the right questions. If strategic voting is beneficial under many commonly used voting rules, it does not seem very fruitful to seek strategy-proof voting mechanisms or to find out which voting rules are least susceptible to strategic voting. The relevant question concerns how much strategic voting increases (or perhaps decreases) utilitarian efficiency under various voting rules under different assumptions on the voters' willingness to take risks, preference intensities and interpersonal comparisons. There are significant differences between different voting rules in these respects.

## References

- Alchian, Armen A. 1953. "The meaning of utility measurement." *The American Economic Review* 43(1):26–50.
- Arrow, Kenneth J. 1951. *Social choice and individual values*. 1 ed. New York, London: Wiley, Chapman and Hall.
- Arrow, Kenneth J. 1963. *Social choice and individual values*. 2 ed. New York: Wiley.
- Arrow, Kenneth J. 1973a. Formal theories of social welfare. In *Dictionary of the history of ideas*, ed. Philip Wiener. Vol. 4 New York: Charles Scribner's Sons pp. 276–284.
- Arrow, Kenneth J. 1973b. "Some ordinalist-utilitarian notes on Rawls' theory of justice." *Journal of Philosophy* 70:245–263.
- Arrow, Kenneth J. 1977b. "Current developments in theory of social choice." *Social Research* 44(4):607–622.
- Arrow, Kenneth J. 1983 [1967]. Values and collective decision making. In *Collected papers of Kenneth Arrow, Vol 1; Social Choice and Justice*, ed. Kenneth J. Arrow. Cambridge Massachusetts: Belknap Press of Harvard University Press chapter 4, pp. 59–77. Appeared originally in P. Laslett and W.G. Runciman, eds., *Philosophy, Politics and Society* 1967, Oxford, Basil Blackwell, pp. 215–232.
- Arrow, Kenneth J. 1983 [1973]c. Some ordinalist-utilitarian notes on Rawls' theory of justice. In *Collected papers of Kenneth J. Arrow: social choice and justice*, ed. Kenneth J. Arrow. Belknap Press of Harvard University Press. Published originally in 1973, *Journal of Philosophy*, 70, pp. 245–263.
- Arrow, Kenneth J. 1983 [1977]a. Current developments in theory of social choice. In *Collected Papers of Kenneth J. Arrow: social choice and justice*, ed. Kenneth J. Arrow. Belknap Press of Harvard University Press chapter 12, pp. 162–174. Published originally in *Social Research*, vol. 44, no. 4, pp. 607–622.

- Arrow, Kenneth J. 1983[1952]. The principle of rationality in collective decisions. In *Collected Papers of Kenneth J. Arrow: social choice and justice*, ed. Kenneth J. Arrow. Belknap Press of Harvard University Press chapter 1.
- Arrow, Kenneth J. 1987. Reflections on the essays. In *Arrow and the foundations of the theory of economic policy*, ed. G. Feiwel. U.K.: Macmillan pp. 727–734.
- Arrow, Kenneth J. 1997. The functions of social choice theory. In *Social choice re-examined, Proceedings of the IEA Conference held at Schloss Herstein, Berndorf, near Vienna*. Vol. 1 pp. 3–9.
- Baumol, William J. 1958. “The cardinal utility which is ordinal.” *The Economic Journal* 68(272):665–672.
- Bordes, Georges & Nikolaus Tideman. 1991. “Independence of irrelevant alternatives in the theory of voting.” *Theory and Decision* 30(2):163–186.
- Buchanan, James M. & Yong J. Yoon. 2006. “All voting is strategic.” *Public Choice* 129:159–167.
- Campbell, Donald E. 1973. “Social choice and intensity of preference.” *Journal of Political Economy* 81(1):211–18.
- Coleman, Jules. 1966. “The possibility of a social welfare function.” *American Economic Review* 56:1105–1122.
- Dahl, Robert A. 1956. *A preface to democratic theory*. Charles R. Walgreen Foundation lectures Chicago: University of Chicago Press.
- Dowding, Keith M. 2006. “Can populism be defended? William Riker, Gerry Mackie and the interpretation of democracy.” *Government and Opposition* 41(3):327–346.
- Enelow, James. 1981. “Saving amendments, killer amendments, and an expected utility theory of sophisticated voting.” *The Journal of Politics* 43(4):1062–1089.
- Farquharson, Robin. 1969. *Theory of voting*. UK: Hodge.
- Fishburn, Peter C. 1989. “Retrospective on the utility theory of von Neumann and Morgenstern.” *Journal of Risk and Uncertainty* 2(2):127–158.
- Frohlich, Norman & Joe A. Oppenheimer. 1999. Kenneth J. Arrow, welfare aggregation, and progress in political theory. In *Competition and cooperation: Conversations with Nobelists about economics and political science*, ed. E. Alt, James, Margaret Levi & Elinor Ostrom. New York: Russell Sage Foundation pp. 4–32.
- Gibbard, Allan. 1973. “Manipulation of voting schemes, a general result.” *Econometrica* 41:587–60.



- Hammond, Peter J. 1991. "Independence of irrelevant interpersonal comparisons." *Social Choice and Welfare* 8(1):1–19.
- Hansson, Bengt. 1973. "The independence condition in the theory of social choice." *Theory and Decision* 4:25–49.
- Harsanyi, John C. 1953. "Cardinal utility in welfare economics and in the theory of risk-taking." *Journal of Political Economy* 61:434–435.
- Harsanyi, John C. 1955. "Cardinal welfare, individualistic ethics, and interpersonal comparisons of utility." *The Journal of Political Economy* 63(4):309–321.
- Harsanyi, John C. 1977. *Rational behavior and bargaining equilibrium in games and social situations*. Cambridge Eng. ; New York: Cambridge University Press.
- Harsanyi, John C. 1978. "Bayesian decision-theory and utilitarian ethics." *American Economic Review* 68(2):223–228.
- Harsanyi, John C. 1979. "Bayesian decision-theory, rule utilitarianism, and Arrow impossibility theorem." *Theory and Decision* 11(3):289–317.
- Harsanyi, John C. 1987. Von Neumann-Morgenstern utilities, risk taking, and welfare. In *Arrow and the ascent of modern economic theory*, ed. George Feiwel. Oxford: Macmillan pp. 545–558.
- Harsanyi, John C. 1992. Game and decision theoretic models in ethics. In *Handbook of game theory with economic applications*, ed. Robert Aumann & Sergiu Hart. Vol. 1 Amsterdam, London and Tokyo: North-Holland pp. 669–707.
- Harsanyi, John C. 1993. Normative validity and meaning of von Neumann-Morgenstern utilities. In *Frontiers of game theory*, ed. Ken Binmore, Alan P. Kirman & Piero Tani. London: The MIT Press pp. 307–320.
- Hildreth, Clifford. 1953. "Alternative conditions for social orderings." *Econometrica* 21:81–94.
- Kalai, Ehud & David Schmeidler. 1977. "Aggregation procedure for cardinal preferences: A formulation and proof of Samuelson's impossibility conjecture." *Econometrica: Journal of the Econometric Society* 45(6):1431–1438.
- Kelly, Jerry S. 1988. *Social choice theory: an introduction*. Berlin, New York: Springer-Verlag. Jerry S. Kelly.; Includes index.
- Kemp, Murray C. & Yew-Kwang Ng. 1987. Arrow's independence condition and the Bergson-Samuelson tradition. In *Arrow and the foundations of the theory of economic policy*, ed. R. Feiwel, George. New York University Press pp. 223–241.

- Kolm, Serge-Christophe. 1996. *Modern theories of justice*. Cambridge, MA: MIT Press.
- Lehtinen, Aki. 2006. "The welfare consequences of strategic voting in plurality and runoff rules." Unpublished manuscript, University of Helsinki, Department of social and moral philosophy.  
**URL:** <http://www.valt.helsinki.fi/staff/alehtine/pandr.pdf>
- Lehtinen, Aki. 2007. "The welfare consequences of strategic voting in two commonly used parliamentary agendas." *Theory and Decision* 63(1):1–40.
- Lehtinen, Aki. forthcoming. "The Borda rule is also intended for dishonest men." *Public Choice* .
- Mackay, Alfred. 1980. *Arrow's theorem: The paradox of social choice, a case study in the philosophy of economics*. London: Yale University Press.
- Mackie, Gerry. 2003. *Democracy defended*. Contemporary political theory Cambridge: Cambridge University Press. Gerry Mackie.; Includes bibliographical references.
- McKelvey, Richard & Richard Niemi. 1978. "A multistage game representation of sophisticated voting for binary procedures." *Journal of Economic Theory* 18:1–22.
- McLean, Iain. 1987. *Public choice: an introduction*. New York: Basil Blackwell.
- Miller, Nicholas R. 1995. *Committees, agendas, and voting*. Vol. 58 of *Fundamentals of pure and applied economics* Chur, Switzerland: Harwood Academic Publishers.
- Mongin, Philippe. 2001. "The impartial observer theorem of social ethics." *Economics and Philosophy* 17(2):147–179.
- Moulin, Hervé. 1979. "Dominance solvable voting schemes." *Econometrica* 47(6):1337–1352.
- Ng, Yew-Kwang. 1979. *Welfare economics : introduction and development of basic concepts*. London: Macmillan.
- Ng, Yew-Kwang. 1999. "Utility, informed preference, or happiness: Following Harsanyi's argument to its logical conclusion." *Social Choice and Welfare* 16(2):197–216.
- Ordeshook, Peter & Thomas Palfrey. 1988. "Agendas, strategic voting, and signaling with incomplete information." *American Journal of Political Science* 32:441–466.
- Pattanaik, Prasanta K. 1968. "Risk, impersonality, and the social welfare function." *Journal of Political Economy* 76(6):1152–1169.

- Plott, Charles R. 1976. "Axiomatic social choice theory: An overview and interpretation." *American Journal of Political Science* 20(3):511–596.
- Rawls, John. 1971. *A theory of justice*. Cambridge, Mass.: Belknap Press of Harvard University Press.
- Ray, Paramesh. 1973. "Independence of irrelevant alternatives." *Econometrica* 41(5):987–91.
- Riker, William H. 1982. *Liberalism against populism: A confrontation between the theory of democracy and the theory of social choice*. San Francisco: W.H. Freeman.
- Risse, Mathias E. 2001. "Arrow's theorem, indeterminacy, and multiplicity reconsidered." *Ethics* 111(4):706–734.
- Risse, Mathias E. 2005. "Why the count de Borda cannot beat the marquis de Condorcet." *Social Choice and Welfare* 25(1):95–113.
- Roemer, John E. 1996. *Theories of distributive justice*. Cambridge, Mass: Harvard University Press.
- Rothenberg, Jerome. 1961. *The measurement of social welfare*. Englewood Cliffs, N.J.: Prentice-Hall.
- Saari, Donald G. 1990. "Susceptibility to manipulation." *Public Choice* 64(1):21–41.
- Saari, Donald G. 1995. Inner consistency or not inner consistency: A reformulation is the answer. In *Social Choice, Welfare, and Ethics*, ed. William A. Barnett, Hervé Moulin, Maurice Salles & Norman Schofield. Cambridge: Cambridge University Press pp. 187–212.
- Saari, Donald G. 1998. "Connecting and resolving Sen's and Arrow's theorems." *Social Choice and Welfare* 15(2):239–261.
- Saari, Donald G. 2001. *Decisions and elections: Explaining the unexpected*. Cambridge: Cambridge University Press.
- Saari, Donald G. 2003a. "Capturing the 'Will of the People'." *Ethics* 113(2):333–349.
- Saari, Donald G. 2003b. "Unsettling aspects of voting theory." *Economic Theory* 22(3):529–555.
- Saari, Donald G. 2006. "Which is better: the Condorcet or Borda Winner?" *Social Choice and Welfare* 26(1):107–129.
- Saari, Donald G. & Katri K. Sieberg. 2001. "The sum of the parts can violate the whole." *The American Political Science Review* 95(2):415–433.

- Saari, Donald G. & Katri K. Sieberg. 2004. "Are partwise comparisons reliable?" *Research in Engineering Design* 15:62–71.
- Saari, Donald G. & Vincent R. Merlin. 2000. "A geometric examination of Kemeny's rule." *Social Choice and Welfare* 17:403–438.
- Satterthwaite, Mark. 1975. "Strategy-proofness and Arrows conditions: Existence and correspondence theorems for voting procedures and social welfare functions." *Journal of Economic Theory* 10:187–217.
- Sen, Amartya K. 1970. *Collective choice and social welfare*. Vol. 5 of *Mathematical economics texts* San Francisco: Holden-Day.
- Sen, Amartya K. 1976. "Welfare inequalities and Rawlsian axiomatics." *Theory and Decision* 7(4):243–262.
- Sen, Amartya K. 1977. Non-linear social welfare functions: a reply to professor Harsanyi. In *Foundational Problems in the Special Sciences*, ed. R Butts & J. Hintikka. Dordrecht: Reidel pp. 297–302.
- Sen, Amartya K. 1986. Social choice theory. In *Handbook of mathematical economics*, ed. Kenneth Arrow & Michael Intriligator. Vol. 3 Amsterdam: Elsevier Science publishers B.V. pp. 1073–1181.
- Strasnick, Steven. 1976. "The problem of social choice: Arrow to Rawls." *Philosophy and Public Affairs* 5(3):241–273.
- Van Hees, Martin & Keith Dowding. forthcoming. "In praise of manipulation." *British Journal of Political Science* . available at <http://med.ces.free.fr/pdf/vanHees.pdf>.
- Weymark, John A. 1991. A reconsideration of the Harsanyi-Sen debate on utilitarianism. In *Interpersonal comparisons of well-being*, ed. Jon Elster & John Roemer. Cambridge: Cambridge University Press.